A Network Interference Service Model for Wireless Networks

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Abstract

Taking cue from the role of a Domain Name Server (DNS) in the context of the wired Internet, this paper proposes a Network Interference Service (NIS) model capable of providing information about the neighborhood of a wireless node. All nodes within the network that partake of this directory service provide a log of their transmission pattern to the NIS through reliable control channels. A wireless node can request neighborhood information by sending a record of its received signals, and the NIS correlates this sequence of attenuated signal strengths with the uncorrupted data received by itself. The set of linear equations thus set up is solved to estimate the channel coefficients, and hence the "radio distance" between nodes. A simple simulation was performed for an 802.11b network, and the channel coefficients were estimated to a high degree of accuracy. The problem is then reformulated in vector notation and a least–squares estimate for the channel coefficients is obtained.

Contents

1 Introduction

The role played by a Domain Name Server (DNS) in the context of the Internet is well–known. It translates the computer name specified by a Uniform Resource Locator (URL) into an IP address, thus making route determination possible. A similar application to determine local network topology in the wireless domain is being proposed in this article. The system utilises a centralized Network Interference Service (NIS) provider that identifies wireless nodes from a study of the interference pattern and the correlation between transmitted and received sequences of data signals. The Network Interference Server is modeled as an intelligent information–gathering source that maintains a database of transmissions in its neighborhood through reliable control channel connections (thus eliminating the possibility of any error) with transmitting nodes. Whenever a wireless node within the network requires neighborhood information, it sends a request to the NIS along with a trace of transmissions it has received in the past. The NIS identifies the requesting node and determines the nodes that lie within its "radio–distance" vicinity by attempting a correlation between the original information sequence and the attenuated sequence received by the requesting node from its neighbors.

The problem at hand encapsulates ideas from channel estimation and equalization, modulation techniques and linear algebra methods. The following section introduces a discrete–time representation of signals with flat fading channel characteristics that is used in the analysis, while Section 3 outlines the algorithm proposed, as well as the simulation results obtained with MATLAB. The paper concludes with a discussion of the results and the scope for future work.

2 Problem

2.1 Assumptions

The Network Interference Service (NIS) model comprises m nodes, all within transmission range of each other, which communicate with the NIS node through reliable control channels. Transmissions occur in discrete time-slots and the transmitted power over a number of time-slots, later defined as a *frame*, is reported to the NIS. The nodes also maintain a record of received power in each time-slot.

Some simplifying assumptions used in the analysis are as follows:

1. Nodes cannot transmit and receive simultaneously.
2. Receiver noise of known power exists.
3. Channel coefficients remain constant over the duration of the analysis.
4. Transmission from node to NIS is error–free.

A discrete–time signal representation is used to describe the channel model that satisfies the above assumptions. For such a system, the i-th user is assigned a finite energy signature waveform, $\{s_i(t), t \in [0, T]\}$, and it transmits a string of bits by modulating that waveform antipodally. To assure synchronization and reduce the information overhead at the NIS, strings of K bits are grouped together as frames at the transmitter end, such that the state of the transmitter remains unchanged within the space of a frame. An activity factor $\alpha_j[l]$ is also introduced that keeps track of the state of the transmitter j in the l-th frame according to the following rule:

$$\alpha_j[l] \triangleq \begin{cases} 1, & \text{if transmitter } j \text{ is on during the } l\text{th frame} \\ 0, & \text{if transmitter } j \text{ is off during the } l\text{th frame} \end{cases} \tag{1}$$

This frame activity factor $\alpha_j[l]$ is reported to the NIS by each transmitter j at each frame interval l.

2.2 Algebraic Analysis

The receiver correlates for each bit in the frame and therefore the initial formulation is in terms of individual bits. This result will later be expressed in terms of frames. However, the frame activity factor $\alpha_j[l]$ is still employed, with the understanding that all bits within the same frame correspond to the same activity factor. Thus, assuming signal synchronization is maintained and users share a white Gaussian multiple-access channel, the signal received by the ith user at time t corresponding to the k-th bit interval is given by [11]:

$$\begin{aligned} r_i(t) &= \sum_{j=1}^{m} \alpha_j[l]\sqrt{p_j(k)}\sqrt{h_{ij}}b_j(k)s_j(t-kT) + n_i(t), \quad t \in [kT,(k+1)T] \\ &= \sum_{j=1}^{m} \alpha_j[l]\sqrt{q_{ij}(k)}b_j(k)s_j(t-kT) + n_i(t), \quad t \in [kT,(k+1)T] \end{aligned} \tag{2}$$

where,

h_{ij}: constant channel coefficient between node i and node j

b_j: jth user information sequence where $b \in \{-1, 1\}$

s_j: signature waveform assigned to user j

n_i: additive white gaussian noise (AWGN) with zero–mean and variance σ^2

$q_{ij}(k) = h_{ij}p_j(k)$ is the power of node j received at node i

2.2.1 Receiver Output

The output of matched filter f at receiver i can be obtained by integrating over the interval $[kT, (k+1)T]$. However, if all information sequences are equally likely, it suffices to restrict attention to a specific symbol interval [2]. For notational simplicity, $k = 0$ is chosen without any loss of generality. Thus, the matched filter output may be expressed as:

$$\begin{aligned} y_i^{(f)} &= \int_0^T r_i(t)s_f(t)dt \\ &= \sum_{j=1}^{m} \alpha_j[l]\sqrt{q_{ij}}b_j \int_0^T s_j(t)s_f(t)dt + \int_0^T n_i(t)s_f(t)dt \end{aligned} \tag{3}$$

Defining

$$\begin{aligned} \rho_{jf} &= \int_0^T s_j(t)s_f(t)dt, \quad \rho_{jj} = 1 \\ n_{if} &= \int_0^T n_i(t)s_f(t)dt, \end{aligned} \tag{4}$$

equation (3) is rewritten as:

$$y_i^{(f)} = \sum_{j=1}^{m} \alpha_j[l]\sqrt{q_{ij}} b_j \rho_{jf} + n_{if} \tag{5}$$

Conventional receivers consist of matched filters that are matched to the signature sequences of the users, and squares of the matched filter outputs are unbiased estimates for the received energies in the sense that the expected value of the square of a matched filter output is equal to the received energy through the matched filter [10]. The randomness over which the expectation is taken is due to the randomness of the transmitted bit as well as the channel noise. The square of the matched filter output as well as its theoretical mean and variance are as follows:

$$\left\{y_i^{(f)}\right\}^2 = \sum_{j=1}^{m}\left[\alpha_j[l]q_{ij}\rho_{jf}^2 + \sum_{n=1,n\neq j}^{m}\alpha_j[l]\alpha_n[l]\sqrt{q_{ij}}\sqrt{q_{in}}b_j b_n \rho_{jf}\rho_{nf}\right] + n_{if}^2 + 2n_{if}\sum_{j=1}^{m}\alpha_j[l]\sqrt{q_{ij}}b_j\rho_{jf} \tag{6}$$

$$\mathrm{E}\left[\left\{y_i^{(f)}\right\}^2\right] = \sum_{j=1}^{m}\alpha_j[l]q_{ij}\rho_{jf}^2 + \sigma^2 \tag{7}$$

$$\begin{aligned}
\mathrm{var}\left[\left\{y_i^{(f)}\right\}^2\right] &= \mathrm{E}\left[\left\{y_i^{(f)}\right\}^4\right] - \mathrm{E}^2\left[\left\{y_i^{(f)}\right\}^2\right] \\
&= \sum_{j=1}^{m}\left[\alpha_j[l]q_{ij}^2\rho_{jf}^4 + 2\sum_{n=1,n\neq j}^{m}\alpha_j[l]\alpha_n[l]q_{ij}q_{in}\rho_{jf}^2\rho_{nf}^2\right] + 3\sigma^4 + 4\sigma^2\sum_{j=1}^{m}\alpha_j[l]q_{ij}\rho_{jf}^2 \\
&\quad - \sum_{j=1}^{m}\left[\alpha_j[l]q_{ij}^2\rho_{jf}^4 + \sum_{n=1,n\neq j}^{m}\alpha_j[l]\alpha_n[l]q_{ij}q_{in}\rho_{jf}^2\rho_{nf}^2\right] - \sigma^4 - 2\sigma^2\sum_{j=1}^{m}\alpha_j[l]q_{ij}\rho_{jf}^2 \\
&= \sum_{j=1}^{m}\sum_{n=1,n\neq j}^{m}\alpha_j[l]\alpha_n[l]q_{ij}q_{in}\rho_{jf}^2\rho_{nf}^2 + 2\sigma^4 + 4\sigma^2\sum_{j=1}^{m}\alpha_j[l]q_{ij}\rho_{jf}^2
\end{aligned} \tag{8}$$

Equation (7) follows from the assumption that $\mathrm{E}[b_f(k)] = 0$, $\mathrm{E}[n_{il}(k)] = 0$ and bit–sequences for different nodes are independent.

Assuming perfect frame synchronicity, the received signal is now also grouped into frames of K bits. Since the activity state of each transmitter remains unchanged over a frame, the signal power received by the ith node over a frame may simply be obtained by averaging the received power associated with each bit in the frame. Thus, the power in the fth matched filter of the ith receiver over frame l, denoted by $x_i^{(f)}[l]$, is defined as:

$$x_i^{(f)}[l] = \frac{1}{K}\sum_{k=(l-1)K+1}^{lK}\left\{y_i^{(f)}(k)\right\}^2 \tag{9}$$

Also, the expected value of the received power over the frame l is:

$$\begin{aligned}\mathrm{E}\left[x_i^{(f)}[l]\right] &= \mathrm{E}\left[\left\{y_i^{(f)}(k)\right\}^2\right] \\ &= \sum_{j=1}^{m} \alpha_j[l] q_{ij} \rho_{jf}^2 + \sigma^2\end{aligned} \tag{10}$$

It may be pointed out as an aside that in the case of IEEE 802.11, each transmitter uses the same signature waveform (*i.e.*, $\rho_{jf}(k) = 1, \forall j, f$); in that case, the equation simplifies further to:

$$\mathrm{E}\left[x_i^{(f)}[l]\right] - \sigma^2 = \sum_{j=1}^{m} \alpha_j[l] q_{ij} \tag{11}$$

As has been discussed in [4], measurements at the receiver include the total received signal power $x_i^{(f)}[l]$ and the background receiver noise σ^2. Thus, a procedure will now be outlined to evaluate the channel coefficients h_{ij}, provided there is a large number of received power measurements.

Assuming a sufficiently large number of bits in each frame, equations (9) and (10) may be approximated as:

$$x_i^{(f)}[l] - \sigma^2 = \sum_{j=1}^{m} \alpha_j[l] q_{ij} \rho_{jf}^2[l] \tag{12}$$

In matrix notation,

$$\begin{bmatrix} x_1^{(f)}[l] - \sigma^2 \\ x_2^{(f)}[l] - \sigma^2 \\ \vdots \\ x_m^{(f)}[l] - \sigma^2 \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & q_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mm} \end{bmatrix} \begin{bmatrix} \alpha_1[l]\rho_{1f}^2[l] \\ \alpha_2[l]\rho_{2f}^2[l] \\ \vdots \\ \alpha_m[l]\rho_{mf}^2[l] \end{bmatrix} \tag{13}$$

According to the model, the NIS has access to the transmission sequences $\alpha_j[l]$ for all nodes j over all frame-intervals l. However, it receives the temporal received power values $x_i^{(f)}[l] - \sigma^2$ from only one node i. This dynamic behavior for a single node i for n frame–intervals may be represented by:

$$\begin{bmatrix} x_i^{(f)}[1] - \sigma^2 \\ x_i^{(f)}[2] - \sigma^2 \\ \vdots \\ x_i^{(f)}[n] - \sigma^2 \end{bmatrix} = \begin{bmatrix} \alpha_1[1]\rho_{1f}^2[1] & \alpha_2[1]\rho_{2f}^2[1] & \cdots & \alpha_m[1]\rho_{mf}^2[1] \\ \alpha_1[2]\rho_{1f}^2[2] & \alpha_2[2]\rho_{2f}^2[2] & \cdots & \alpha_m[2]\rho_{mf}^2[2] \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1[n]\rho_{1f}^2[n] & \alpha_2[n]\rho_{2f}^2[n] & \cdots & \alpha_m[n]\rho_{mf}^2[n] \end{bmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \\ \vdots \\ q_{im} \end{bmatrix} \tag{14}$$

The channel coefficients h_{ij} are subsequently obtained by solving this set of equations using the algorithm outlined in the following section. Finally, if the path–loss coefficient is known, the inter–node distances with respect to node i may be estimated.

2.2.2 Discussion

In the study of solutions to linear equations [8, 5], three different cases are identified based on the dimensions of matrices.

1. *Overdetermined case:* When the number of measurements exceeds the number of free parameters, an exact fit of the model to the data is not possible. In this case, the number of columns in the transmission matrix $\mathbf{S}$ is less than the number of rows.

2. *Determined case:* When the number of free parameters equals the number of measurements, the measurements can be fitted exactly as long as there exists no linear dependence among the rows of the square matrix $\mathbf{S}$.

3. *Underdetermined case:* When the number of free parameters exceeds the number of measurements, there is not enough data for a unique solution. In this case, the number of columns in $\mathbf{S}$ is greater than the number of rows.

Thus, the channel coefficents h_{ij} for node i may be uniquely determined when a full–rank matrix $\mathbf{S}$ of dimension $m \times m$ is obtained.

However, for rank–deficient matrices it is not possible to compute the channel coefficient vector deterministically. Instead, techniques need to be borrowed from estimation theory to obtain expected values of the channel coefficients and hence expected inter–node distances. Several estimation techniques are outlined in the literature that allow for blind equalization of a communication channel. The most commmon ones include maximum likelihood (ML) and minimum mean squared error estimates (MMSE) [6], both of which are based on the expectation maximization (EM) algorithm [1].

2.3 Vector Analysis

A similar analysis can be performed in terms of a correlation receiver, as in [9], instead of the matched filter implementation. Assuming that each transmitter j has a pre-assigned unique signature sequence given by $s_j(t)$, the waveform transmitted by it in a single frame–interval is given by:

$$\begin{aligned} x_j(t) &= \alpha_j[l]\sqrt{p_j}s_j(t), \quad t \in [lKT, (l+1)KT] \\ \text{where, } l &= \text{frame number}, \\ s_j(t) &= \text{signature sequence of transmitter } j, \\ K &= \text{number of bits in each frame}, \\ T &= \text{bit interval} \end{aligned} \tag{15}$$

The baseband received signal, $r_i(t)$, in one bit–interval at the front end of the receiver filters at the assigned base of user i is thus given by:

$$\begin{aligned} r_i(t) &= \sum_{j=1;j\neq i}^{m} \alpha_j[l]\sqrt{p_j h_{ij}}s_j(t) + n(t) \\ \text{where, } h_{ij} &= \text{channel coefficient between nodes } i \text{ and } j \\ p_j &= \text{transmission power of user } j \\ n(t) &= \text{additive white Gaussian noise process} \end{aligned} \tag{16}$$

The notation may be simplified by dropping the receiving filter's index and the time dependence. The resulting formula may be expressed in vector form (assuming m users and N dimensions):

$$\begin{aligned} \mathbf{r} &= \sum_{j=1}^{m} \alpha_j \sqrt{q_j}\mathbf{s}_j + \mathbf{n} \\ &= \mathbf{S}\boldsymbol{\lambda} + \mathbf{n} \end{aligned} \tag{17}$$

where,

$$q_j = h_{ij} p_j \tag{18}$$

$$\mathbf{S} = \begin{bmatrix} s_{11} & s_{21} & \cdots & s_{m1} \\ s_{12} & s_{22} & \cdots & s_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ s_{1N} & s_{2N} & \cdots & s_{mN} \end{bmatrix} \tag{19}$$

$$\begin{aligned} \text{and, } \boldsymbol{\lambda} &= \begin{bmatrix} \alpha_1 \sqrt{q_1} \\ \alpha_2 \sqrt{q_2} \\ \vdots \\ \alpha_m \sqrt{q_m} \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 \\ 0 & \alpha_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_m \end{bmatrix} \begin{bmatrix} \sqrt{q_1} \\ \sqrt{q_2} \\ \vdots \\ \sqrt{q_m} \end{bmatrix} \\ &= \mathbf{A}\mathbf{q} \end{aligned} \tag{20}$$

2.3.1 Receiver Output

If $\mathbf{c}_i$ is now defined to be the receiver filter for user i at its assigned base station, then the receiver filter output of user i will be:

$$\begin{aligned} y_i &= \mathbf{r}^\top \mathbf{c}_i \\ &= \boldsymbol{\lambda}^\top \mathbf{S}^\top \mathbf{c}_i + \mathbf{n}^\top \mathbf{c}_i \end{aligned} \tag{21}$$

The received energy may be obtained by squaring the output of the filter:

$$y_i^2 = \mathbf{c}_i^\top \left(\mathbf{S}\boldsymbol{\lambda}\boldsymbol{\lambda}^\top \mathbf{S}^\top + \mathbf{n}\mathbf{n}^\top + 2\mathbf{S}\boldsymbol{\lambda}\mathbf{n}^\top \right) \mathbf{c}_i \tag{22}$$

Then,

$$\begin{aligned} \mathrm{E}\left[y_i^2\right] &= \mathbf{c}_i^\top \left(\mathrm{E}\left[\mathbf{S}\boldsymbol{\lambda}\boldsymbol{\lambda}^\top \mathbf{S}^\top\right] + \sigma^2 \mathbf{I} \right) \mathbf{c}_i \\ &= \mathbf{c}_i^\top \mathrm{E}\left[\mathbf{S}\mathbf{Q}\mathbf{S}^\top\right] \mathbf{c}_i + \sigma^2 \left\|\mathbf{c}_i\right\|^2 \end{aligned} \tag{23}$$

$$\text{where, } \mathbf{Q} = \boldsymbol{\lambda}\boldsymbol{\lambda}^\top \tag{24}$$

Proceeding in the same manner,

$$\mathrm{var}\left(y_i^2\right) = \mathrm{var}\left(\mathbf{c}_i^\top \mathbf{S}\mathbf{Q}\mathbf{S}^\top \mathbf{c}_i\right) + 4\sigma^2 \left\|\mathbf{c}_i\right\|^2 \mathrm{E}\left[\mathbf{c}_i^\top \mathbf{S}\mathbf{Q}\mathbf{S}^\top \mathbf{c}_i\right] + 2\sigma^4 \left\|\mathbf{c}_i\right\|^4 \tag{25}$$

2.3.2 Discussion

The purpose of this exercise was to establish that although matched filters had been employed in the earlier analysis (Section 2.2), this does not result in any loss of generality, and any other receiver scheme could equally well be used. However, the receiver is assumed to know the signature sequences of all transmitters, which may be justified by the use of a known training sequence prior to the transmission of data, so that the receiver can estimate the individual signature waveforms. It now has to be established that (21) can be expressed in a form similar to (14) so that solving the set of linear equations would give the channel coefficients.

This is possible if it can be demonstrated that $\operatorname{var}\left(y_i^2\right)$ is sufficiently small. For the worst case scenario, this is satisfied if the maximum value of $\operatorname{var}\left(y_i^2\right)$, over all choice of $\mathbf{S}$, is within an acceptable bound.

Defining $\boldsymbol{\rho}_i = \mathbf{S}^\top \mathbf{c}_i$, equation (24) may be rewritten as:

$$\begin{aligned}
\operatorname{var}\left(y_i^2\right) &= \operatorname{var}\left(\boldsymbol{\rho}_i^\top \boldsymbol{\lambda}\boldsymbol{\lambda}^\top \boldsymbol{\rho}_i\right) + 4\sigma^2 \left\|\mathbf{c}_i\right\|^2 \mathrm{E}\left[\boldsymbol{\rho}_i^\top \boldsymbol{\lambda}\boldsymbol{\lambda}^\top \boldsymbol{\rho}_i\right] + 2\sigma^4 \left\|\mathbf{c}_i\right\|^4 \\
&= \operatorname{var}\left(\boldsymbol{\rho}_i^\top \boldsymbol{\lambda}\right)^2 + 4\sigma^2 \left\|\mathbf{c}_i\right\|^2 \mathrm{E}\left[\boldsymbol{\rho}_i^\top \boldsymbol{\lambda}\right]^2 + 2\sigma^2 \left\|\mathbf{c}_i\right\|^4 \\
&= \operatorname{var}\left(R_i^2\right) + 4\sigma^2 \left\|\mathbf{c}_i\right\|^2 \mathrm{E}\left[R_i^2\right] + 2\sigma^4 \left\|\mathbf{c}_i\right\|^4
\end{aligned} \tag{26}$$

where the following notation is adopted for brevity:

$$\begin{aligned}
R_i &= \langle \boldsymbol{\rho}_i, \boldsymbol{\lambda} \rangle \\
R_i^2 &= \left(\langle \boldsymbol{\rho}_i, \boldsymbol{\lambda} \rangle\right)^2
\end{aligned} \tag{27}$$

To maximize $\operatorname{var}\left(y_i^2\right)$, the following statistics need to be maximized:

1. $\mathrm{E}\left[R_i^2\right]$
2. $\operatorname{var}\left(R_i^2\right)$

Using Cauchy–Schwarz inequality,

$$\left(\langle \boldsymbol{\rho}_i, \boldsymbol{\lambda} \rangle\right)^2 \leq \left\|\boldsymbol{\rho}_i\right\|^2 \left\|\boldsymbol{\lambda}\right\|^2 \tag{28}$$

Thus,

$$\begin{aligned}
\mathrm{E}\left[R_i^2\right] &= \mathrm{E}\left[\left(\langle \boldsymbol{\rho}_i, \boldsymbol{\lambda} \rangle\right)^2\right] \\
&\leq \mathrm{E}\left[\left\|\boldsymbol{\rho}_i\right\|^2 \left\|\boldsymbol{\lambda}\right\|^2\right] \\
&= \mathrm{E}\left[\left\|\boldsymbol{\rho}_i\right\|^2\right] \mathrm{E}\left[\left\|\boldsymbol{\lambda}\right\|^2\right] \\
&= \mathrm{E}\left[\left\|\boldsymbol{\rho}_i\right\|^2\right] \mathrm{E}\left[\sum_{i=1}^{m} \alpha_i^2 q_i\right]
\end{aligned} \tag{29}$$

In most of the literature, $\mathbf{s}_j$ and $\mathbf{c}_i$ are assumed to be unit vectors.
i.e.,

$$\begin{aligned}
\left\|s_j(t)\right\|^2 &= \int_0^T s_i(t) s_j(t) dt = 1 \\
\left\|c_i(t)\right\|^2 &= \int_0^T c_i(t) c_j(t) dt = 1
\end{aligned} \tag{30}$$

Thus, applying Cauchy–Schwarz inequality to each of the individual correlation terms $\rho_{ij} = \mathbf{s}_j^\top \mathbf{c}_i$,

$$\begin{aligned}
\rho_{ij} &\leq \left\|\mathbf{s}_j\right\| \left\|\mathbf{c}_i\right\| \\
&= 1
\end{aligned} \tag{31}$$

Hence,

$$\begin{aligned}
\left\|\boldsymbol{\rho}_i\right\| &= \sqrt{\rho_{i1}^2 + \rho_{i2}^2 + \cdots + \rho_{im}^2} \\
&\leq \sqrt{m}
\end{aligned} \tag{32}$$

and,

$$\max_{R_i} \mathrm{E}\left[R_i^2\right] = m \sum_{i=1}^{m} \mathrm{E}\left[q_i\right] \tag{33}$$

In a similar manner,

$$\begin{aligned}
\mathrm{var}\left(R_i^2\right) &= \mathrm{var}\left(\langle \boldsymbol{\rho}_i, \boldsymbol{\lambda} \rangle^2\right) \\
&= \mathrm{E}\left[\langle \boldsymbol{\rho}_i, \boldsymbol{\lambda} \rangle^4\right] - \mathrm{E}^2\left[\langle \boldsymbol{\rho}_i, \boldsymbol{\lambda} \rangle^2\right] \\
&\leq \mathrm{E}\left[\left\|\boldsymbol{\rho}_i\right\|^4 \left\|\boldsymbol{\lambda}\right\|^4\right] - \mathrm{E}^2\left[R_i^2\right] \\
&= \mathrm{E}\left[\left\|\boldsymbol{\rho}_i\right\|^4\right] \mathrm{E}\left[\left\|\boldsymbol{\lambda}\right\|^4\right] - \mathrm{E}^2\left[R_i^2\right] \\
&\leq m^2 \mathrm{E}\left[\sum_{i=1}^{m} \alpha_i^4 q_i^2\right] - \mathrm{E}^2\left[R_i^2\right]
\end{aligned} \tag{34}$$

Hence,

$$\begin{aligned}\max_{R_i} \operatorname{var}\left(R_i^2\right) &= m^2 \sum_{i=1}^{m} \mathrm{E}\left[q_i^2\right] - \min_{R_i} \mathrm{E}^2\left[R_i^2\right] \\ &= m^2 \sum_{i=1}^{m} \mathrm{E}\left[q_i^2\right], \quad \text{when } \mathbf{S} \text{ and } \mathbf{c}_i(t) \text{ are orthogonal}\end{aligned} \tag{35}$$

Thus, substituting (32) and (34) in (25), a crude upper bound for $\operatorname{var}\left(y_i^2\right)$ is obtained.

$$\max_{R_i} \operatorname{var}\left(y_i^2\right) = m^2 \sum_{i=1}^{m} \mathrm{E}\left[q_i^2\right] + 4m\sigma^2 \sum_{i=1}^{m} \mathrm{E}\left[q_i\right] + 2\sigma^4 \tag{36}$$

2.4 Least Squares Estimate

This section attempts to estimate the channel coefficients using a simple least–squares solution, from the signal before and after the receiver filter. Subsequently, the performance of the estimates are compared on the basis of MATLAB simulations.

2.4.1 Before Correlation

Combining (17) and (20) results in:

$$\begin{aligned} \mathbf{r} &= \mathbf{SAq} + \mathbf{n} \\ &= \mathbf{Hq} + \mathbf{n} \end{aligned} \tag{37}$$

$$\text{where, } \mathbf{H} = \mathbf{SA} \text{ is a known matrix of dimension } N \times m$$

Denoting the least–squares estimate of the channel coefficient as $\hat{q}$, the problem reduces to minimizing the following expression:

$$\begin{aligned} \mathcal{L} &= \left(\mathbf{r} - \mathbf{H}\hat{\mathbf{q}}\right)^\top \left(\mathbf{r} - \mathbf{H}\hat{\mathbf{q}}\right) \\ &= \mathbf{r}^\top\mathbf{r} - \mathbf{r}^\top\mathbf{H}\hat{\mathbf{q}} - \hat{\mathbf{q}}^\top\mathbf{H}^\top\mathbf{r} + \hat{\mathbf{q}}^\top\mathbf{H}^\top\mathbf{H}\hat{\mathbf{q}} \end{aligned} \tag{38}$$

$$\begin{aligned} \Rightarrow \frac{\partial\mathcal{L}}{\partial\hat{q}} &= 0 - 2\mathbf{H}^\top\mathbf{r} + 2\mathbf{H}^\top\mathbf{H}\hat{\mathbf{q}} \\ &= 0, \text{ for minimization} \end{aligned} \tag{39}$$

Thus,

$$\hat{\mathbf{q}} = \left(\mathbf{H}^\top\mathbf{H}\right)^{\mathbf{-1}}\mathbf{H}^\top\mathbf{r} \tag{40}$$

2.4.2 After Correlation

Combining (21) and (20) results in:

$$\begin{aligned} y_i &= \mathbf{q}^\top\mathbf{A}^\top\mathbf{S}^\top\mathbf{c}_i + \mathbf{n}^\top\mathbf{c}_i \\ &= \mathbf{q}^\top\mathbf{h}_i + n_i \\ \text{where, } \mathbf{h}_i &= \mathbf{A}^\top\mathbf{S}^\top\mathbf{c}_i \text{ is a known vector of size } m \\ \text{and, } n_i &\quad \text{is the receiver noise} \end{aligned} \tag{41}$$

Assuming that the received signal in (41) is obtained in the l-th frame, the equation may be rewritten as:

$$y_i[l] = \mathbf{q}^\top\mathbf{h}_i[l] + n_i \tag{42}$$

If measurements are made over L frame–intervals, the resulting observation vector may be written as:

$$\begin{bmatrix} y_i[1] \\ y_i[2] \\ \vdots \\ y_i[L] \end{bmatrix} = \begin{bmatrix} h_{i1}[1] & h_{i2}[1] & \cdots & h_{im}[1] \\ h_{i1}[2] & h_{i2}[2] & \cdots & h_{im}[2] \\ \vdots & \vdots & \ddots & \cdots \\ h_{i1}[L] & h_{i2}[L] & \cdots & h_{im}[L] \end{bmatrix} \begin{bmatrix} \sqrt{q_1} \\ \sqrt{q_2} \\ \vdots \\ \sqrt{q_m} \end{bmatrix} + \begin{bmatrix} n_i[1] \\ n_i[2] \\ \vdots \\ n_i[L] \end{bmatrix} \tag{43}$$

$$\text{or, } \mathbf{y_i} = \mathbf{H_i q} + \mathbf{n}$$

Two important observations may be made at this point:

1. In the equation $\mathbf{y_i} = \mathbf{H_i q} + \mathbf{n}$, the vectors do not represent signal–space, but rather, correspond to observations made over multiple frames.

2. It is also worth noticing that the form of (41) is almost identical to equation (14), the result obtained for the algebraic analysis of the matched filter formulation.

Following the least–squares estimate technique introduced in Section 2.4.1, the estimate for the channel coefficient is:

$$\hat{\mathbf{q}} = \left(\mathbf{H}^\top \mathbf{H}\right)^{\mathbf{-1}} \mathbf{H}^\top \mathbf{y} \tag{44}$$

2.4.3 Discussion

The channel estimates shown in (40) and (44) were run in MATLAB for both scenarios and the results are outlined below. Equation (40) provided very accurate estimates for the channel coefficients when the rank of $\mathbf{H}$ was equal to the dimension of the estimation vector (*i.e.*, the number of receivers in the system). Choosing the activity matrix $\mathbf{A}$ as an identity, biased $\mathbf{H}$ in that direction. Equation (44) also provided very accurate results when the rank of $\mathbf{H}$ was equal to the dimension of the estimation vector (*i.e.*, the number of receivers in the system). This could be encouraged by choosing a larger number of observations (*i.e.*, increasing L) so as to obtain an overdetermined system.

3 Algorithm

The proposed model for the Network Interference Server (NIS) is theoretically capable of obtaining inter–node distances for any network where a model for the transmission activity is known. However, with increase in the number of nodes within the network, the behavior of the algorithm quickly deteriorates. Since a typical network may potentially contain hundreds of nodes, there should be a way to eliminate distant nodes that do not contribute much interference to the receiver at hand. Such a technique is now described.

Since the frame activity sequence of each of the nodes, *i.e.*, $\alpha_j[l]$, is known to the NIS, it can evaluate the correlation between the received signal over a certain number of frame–intervals and the activity of each of the other $m-1$ nodes. The resulting column matrix of $m-1$ elements gives an indication of the effect of the transmitter nodes at the receiver since the nearest nodes will have the maximum correlation with the received power.

Using the approximation for the received power introduced in (12), the covariance between the sample mean of received power at requesting node i over L frames and the activity of the jth node over the same number of frames is given by:

$$\begin{aligned}\operatorname{cov}\left[x_i^{(f)}, \alpha_j\right] &= \mathrm{E}\left[\left\{x_i^{(f)} - \mathrm{E}\left(x_i^{(f)}\right)\right\}\left\{\alpha_j - \mathrm{E}\left(\alpha_j\right)\right\}\right] \\ &\approx \frac{1}{L}\sum_{u=l-L}^{l}\left[\left(x_i^{(f)}[u] - \overline{x_i^{(f)}[l]}\right)\left(\alpha_j[u] - \overline{\alpha_j[l]}\right)\right]\end{aligned} \tag{45}$$

where,

$$\begin{aligned}\overline{x_i^{(f)}[l]} &\triangleq \frac{1}{L}\sum_{u=l-L}^{l} x_i^{(f)}[u] \\ \overline{\alpha_j[l]} &\triangleq \frac{1}{L}\sum_{u=l-L}^{l} \alpha_j[u]\end{aligned} \tag{46}$$

Then, the corresponding correlation coefficient is defined as:

$$\operatorname{cor}\left[x_i^{(f)}[l], \alpha_j[l]\right] = \frac{\operatorname{cov}\left[x_i^{(f)}[l], \alpha_j[l]\right]}{\sqrt{\operatorname{var}(x_i^{(f)}[l])}\sqrt{\operatorname{var}(\alpha_j[l])}} \tag{47}$$

where,

$$
\begin{aligned}
\mathrm{var}(x_i^{(f)}[l]) &\triangleq \mathrm{E}\left[\left\{x_i^{(f)} - \mathrm{E}\left(x_i^{(f)}\right)\right\}^2\right] \\
&\approx \frac{1}{L}\sum_{u=l-L}^{l}\left[\left\{x_i^{(f)}[u] - \overline{x_i^{(f)}[l]}\right\}^2\right] \\
\mathrm{var}(\alpha_j[l]) &\triangleq \mathrm{E}\left[\left\{\alpha_j - \mathrm{E}\left(\alpha_j\right)\right\}^2\right] \\
&\approx \frac{1}{L}\sum_{u=l-L}^{l}\left[\left\{\alpha_j[u] - \overline{\alpha_i[l]}\right\}^2\right]
\end{aligned}
\tag{48}
$$

and the sample means, $\overline{x_i^{(f)}[l]}$ and $\overline{\alpha_i[l]}$, are defined in (46).

The correlation vector between the power received at the ith node and the activity of each of the m transmitters over the interval corresponding to L frames is then denoted by the following $m \times 1$ matrix:

$$
\left[\mathrm{cor}\left\{x_i^{(f)}[l], \alpha_1[l]\right\}, \mathrm{cor}\left\{x_i^{(f)}[l], \alpha_2[l]\right\}, \ldots, \mathrm{cor}\left\{x_i^{(f)}[l], \alpha_m[l]\right\}\right]^\top \tag{49}
$$

The correlation coefficients may now be sorted and only the nodes corresponding to correlation coefficient values that satisfy a pre-defined threshold are considered for neighborhood estimation.

Thus, the steps involved in obtaining the inter–node distances with respect to a requesting node i may be summarised as follows:

1. Each node records the average received power in the frames in which it itself is not transmitting.
2. The sequence of activity periods for each of the nodes is recorded by the NIS through reliable control channels.
3. When a wireless node needs a neighborhood map, it sends a request to the NIS with a history of received powers.
4. After identifying the requesting node, the NIS correlates the requesting node's received power values with the activity log of all nodes in its database to estimate a subset of nodes that affect the power measurements at the receiver and hence lie within a certain radio–distance from the receiver.
5. Finally, the NIS solves the channel coefficients (and hence obtains the distances from the requesting node) as long as the number of independent and non–zero measurements sent by the requesting node is equal to the number of other nodes in the sub–network evaluated in the previous step, *i.e.*, the matrix in (14) is full-ranked and forms a determinable system.

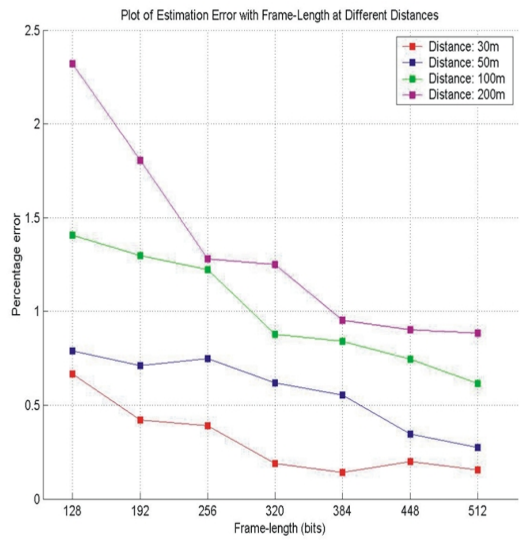

Figure 1: MATLAB plot of percentage error in estimating the channel coefficient against different frame–lengths for different inter–node distances.

4 Simulation

To verify the operation of the NIS algorithm, a simple simulation experiment was designed. This comprised a single NIS server and five nodes uniformly distributed in a square of side 200 m. Nodes were assumed to transmit data packets at a constant bit-rate (*i.e.*, CBR traffic) of 1 Mbps, and contention was decided by the CSMA/CA (carrier sense multiple-access/collision avoidance) mechanism of the IEEE 802.11b protocol [7].

The distribution of the activity factors was obtained by running the network simulator [3] and these transmission time values were then used to execute the NIS algorithm in MATLAB. Since nodes were placed at different distances from the NIS, it was possible to estimate the channel coefficients as well as the inter–node distances with different degrees of accuracy. Fig. 1 shows the variation in channel coefficient estimation error (in percentage) with the increase in frame-length at different inter–node distances. The signal-to-noise ratio (SNR) was fixed at 40 dB at a distance of 1 m from the transmitter.

It was also observed that the estimation improved with longer frame–lengths. This follows from the fact that the channel coefficients are obtained by solving the set of deterministic simultaneous equations (14), which was

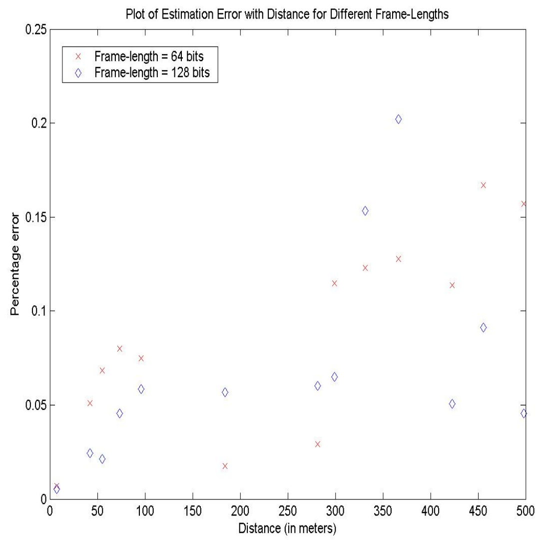

Figure 2: MATLAB plot of percentage error in estimating the channel coefficient with distance for different frame–lengths.

approximated from (12) by assuming that each frame contained a large number of bits.

The experiment was repeated for a larger number of nodes, classified into "near–nodes" and "far–nodes". Five nodes were assumed to be within 100 m of the information–requesting node, and another eight nodes were placed randomly between 150 m and 500 m from the receiver. Results for frames of length 64 bits and 128 bits have been obtained so far, and it is observed that there is a fall in the channel estimation error with distance (Fig. 2).

5 Conclusion

This paper describes a theoretical framework to obtain inter–node distances within a network where a centralized Network Interference Server is allowed to keep track of the transmission activity of all nodes that use its service. Ideal channel coefficients are assumed in the analysis, including a known background noise variance and time–invariant channel coefficients. An improvement of the current thesis over previous formulations is that it roughly estimates the node neighborhood from the correlation between received power and transmitter activity *before* evaluating the channel coefficients, thus allowing the algorithm to be applied even to a densely populated network.

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