Network Tomography
4 node problem

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Contents

1 Notations

p_{01} - probability of a packet sent from 0, successfully arriving at 1.
p_{12}- probability of a packet sent from 1, successfully arriving at 2.
p_{23} - probability of a packet sent from 2, successfully arriving at 3.
N_{0k} - Number of packets sent from node 0 to node K
M_{0k} - Number of packets received at node K
R_{0k} - Number of packets recorded at node K
β_j - Probability with which node j records a packet identifier.
L_{0k} - List of packets recorded at node k.
The quantities with primes are the refined estimates.
$\bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet$

2 Estimation

2.1 Step1

Estimate p_{03} by

$$\begin{aligned} \hat{p_{03}} &= M_{03}/N_{03} \\ &= R_{03}/\beta_3 N_{03} \end{aligned}$$

Estimate p_{23} by

$$\hat{p_{23}} = |L_{02} \cap L_{03}|/\beta_3 R_{02}$$

2.2 Step2

Estimate p_{02} by

$$\hat{p_{02}} = R_{02}/\beta_2 N_{02} \tag{1}$$

But we can make a better estimate of p_{02} by extracting the mutual information between R_{02} and R_{03}. This comes from the fact that those packets which were not recorded by 2 but were recorded by 3, surely made it through 2. This gives us a better estimate of p_{02}.

Hence

$$\begin{aligned} L'_{02} &= L_{02} + \textit{packets recorded by } 3 \textit{ but not by } 2. \\ &= L_{02} + (L_{03} - L_{02}) \end{aligned}$$

$$\begin{aligned} R'_{02} &= |L'_{02}| \\ &= R_{02} + |L_{03} - L_{02}| \\ &= R_{02} + \{R_{03} - R_{02} p_{23} \beta_3\} \\ &= \beta_2 M_{02} + \{\beta_3 M_{03} - \beta_2 M_{02} \beta_3 p_{23}\} \\ &= \beta_2 M_{02} + \{\beta_3 M_{02} p_{23} - \beta_2 M_{02} \beta_3 p_{23}\} \\ &= M_{02}\{\beta_2 + \beta_2 p_{23}(1 - \beta 2)\} \end{aligned}$$

$$\begin{aligned} & Now, \\ R'_{02} &= \beta'_2 M_{02} \\ \Rightarrow \quad \beta'_2 &= \beta_2 + \beta_2 p_{23}(1 - \beta 2) \end{aligned}$$

Therefore we get a refined estimate of p_{02} by

$$\widehat{p'_{02}} = R'_{02}/\beta'_2 N_{02} \tag{2}$$

2.3 Step3

Now we re-estimate p_{23} by

$$\widehat{p_{23}} = \widehat{p_{03}}/\widehat{p'_{02}} \tag{3}$$

2.4 Step4

Initial Estimate of p_{01}

$$\widehat{p_{01}} = R_{01}/\beta_1 N_{01} \tag{4}$$

Initial Estimate of p_{12}

$$\widehat{p_{12}} = |L_{01} \cap L'_{02}|/\beta_2 R_{01} \tag{5}$$

Now we update the list L_{01}

$$\begin{aligned} L'_{01} &= L_{01} + (L'_{02} - L_{01}) \\ \\ R'_{01} &= R_{01} + |L'_{02} - L_{01}| \\ &= R_{01} + \{R'_{02} - R_{01} p_{12} \beta'_2\} \\ &= R_{01} + \{\beta'_2 M'_{02} - \beta_1 \beta'_2 M_{01} p_{12}\} \\ &= \beta_1 M_{01} + M_{01} p_{12} (\beta'_2 - \beta_1 \beta'_2) \\ &= M_{01}\{\beta_1 + p_{12}\beta'_2(1 - \beta_1)\} \\ Now R'_{01} &= \beta'_1 M_{01} \\ Therefore \beta'_1 &= \beta_1 + p_{12}\beta'_2(1 - \beta_1) \end{aligned}$$

Now we can recalculate p_{01} by

$$\widehat{p'_{01}} = R'_{01}/\beta'_1 N_{01} \tag{6}$$

2.5 Step5

Re-estimate p_{12} by

$$\widehat{p'_{12}} = \widehat{p'_{02}}/\widehat{p'_{01}} \tag{7}$$