Throughput of an Ad-Hoc Network with Wired Access Points

Abstract

This treatise aims at extending the results obtained by Gupta and Kumar[1] for the throughput of an ad-hoc network containing n wireless nodes, by assuming that a fraction q of the nodes have wired connections and can behave as access points. The problem is modelled as a collection of qn clusters, each of which has a wired node as its master. The net thoughput of the network is obtained by combining the transport capacity of each of the clusters, and is found to be independent of the total number of nodes. The throughput of such a hierarchical network compares favourably with the results obtained by Gupta and Kumar, namely that the throughput falls as $\sqrt{n}$.

1 Introduction

In their celebrated paper, Gupta and Kumar have proved that node throughput decreases as $\sqrt{n}$ for a classical ad-hoc network with identical radios. However, in a hierarchical network, it is feasible that some of the nodes would be interconnected through hard-wires to enable data transfer among themselves and the ability to connect to the Internet if necessary. In this analysis it is assumed that a fraction q of the nodes have wires (*i.e.* can function as access points) and that a packet that reaches an access point tunnels through to another access point closest to the destination. It is further assumed that an access point is never the initiator of a data transmission, but merely a relay node that acts as a gateway between various clusters. Thus, the traffic handled by the access points is a direct indicator of the traffic through the wired backbone. The nodes behave in the same way as in the Gupta–Kumar model in every other sense.

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[1] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks", *IEEE Transactions on Information Theory*, vol. 46, pp. 388–404, March 2000.

Assuming that each of the wireless nodes is *closest* to only *one* access point, it is possible to subdivide the entire network into qn mutually exclusive clusters, with an access point behaving as the master within the cluster. While data transmission within a single cluster can be carried out without the mediation of the access point, transmission to nodes in other clusters is carried out more efficiently by routing the data via the access point and hence through the wired backbone. Interference between adjacent cells may be reduced by employing frequency reuse, as in the case of a cellular network.

2 Analysis

For the sake of completeness, the throughput of a wireless network, as obtained by Gupta and Kumar, is discussed in this section. It is assumed that n nodes are arbitrarily located in a disk of unit area in the plane. The *physical model* is used to describe a successful reception of a transmission over one hop. In the analysis that follows, X_i denotes the location of the ith node. The number of wireless nodes is $(1-q)n$, while the number of access points is qn.

Let $\Gamma(m,s)$ denote the set of all nodes transmitting simultaneously over sub-channel m in time-slot s. Let P_k be the power level chosen by node $k \in \Gamma(m,s)$. Then the transmission from the node located at X_i, $i \in \Gamma(m,s)$, is successfully received by a node at X_j if the following inequality is satisfied:

$$\frac{\frac{P_i}{|X_i - X_j|^\alpha}}{N + \sum_{k \in \Gamma(m,s); k \neq i} \frac{P_k}{|X_k - X_j|^\alpha}} \geq \beta \qquad (1)$$

This models a situation where a minimum signal to interference ratio (SIR) of β is necessary for successful reception, the ambient noise power level is N, and signal power decays as the α-th power of the distance.

The main assumptions for the determination of throughput are listed below:

1. There are n nodes (including access points) arbitrarily located in a disk of area A on the plane.
2. The network transports $\lambda n T$ bits over T seconds, where λ is the average throughput of each node for a randomly chosen destination.

3. The average distance between the source and destination of a bit is L. In combination with the above assumption, this means that a transport capacity of $\lambda n L$ bit-metre/second is achieved.

4. Each node can broadcast wirelessly over any subset of M sub-channels with capacities W_m bit/sec, $1 \leq m \leq M$, where $\sum_{m=1}^{M} W_m = W$. Thus, W is the maximum achievable transmission rate over the common wireless channel.

5. Transmissions are synchronised into time-slots of length τ seconds.

6. The number of hops required by the bth bit to reach the destination from the source is $h(b)$, and $r(h, b)$ denotes the distance travelled by the bth bit in the hth hop.

It has already been established that for a successful transmission from a node $i \in \Gamma(m, s)$ to a node j, condition (1) must be satisfied. If the signal power of i is also included in the denominator of (1), then the signal-to-interference requirement may be rewritten as:

$$\frac{\frac{P_i}{|X_i - X_j|^\alpha}}{N + \sum_{k \in \Gamma(m,s)} \frac{P_k}{|X_k - X_j|^\alpha}} \geq \frac{\beta}{\beta + 1} \tag{2}$$

where $\Gamma(m, s)$ is the set of nodes (wireless nodes *and* access points) transmitting simultaneously in the mth sub-channel and sth time-slot.

$$\Rightarrow |X_i - X_j|^\alpha \leq \frac{\beta + 1}{\beta} \frac{P_i}{N + \sum_{k \in \Gamma(m,s)} \frac{P_k}{|X_k - X_j|^\alpha}} \tag{3}$$

$$\leq \frac{\beta + 1}{\beta} \frac{P_i}{N + \left(\frac{\pi}{4A}\right)^{\frac{\alpha}{2}} \sum_{k \in \Gamma(m,s)} P_k} \tag{4}$$

The result $|X_k - X_j| \leq \sqrt{\frac{4A}{\pi}}$ follows from the fact that the diameter of a circle with area A is $\sqrt{\frac{4A}{\pi}}$ and that is the maximum possible separation between two nodes.

Summing over all transmitter–receiver pairs,

$$\sum_{i\in\tau} |X_i - X_j|^\alpha \le \frac{\beta+1}{\beta} \frac{\sum_{i\in\Gamma(m,s)} P_i}{N + \left(\frac{\pi}{4A}\right)^{\frac{\alpha}{2}} \sum_{k\in\Gamma(m,s)} P_k} \tag{5}$$

$$\le \left(\frac{\beta+1}{\beta}\right)\left(\frac{4A}{\pi}\right)^{\frac{\alpha}{2}} \tag{6}$$

Summing over all slots and sub-channels (and noting that there can be no more than $\frac{T}{\tau}$ slots in T seconds),

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} r^\alpha(h,b) \le \sum_{s=1}^{\frac{T}{\tau}}\sum_{m=1}^{M} \left(\frac{\beta+1}{\beta}\right)\left(\frac{4A}{\pi}\right)^{\frac{\alpha}{2}} \tag{7}$$

$$= \left(\frac{\beta+1}{\beta}\right)\left(\frac{4A}{\pi}\right)^{\frac{\alpha}{2}} \frac{T}{\tau}M \tag{8}$$

$$= \left(\frac{\beta+1}{\beta}\right)\left(\frac{4A}{\pi}\right)^{\frac{\alpha}{2}} WT \tag{9}$$

where equation (9) follows from Assumption 4.

To reduce the result in terms of $r(h,b)$, the convexity of r^α has to be exploited. In order to do that, a function H is defined that sums the number of hops for every bit b:

$$H = \sum_{b=1}^{\lambda nT} h(b) \tag{10}$$

Since at most $\frac{n}{2}$ nodes can transmit over any sub-channel in the same time-slot, the upper bound of H is given by:

$$H \le \frac{WTn}{2} \tag{11}$$

By dividing both sides of (9) by H, the remaining steps in the derivation follow directly from Gupta–Kumar's paper:

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H} r^\alpha(h,b) \le \left(\frac{\beta+1}{\beta}\right)\left(\frac{4A}{\pi}\right)^{\frac{\alpha}{2}} \frac{WT}{H} \tag{12}$$

Since this function is convex,

$$\left[\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}r(h,b)\right]^{\alpha} \leq \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}r^{\alpha}(h,b) \tag{13}$$

$$\leq \left(\frac{\beta+1}{\beta}\right)\left(\frac{4A}{\pi}\right)^{\frac{\alpha}{2}}\frac{WT}{H} \tag{14}$$

where (14) follows by substituting (12) into (13).

$$\Rightarrow \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}\frac{1}{H}r(h,b) \leq \left[\left(\frac{\beta+1}{\beta}\right)\left(\frac{4A}{\pi}\right)^{\frac{\alpha}{2}}\frac{WT}{H}\right]^{\frac{1}{\alpha}} \tag{15}$$

$$\Rightarrow \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}r(h,b) \leq \left[\left(\frac{\beta+1}{\beta}\right)\left(\frac{4A}{\pi}\right)^{\frac{\alpha}{2}}WTH^{\alpha-1}\right]^{\frac{1}{\alpha}} \tag{16}$$

However, if a bit b ($1 \leq b \leq \lambda nT$) moves from its origin to a destination in a sequence of $h(b)$ hops, where the h-th hop traverses a distance of $r(h,b)$, then from Assumption 3, it follows that:

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)}r(h,b) \geq \lambda nTL \tag{17}$$

$$\Rightarrow \lambda nTL \leq \left[\left(\frac{\beta+1}{\beta}\right)\left(\frac{4A}{\pi}\right)^{\frac{\alpha}{2}}WTH^{\alpha-1}\right]^{\frac{1}{\alpha}} \tag{18}$$

$$= \frac{2\sqrt{A}}{\sqrt{\pi}}\left[\left(\frac{\beta+1}{\beta}\right)WTH^{\alpha-1}\right]^{\frac{1}{\alpha}} \tag{19}$$

$$\leq \frac{2\sqrt{A}}{\sqrt{\pi}}\left[\left(\frac{\beta+1}{\beta}\right)\frac{W^{\alpha}T^{\alpha}n^{\alpha-1}}{2^{\alpha-1}}\right]^{\frac{1}{\alpha}} \tag{20}$$

$$= \frac{2\sqrt{A}}{2\sqrt{\pi}}\left[\left(\frac{2\beta+2}{\beta}\right)n^{\alpha-1}\right]^{\frac{1}{\alpha}}WT \tag{21}$$

where inequality (20) follows from (11).

The throughput is obtained by dividing both sides of (21) by T:

$$\lambda nL \leq \frac{W\sqrt{A}}{\sqrt{\pi}}\left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}}n^{\frac{\alpha-1}{\alpha}} \tag{22}$$

Hence,

$$\text{Transport capacity} \leq \frac{W\sqrt{A}}{\sqrt{\pi}} \left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}} n^{\frac{\alpha-1}{\alpha}} \tag{23}$$

$$\text{Throughput / node} \leq \frac{W\sqrt{A}}{\sqrt{\pi}} \left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}} n^{-\frac{1}{\alpha}} \tag{24}$$

It may be pointed out that when the disc that contains the nodes is of unit area (*i.e.*, $A = 1$), the upper bounds in (23) and (24) reduce to the results obtained by Gupta and Kumar.

3 Results

As has been mentioned in Section 1, the aim of this analysis is to subdivide the modified network containing $(1-q)n$ wireless nodes into qn clusters with an access point as the master node in each cluster. Subsequently, a scaled version of equation (22) (both in terms of the reduced area *and* the reduced number of nodes) is applied to each of the clusters to obtain the individual throughputs. Finally, the net throughput is obtained by combining the contributions from all qn clusters.

It may be added that the effect of interference between nodes in different clusters may be reduced by assuming that adjacent clusters have different frequencies of operation assigned to them, as in cellular networks.

Since each access point corresponds to a cluster, there are qn clusters in the network. Thus, the number of wireless nodes in each cluster is $\frac{n}{qn} = \frac{1}{q}$.

Moreover, the average area of each of the clusters is $\frac{A}{qn}$.

Applying equation (23) to each of the clusters yields the average cluster capacity, measured in *bit m* s^{-1}:

$$\text{Cluster capacity} \leq \frac{W\sqrt{\frac{A}{qn}}}{\sqrt{\pi}} \left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}} \left(\frac{1}{q}\right)^{\frac{\alpha-1}{\alpha}} \tag{25}$$

$$= \frac{W\sqrt{A}}{\sqrt{\pi qn}} \left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}} \left(\frac{1}{q}\right)^{\frac{\alpha-1}{\alpha}} \tag{26}$$

Since there are qn clusters, the upper bound for the average throughput over all clusters is:

$$\text{Total wireless capacity} \leq \frac{Wqn\sqrt{A}}{\sqrt{\pi qn}}\left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}}\left(\frac{1}{q}\right)^{\frac{\alpha-1}{\alpha}} \tag{27}$$

$$= \frac{W\sqrt{qnA}}{\sqrt{\pi}}\left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}}\left(\frac{1}{q}\right)^{\frac{\alpha-1}{\alpha}} \tag{28}$$

It may be pointed out that when the propagation constant, α, is chosen as 2, the wireless throughput of the cluster network becomes independent of q and, in fact, reduces to the results obtained by Gupta and Kumar for a disc of area A. Since one of the most common choices of α is indeed 2, it might appear that no advantage was obtained by choosing a hierarchical network.

However, the improvement in capacity becomes obvious when careful cognizance is taken of the unit of measurement ($bit\ m\ s^{-1}$) of the throughput. In the cluster model, the distance travelled by each packet is much less, on the average, than in the flat network model. Thus the capacity, in terms of $bit\ s^{-1}$, is vastly improved as will now be demonstrated.

The average distance between nodes in a disc of area A is of the order of $l = O\left(\sqrt{A}\right)$. Thus, normalising the transport capacity obtained in (23) to $bit\ s^{-1}$ leads to a capacity of the order of:

$$\lambda n \leq \frac{W}{\sqrt{\pi}}\left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}} n^{\frac{\alpha-1}{\alpha}} \tag{29}$$

$$\text{Hence, throughput / node} \leq \frac{W}{\sqrt{\pi}}\left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}} n^{-\frac{1}{\alpha}} \tag{30}$$

However, when the cluster model is considered, the average distance travelled by data packets reduces to the order of the square root of the area of a single cluster. Since there are qn clusters, the average area of a cluster is $\frac{A}{qn}$, and the average distance between nodes is $l = O\left(\frac{A}{\sqrt{qn}}\right)$. Thus, the

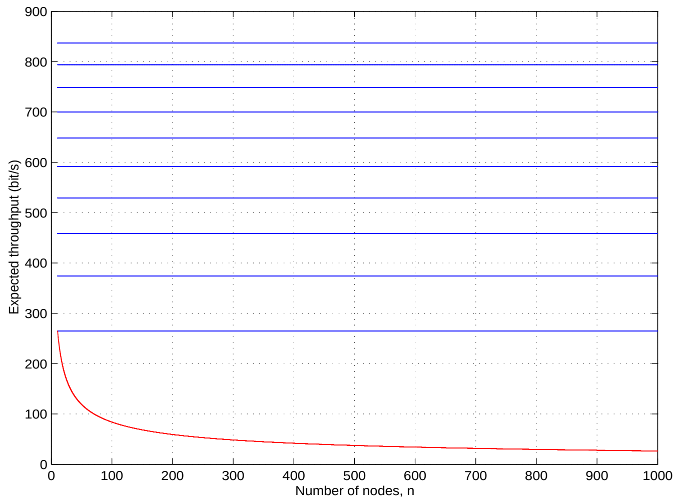

Figure 1: MATLAB plot of expected throughput as a function of the number of nodes n for $W = 1000$, $\beta = 10$, $\alpha = 2$ and q being varied from 0.1 (lowest blue line) to 1.0 (highest blue line) in steps of 0.1, for nodes distributed uniformly within the entire network. This is compared with the throughput of a flat wireless network (red line).

network throughput in $bit\ s^{-1}$ is of the order of:

$$\lambda n \leq \frac{Wqn}{\sqrt{\pi}} \left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}} \left(\frac{1}{q}\right)^{\frac{\alpha-1}{\alpha}} \tag{31}$$

$$\text{Throughput / node} \leq \frac{Wq}{\sqrt{\pi}} \left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}} \left(\frac{1}{q}\right)^{\frac{\alpha-1}{\alpha}} \tag{32}$$

$$= \frac{W}{\sqrt{\pi}} \left(\frac{2\beta+2}{\beta}\right)^{\frac{1}{\alpha}} q^{\frac{1}{\alpha}} \tag{33}$$

This leads to the remarkable result that the throughput per node is independent of the number of nodes and is constant for a fixed value of q, the fraction of access points. The scaling property of such a hierarchical network *vis-a-vis* a flat network model is illustrated in Fig. 1. The behaviour of the network throughput with the fraction q of access points is illustrated in Fig. 2.

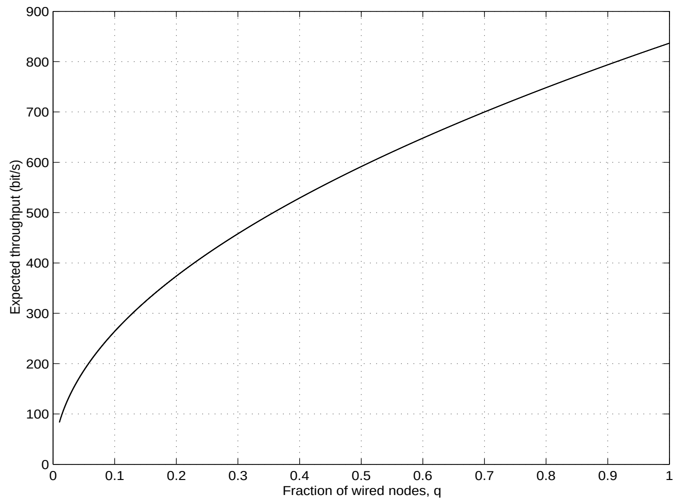

Figure 2: Matlab plot of expected throughput as a function of the fraction q of wired nodes for $W = 1000$, $\beta = 10$ and $\alpha = 2$. The throughput is independent of the number of nodes, n.

4 Conclusion

This analysis extends Gupta–Kumar's results to obtain network throughput when the scope of an ad-hoc network is increased to include wired access points. The crux of the analysis is the subdivision of the network into a number of clusters, with the wired access points serving as the gateway between different clusters. Simple plots with matlab suggest that the throughput of such a hierarchical network remains constant with increasing number of nodes n for a fixed fraction of access points, unlike the throughput of a flat network that scales to zero as the square root of n. Furthermore, the throughput of the network may effectively be doubled if transmission and reception occur at different frequencies. Finally, if the access points are also allowed to initiate data for transmission to other access points, the throughput of the network will be further increased by their rate of packet generation since the wired line capacity may be considered to be boundless.