Demand Responsive Pricing and Competitive Spectrum Allocation via a Spectrum Server

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Abstract— **In this paper we develop a framework for competition of future operators likely to operate in a mixed commons/property-rights regime under the regulation of a spectrum policy server (SPS). The operators dynamically compete for customers as well as portions of available spectrum. The operators are charged by the SPS for the amount of bandwidth they use in their services. Through demand responsive pricing, the operators try to come up with convincing service offers for the customers, while trying to maximize their profits. We first consider a single-user system as an illustrative example. We formulate the competition between the operators as a non-cooperative game and propose an SPS-based iterative bidding scheme that results in a Nash equilibrium of the game. Numerical results suggest that, competition increases the user's (customer's) acceptance probability of the offered service, while reducing the profits achieved by the operators. It is also observed that as the cost of unit bandwidth increases relative to the cost of unit infrastructure (fixed cost), the operator with superior technology (higher fixed cost) becomes more competitive. We then extend the framework to a multiuser setting where the operators are competing for a number of users at once. We propose an SPS-based bandwidth allocation scheme in which the SPS optimally allocates bandwidth portions for each user-operator session to maximize its overall expected revenue resulting from the operator payments. Comparison of the performance of this scheme to one in which the bandwidth is equally shared between the user-operator pairs reveals that such an SPS-based scheme improves the user acceptance probabilities and the bandwidth utilization in multiuser systems.**

I. INTRODUCTION

The emergence of innovative communication technologies operating in the unlicensed bands has started an ongoing debate regarding the efficiency of spectrum utilization and its relationship to usage policies enforced by the US FCC [1].

The US FCC regulates the available spectrum by allocating portions of it for predefined purposes. Nearly 75% of the UHF Band (300 MHz - 3 GHz) is allocated as command and control bands in which spectrum access is under strict govermental control. Also, 7% is dedicated to usage under a property-rights regime, in which the licensed cellular and PCS services are offered. Only 4% is allocated as unlicensed bands, i.e., the commons regime [2].

Such an allocation is typically long term and leads to the underutilization of spectrum. For example, [3] reports that even during the high demand period of a political convention such as the one held between August 31 and September 1 of 2004 in New York City, only about 13% of the spectrum opportunities were utilized.

Among the proposed alternative spectrum management policies, the mixed commons/property-rights overseen by a regulator seems to be a realistic approach for the near future [4]. In light of this, it seems prudent to expect that future communication systems will likely consist of heterogenous devices/technologies, dynamically sharing the locally available bandwidth under the supervision, or assistance of a local authority. The dynamic allocation of spectrum will most likely be on a much shorter time scale as opposed to current long term licences, which typically expire after 10 years [1].

The mixed commons/property-rights approach raises the issue of enabling architectures for coordinated spectrum access. In [5], this issue is addressed via the introduction of a (*spectrum policy server*) SPS. The SPS is a server that performs the centralized allocation of available spectrum, in a specified geographical region (locality). It mediates spectrum sharing among communicating devices, and also monitors the relevant spectrum. Reference [6] proposes a similar framework for coordinating dynamic spectrum access among service providers. The proposed scheme relies on a per-domain spectrum broker that controls the allocation of spectrum among the spectrum requesting operators.

In this paper, we develop a framework that can be used as a model for competition among future operators likely to operate in a mixed commons/property-rights regime, under the regulation of an SPS. We consider the case of operators dynamically competing for the spectrum as well as the potential customers. Portions of spectrum are devoted to any operator that provides service to a user. The operators in return pay the SPS for their usage of the spectrum. The operators try to attract customers through demand responsive pricing [7], [8]. They have differing service spectral efficiencies r [bps/Hz], and offer a rate R [bps] as well as a total price P [\$]. The customer's response to each offer is modelled through an *acceptance probability* $A(R,P)$ which reflects its willingness to buy the offered service at the asked price. Each offer invokes an *expected profit* for the operator making the offer. This profit is related to the associated $A(R,P)$ as well as the price asked

P, the related fixed costs (independent of the offered rate R) and variable costs that depend on the actual spectrum usage. The operators compete with each other in order to ensure that the user accepts their service offer with the highest probability. We formulate the operator competition as a non-cooperative game and propose an SPS-centered iterative bidding scheme that achieves a Nash equilibrium of the operator game.

Specifically, the user, upon entering the system gets connected to the SPS[1]. The SPS first collects user specific information such as its location and acceptance probability profile. It then mediates a bidding process between the operators to determine the one that could offer service to the user. Each operator adjusts its service offer in order to maximize its expected profit in the presence of competition. If the user accepts the service offer of the winning operator, the required portion of the spectrum is allocated to the operator for the duration of the session. We believe this reflects operation in a mixed commons/property-rights regime.

In the last section of the paper, we extend this framework to include the cases where the operators are competing for a number of users at once. We develop a simple extension of the single-user scheme where the SPS mediates bidding processes for many users simultaneously in parallel. We then propose an SPS-based bandwidth allocation scheme where the SPS optimally partitions the total available bandwidth among the different sessions in order to maximize its expected revenue.

II. Model

We consider a limited geographical region for which spectrum management is under the control of a local SPS. Two operators, Operator 1 and Operator 2, provide services to a user within the specified region. Each operator has a number of base stations throughout the region. For the sake of simplicity, we limit our discussions to a single user. The multiuser case will be addressed later in text. The SPS keeps track of the vacant spectrum W_{A} that would be available for usage of Operator 1 or Operator 2. We assume that the available bandwidth W_{A} is finite. Here, W_{A} could be exclusively or partially devoted to an operator, for it to offer service to the user. The allocation would be valid for the whole duration of the session established between the specified user and the operator. In this paper we assume that all sessions established between the user and the operators are of fixed duration. Thus the time parameter is not included in our formulations. We consider an "interference free" system in which the spectrum allocations are valid throughout the region and no two sessions can occupy the same frequency band, i.e., at each point in the frequency spectrum, there is at most one user-operator pair communicating at a time. In the following, we describe in detail the user acceptance model and the operator profit model that will govern the competitive spectrum allocation.

[1]This initial connection can be considered to be analogous to the operation of either domain name server (DNS) or dynamic host configuration protocol (DHCP) in internet engineering.

A. Users' Acceptance Model

In demand responsive pricing [7], [8], it is important to take into account the users' responses to the pricing strategy of the operator. From the user point of view, the service offer made by the operator is acceptable only if the price asked is reasonable.

Specifically, this consideration is addressed by introducing an acceptance probability $A(u,P)$ where u is the utility of the user and P is the associated price.

Intuitively, the acceptance probability $A(u,P)$ should have the following qualitative properties. It should be an increasing function of u for a fixed P while decreasing in P for fixed u. Mathematically, these properties are formulated as:

$$\begin{aligned} &\frac{\partial A}{\partial u} \geq 0, \qquad \frac{\partial A}{\partial P} \leq 0, \\ \forall P > 0, \quad &\lim_{u\to 0} A(u,P) = 0, \\ &\lim_{u\to \infty} A(u,P) = 1, \\ \forall u > 0, \quad &\lim_{P\to 0} A(u,P) = 1, \\ &\lim_{P\to \infty} A(u,P) = 0. \end{aligned} \tag{1}$$

While there are several candidate choices for the function $A(u,P)$, we will follow [7], [8] and choose

$$A(u,P) = 1 - e^{-Cu^{\mu}P^{-\epsilon}} \tag{2}$$

where μ is the utility sensitivity of the user, ϵ is the price sensitivity, and C is an appropriate constant. Note that the acceptance probability function can be differentiated among users through the above parameters. The above acceptance model is similar to the Cobb-Douglas demand curves that are used in economics [9].

In this paper, for simplicity, we ignore the role of transmit power in the user utility and instead parameterize u as a function of offered rate R only[2]. The specific model for the utility function chosen here is one that obeys a *law of diminishing returns* such as [7]–[9]:

$$u(R) = \frac{(R/K)^{\zeta}}{1 + (R/K)^{\zeta}} \tag{3}$$

where K and ζ are parameters that determine the exact shape of the above sigmoid function. Note that the above expression gives normalized utility values in the interval $[0,1)$ with the rate $R = K$ yielding a utility of $1/2$.

As $u(R)$ is a function of R, we simplify the notation by representing the acceptance probability as $A(R,P)$ in the rest of the paper.

In Fig. 1, we show the acceptance probability as a function of the offered rate R and price P. It is an illustration for the following choice of parameters: $K = 5\times10^{6}$, $\zeta = 10$, $C = 1$, $\epsilon = 4$, $\mu = 4$. As expected, the acceptance probability is decreasing with price. We also note the effect of diminishing returns. Specifically, even for a low price, if the user is offered

[2]It is possible to consider the role of transmit power in the user utility as has been done in [10].

rates in excess of 5 Mbps its acceptance probability will not increase in this example.

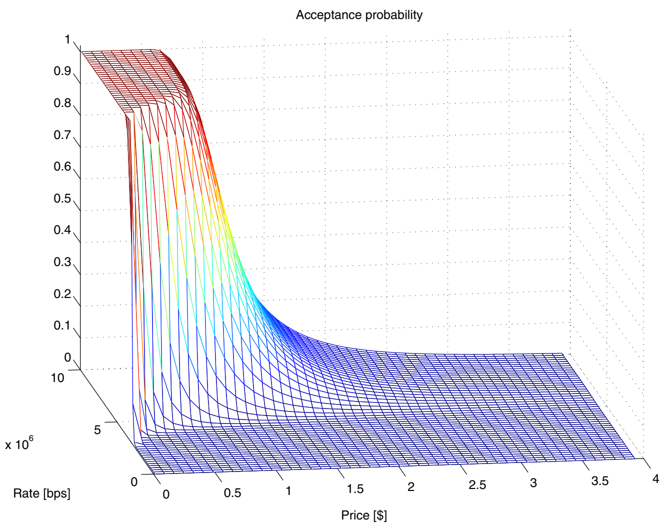

Fig. 1. The acceptance probability for $K = 5 \times 10^6$, $\zeta = 10$, $C = 1$, $\epsilon = 4$, $\mu = 4$.

B. Operator Profit Model

From the operator's point of view, the service is worth providing only if the achievable revenue is high enough to compensate for the costs associated with providing the service. The two operators considered in the model, Operator 1 and Operator 2, are able to provide spectral efficiencies of r_i [bps/Hz] to a specified user, where $i \in \{1, 2\}$ is the index denoting the operator. The spectral efficiency may depend on various parameters like the technology used by the operator, the density of the base stations belonging to the operator in the considered geographical region, and the location of the user. For a specific rate offered R_i, the bandwidth required $W_i(R_i)$ by the operator is inversely proportional to the spectrum efficieny: $W_i(R_i) = R_i/r_i$.

For the offered rate R_i and price P_i, the profit $Q_i(R_i, P_i)$ can be expressed as:

$$Q_i(R_i, P_i) = P_i - F_i - V_i R_i / r_i \tag{4}$$

where F_i [\$] is the fixed cost incurred by the operator, and V_i [\$/Hz] is the price per unit bandwidth that the SPS charges Operator i. The last term denotes the usage-based variable cost for the operator. Note that in most cases, the fixed cost F_i is implicitly related to the efficiency r_i. One would expect operators with higher fixed cost to be able to sustain greater efficiencies resulting from superior infrastructure [11].

Considering the user's acceptance probability, the *expected profit* for Operator i is

$$\overline{Q_i}(R_i, P_i) = A(R_i, P_i)\, Q_i(R_i, P_i). \tag{5}$$

Note that for fixed r_i, the acceptance probability $A(R_i, P_i)$ is increasing in R_i and decreasing in P_i while the profit $Q_i(R_i, P_i)$ is decreasing in R_i and increasing in P_i.

C. Operator Interaction as a Non Cooperative Game

The user response to an offer (R, P) is modelled as in (2). If two offers (R_1, P_1) and (R_2, P_2) are made by Operators 1 and 2 respectively, the offer for which $A(R, P)$ is lower is ignored by the user. The other offer is then accepted with the associated acceptance probability. When the offers made invoke equal acceptance probabilities, $A(R_1, P_1) = A(R_2, P_2)$, we assume that each offer is equally likely to be accepted.

In the context of operators competing for resources and the user preference, the game can be represented by $G = [N, \{Si\}, \beta_i]$ where $N = \{1, 2\}$ is the index set of the players (operators), S_i is the strategy space available to Operator i, and $\beta_i(.)$ is the resulting expected profit associated with the operator with index i. The strategy space S_i for Operator i consists of all (R, P) pairs which satisfy the bandwidth constraint:

$$S_i = \left\{ \forall (R, P) \,|\, \begin{array}{l} F_i + V_i R_i / r_i \le P \le P_{\max}, \\ 0 \le R \le W_A \times r_i. \end{array} \right\} \tag{6}$$

where $P_{\max}$ is the maximum price the operator is permitted to charge. We further impose the constraint that the profits are necessarily nonnegative.

The *resulting expected profit* β_i of operator i given the strategy of the opponent operator j is

$$\beta_i(R_i, P_i, R_j, P_j) = \begin{cases} 0 & \text{if } A(R_i, P_i) < A(R_j, P_j), \\ \frac{1}{2}\overline{Q_i}(R_i, P_i) & \text{if } A(R_i, P_i) = A(R_j, P_j), \\ \overline{Q_i}(R_i, P_i) & \text{if } A(R_i, P_i) > A(R_j, P_j). \end{cases} \tag{7}$$

The non-cooperative operator game can now be formally stated as

$$\max_{(R_i, P_i) \in S_i} \beta_i(R_i, P_i, R_j, P_j) \quad i \in \{1, 2\} \tag{8}$$

We now state the following theorem regarding the above game.

Theorem 2.1: At any Nash equilibrium for the game *G*, at least one of the operators has zero expected profit.

Proof: By contradiction: Assume there exist the equilibrium strategies (R_1^*, P_1^*) and (R_2^*, P_2^*) for which $\beta_1(.) > 0$ and $\beta_2(.) > 0$. Considering (7), the only way this can be achieved is to have equality between the achieved acceptance equalities; $A(R_1^*, P_1^*) = A(R_2^*, P_2^*)$. In this situation, in accordance with (7), the corresponding payoffs would be $\beta_1(R_1^*, P_1^*) = \frac{1}{2}\overline{Q_1}(R_1^*, P_1^*)$ and $\beta_2(R_2^*, P_2^*) = \frac{1}{2}\overline{Q_2}(R_2^*, P_2^*)$. Note that the assumption of non-zero profits implies that $\frac{1}{2}\overline{Q_1}(R_1^*, P_1^*) > 0$ and $\frac{1}{2}\overline{Q_2}(R_2^*, P_2^*) > 0$. Consider Operator 1 without loss of generality. If Operator 1 were now to deviate from the strategy (R_1^*, P_1^*) to $(R_1^*, P_1^* - \Delta_P)$ by lowering its price offer by an infinitesimal amount Δ_P, then it follows that $A(R_1^*, P_1^* - \Delta_P)$ is greater than $A(R_2^*, P_2^*)$. Further, from (7) it follows that the resulting expected profit for Operator 1 is $\overline{Q_1}(R_1^*, P_1^* - \Delta_P)$. By continuity of the profit function, it follows that $|\overline{Q_1}(R_1^*, P_1^* - \Delta_P) - \overline{Q_1}(R_1^*, P_1^*)| < \delta$ for arbitrarily small $\delta > 0$. We can thus bound the change in payoff of Operator 1, i.e., $\overline{Q_1}(R_1^*, P_1^* - \Delta_P) - \frac{1}{2}\overline{Q_1}(R_1^*, P_1^*)$

as $\frac{1}{2}\overline{Q_1}\left(R_1^*, P_1^*\right) - \delta < \overline{Q_1}\left(R_1^*, P_1^* - \Delta_P\right) - \frac{1}{2}\overline{Q_1}\left(R_1^*, P_1^*\right) < \frac{1}{2}\overline{Q_1}\left(R_1^*, P_1^*\right) + \delta$. Given that $\frac{1}{2}\overline{Q_1}\left(R_1^*, P_1^*\right) > 0$ and δ is arbitrarily small, it follows that the change in payoff for Operator 1 is strictly positive. Therefore the strategy $\left(R_1^*, P_1^*\right)$ can never be the best response of Operator 1. This contradicts the initial assumption that at equilibrium $\beta_1(.)$ and $\beta_2(.)$ are greater than zero. ■

III. SPS as a Mediator in Iterative Bidding

In this section we propose an iterative bidding process to implement the operator game *G*. The SPS mediates the bidding process on behalf of the user. Such an SPS based scheme is more practical as it reduces the amount of overhead and control information transmission to and from the user.

The scheme is composed of three steps (Fig. 2):

- Step 1: *A new user gets connected to the SPS. User specific information, e.g.,* $A(R, P)$*,* r_1*,* r_2*, is communicated to the SPS.*
- Step 2: *The iterative bidding process between the operators is undergone and the winning operator is declared by the SPS.*
- Step 3: *The winning operator offers the winning bid* (R_{winner}, P_{winner})*. The user decides to accept the service with probability* $A(R_{winner}, P_{winner})$*.*

Note that at the end of Step 1, the SPS has all the relevant information regarding the user so it can act on the user's behalf. Consequently, in Step 2, during the iterative bidding, only the SPS and the operators are involved. In Step 3, the user makes the final decision whether or not to take the service offer of the winning operator.

A. Iterative Bidding

In the iterative bidding process, the operators make offers in each iteration. The strategy of each operator is to make the offer such that $A(R, P)$ associated with its offer is greater than the one associated with its opponent's offer while simultaneously maximizing the resulting expected profit.

The iterative bidding is initialized by allowing the operators to choose their service offers without consideration of the opponent strategy.

It is clear from the structure of $\beta_i(.)$ in (7) that the iteration process is terminated when a zero value for expected profit is declared by at least one operator. The opportunity to offer service to the user is then given to the operator that wins. The winning operator uses its most recent bid $(R_{\text{winner}}, P_{\text{winner}})$, as a service offering to the user. Note that from Theorem 2.1, the iterative bidding process by definition should converge to a Nash equilibrium of the game *G*. If both operators declare zero expected profit at the same iteration, both are dismissed. This degenerate situation can happen when both operators have identical fixed costs and the user is located in a geographical location where the spectral efficiencies of both operators are identical. Such an operating point is also a Nash equilibrium. In such a case, we assume that the SPS randomly selects one of them to offer service.

It is assumed that the offers made to any user are final and the operators can not update any offers they have made to a specific user after the competition is over. The winning operator is obliged to provide the transmission rates they have offered. This is considered as part of the regulations enforced on the operators.

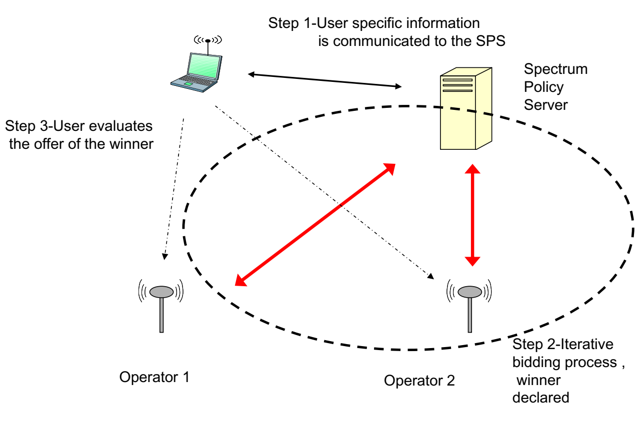

Fig. 2. SPS as a mediator.

IV. Numerical Results for Single User Systems

In this section, we present numerical results that correspond to a linear geographical region with two operators. The system and the base station locations are depicted in Fig. 3. We assume that both operators use the same technology, with the only difference being in the infrastructure density. Operator 1 has two base stations while Operator 2 only one. Consequently, we assume the associated fixed cost for Operator 1 will be twice the fixed cost of Operator 2, i.e., $F_1 = 2F_2$. Note that the fixed cost per base station is the same. We also assume that the SPS will be charging both operators at the same variable cost rate V [$/Hz].

The spectral efficiency between base station *k* and the user's mobile terminal is determined as

$$r_k = \log_2\left[1 + \frac{P_\text{s}}{N_o}\left(\frac{d_k}{L/4}\right)^{-2}\right], \quad (9)$$

where P_s is the signal power, N_o is the AWGN variance, d_k is the distance between the base station k and the terminal, and L is the total length of the linear region in Fig. 3 ($L = 1000$ m). We set $P_\text{s} = 2N_o$, which guarantees a SNR $= 3$ dB at the distance of $L/4 = 250$ m from the base station. Note that Operator 1 always selects a base station that provides higher spectral efficiency to serve the user (i.e., the base station that is closer to the user's mobile terminal).

The available bandwidth is $W_\text{A} = 10$ MHz, and the user acceptance probability and the corresponding parameters are selected as in Fig. 1. We characterize the cost structure with

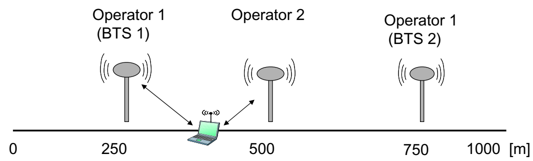

Fig. 3. Geographical region with two operators.

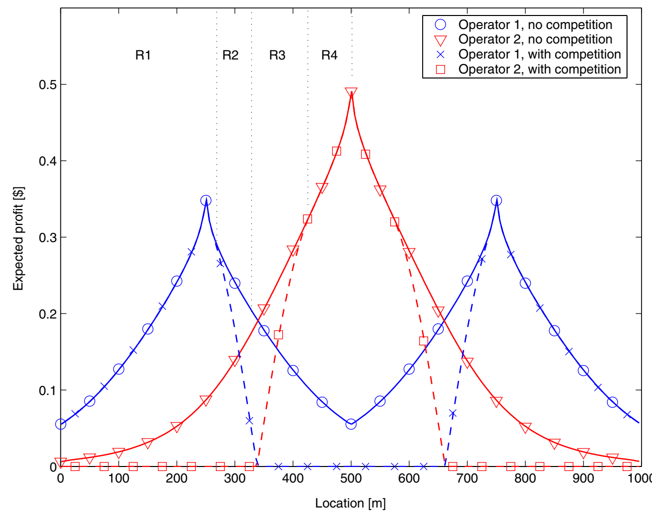

Fig. 4. The expected profit versus the location of the user within the linear region for $\eta = 1 \times 10^{-6}$, $W_{\rm A} = 10$ MHz, and $F + VW_{\rm A} = 2$ \$.

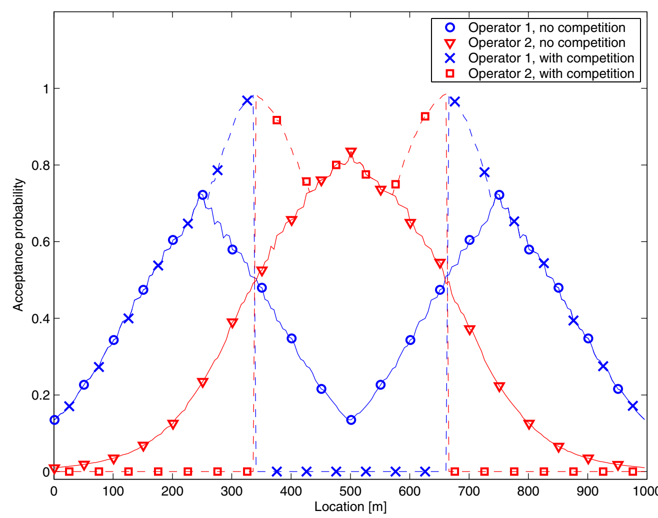

Fig. 5. The acceptance probability versus the location of the user within the linear region for $\eta = 1 \times 10^{-6}$, $W_{\rm A} = 10$ MHz, and $F + VW_{\rm A} = 2$ \$.

the ratio $\eta = V/F$ between the variable cost V [\$/Hz] versus the fixed cost per base station $F = F_1/2 = F_2$ [\$]. Lower values of η correspond to the spectrum being less expensive than the infrastructure. Furthermore the absolute values for F and V are selected such that $F + VW_{\rm A} = 2$ \$, while $F_1 = 2F$ and $F_2 = F$.

In Fig. 4 we present the expected profit versus the location of the user within the linear region for $\eta = 1 \times 10^{-6}$. The solid lines correspond to the case when only one of the operators is present. In that case the operator is offering the price and rate such that its expected profit is maximized without a competitor being present. The dashed line corresponds to the case when both operators are present and do compete for the user (as described in the previous sections). From these results, we note that the case with no competition provides an upper bound on expected profit for the case with competition. The corresponding acceptance probability is presented in Fig. 5.

Furthermore, depending on the user's location we observe the following behavior. In Region 1 (denoted as R1 in Fig. 4), the spectral efficiency of Operator 1 is much higher than that of Operator 2. Consequently Operator 1 is superior in the given region and can drive its expected profit up to the upper bound (case of no competition). In Region 2, the superiority of Operator 1 is diminishing and it is forced to lower its expected profit and out-compete Operator 2. In Region 3, Operator 2 is winning the competition (due to higher spectral efficiency while lowering its expected profit to become more competitive). In Region 4 the user is very close to the Operator 2 base station and its spectral efficiency is much higher than that of Operator 1. Now Operator 2 becomes superior in the given region and can drive its expected profit up to the upper bound (case of no competition). Beyond $L/2$, the situation is symmetric to the discussed regions. Complementary analysis can be presented for the acceptance probability versus the user location.

In the following figures, we present the results as functions of the ratio $\eta = V/F$ between the variable cost V [\$/Hz] versus the fixed cost per base station $F = F_1/2 = F_2$ [\$].

In Fig. 6, we present the expected profit, which is averaged over all locations of the user for a given ratio η. We assume a uniform distribution of the user locations. For the lower values of η, Operator 2, which has lower infrastructure density and lower total fixed cost, realizes higher expected profit. When the price of spectrum is increased, Operator 1 realizes higher expected profit because it is able to offer higher spectral efficiency (requiring a smaller portion of the spectrum), while its infrastructure cost is lower in relative terms.

In Fig. 7, we present the acceptance probability, averaged over all locations, in the case when one of the two operators is present and in the case when both operators are present and do compete for the user. Clearly, in the case of competition between the operators, the acceptance probability is higher. This points out that the operator competition is beneficial to the user.

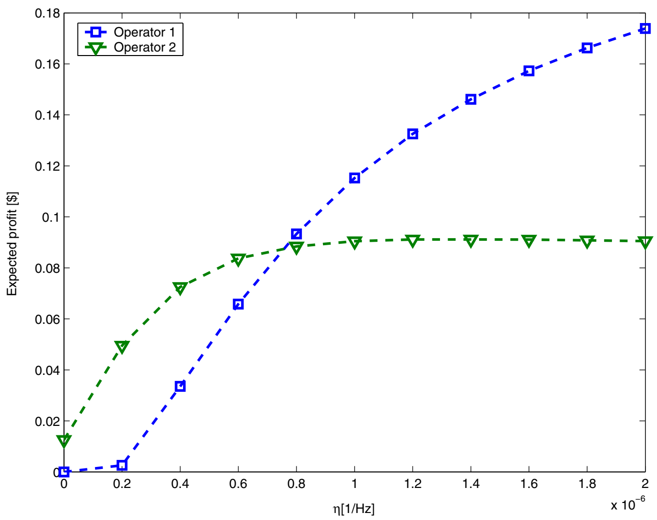

Fig. 6. The expected profit averaged over all locations versus the ratio $\eta = V/F$ between variable cost V \$/Hz and fixed cost per base station F \$, for $W_\mathrm{A} = 10$ MHz and $F + VW_\mathrm{A} = 2$ \$.

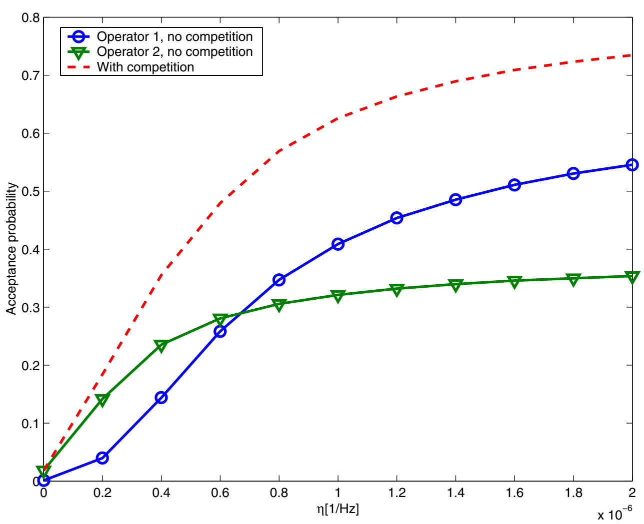

Fig. 7. The acceptance probability averaged over all locations versus the ratio between variable cost V [\$/Hz] versus the fixed cost per base station F [\$], for $W_\mathrm{A} = 10$ MHz and $F + VW_\mathrm{A} = 2$ \$.

In Fig. 8, we present the expected profit averaged over all locations, as a function of the available bandwidth W_A. The ratio between the variable and fixed cost is set to $\eta = 2 \times 10^{-6}$. Note that for fixed η, the cost of the total available bandwidth increases relative to the fixed cost with increasing W_A. The absolute values of F and V are again selected such that $F + VW_\mathrm{A} = 2$ \$. On average, Operator 2 is able to support lower spectral efficiency (due to the lower density of its infrastructure) than Operator 1. Consequently, the competitiveness of Operator 2 is diminishing with W_A, while Operator 1 realizes higher expected profit.

In Fig. 9 we present the acceptance probability averaged over all locations as a function of the available bandwidth W_A (for $\eta = 2 \times 10^{-6}$). Clearly, the acceptance probability increases with the available bandwidth, while competition provides additional gains.

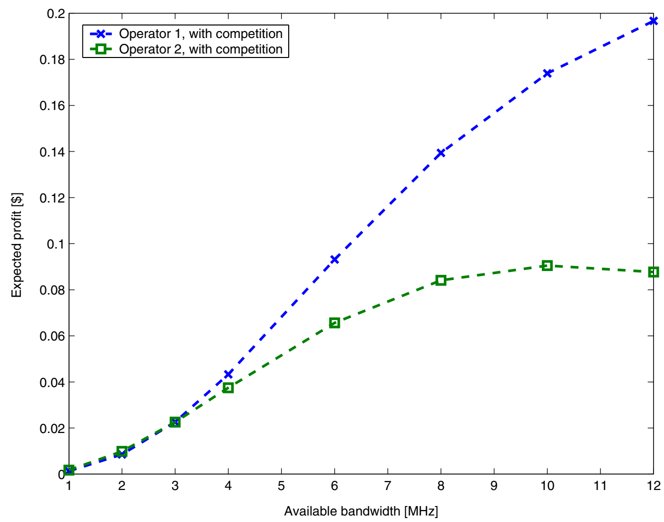

Fig. 8. The expected profit averaged over all locations versus the available bandwidth W_A for $\eta = 2 \times 10^{-6}$ and $F + VW_\mathrm{A} = 2$ \$.

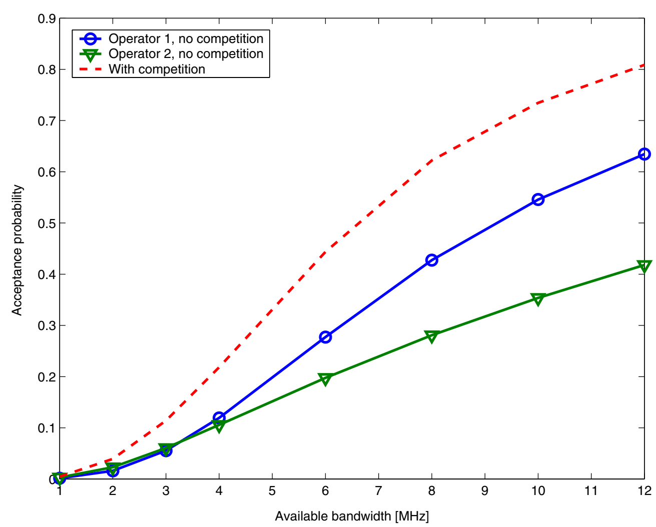

Fig. 9. The acceptance probability averaged over all locations versus the available bandwidth W_A for $\eta = 2 \times 10^{-6}$ and $F + VW_\mathrm{A} = 2$ \$.

V. Multiuser Systems

In this section we consider systems in which two operators are competing for spectrum and a number N of users with $N > 1$. The operators compete for each user individually. For each user, the SPS mediates the operator competition in accordance with the approach developed in Sec. III. We assume that as the result of the competition each user chooses a single operator as the service provider and further accepts the service with a certain acceptance probability.

We further assume that the sessions corresponding to the user-operator pairs are held in nonoverlapping spectrum portions thus leading to interference free transmission. The total available bandwidth $W_{\rm A}$ is partitioned among these sessions by the SPS as will be described later in the text.

It is crucial that the total available bandwidth $W_{\rm A}$ is sufficient to support all the winning offers. Note that operators, if not assisted by the SPS, can not keep track of the winning bids and their bandwidth requirements for each user, and thus can end up making unrealizable offers.

Thus, in keeping with the spectrum server role of the SPS, we propose an SPS-based resource allocation scheme in which the SPS sets the upper limits on bandwidth usage for each user-operator session. The SPS determines these limits in the context of an optimization problem where it maximizes its expected revenue which is the sum of the expected payments of the operators for their spectrum utilization.

A. SPS-based Resource Allocation Model

In the proposed scheme, the SPS maximizes its expected revenue $R_{SPS}(.,\mathbf{W})$ with respect to bandwidth allocation vector $\mathbf{W} = [W_1 W_2 ... W_N]^T$. The operators competing for user n are subject to the constraint that they must not make offers that require bandwidths greater than W_n.

The SPS maximizes its expected revenue subject to the constraint that the total allocated bandwidth does not exceed the total available bandwidth $W_{\rm A}$. Consequently, the SPS optimization problem can be expressed as:

$$\max_{\mathbf{W}} R_{SPS}\left(.,\mathbf{W}\right) \qquad \text{st.} \sum_{n=1}^{N} W_n \leq W_{\rm A}. \tag{10}$$

Note that the expected revenue $R_{SPS}(.,\mathbf{W})$ is defined as the sum of the expected bandwidth utilizations of the users scaled by the variable cost per bandwidth V [\$/Hz]. In this sense, it is a function of the bandwidth allocation vector $\mathbf{W}$ as well as the user locations and the cost parameters in the system:

$$R_{SPS}\left(\mathbf{r_1},\mathbf{r_2},F_1,F_2,V,\mathbf{W}\right) = \sum_{n=1}^{N} V A_n^f\left(.,\mathbf{W}\right) W_n^f\left(.,\mathbf{W}\right). \tag{11}$$

In the above equation, $\mathbf{r_1},\mathbf{r_2}$ are the N dimensional spectral efficiency vectors for Operator 1 and Operator 2, respectively. Each element of the vectors denote the service spectral efficiency the operators enjoy while providing service to a specified user. Note that these vectors depend on the exact locations of the users. F_1 and F_2 denote the fixed costs of Operator 1 and Operator 2, respectively. A_n^f and W_n^f refer to the winning bid acceptance probability and bandwidth usage achieved as a result of the operator competition over user n. W_n^f depends on the winning rate offer and the winning operator's spectral efficiency through the relation $W_n^f = R_{winner}/r_{winner}$.

When maximizing the expected revenue $R_{SPS}(.,\mathbf{W})$ over $\mathbf{W}$, the SPS performs a centralized optimization whose result is a vector $\mathbf{W}^*$ that maximizes the total expected revenue. In order to determine $\mathbf{W}^*$, the SPS performs an exhaustive search in which it declares all possible $\mathbf{W}$s one at a time. For any declared allocation vector $\mathbf{W}$, the operators compete with each other considering the bandwidth constraints imposed by the allocation vector, as illustrated in Fig. 10. The SPS then computes the resulting $R_{SPS}(.,\mathbf{W})$ given the competition results for each user. It then selects the vector which achieves the greatest revenue among all $\mathbf{W}$s as the optimum allocation vector $\mathbf{W}^*$.

It is interesting to note that the SPS optimization in (10) is equivalent to the maximization of the expected bandwidth utilization $\sum_{n=1}^{N} A_n^f W_n^f$. This demonstrates that a revenue seeking SPS will actually be maximizing the total bandwidth utilization in the system.

Note that, besides the expected SPS revenue $R_{SPS}(.,\mathbf{W})$, the SPS can have a number of different criteria for determining the optimum bandwidth allocation vector. It could, for instance maximize the average user acceptance probability, as a social goal, or it could try to maximize the total expected profits of the operators.

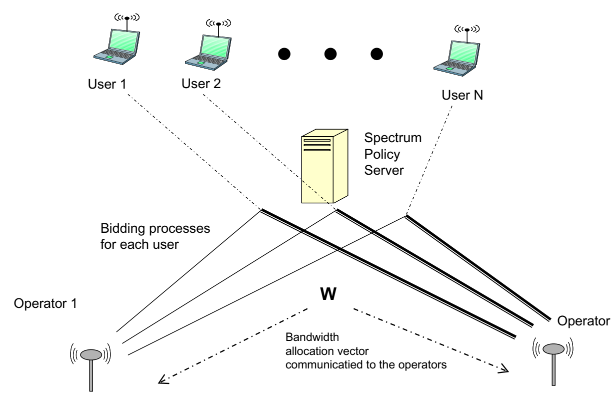

Fig. 10. SPS mediating iterative bidding processes for *N* users.

B. Numerical Results for Multiuser Systems

In this section we present numerical results regarding a multiuser setting in the same linear geometry illustrated in Fig. 3. We consider the case for the variable cost/fixed cost ratio fixed at $\eta = 2 \times 10^{-6}$. All other parameters regarding the cost structure, the total available bandwidth $W_{\rm A}$ and the calculation of the spectral efficiency are same as in Sec. IV.

In order to keep the exhaustive search tractable, the bandwidth is quantized to be made of basic units of 400 kHz wide. The SPS optimization problem is then solved using a brute force search method in which all combinations of bandwidth allocations among users are tested and the one which achieves the greatest expected revenue for the SPS is chosen as the optimum allocation.

In the following figures, we compare the proposed SPS-based allocation scheme to the one in which the available bandwidth is partitioned equally among all N user-operator sessions, with each one getting $W_{\rm A}/N$ [Hz.].

We present the results as functions of the number of users N, in the system. For each value of N, 100 different realizations of the user geometry are tested by randomly locating each user along the linear region, assuming a uniform distribution of the user locations. The results are then averaged over all 100 different realizations. The presented plots are the curves referring to the achieved average values.

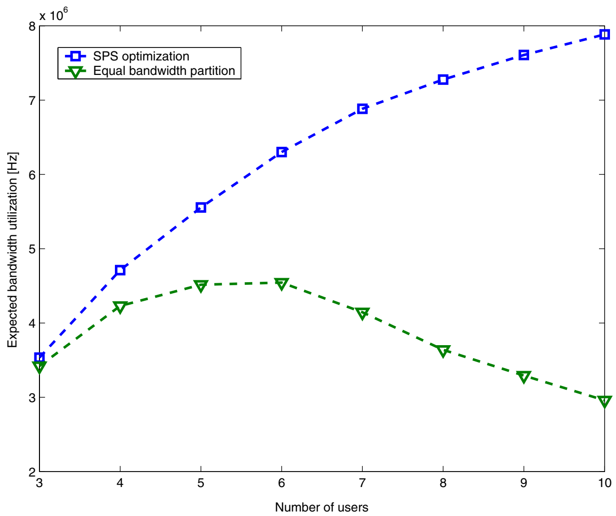

Fig. 11. The expected bandwidth utilization averaged over all realizations versus the number of users in the system N for $\eta = 2 \times 10^{-6}$.

In Fig. 11, we present the total expected bandwidth utilization as a function of the number of users in the system. It is observed that the proposed scheme achieves a greater bandwidth utilization and thus a higher SPS revenue, for all number of users tested. It is also clear that as the number of users increases beyond a threshold, the equal bandwidth partition scheme utilizes the available bandwidth poorly.

In Fig. 12, we present the average values for the mean acceptance probabilities of the users. The mean acceptance probabilities for each realization is computed by dividing the sum of the acceptance probabilities by the number of users. These mean acceptance probabilities are then averaged over all the 100 realizations. It is noted that the average acceptance probability is decreasing in the number of users. Our proposed scheme achieves a higher average user acceptance probability as opposed to the equal bandwidth allocation scheme.

In Figs. 13 and 14, we present the total expected bandwidth utilization and average acceptance probability, respectively, both as functions of η. Note that the SPS optimization scheme achieves greater bandwidth utilization and average acceptance probability for all values of η tested.

VI. Conclusions

In this paper we developed a framework for competition of future operators likely to operate in a mixed commons/property-rights regime under the regulation of a spectrum policy server (SPS). The operators dynamically compete for customers as well as portions of available spectrum.

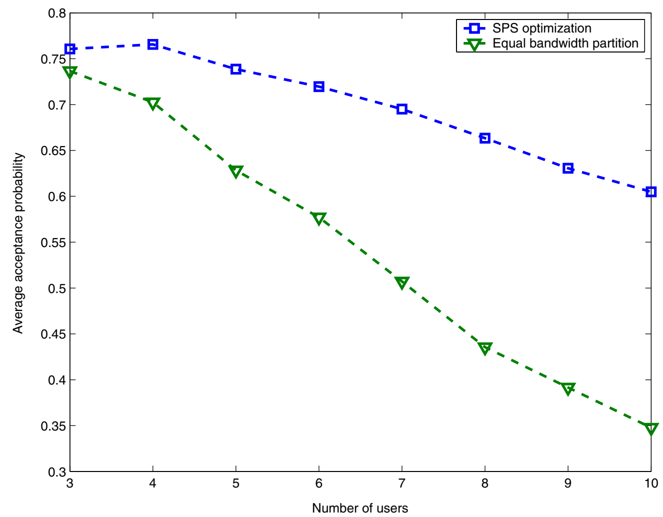

Fig. 12. The average acceptance probability per user averaged over all realizations versus the number of users in the system N for $\eta = 2 \times 10^{-6}$.

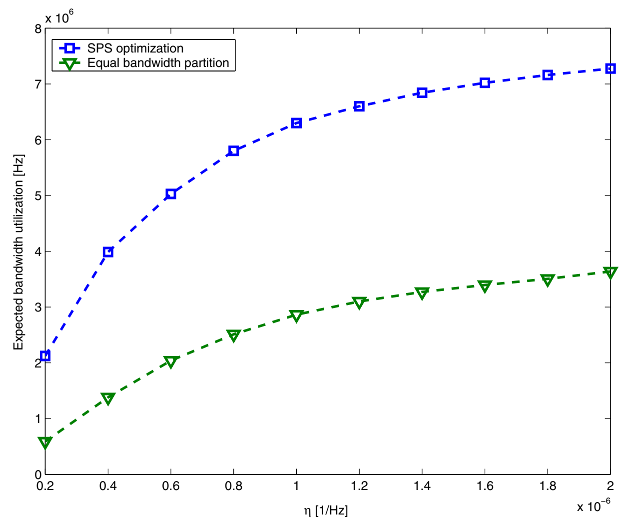

Fig. 13. The expected bandwidth utilization in 8 user system versus $\eta = V/F$, for $W_\mathrm{A} = 10$ MHz and $F + VW_\mathrm{A} = 2$ \$.

The operators are charged by the SPS for the amount of bandwidth they use in their services. Through demand responsive pricing, the operators try to come up with convincing service offers for the customers, while trying to maximize their profits. We first considered a single-user system as an illustrative example. We formulated the competition between the operators as a non-cooperative game and propose an SPS-based iterative bidding scheme that results in a Nash equilibrium of the game. Numerical results suggested that, competition increases the user's (customer's) acceptance probability of the offered service, while reducing the profits achieved by the operators. It was also observed that as the cost of unit bandwidth increases relative to the cost of unit infrastructure (fixed cost), the operator with superior technology (higher fixed cost) becomes more competitive. We then extended the

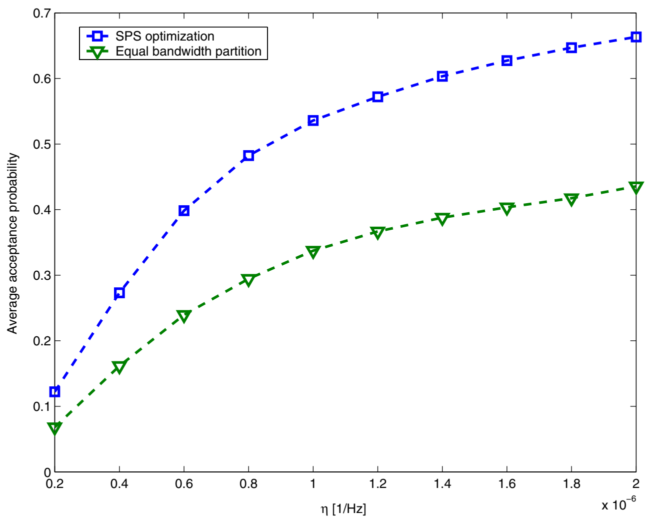

Fig. 14. The average acceptance probability per user in 8 user system versus $\eta = V/F$, for $W_{\mathrm{A}} = 10$ MHz and $F + VW_{\mathrm{A}} = 2$ \$.

framework to a multiuser setting where the operators were competing for a number of users at once. We proposed an SPS-based bandwidth allocation scheme in which the SPS optimally allocates bandwidth portions for each user-operator session to maximize its overall expected revenue resulting from operator payments. Comparison of the performance of this scheme to one in which the bandwidth is equally shared between the user-operator pairs revealed that such an SPS-based scheme improves the user acceptance probabilities and the bandwidth utilization in multiuser systems.

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References

[1] J. M.Peha, "Approaches to spectrum sharing," *IEEE Communications Magazine*, pp. 10–12, February 2005.

[2] E. Kwerel, "Spectrum policy," in *The Conference on the Economics, Technology and Policy of Unlicensed Spectrum, East Lansing, MI*, 2005. [Online]. Available: http://quello.msu.edu/conferences/spectrum/program.htm

[3] M. McHenry and D. McCloskey, "New York City spectrum occupancy measurements september 2004," Tech. Rep., December 2004. [Online]. Available: http://www.sharedspectrum.com/inc/content/measurements/nsf/NYC_report.pdf

[4] G. R. Faulhaber, "The question of spectrum: Technology, management and regime change," in *The Conference on the Economics, Technology and Policy of Unlicensed Spectrum, East Lansing, MI*, 2005. [Online]. Available: http://quello.msu.edu/conferences/spectrum/program.htm

[5] N. Mandayam, "Cognitive algorithms and architectures for open access to spectrum," in *The Conference on the Economics, Technology and Policy of Unlicensed Spectrum, East Lansing, MI*, 2005. [Online]. Available: http://quello.msu.edu/conferences/spectrum/program.htm

[6] M. M. Buddhikot, P. Kolodzy, S. Miller, K. Ryan, and J. Evans, "Dimsumnet: New directions in wireless networking using coordinated dynamic spectrum access," in *IEEE WoWMoM05*, June 2005.

[7] M. Lindstrom, "Demand responsive resource management for cellular networks," Ph.D. dissertation, KTH Royal Institute of Technology, Stockholm, Sweden, 2005.

[8] L. Badia, M. Lindstrom, J. Zander, and M. Zorzi, "Demand and pricing effects on the radio resource allocation of multimedia communication systems," in *Globecom 2003*, 2003.

[9] H. R. Varian, *Intermediate Microeconomics: A Modern Approach*. New York, W.W. Norton, 1987.

[10] C. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. on Communications*, vol. 50, no. 2, pp. 291–303, February 2002.

[11] K. Johansson, A. Furuskar, P. Karlsson, and J. Zander, "Relation between cost structure and base station characteristics in cellular systems," in *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, September 2005.