Throughput in an Ad-Hoc Network with Wired Access Points

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Abstract

This treatise provides a solution for the throughput of an ad-hoc network containing n nodes, a fraction q of which have wired connections and can behave as access points. The problem attempts to build on the results obtained by Gupta and Kumar[1] for an ad-hoc network containing n wireless nodes, namely, that the throughput falls as $\sqrt{n}$.

1 Problem Statement

Gupta and Kumar have proved that node throughput decreases as $\sqrt{n}$ for a classical ad-hoc network with identical radios.

Now suppose that some fraction q of the nodes have wires (*i.e.* can function as access points). A packet that reaches an access point is assumed to tunnel through to another access point close to the destination. Otherwise, the radios behave in the same way as in the Gupta–Kumar model.

Find a similar equation for node throughput as a function of n and q. Does the system scale linearly for any value of q?

2 Preliminaries

Let us state some of assumptions and notations for an arbitrary ad-hoc network that will be used in the course of this derivation. We assume, after

[1] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks", *IEEE Transactions on Information Theory*, vol. 46, pp. 388–404, March 2000.

Gupta and Kumar, that n nodes are arbitrarily located in a disk of unit area in the plane. The *physical model* is going to be used to describe a successful reception of a transmission over one hop. In the analysis to follow, X_i denotes the location of a wireless node, while X_i^{AP} represents the position of a wired access point. The number of wireless nodes is $(1-q)n$, while the number of access points is qn.

Let $\{X_k; k \in \tau\}$ be the positions of the subset k of nodes simultaneously transmitting at some time instant over a certain sub-channel. Let P_k be the power level chosen by node X_k for $k \in \tau$. Then the transmission from a node X_i, $i \in \tau$, is successfully received by a node X_j if the following inequality is satisfied:

$$\frac{\frac{P_i}{|X_i - X_j|^2}}{N + \sum_{k\in\tau; k\neq i} \frac{P_k}{|X_k - X_j|^2}} \geq \beta \tag{1}$$

This models a situation where a minimum signal to interference ratio (SIR) of β is necessary for successful receptions, the ambient noise power level is N, and signal power decays with inverse square of the distance.

The throughput of the network is defined as the time average of the number of bits per second that can be transmitted by every node to its destination.

Thus, the main assumptions may be summarised as follows:

1. There are n nodes (including access points) arbitrarily located in a disk of unit area on the plane.

2. The network transports λnT bits over T seconds.

3. The average distance between the source and destination of a bit is L. In combination with the above assumption, this means that a transport capacity of λnL bit-metre/second is achieved.

4. Each node can broadcast wirelessly over any subset of M sub-channels with capacities W_m bit/sec, $1 \leq m \leq M$, where $\sum_{m=1}^{M} W_m = W$. In addition, the access points can transmit data to another access point through the wired connection with a throughput of W.

3 Analysis

To obtain the throughput of an arbitrary ad-hoc network, we shall consider the following four possibilities that correspond to different combinations of the transmitter and receiver:

1. Node-to-node transmission.
   The probability of this happening is $= \frac{(1-q)n}{n}\frac{(1-q)n}{n} = (1-q)^2$.

2. Node-to-access point transmission.
   The probability of this happening is $= \frac{(1-q)n}{n}\frac{qn}{n} = q(1-q)$.

3. Access point-to-node transmission.
   The probability of this happening is $= \frac{qn}{n}\frac{(1-q)n}{n} = q(1-q)$.

4. Access point-to-access point transmission.
   The probability of this happening is $= \frac{qn}{n}\frac{qn}{n} = q^2$.

For simplicity, it is assumed that the transmissions in the network are slotted into synchronized slots of length τ s. The other parameters of the system are formally defined below:

- λ: Average throughput of each node for a randomly chosen destination
- L: Mean distance between node and randomly chosen destination
- W: Maximum transmission rate of each node over common wireless channel
- $h(b)$: number fo hops required by bth bit to reach destination from source
- $r(h, b)$: distance traveled by bth bit in hth hop

3.1 Throughput determination for node-to-node transmission

It has already been established that for a successful transmission from a node X_i, $i \in \tau$ to a node X_j, condition (1) must be satisfied. If the signal power of X_i is also included in the denominator, then the signal-to-interference requirement may be rewritten as:

$$\frac{\frac{P_i}{|X_i - X_j|^2}}{N + \sum_{k \in \tau} \frac{P_k}{|X_k - X_j|^2}} \geq \frac{\beta}{\beta + 1} \tag{2}$$

where τ is the set of simultaneous transmitters (wireless nodes *and* access points).

$$\Rightarrow |X_i - X_j|^2 \leq \frac{\beta + 1}{\beta} \frac{P_i}{N + \sum_{k \in \tau} \frac{P_k}{|X_k - X_j|^2}} \tag{3}$$

$$\leq \frac{\beta + 1}{\beta} \frac{P_i}{N + (\frac{\pi}{4}) \sum_{k \in \tau} P_k} \tag{4}$$

The result $|X_k - X_j| \leq \sqrt{\frac{4}{\pi}}$ follows from the fact that the diameter of a unit area circle is $\sqrt{\frac{4}{\pi}}$ and that is the maximum possible separation between two nodes.

Summing over all transmitter–receiver pairs,

$$\sum_{i \in (1-q)\tau} |X_i - X_j|^2 \leq \frac{\beta + 1}{\beta} \frac{\sum_{i \in (1-q)\tau} P_i}{N + (\frac{\pi}{4}) \sum_{k \in (1-q)\tau} P_k} \tag{5}$$

$$\leq \left(\frac{\beta + 1}{\beta}\right) \left(\frac{4}{\pi}\right) \tag{6}$$

Summing over all slots and channels,

$$\sum_{b=1}^{\lambda(1-q)nT} \sum_{h=1}^{h(b)} r^2(h, b) \leq \left(\frac{\beta + 1}{\beta}\right) \left(\frac{4}{\pi}\right) WT \tag{7}$$

To reduce the result in terms of $r(h, b)$, convexity of the quadratic equation has to be invoked. In order to do that, a function H is defined that sums the number of hops for every bit b:

$$H = \sum_{b=1}^{\lambda n(1-q)T} h(b) \tag{8}$$

Since at most $\frac{n(1-q)}{2}$ nodes can transmit over any channel in the same slot, the upper bound of H is given by:

$$H \leq \frac{WT(1-q)n}{2} \tag{9}$$

By dividing both sides of (7) by H, the remaining steps in the derivation follow directly from Gupta–Kumar's paper:

$$\sum_{b=1}^{\lambda(1-q)nT} \sum_{h=1}^{h(b)} \frac{1}{H} r^2(h,b) \leq \left(\frac{\beta+1}{\beta}\right)\left(\frac{4}{\pi}\right)\frac{WT}{H} \tag{10}$$

Since the quadratic function is convex,

$$\left[\sum_{b=1}^{\lambda(1-q)nT} \sum_{h=1}^{h(b)} \frac{1}{H} r(h,b)\right]^2 \leq \sum_{b=1}^{\lambda(1-q)nT} \sum_{h=1}^{h(b)} \frac{1}{H} r^2(h,b) \tag{11}$$

$$\Rightarrow \left[\sum_{b=1}^{\lambda(1-q)nT} \sum_{h=1}^{h(b)} \frac{1}{H} r(h,b)\right]^2 \leq \left(\frac{\beta+1}{\beta}\right)\left(\frac{4}{\pi}\right)\frac{WT}{H} \tag{12}$$

$$\Rightarrow \sum_{b=1}^{\lambda(1-q)nT} \sum_{h=1}^{h(b)} r(h,b) \leq \frac{2}{\sqrt{\pi}}\sqrt{\left(\frac{\beta+1}{\beta}\right)\left(\frac{4}{\pi}\right)WTH} \tag{13}$$

However, if a bit b ($1 \leq b \leq \lambda nT$) moves from its origin to a destination in a sequence of $h(b)$ hops, where the h-th hop traverses a distance of $r(h,b)$, then from Assumption 3, we must have:

$$\sum_{b=1}^{\lambda(1-q)nT} \sum_{h=1}^{h(b)} r(h,b) \geq \lambda(1-q)nTL \tag{14}$$

$$\Rightarrow \lambda(1-q)nTL \leq \frac{2}{\sqrt{\pi}}\sqrt{\left(\frac{\beta+1}{\beta}\right)\left(\frac{4}{\pi}\right)WTH} \tag{15}$$

The throughput is obtained by substituting (9) in (15):

$$\lambda(1-q)nL \leq \frac{2}{\sqrt{\pi}}\sqrt{\left(\frac{\beta+1}{\beta}\right)\left(\frac{1-q}{T}\right)W\frac{WTn}{2}} \tag{16}$$

Hence,

$$Throughput \leq \frac{W}{\sqrt{\pi}}\sqrt{\left(\frac{2\beta+2}{\beta}\right)(1-q)n} \tag{17}$$

This is merely scaled form of the result obtained by Gupta–Kumar for a system with n nodes.

3.2 Throughput determination for node-to-access point transmission

Since the number of transmitters in this case is still $(1-q).n$, the expression for the throughput may be written as:

$$Throughput \leq \frac{W}{\sqrt{\pi}}\sqrt{\left(\frac{2\beta+2}{\beta}\right)(1-q)n} \tag{18}$$

3.3 Throughput determination for access point-to-node transmission

Since the number of access points is different from the wireless nodes, the throughput expression for wireless transmission must be scaled accordingly:

$$Throughput \leq \frac{W}{\sqrt{\pi}}\sqrt{\left(\frac{2\beta+2}{\beta}\right)qn} \tag{19}$$

3.4 Throughput determination for access point-to-access point transmission

It follows from assumption 4, that for wired connection:

$$\lambda qnL = WqnL \tag{20}$$

But, $L \leq \frac{2}{\sqrt{\pi}}$ since a unit area circle is being considered. Thus,

$$throughput \leq Wqn\frac{2}{\sqrt{\pi}} \tag{21}$$

3.5 Expected throughput

The expected throughput is now obtained as follows:

$$\begin{aligned}
E[throughput] &\leq (1-q)^2\frac{W}{\sqrt{\pi}}\sqrt{\left(\frac{2\beta+2}{\beta}\right)(1-q)n} \\
&\quad +q(1-q)\frac{W}{\sqrt{\pi}}\sqrt{\left(\frac{2\beta+2}{\beta}\right)(1-q)n} \\
&\quad +q(1-q)\frac{W}{\sqrt{\pi}}\sqrt{\left(\frac{2\beta+2}{\beta}\right)qn} \\
&\quad +q^2Wqn\frac{2}{\sqrt{\pi}} \qquad (22) \\
&= (1-q)\frac{W}{\sqrt{\pi}}\sqrt{\left(\frac{2\beta+2}{\beta}\right)(1-q)n} \\
&\quad +q(1-q)\frac{W}{\sqrt{\pi}}\sqrt{\left(\frac{2\beta+2}{\beta}\right)qn} \\
&\quad +q^2Wqn\frac{2}{\sqrt{\pi}} \qquad (23)
\end{aligned}$$

4 Discussion

This analysis borrowed heavily from the paper by Gupta and Kumar to obtain the throughput result when the scope of the ad-hoc network is increased to include wired access points. The expression obviously reduces to the original Gupta–Kumar result when the fraction q of access points is considered to be zero. Simple plots with MATLAB suggest that the expected value of the throughput increases exponentially with q for a constant n. The following figures illustrate the expected throughput as a function of q (for different values of n). The other fixed parameters are:

- $\beta = 10$ (*i.e.* 10 dB)
- $W = 10$

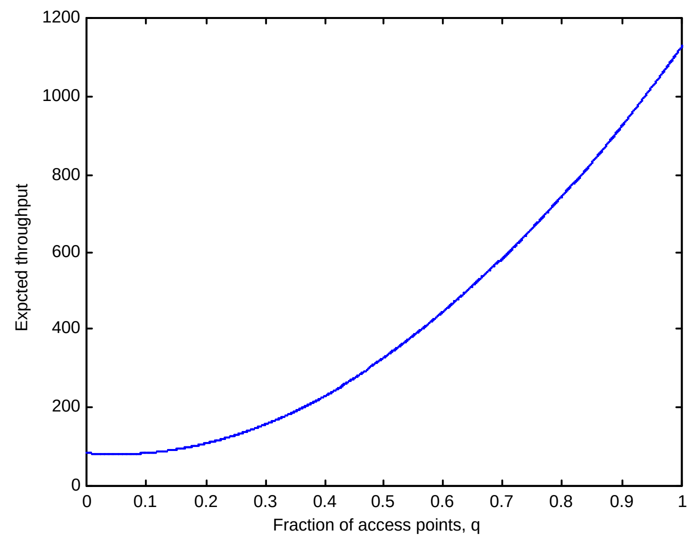

Figure 1: MATLAB plot of expected throughput as a function of q for $n = 100$.

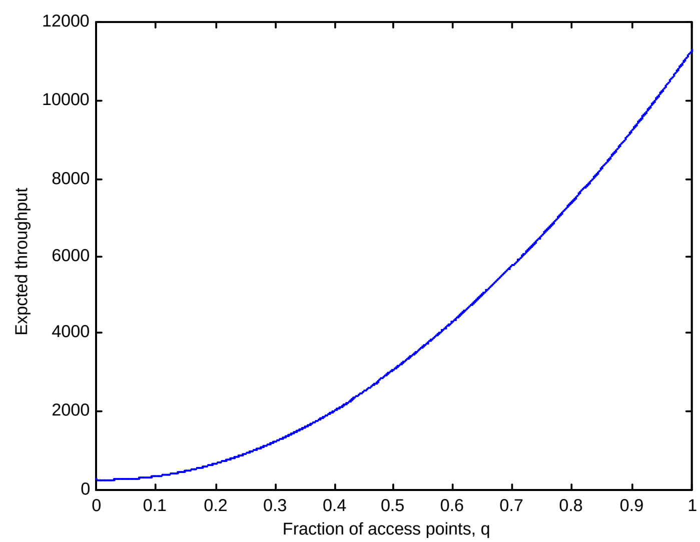

Figure 2: MATLAB plot of expected throughput as a function of q for $n = 1000$.