A Framework for Dynamic Spectrum Sharing between Cognitive Radios

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***Abstract*— We consider a cognitive radio system like the future 802.22 networks where license-exempt service providers (SPs) will share a fixed spectrum in a non-interference basis to each other and also to the licensed users in that spectrum. The percentage of spectrum utilized by one SP depends on how many users it is serving and how much spectrum each user application demands. We assume that an user can obtain service from all the SPs. The quality of service depends on system parameters such as number of users and SPs, the channel conditions between the users and SPs and the total power available at each user. We adopt an user utility maximization framework to analyze this system. Given the user utility functions, and the above mentioned system parameters we derive optimal values of spectrum that the users should obtain from the SPs. We also introduce the notion of spectrum price and use it to demonstrate several key results about spectrum allocation. The spectrum price proves to be the regulatory mechanism that brings about coordination amongst the SPs with minimal control messaging. Our approach thus strikes a balance between a centralized network and a fully uncoordinated open access network.**

I. Introduction

In the 1950s, the FCC in USA sold licenses for 330 MHz of spectrum for UHF television. This experiment never took off leading to considerable bands of unused spectrum between VHF and UHF broadcast channels from 54 to 865 MHz. Recently in October 2004, the IEEE set up a working group to develop the 802.22 standard that would employ these unused spectrum to offer wireless broadband services to areas not well served by alternatives such as cable or DSL [1].

In the 802.22 draft it has been decided that fixed wireless access will be provided in these bands [7] by professionally installed Wireless Regional Area Network (WRAN) base stations to WRAN user terminals. A service provider (SP) operating a base station will not have to pay any licensing fees. It would share a total spectrum with other SPs in a geographical region and further allocate this spectrum to users efficiently. Since the spectrum within the 54 to 865 MHz range where the TV broadcasters operate varies within location, a WRAN SP would have to sense the presence of digital televisions (DTVs) and NTSC and also other SPs before deciding how to share the spectrum with those SPs. Hence the term *cognitive radio* is often used to denote the WRAN base stations and user devices. Fundamental issues associated with sensing can be found in [12].

The physical layer of 802.22 cognitive systems is based on OFDMA [15]. The task of spectrum sharing is simplified as spectrum allocated to an user directly translates to a set of OFDM subchannels.

Any allocation solution in a cognitive radio setting should be decentralized and easy to implement. Thus we can rule out strict centralized cellular network type allocations by the SP. Another option is to consider an information theoretic setting and let the users operate over the entire spectrum and waterfill over the noise plus interference power spectra of other users [6]. But this leads to complicated encoder-decoder structures. Both these approaches assumed non-strategic behavior of users. Thus another approach is to consider selfish user behavior and let users utilize the entire spectrum. This would simplify the implementation aspect but can lead to arbitrary low rates [8].

Thus we need to consider approaches where users follow a simple protocol that leads to distributed spectrum allocation and also to QoS (e.g. achievable rate) guarantees. Similar views are also echoed by the authors of [4], [5].

A. Our Contribution

Given the interest around dynamic spectrum allocation for cognitive radios, it becomes important to study this problem from an analytical framework. We consider a two tiered spectrum allocation scheme as shown in Figure 1. There is some net spectrum C available in a geographical area which are allocated to the users through the SPs. The users could potentially obtain spectrum from all the SPs. We assume that the users obtain non-overlapping chunks of spectrum from the SPs to avoid interference. We assume that each user application has an associated utility expression as a function of spectrum obtained and adopt an utility maximization framework [10] to analyze the system. Given user utility functions, channel coefficients between users and SPs and user power constraints, our aim is to derive how much spectrum should a user obtain from an SP and what power should he allocate for sending his information to the SPs. We allow for simple SP coordination to share the spectrum C amongst themselves where the amount of spectrum utilized by a SP depends on how much spectrum it has to allocate to the users. This is facilitated by a spectrum

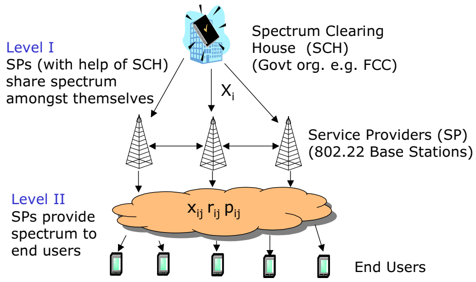

Fig. 1. The network topology

clearing house (SCH), akin to an FCC-controlled regional spectrum broker [4].

Based on our analysis we propose a simple spectrum allocation protocol based on the notion of *spectrum price* which SPs announce to the users. Given the spectrum price each user decides distributively how much spectrum to use and conveys this back to the SPs. For utility functions that are linear in transmission rate we derive several key results about the allocated spectrum.

B. Related Work in Spectrum Allocation

Buddhikot et. al. [4], [5] propose two and three tiered hierarchical spectrum allocation systems. They introduce the notion of Coordinated Access Bands of Spectrum which are dynamically assigned to various SPs by a central entity called the Regional Spectrum Information Manager. They focus their attention on the network architecture and protocol design for the system. The two tiered model is also considered by the authors in [9] who consider a game-theoretic framework where two service providers dynamically compete for customers as well as portions of available spectrum from a central Spectrum Policy Server. As noted in [5], a multi-tiered model enables unbundling of providers and creates competition.

Our work is also related to multi-user uplink vector channel allocation problems arising from ISI or FDMA-based transmissions [6], [16], [11]. In these works there is a single receiver (like a SP) who operates the entire spectrum which is discretized into bins. In [6] the users are allowed to interfere and water-fill over the interference plus noise spectra. In [16] optimal frequency partitioning is considered for two users. Our setting is different as it allows for multiple SPs with different efficiencies of transmission thus presenting the user with more choice. However since our channel model is flat we are able to show similarities between some of our results and established results. Also most of the prior work are strictly centralized allocations because of the underlying FDMA application, whereas our work also focuses on the distributed nature by developing the spectrum price concept.

II. System Model

Consider the network topology shown in Figure 1. Assume that there are N SPs and L end users and one central Spectrum Clearing House (SCH). Based on the demand for spectrum, Service Provider i provides X_i units to the L users or a subset of them. Let x_{ij} be the amount of spectrum obtained by user j from SP i. Subsequently, user j transmits his data to SP i over this spectrum at rate r_{ij} and with power p_{ij}. Each user has a total transmit power constraint. The channel between SP i and user j is characterized by the link gain coefficient h_{ij}, which remains constant during the period of spectrum allocation and subsequent transmission to the SP. We assume that h_{ij} is flat over frequencies and thus no matter between what bands x_{ij} lies, h_{ij} is same. The coefficients h_{ij} are assumed to be known at both the users and the SPs. We also assume that the background additive Gaussian noise is of unit power spectral density. The maximum value of r_{ij} is decided by x_{ij}, p_{ij} and h_{ij} as per the Shannon capacity expression $x_{ij}\log(1+h_{ij}p_{ij}/x_{ij})$ scaled by a term η_i which is the fraction of the Shannon capacity that can be reliably guaranteed by SP i to an user. We call this the SP efficiency term and introduce it to differentiate between SPs in terms of offered QoS. A possible example could be SP i, who has invested more in deploying a better decoder (a Turbo decoder with more iterations or better interleaver design) has a higher η_i than an SP with a conventional Viterbi decoder, with the encoder being same in both cases. Thus the total rate at which user j can transmit reliably is thus $R_j = \sum_{i=1}^{N} r_{ij}$. We assume that there is a utility function $U_j(R_j)$ associated with user j which is concave and increasing in R_j [14]. The user welfare optimization problem is thus,

$$\max_{x_{ij},p_{ij},X_i} \sum_{j=1}^{L} U_j\left(\sum_{i=1}^{N} r_{ij}\right) \tag{1}$$

$$\text{s.t. } r_{ij} = \eta_i x_{ij}\log\left(1+\frac{h_{ij}p_{ij}}{x_{ij}}\right) \tag{2}$$

$$\sum_{j=1}^{L} x_{ij} \le X_i,\ 1 \le i \le N \tag{3}$$

$$\sum_{i=1}^{N} p_{ij} \le P_j,\ 1 \le j \le L \tag{4}$$

$$\sum_{i=1}^{N} X_i \le C \tag{5}$$

$$x_{ij}, p_{ij} \ge 0,\ 1 \le i \le N,\ 1 \le j \le L \tag{6}$$

$$X_i \ge 0,\ 1 \le i \le N. \tag{7}$$

As shown in (3), X_i is the spectrum utilized by SP i which is equal to the spectrum it has to allocate to the users. User j transmits with power p_{ij} to SP i and as (4) shows there is a constraint P_j on the total transmit power. The total amount of available spectrum is C.

A. Distributed Solution and Pricing

We show that the spectrum allocation problem (1) - (7) is distributed at the user end. For this we form the partial Lagrangian $\mathcal{L}$ [3] by relaxing the constraints (3) and (5) in the objective function to obtain,

$$\begin{aligned} \mathcal{L}(x_{ij}, p_{ij}, X_i, \boldsymbol{\lambda}, \mu) &= \sum_{j=1}^{L} U_j \left(\sum_{i=1}^{N} r_{ij} \right) \\ + \sum_{i=1}^{N} \lambda_i \left(X_i - \sum_{j=1}^{L} x_{ij} \right) &+ \mu \left(C - \sum_{i=1}^{N} X_i \right), \end{aligned} \tag{8}$$

where $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_N]^T$. The stationarity conditions w.r.t. X_i can be expressed as,

$$\frac{\partial \mathcal{L}}{\partial X_i} = \lambda_i - \mu \begin{cases} = 0 & \text{if } X_i > 0. \\ < 0 & \text{if } X_i = 0. \end{cases} \tag{9}$$

Interpreting the Lagrange multipliers λ_i, μ as shadow prices for spectrum [10], we see that the social utility maximization problem has a unique spectrum price. With slight abuse of notation, we replace $\lambda_i = \mu$ for all i, in (8) and denote the new Lagrangian as,

$$\mathcal{L}(x_{ij}, p_{ij}, \mu) = \sum_{j=1}^{L} U_j \left(\sum_{i=1}^{N} r_{ij} \right) - \sum_{i=1}^{N} \mu \sum_{j=1}^{L} x_{ij} + \mu C. \tag{10}$$

The dual $g(\mu)$ is,

$$g(\mu) = \max_{x_{ij}, p_{ij}} \mathcal{L}(x_{ij}, p_{ij}, \mu) \tag{11}$$

$$\text{s.t. } \sum_{i=1}^{N} p_{ij} \leq P_j, \ 1 \leq j \leq L \tag{12}$$

$$x_{ij}, p_{ij} \geq 0. \tag{13}$$

From $g(\mu)$ we see that the optimization in (1) - (7) decomposes into separate optimization problems [3] for the users and the SPs. The optimization subproblem for user j is,

$$f_j^*(\mu) = \max_{x_{ij}, p_{ij}} U_j \left(\sum_{i=1}^{N} r_{ij} \right) - \sum_{i=1}^{N} \mu x_{ij} \tag{14}$$

$$\text{s.t. } \sum_{i=1}^{N} p_{ij} \leq P_j \tag{15}$$

$$r_{ij} = \eta_i x_{ij} \log \left(1 + \frac{h_{ij} p_{ij}}{x_{ij}} \right) \tag{16}$$

$$x_{ij}, p_{ij} \geq 0, \ 1 \leq i \leq N. \tag{17}$$

The spectrum price μ is set jointly by the SPs and the SCH by minimizing the dual $g(\mu) = \sum_{j=1}^{L} f_j^*(\mu) + \mu C$, for $\mu \geq 0$. An intuition about how the prices are set can be obtained by writing the distributed price update as [2],

$$\mu(t+1) = \left[\mu(t) - \alpha \left(C - \sum_{i=1}^{N} \sum_{j=1}^{L} x_{ij}(\mu(t)) \right) \right]^+, \tag{18}$$

where $x_{ij}(\mu)$ is the spectrum obtained by user j from SP i, for a given value of μ and α is a positive step size. Thus we see that if the spectrum is underutilized, $C - \sum_{i=1}^{N} \sum_{j=1}^{L} x_{ij}(\mu(t))$ is positive and thus the price decreases to facilitate more utilization of spectrum. Similarly if spectrum is over utilized, the price increases. We can thus state the following Lemma

Lemma 1: There is a global spectrum price μ charged by all the SPs and it is set such that the entire spectrum is utilized.

Thus the distributed spectrum allocation mechanism is,

Distributed Mechanism

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1) At time t SPs broadcast price $\mu(t)$.
2) Each user j solves (14)-(17) and calculates $x_{ij}(\mu(t))$ for all i SPs.
3) All users inform the SPs about the $x_{ij}(\mu(t))$ values.
4) SPs calculate $\mu(t+1)$ from (18).

From [2], $\mu(x(t))$ converges to the equilibrium price μ.

B. $U_j(R_j) = R_j$

In this work we mostly deal with $U_j(R_j) = R_j$. Extending notion of spectrum price for more general concave utility functions is a scope for future work. For $U_j(R_j) = R_j$ the user optimization problem (14) - (17) becomes,

$$\max_{x_{ij}, p_{ij}} \sum_{i=1}^{N} \eta_i x_{ij} \log \left(1 + \frac{h_{ij} p_{ij}}{x_{ij}} \right) - \sum_{i=1}^{N} \mu x_{ij} \tag{19}$$

$$\text{s.t. } \sum_{i=1}^{N} p_{ij} \leq P_j \tag{20}$$

$$x_{ij}, p_{ij} \geq 0, \ 1 \leq i \leq N. \tag{21}$$

To arrive at the optimal solution we first write the Lagrangian for the problem (19) - (21),

$$\begin{aligned} \mathcal{L}_j &= \sum_{i=1}^{N} \eta_i x_{ij} \log \left(1 + \frac{h_{ij} p_{ij}}{x_{ij}} \right) - \sum_{i=1}^{N} \mu x_{ij} \\ &+ \gamma_j \left(P_j - \sum_{i=1}^{N} p_{ij} \right) + \sum_{i=1}^{N} \alpha_{ij} x_{ij} + \sum_{i=1}^{N} \beta_{ij} p_{ij}, \end{aligned} \tag{22}$$

where all Lagrange multipliers are positive. The stationarity conditions for the Lagrangian are,

$$\frac{\partial \mathcal{L}_j}{\partial x_{ij}} = \eta_i \log \left(1 + \frac{h_{ij} p_{ij}}{x_{ij}} \right) - \frac{\eta_i h_{ij} p_{ij}}{x_{ij} + h_{ij} p_{ij}} - \mu + \alpha_{ij} = 0 \tag{23}$$

$$\frac{\partial \mathcal{L}_j}{\partial p_{ij}} = \frac{\eta_i h_{ij} x_{ij}}{x_{ij} + h_{ij} p_{ij}} - \gamma_j + \beta_{ij} = 0. \tag{24}$$

We are now in a position to state the following lemma,

Lemma 2: In the optimal solution $p_{ij} = 0$ if and only if $x_{ij} = 0$.

Proof: The result follows immediately by re-writing the (24) as,

$$p_{ij} = x_{ij} \left(\frac{\eta_i}{\gamma_j - \beta_{ij}} - \frac{1}{h_{ij}} \right), \tag{25}$$

which holds for all values of p_{ij} and x_{ij}. ■

Lemma 2 has an intuitive explanation. If no power is put in the chunk of spectrum bought from SP i, the utility term $\eta_i x_{ij} \log(1 + h_{ij} p_{ij}/x_{ij}) = 0$. If $x_{ij} \neq 0$, then user j still pays the price μx_{ij}, which clearly brings down his net utility and hence he sets $x_{ij} = 0$. Alternatively if no spectrum is bought from SP i, i.e if $x_{ij} = 0$, then $p_{ij} \neq 0$ leads to power wastage. Henceforth we call the SPs for which $x_{ij} > 0$ and $p_{ij} > 0$ as *active* SPs.

Consider the case when $x_{ij} > 0$ and $p_{ij} > 0$, which implies that $\alpha_{ij} = 0$ and $\beta_{ij} = 0$. From (25) we obtain,

$$p_{ij} = x_{ij} \left(\frac{\eta_i}{\gamma_j} - \frac{1}{h_{ij}} \right). \tag{26}$$

Substituting for $h_{ij} p_{ij}/x_{ij}$ from (26) in (23) we obtain,

$$\frac{\gamma_j/\eta_i}{h_{ij}} = \log\left(\frac{\gamma_j/\eta_i}{h_{ij}} \right) + \frac{\mu}{\eta_i} + 1. \tag{27}$$

From (27) we again see that the effect of η_i is to modify the spectrum price μ and power level parameter γ_j to μ/η_i and γ_j/η_i respectively. We now state our second lemma,

Lemma 3: In the optimal solution only one SP is active per user almost surely.

Proof: Assume that for user j, two SPs are active and are indexed by $i = 1, 2$. Hence from (27) we obtain,

$$\frac{\gamma_j/\eta_1}{h_{1j}} - \log\left(\frac{\gamma_j/\eta_1}{h_{1j}} \right) - \frac{\mu}{\eta_1} = \frac{\gamma_j/\eta_2}{h_{2j}} - \log\left(\frac{\gamma_j/\eta_2}{h_{2j}} \right) - \frac{\mu}{\eta_2}.$$

Since h_{ij} is a continuous random variable the probability of the above event is zero. Hence only one SP is active *almost surely*. ■

Various flavors of this result are also observed in [16], [11]. Let this SP be indexed by $i^*(j)$. Then $p_{i^*(j)j} = P_j$, i.e. as user j is transmitting to only one SP he transmits with full power. Thus from (19), $i^*(j)$ is given by the solution of,

$$i^*(j) = \arg\max_i \left[\max_{x_{ij} \geq 0} \left\{ \eta_i x_{ij} \log\left(1 + \frac{h_{ij} P_j}{x_{ij}} \right) - \mu x_{ij} \right\} \right].$$

To simplify notation we denote $x_{i^*(j)j}$ by x_j^*, $h_{i^*(j)j}$ by h_j^* and $\eta_{i^*(j)}$ by η_j^*. Thus the optimization problem in (19) can be re-written by considering only $i = i^*(j)$ as,

$$\mathcal{U}(x_j^*) = \max_{x_j^* \geq 0} \eta_j^* x_j^* \log\left(1 + \frac{h_j^* P_j}{x_j^*} \right) - \mu x_j^*. \tag{28}$$

The value of $\mathcal{U}(x_j^*)$ can be derived by considering (23) for $i = i^*(j)$. The result is,

$$\mathcal{U}(x_j^*) = \frac{\eta_j^* h_j^* x_j^* P_j}{x_j^* + h_j^* P_j}. \tag{29}$$

We now characterize the spectrum x_j^* of user j and his signal to noise ratio which is $\mathsf{snr}_j^* = h_j^* P_j / x_j^*$.

Lemma 4: For given channel coefficients h_j^*, power P_j and efficiency η_j^*, user j operates at a unique snr_j^* which is given by the solution of,

$$\Phi(\mathsf{snr}_j^*) = \log\left(1 + \mathsf{snr}_j^*\right) - \frac{\mathsf{snr}_j^*}{1 + \mathsf{snr}_j^*} = \frac{\mu}{\eta_j^*}. \tag{30}$$

Proof: (30) can be directly derived by writing down (23) for $i = i^*(j)$ and substituting $\mathsf{snr}_j^* = h_j^* P_j / x_j^*$. We have to show that it has a unique solution in snr_j^*. This is proved in Appendix I.

From $\Phi\left(\mathsf{snr}_j^*\right) = \Phi\left(h_j^* P_j / x_j^*\right) = \mu/\eta_j^*$, we obtain,

$$x_j^* = \frac{h_j^* P_j}{\Phi^{-1}\left(\mu/\eta_j^*\right)}. \tag{31}$$

We have proved in Appendix I that $\Phi(\cdot)$ is an increasing and one-to-one function; hence $\Phi^{-1}(\cdot)$ is also increasing and one-to-one. Thus there is a unique allocated spectrum x_j^*.

We note that (3) and (5) can be jointly written as $\sum_{j=1}^{L} x_j^* \leq C$ or from (31) as,

$$\sum_{j=1}^{L} \frac{h_j^* P_j}{\Phi^{-1}\left(\mu/\eta_j^*\right)} \leq C. \tag{32}$$

Let $\tilde{\mu}$ be the solution when equality is assumed in (32). Then the optimal spectrum price is given by $\mu = \min\{\tilde{\mu}, 0\}$. When $\tilde{\mu} < 0$, it implies that constraints (3) and (5) are not tight and the entire spectrum C is not being utilized by the users. ■

Note that (32) is the centralized one-shot way of calculating μ. In our system the same value of μ is calculated via a distributed update process as was shown in (18).

1) Special Case: $\eta_j^* = 1$ for all j: For this case we can simplify the solutions. From (31), we obtain,

$$\Phi^{-1}(\mu) = \frac{h_1^* P_1}{x_1^*} = \cdots = \frac{h_L^* P_L}{x_L^*} = \frac{\sum_{k=1}^{L} h_k^* P_k}{C}. \tag{33}$$

Thus x_j^* is given by,

$$x_j^* = \frac{h_j^* P_j}{\sum_{k=1}^{L} h_k^* P_k} C. \tag{34}$$

Thus we see that the allocated spectrum is directly proportional to the received signal power. Substituting for x_j^* in (29), we obtain,

$$\mathcal{U}(x_j^*) = \frac{h_j^* P_j}{1 + \sum_{k=1}^{L} h_k^* P_k / C}. \tag{35}$$

It can be shown that (35) is an increasing function of h_j^*. Thus for $\eta_j^* = 1$, the active SP for user j is the one which has the best channel to user j, i.e. $i^*(j) = \arg\max_i h_{ij}$. To understand this better consider a system with two users indexed by $1, 2$ and h_2^* is higher than h_1^* by 8dB. The total spectrum varies from 500 Khz to 100 MhZ. The operating SNR and rates are plotted in 2. We see that both users go from a high SNR regime where the rates increase linearly to a low SNR regime where the rates saturate and the difference determined by the channel coefficients.

We now see how change in users power P_j or channel h_j^* affects price μ and x_j^*.

Lemma 5: With increase in available power the spectrum price increases, an user obtains more spectrum and derives a higher utility.

Proof: Consider equality in (32) and focus on user 1. Without loss in generality we assume that P_1 increases to $P_1' > P_1$

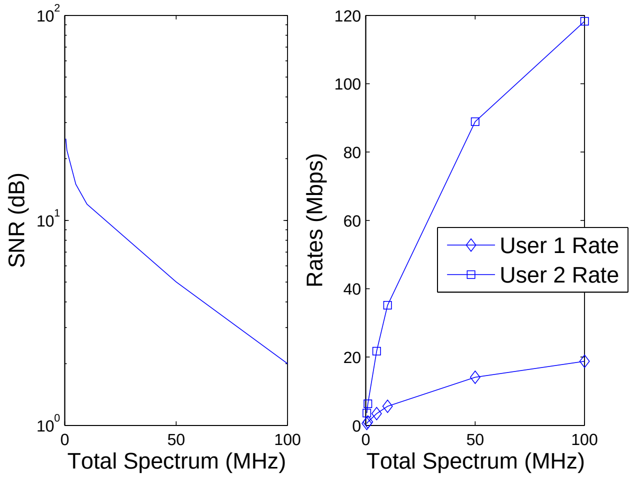

Fig. 2. Operating SNR and Rates vs Total Spectrum

and P_2 to P_L and h_1^* to h_L^* stay the same. Now μ can either increase, decrease or stay the same. If μ decreases, so does the term $\Phi^{-1}(\mu/\eta_i)$ (as $\Phi^{-1}(\cdot)$ is an increasing function) and each term in the LHS of (32) increases. The first term has a further increase as $P_1' > P_1$. Thus equality can't be maintained as RHS of (32) is still C. If μ stays the same, the second to L^{th} terms in the LHS of (32) stays the same but the first term still increases as $P_1' > P_1$. Hence μ must increase to $\mu' > \mu$. Thus the allocated spectrum to users 2 to L decreases as per (31). Since allocated spectrum for all other users decrease, the allocated spectrum for user 1, i.e. x_1^* must increase to satisfy (32).

To prove the second part of the lemma, consider (29). Let $x_j^* = x_a$ correspond to $P_j = P_a$ and $x_j^* = x_b$ to $P_j = P_b$, where $P_b > P_a$ and hence $x_b > x_a$. We can easily show that,

$$\mathcal{U}(x_b) - \mathcal{U}(x_a) = \eta_j^* h_j^* \frac{(P_b - P_a)x_b x_a + h_j^*(x_b - x_a)P_b P_a}{(x_a + h_j^* P_a)(x_b + h_j^* P_b)},$$

which is positive. ■

Note that as P_j changes from P_a to P_b so does the spectrum price μ, but the Lemma still holds as the expression for the optimum user utility in (29) is independent of μ.

Lemma 6: As channel condition to the active SP becomes better the spectrum price increases, an user obtains more spectrum and derives a higher utility.

Proof: The proof is exactly similar to proof of Lemma 5, with h_j^* now being the variable instead of P_j. ■

We now consider how the dynamics of the system changes as more users or SPs are added to it.

Lemma 7: As more users are added to the system, the spectrum price increases.

Proof: Assume that the system is in equilibrium and one more user joins in the system. In the new equilibrium, he is allocated some chunk of spectrum and thus there is one additional term in (32). Since the h_j^*, P_j values for the other users stay the same and arguing similarly as in Lemma 5 we can show that the value of spectrum price μ increases. Addition of an extra user increases the demand for spectrum thus raising the price.■

Lemma 8: As one more SP is added to the system the spectrum price either stays the same or increases.

Proof: Assume that the system is in equilibrium and a new SP (SP $N+1$) joins in the system. If it offers no better channel to any of the users than their existing ones, i.e. if $h_j^* > h_{(N+1)j}$ for all j, then no user engages itself to the SP and the optimal solution (spectrum price, spectrum allocated etc) is the same as before.

On the other hand if for some user j, the new SP provides a better channel coefficient, i.e $h_j^* < h_{(N+1)j}$, then user j engages itself to SP $N+1$ and adjusts its engaged SP index to $i^*(j) = N+1$ and channel coefficient to $h_j^* = h_{(N+1)j}$. Thus user j's channel condition to his active SP has improved and as per Lemma 6, the price goes up. ■

The result might seem surprising as we often associate more service providers with lesser prices. However that happens when *supply* of a commodity (like total available spectrum C) increases with more service providers but in our case C is independent of the number of service providers. Prices are also reduced when we consider competition amongst SPs in attracting the users. However in our setting the prices are set to maximize the social welfare of the users and the effect of more SPs shows up indirectly as they provide users with a bigger set of channel coefficients to choose from.

III. Various Comments

In this section we present some comments about our problem formulation and solution.

A. Dynamic Allocation

In this work, we have assumed that the system parameters like h_{ij}, N and L stay constant for the duration in which the optimization is carried out and for the subsequent transmission time. Whenever there is a significant change in these parameters the optimization can be re-done. The optimization in Section II-A is ideal for static outdoor settings where there is a strong Line-of-Sight component between users and SPs.

B. Orthogonal Spectrum Allocation

Motivated by 802.22 systems we have considered zero interference. In our system users are allocated non-overlapping chunks of spectrum making them are orthogonal in frequency domain. As number of users increase, each user gets lesser spectrum and thus derives lesser utility.

In contrast there are non-orthogonal systems where the available spectrum is shared equally by all users in the system, like Bluetooth and 802.11 sharing the UNII band. As number of users increase each user derives a lesser utility due to interference [13].

Orthogonal spectrum allocation gives up in degrees of freedom to avoid interference and is suited for interference limited settings where there are lot of users in the close vicinity of each other.

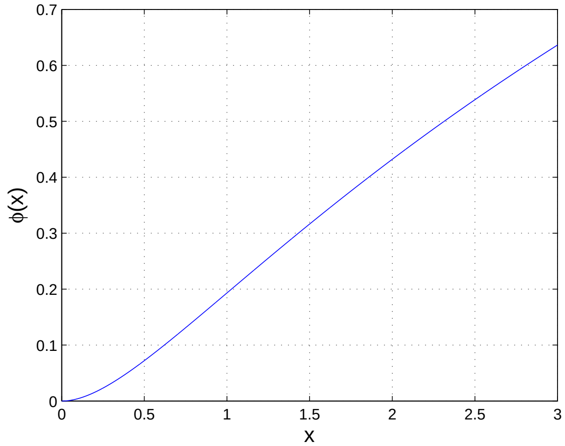

Fig. 3. $\Phi(x)$ vs x

IV. Conclusion

In this work we analyze a two tier spectrum allocation system consisting multiple SPs providing spectrum to users. We model the above system from an user welfare maximization framework. We show that in the optimal policy each user obtains spectrum only from one service provider. We introduce the notion of spectrum price and show that this facilitates distributed spectrum allocation for the users. For utility functions that are linear with transmission rates we completely characterize the behavior of the spectrum allocation solution. We believe that the model presented here forms the starting step to analyze more complex networks where apart from user welfare maximization, service providers also want to maximize their own revenues.

Appendix I
Proof of Uniqueness in Lemma 4

Prove that the equation,

$$\Phi(x) = \log(1+x) - \frac{x}{1+x} = \mu, \tag{36}$$

where $x > 0$ has a unique solution which is increasing in μ.

A. Solution

Taking derivatives we obtain,

$$\Phi'(x) = \frac{x}{(1+x)^2} > 0 \text{ for } x > 0. \tag{37}$$

Thus $\Phi(x)$ is a strictly increasing function. Thus it is also one-to-one, i.e. a value of μ yields a unique value of x. By taking the second derivative we obtain,

$$\Phi''(x) = \frac{1-x}{(1+x)^3}, \tag{38}$$

which shows that there is a point of inflection at $x = 1$. A sample $\Phi(x)$ is plotted in Figure 3

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