Profit Maximizing Pricing Strategies for Dynamic Spectrum Allocation

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***Abstract*— We consider a system where one service provider (SP) allocates spectrum to users subscribed to his network. The SP charges a two part tariff [10] from the users comprising of a fixed subscription price and price per unit spectrum used. We formulate the spectrum allocation problem from the SP profit maximization framework and calculate the resulting expressions for prices and spectrum allocated for different user utilities and SP power allocation strategies.**

I. Introduction

We consider the downlink of a wireless system where a Service Provider (SP) sets up a base station to transmit to a group of users. There is a spectrum band assigned for the underlying application by a Government agency like the FCC in USA. The SP allocates this spectrum to the users who are located in the same geographical region and could potentially interfere if assigned overlapping spectrum. Such a situation models the upcoming 802.22 networks which would operate in the VHF and UHF TV bands. Spectrum allocation between different users is important for ensuring QoS. Traditional approaches like FDMA assigned fixed frequency channels to users. However spectrum usage is dynamic depending upon spatial and temporal factors and fixed spectrum allocation leads to underutilization [8].

This can be remedied by allowing all users to share the spectrum. But the resulting interference affects QoS and may not be suitable for applications with high rate or stringent delay/latency requirements as noted in [4].

Thus to simultaneously minimize spectrum underutilization and maximize application objectives like rate or minimum latency, *dynamic spectrum allocation* is necessary. By dynamic spectrum allocation we refer to a system where the allocated spectrum depends on the end application and the allocation is for short time scales (typically of the order of a session duration like a file transfer or voice call). Such a system necessitates the need for some form of coordination amongst users and/or central control. A popular model for such dynamic spectrum allocation is based on the concept of a central entity (which has been called the Spectrum Policy Server (SPS) in [7]) that regulates spectrum usage amongst multiple SPs on a need-only basis. The SPs then allocate this spectrum to the users subscribed to their respective networks. Such a *multi-tier* spectrum allocation model has been analyzed in [2], [5]. The authors in [3] explore the network architecture design issues of this model.

A. Pricing for Spectrum Allocation

Dynamic spectrum allocation can be implemented by several mechanisms like centralized decision making by the SP or SPS, auctions [11] etc. Another important mechanism is *pricing* where the SP sets a price for spectrum and the users decide how much spectrum to buy based on the price and the utility which the purchased spectrum would yield. The SPs then obtain exactly the same amount of spectrum which they have to allocate to the users thus eliminating spectrum wastage. Pricing for spectrum allocation has been considered in [2], [5], [11].

B. Our Contribution and Related Work

In our previous work [2], we analyze the spectrum allocation problem from a user sum utility maximization framework and show that the spectrum price is set such that the entire spectrum is utilized. However for many networks of practical interest the objective of the SP is to maximize its own revenue because it would have and investments in purchasing the right to offer services in the spectrum and in setting up the equipment to do so (hereafter referred to as the *SP cost*). SP profit maximizing pricing strategies may not necessarily maximize sum utility of the users. In this work we consider a single SP who sets a spectrum price to maximize his profits and users decide how much spectrum to purchase. The SP subsequently provides services to the users over their allocated spectrum. SP profits resulting from pricing are also dealt in [5] and [12]. The authors in [5] consider 2 SPs who offer fixed prices and rates and study the resulting SP revenues based on the probabilities that the users accept their services. The authors in [12] vary the prices for SP revenue maximization based on fixed rates and frame success probabilities of the received packets. In their model, users pay more for a better frame success probability which corresponds to more allocated spectrum. However the relationship between the spectrum price charged from users to the SP cost is not completely explored. In this paper we adopt a microeconomic approach to derive the optimal value of spectrum price as a joint function of SP costs and what the users are willing to pay. Our notion of utility is the achievable rate between the SP and the users which varies as a function of the spectrum allocated to the user. This makes sense for most future data centric systems that are based on OFDM transmission. Spectrum allocated to an user directly translates to the number of OFDM tones over which his data is sent. Unlike the OFDM example, in our work we assume that spectrum is a divisible resource for analytical simplicity. We derive optimal values of the prices and power allocation strategies that maximize the profit of the SP. We use the concepts of pricing from the microeconomics literature [10], [6] and show that there is benefit in applying them to spectrum allocation problems.

C. SP Costs

We assume that the SP has to bear a cost $C(X)$ for the allocation of spectrum X to the users. We could broadly decompose $C(X)$ into,

- Fixed radio deployment costs like cost of setting up an antenna denoted by F.
- Variable spectrum lease cost which arise as the SPs would have purchased rights from the SPS to provide services over some spectrum band. This can be similar to the spectrum licensing costs that the cellular operators pay to the FCC but is on a shorter time scale. A reasonable model for cost is $k_1 k_2 S X$, where S is base spectrum cost in dollars/MHz. k_1 denotes the geographical region in which the SP wants to operate with spectrum more expensive in urban zones. Factor k_2 denotes the band in which the spectrum is leased, with a MHz in the *crowded* bands like 800-900 MHz being more costly than a MHz in the relatively unused bands.
- Thus we consider a cost function $C(X) = F + CX$.

II. System Model

We consider a system with a single SP and L end users. Let user j purchase x_j amount of spectrum from the SP. In the subsequent communication process the SP transmits to user j over his purchased spectrum. The SP transmits with spectral efficiency $\nu(x_j)$ (measured in bps/Hz) to user j, resulting in a received rate of $R_j = \nu(x_j)x_j$ bps. Assume that user j has a utility function $U_j(R_j)$ which is increasing and concave in R_j. The SP charges a two part tariff [10] from user j, consisting of a fixed subscription price κ and a price μ charged per unit of spectrum used. This results in a SP revenue of $\rho(x_j) = \mu x_j + \kappa$ from user j. The user does not have to pay the subscription price κ if he is not receiving any service from the SP, i.e. if $x_j = 0$. Thus

$$\rho(x_j) = \begin{cases} \mu x_j + \kappa, & x_j > 0 \\ 0, & x_j = 0. \end{cases} \tag{1}$$

The SP initially announces a price pair (μ, κ). Given this the user optimizes over how much spectrum to obtain as follows,

$$\max_{x_j} U_j(R_j) - \rho(x_j) = \max_{x_j} V_j(x_j) - \rho(x_j), \tag{2}$$

where

$$V_j(x_j) = U_j(R_j) = U_j(\nu(x_j)x_j). \tag{3}$$

Note that if the price pair (μ, κ) is *high* some of the users may refuse service and hence $x_j = 0$ for these users. After all users perform this optimization they inform the SP about how much spectrum they desire. The SP has to provide a total spectrum of $X = \sum_{j=1}^{L} x_j$. The SP purchases this amount of spectrum from the FCC and has to pay $C(X)$. Given X the SP can further optimize his prices (μ, κ) to maximize his profits $\sum_{j=1}^{L} \rho(x_j) - C(X)$. He performs

$$\max_{\mu,\kappa} \mu X + \kappa N - C(X), \tag{4}$$

where N users accept the service. The SP announces the new prices (μ, κ) and users again optimize over x_j. We note that the optimization problems (2) and (4) are coupled. At the equilibrium there is a unique $(L+2)$-tuple $\{x_1, \cdots, x_L, \mu, \kappa\}$ that satisfy both these problems simultaneously.

III. Monopolistic Pricing

Since there is only one SP the domain of the problem lies in the monopolistic pricing literature of microeconomics [10], [13]. To arrive at the solution consider the following cases,

A. $U_j(R_j)$ is same for all users

Recall that $U_j(R_j) = U_j(\nu(x_j)x_j)$. Thus for $U_j(R_j)$ to be same for all users consider when the SP provides uniform spectral efficiency to all users, i.e $\nu_j = \nu$ and $U_j(\nu x)$ is same for all j. An example could be the SP implementing an OFDM transmitter in the downlink with the same modulation per subcarrier and each user is interested in maximizing his rate. Note that different subcarriers have different channel gains and in order to provide the same ν per subcarrier, the SP has to vary the transmit power, which is similar to perfect power control in CDMA. Thus all users are allocated an equal share of spectrum and the optimal allocation can be obtained by considering any one user. For one user, the optimization problems (2) and (4) become,

$$\max_{x} V(x) - \mu x - \kappa. \tag{5}$$

The SP optimization is,

$$\max_{\mu,\kappa} \mu x + \kappa - C(x). \tag{6}$$

To maximize his profits, the SP has to raise his prices (μ, κ). However if the prices are too high the user might decide not to obtain service from the SP. Thus SP will set the prices such that the user is just *indifferent* between obtaining or not obtaining the service [13]. In other words from (5),

$$\max_{x} V(x) - \mu x - \kappa = 0. \tag{7}$$

The first order conditions of (5) is

$$\mu = V'(x). \tag{8}$$

The graph of (8) is called the *demand function* [13] which shows how the demand for resource x varies with price μ. Substituting this in (7) we obtain,

$$\kappa = V(x) - \mu x = V(x) - xV'(x). \tag{9}$$

Calculating $d\mu/dx$ and $d\kappa/dx$ from (8) and (9) and dividing we obtain,

$$\frac{d\kappa}{d\mu} = -x \;\text{ or }\; \kappa^* = \int_{\mu^*}^{\infty} x(\mu) d\mu, \tag{10}$$

where (μ^*, κ^*) is the solution to (6) for $x = x^*$, where x^* is the solution of (5). The first order conditions of (6) w.r.t μ is,

$$x + \mu\frac{dx}{d\mu} - C'(x)\frac{dx}{d\mu} + \frac{d\kappa}{d\mu} = 0. \tag{11}$$

But the first and last terms cancel out from (10). We thus obtain

$$\mu = C'(x). \tag{12}$$

Thus the optimal usage price is equal to the *marginal cost* evaluated at the optimal spectrum allocated x^*, or the cost borne by the SP to produce one extra unit of spectrum at x^*. Further revenue is then obtained from the subscription fee κ. From (7) $\kappa = V(x) - \mu x$. In the economics literature, κ is called the *consumer's surplus*, which is a measure of how much money the user has to be given in order to make him give up the entire consumption of the good. Thus the SP makes maximum profit by charging the user his surplus. For $C(x) = Cx + F$, the production cost is exactly salvaged by the usage fees μx and the SP's subsequent profit is entirely due to κ.

B. An example of Spectrum Allocation

From (8) and (12) we observe that x is obtained by solving,

$$V'(x) = C'(x). \tag{13}$$

This is a standard result in microeconomics that says that at the optimal production x^* the marginal utility $V'(x)$ is equal to the marginal cost of production $C'(x)$. Thus at optimal production, the cost for providing an additional unit of spectrum is same as the utility (in dollar value) provided by it.

Example 1: As an example consider $V(x) = \log(\nu x)$ (corresponding to $U(R) = \log(R)$) and $C(x) = Cx + F$. From (12), the demand function is $\mu = 1/x$. From (13) the spectrum price is $\mu = C$ and spectrum allocated is $x = 1/C$. The surplus of the user is $\kappa = \log(\nu/C) - 1$.

C. Relative Magnitudes of usage cost and subscription fee

Does the SP profit more by keeping usage cost rate μ high or subscription fee κ high? The answer depends on the shape of $V(x)$. If the optimal value of allocated spectrum is $x = x^*$, we have to compare the values of $\mu x^* = x^* V'(x^*)$ and $\kappa = V(x^*) - x^* V'(x^*)$. Thus

$$\mu x^* \gtrless \kappa^* \text{ iff } x^* V'(x^*) \gtrless V(x^*) - x^* V'(x^*) \tag{14}$$

$$\text{or iff } 2x^* V'(x^*) \gtrless V(x^*). \tag{15}$$

Let us consider the class of general α-fair concave utility functions [9] given by,

$$U_\alpha(x) = \begin{cases} \dfrac{x^{1-\alpha}}{1-\alpha}, & \alpha \neq 1 \\ \log(x), & \alpha = 1 \end{cases} \tag{16}$$

Different values of α lead to solutions that have different notions of fairness [9]. For e.g. $\alpha = 0$ corresponds to throughput maximization utility, $\alpha = 1$ corresponds to proportional fairness and $\alpha \to \infty$ corresponds to max-min fairness. For these class of utilities we can show that whenever $0 \leq \alpha < 0.5$ or $\alpha > 1$, usage cost μx is always greater than subscription fee κ for any value of x. This includes the utilities for throughput maximization and max-min fairness. The relationship between the two costs is reversed for utilities with $0.5 < \alpha < 1$. For $\alpha = 0.5$ they are equal and for $\alpha = 1$, there is no unique relationship between the two for all values of x.

D. $U_j(R_j)$ is different for users

In section III-A the SP transmitted at variable power to enable equal rate to all users. Thus if the channel coefficient between user j and SP was h_j, then the SP transmitted at a power proportional to $1/h_j$. However this might lead to large transmit power requirements. In this section we consider that the SP has a total power constraint P and allocates power P_j to the transmission of user j. The SP is thus performing a joint power and spectrum allocation. The spectral efficiency obtained by user j is now given by, $\nu(x_j) = \log(1+h_j P_j/x_j)$, where we have assumed unit power spectral density of background AWGN. Thus $U_j(R_j)$ is different for different users if their channel coefficients to the SP are different. We assume that h_j is flat over frequencies and thus no matter between what bands x_j lies, h_j is same.

We consider two different types power allocations,

1) The SP serves all the L users with equal power allocation. Thus $x_j \neq 0$ and $P_j = P/L$ for all j.
2) The SP optimizes P_j to maximize his profit. Thus some users not accept service i.e. $x_j = 0, P_j = 0$ because of prices (μ, κ). The loss of revenue from these users can be made up by increased revenue from other users.

Let $U_j(R_j) = \eta R_j$ and thus $V_j(x_j) = \eta x_j \log(1 + h_j P_j / x_j)$ where η is the fraction of the capacity that can be reliably guaranteed by the SP and is a measure of its *efficiency*. Usually the demand function of a user does not depend on number of users L. However in our case the demand function of spectrum as per (8) is,

$$\mu = \eta \log\left(1 + \frac{h_j P_j}{x_j}\right) - \frac{\eta h_j P_j}{x_j + h_j P_j}, \tag{17}$$

which depends on P_j. Since there is a total transmit power constraint P, the demand functions of the users are in fact dependent on each other.

1) SP serves all users: From (4) the SP optimization equation for $N = L$ is $\max_{\mu,\kappa} \mu X + \kappa L - C(X)$. The first order conditions w.r.t μ are,

$$X + \mu\frac{dX}{d\mu} - C'(X)\frac{dX}{d\mu} + L\frac{d\kappa}{d\mu} = 0. \tag{18}$$

If users have different utilities and the SP has to maximize his profits while serving all of them, it will raise the (μ, κ) values to that point when the surplus of some user becomes zero. Increasing price any further would cause this user to refuse the service. This user is referred to as the *marginal user*. Let us denote this user by index m. Thus the price (μ, κ) are set such that (7) and (10) are satisfied for user m. Thus $d\kappa/d\mu = -x_m$ in (18). To simplify the expression further we follow the treatment in [10] and introduce the notations $s = x_m/X$, which indicates the fraction of the total spectrum that

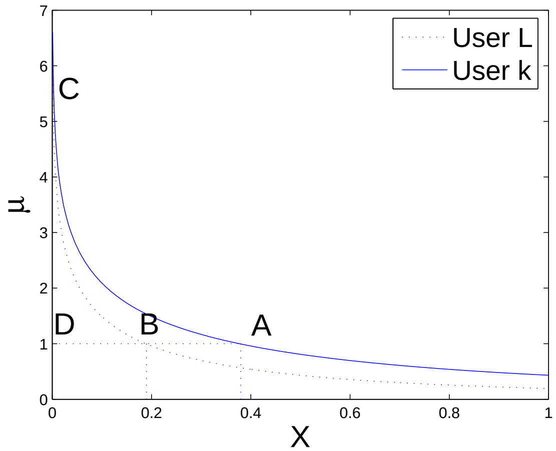

Fig. 1. The demand function for two users

the marginal user obtains and $\epsilon = -(\mu/X)(dX/d\mu)$ which in economics is called the *price elasticity of demand* [13]. This is a measure of how much percentage change in price affects percentage change in consumption. This can be seen by writing ϵ as $-(dX/X)/(d\mu/\mu)$. Note that X decreases with price μ and hence $dX/d\mu$ is negative and ϵ is positive. In terms of these constants we can solve for μ from (18) as,

$$C'(X) = \mu\left(1 - \frac{1 - Ls}{\epsilon}\right). \quad (19)$$

For the single user case considered in Section III-A, $L = 1$ and $s = x_1/X = 1$. Thus (19) reduces to (12) as expected.

We now state and prove the following lemma about spectrum allocation,

Lemma 1: The function

$$\mu = f(z) = \eta \log\left(1 + \frac{1}{z}\right) - \frac{\eta}{1+z} \quad (20)$$

is decreasing and convex for positive z.

Proof: For $z > 0$, $f'(z) < 0$ and $f''(z) > 0$.

Lemma 2: The user with the weakest channel to the SP is the marginal user.

Proof: The demand function of user j is obtained by substituting $P_j = P/L$ in (17),

$$\mu = \eta \log\left(1 + \frac{h_j P}{L x_j}\right) - \frac{\eta h_j P}{L x_j + h_j P}. \quad (21)$$

Assume WLOG that $h_1 > h_2 > \cdots > h_L$. Consider users L and k, where $k < L$. Fix $x_L = x_k = x$. In terms of $z_L = Lx/h_L P$ and $z_k = Lx/h_k P$, (21) reduces to (20). Now $h_k > h_L \Rightarrow z_k < z_L$ and hence by Lemma 1, $f(z_k) > f(z_L)$. This is illustrated in Figure 1 which shows that the graph for user k lies to the right of the graph for user L for all k.

Let the optimal price be μ^*. The surplus of user k is the area of the region DAC while that of user L is the area of region DBC as these are the values of the integral in (10) evaluated for x_k and x_L. Since area of DAC > area of DBC user L has the least surplus amongst all users and is thus the marginal user. ■

Lemma 3: At the optimal solution spectrum price μ is more than the marginal cost of production $C'(X)$.

Proof: Let the solution of (20) be $z = f^{-1}(\mu)$. Since $f(z)$ is decreasing and convex, it is one-to-one and thus $f^{-1}(\mu)$ is also one-to-one. Let the optimal price be $\mu = \mu^*$. Since $z_k = Lx_k/h_k P$, we obtain,

$$x_k = \frac{h_k P}{L} f^{-1}(\mu^*). \quad (22)$$

Thus in (19) we can evaluate,

$$Ls = \frac{Lx_L}{X} = \frac{Lh_L(P/L)f^{-1}(\mu^*)}{\sum_{k=1}^{L} h_k (P/L) f^{-1}(\mu^*)} = \frac{Lh_L}{\sum_{k=1}^{L} h_k} < 1, \quad (23)$$

as $h_L = \min(h_1, h_2, \cdots, h_L)$. Hence Proved. ■

The relationship $\mu > C'(X)$ means that the spectrum cost charged by spectrum regulatory body of a country like FCC to the SP is increased and passed on directly to the customers by the SP.

The parametric expression of x_k in terms of h_k as shown in (22) is very useful for simplifying the results for optimal pricing. For e.g. consider $C(X) = F + CX$, then substituting for x_k in (19) and after some algebraic manipulations we obtain,

$$C = \mu + \frac{1 - Lh_L/\sum_{k=1}^{L} h_k}{\left(1 + f^{-1}(\mu)\right)^2}. \quad (24)$$

Thus given a set of L users (24) can be solved and the optimal value of μ determined. In practice we can assume that before the actual spectrum allocation takes place, each user sends beacon packets to the SP and thus the SP is aware of number of users L and the channel coefficients $h_1 \cdots h_L$. The SP can then solve (24).

Recall that $d\kappa/d\mu = -x_L$. Thus for $\mu = \mu^*$,

$$\kappa^* = \frac{h_L P}{L} \int_{\mu^*}^{\infty} f^{-1}(\mu) d\mu. \quad (25)$$

Observe from (24) and (25) that for $C(X) = F + CX$, μ^* does not depend on transmit power P while κ^* does. In fact (25) shows that κ^* increases linearly with P. Thus it is profitable for the SP to increase his transmit power. Of course we haven't factored in the cost incurred by the SP for increasing his transmit power (for e.g. batteries draining out faster) which might have given the optimal value of P.

2) SP optimizes the transmit powers: In this section we return to the general demand function as given in (17) and ask the following question: *if there are L users and the SP has a total transmit power constraint how should he optimally allocate power to their transmissions so as to maximize his profits?* We mention at this juncture that power allocation has traditionally been used for objectives such as user sum capacity maximization but to the best of our knowledge has never been explored for the SP profit maximization.

First assume that in the optimal solution the SP serves only the first N out of L users. Then the user N is the marginal user. Then (22) is modified to $x_k = h_k P_k f^{-1}(\mu^*)$ and (25) is modified to $\kappa^* = h_N P_N \int_{\mu^*}^{\infty} f^{-1}(\mu) d\mu$. Recall that SP profit $\pi(N) = (\mu^* - C)X + N\kappa^*$. Hence the power allocation problem for profit maximization can be expressed as,

$$
\begin{aligned}
&\max_{N} \max_{P_1,\cdots,P_N} (\mu^* - C) f^{-1}(\mu^*) \sum_{k=1}^{N} h_k P_k + N\kappa^* \\
&\text{s.t. } C = \mu^* + \frac{1}{(1+f^{-1}(\mu^*))^2} \left(1 - \frac{N h_N P_N}{\sum_{k=1}^{N} h_k P_k}\right) \\
&\quad \kappa^* = h_N P_N \int_{\mu^*}^{\infty} f^{-1}(\mu) d\mu \\
&\quad P_1 + \cdots + P_N = P \\
&\quad P_1, \cdots, P_N > 0.
\end{aligned}
$$

This is a complicated non-convex problem. However we can arrive at the optimal solution indirectly. Let us first consider 2 users with $h_1 > h_2$ and derive the optimal power allocation. The result can be generalized to the case when more users are present. From definition of $\rho(x_j)$, the SP revenue from user j (see (1)) we can write the SP objective function is,

$$\pi(2) = \rho(x_1) + \rho(x_2) - C(x_1 + x_2) \quad (26)$$

Recall from the discussion in Section III-D.1 that the entire surplus of the second user is extracted by the SP. Thus utility $V_2(x_2) = \rho(x_2)$. This is true even if no spectrum is allocated to user 2 as then $V_2(x_2) = 0$ and $\rho(x_2) = 0$ by definition. However the first user still has some surplus left and hence $V_1(x_1) > \rho(x_1)$. Thus for all x_1, x_2

$$\pi(2) < V_1(x_1) + V_2(x_2) - C(x_1 + x_2). \quad (27)$$

Thus,

$$\max_{\substack{x_1 \geq 0, x_2 \geq 0 \\ P_1 \geq 0, P_2 \geq 0 \\ P_1 + P_2 = P}} \pi(2) < \max_{\substack{x_1 \geq 0, x_2 \geq 0 \\ P_1 \geq 0, P_2 \geq 0 \\ P_1 + P_2 = P}} \sum_{k=1}^{2} V_k(x_k) - C x_k. \quad (28)$$

But the optimization problem in the RHS of (28) is similar to the sum utility maximization problem considered in [2] with the shadow price being replaced by the spectrum cost C. It has been shown in [2] that the solution of the optimization problem is achieved by allocating all power to the user with the best channel. Since $h_1 > h_2$, the optimal power vector is $[P, 0]$. But for this power vector both the LHS and RHS optimization problems in (28) become the same problem. This is a optimizing x_1 for $P_1 = P$. Thus for $[P, 0]$ the optimization of $\pi(2)$ touches the maximum value of its upper bound and hence this is the optimal power allocation strategy.

Thus to maximize profits the SP maximizes the sum utility of the system as he can then extract dollar revenues proportional to the sum utility.

E. Numerical Results

We now numerically evaluate how the SP profit varies with the number of users served under a uniform power allocation policy. Let the SP serve N users with the largest value of channel gains. The users are indexed from 1 to N and each transmission takes place at power P/N. Cases considered in Sections III-D.1 and III-D.2 correspond to $N = L$ and $N = 1$ respectively.

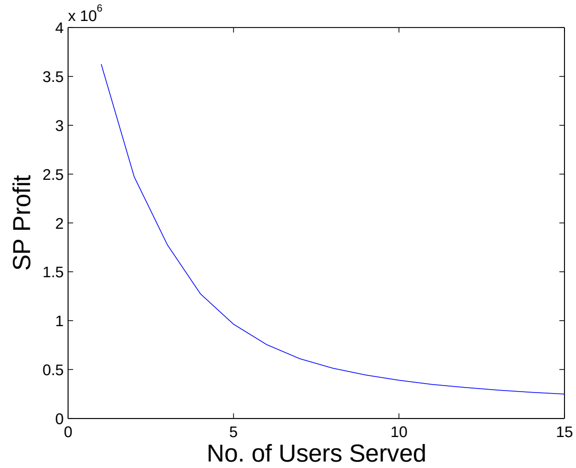

Fig. 2. The SP profit as the number of served users varies

No. of Users, L	15
Path Loss Coeff	3.7
Reference Distance	100m
Cell Radius	1km
Transmit Power	5 Watt
Efficiency, η	0.08
Fading Margin	44 dB

TABLE I
PARAMETERS FOR SPECTRUM ALLOCATION

We assume $L = 15$ users are distributed in a cellular area of 1 km. The channel coefficients originate from a distance based path loss model. We also assume that due to shadow fading the received SNR is reduced by a constant fading margin. As mentioned earlier the the channel coefficients are flat over frequencies and depend only upon the user locations. The values of μ and κ are calculated from (24) and (25) respectively. The simulation parameters are explained in Table I. The SP profit depends on how the function $g(N) = N h_N / \sum_{k=1}^{N} h_k$, varies with N. Now $g(1) = 1$ and for $N_1, N_2 > 1$, $g(N_1), g(N_2) < 1$, but their relative order depends on the values of the coefficients h_k. Thus for each $N > 1$, we generate 10000 instances of channel vector $[h_1, \cdots, h_N]$ and calculate the average value of SP profit. The result is shown in Figure 2. We see that the SP profit decreases as the number of users increase. It is most profitable for the SP to serve only one user as was proved in Section III-D.2.

Figure 3 plots the breakup of the SP revenue from subscription cost κ and usage cost μx for the marginal user and the user with the best channel to the SP (referred to as the best user), for $C = 0.5$. The majority of the revenue comes from the usage cost of the best user. Least revenue comes from the subscription cost. The demand function graphs as given in Figure 1 give us the intuition that for lower values of C, the spectrum purchased and the surplus κ is more. This is observed in Figure 4 where the value of C is lowered to 0.05. Lastly Figure 5 shows that the SP profit reduces exponentially with production cost C.

Note that the absolute values of the various parameters

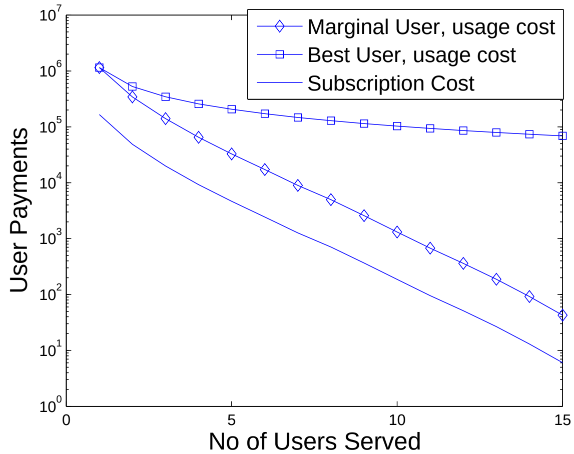

Fig. 3. The SP revenue from different users for $C = 0.5$

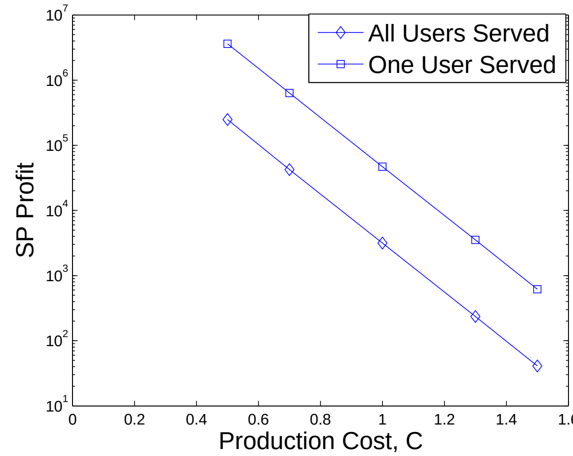

Fig. 5. The SP profit as spectrum production cost C varies

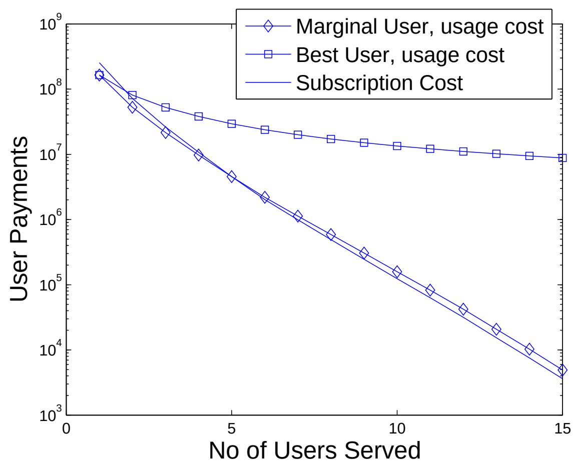

Fig. 4. The SP revenue from different users for $C = 0.05$

shown in the figures should not be interpreted literally. For e.g. we have $C = 0.5, 0.05$ but profit values which are bigger by several orders of magnitude. This is because for simulation purposes, the various systems equations like $\mu = V'(x) = C'(x)$ haven't been normalized. One *dollar*, the unit of $C'(x)$ is not equivalent to one *bps*, the unit of $V'(x)$. The results in this paper are true within bounds of proper scaling.

In passing we note that the role of the marginal user in profit maximizing pricing strategies have also been studied in [1] for a communications system with only fixed subscription costs κ and where the SP allocates power to a group of downlink nodes. The results are slightly different due to the non inclusion of usage based cost in the problem formulation.

IV. CONCLUSION

In this paper we considered a system where a single SP allocated spectrum to users subscribed to its network and charged a two part tariff consisting of a fixed subscription price and variable usage cost to maximize its profits. We showed that the usage price is determined by the cost incurred by the SP in providing services over the spectrum and the subscription price is determined by the excess surplus of the marginal user. We also showed that the optimal power allocation policy that maximizes SP profit is to allocate all resources to the user with the highest channel gain which is also the policy that solves the sum utility maximization problem of the users.

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