Dynamic Spectrum Allocation for Uplink Users with Heterogeneous Utilities

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***Abstract*— Dynamic allocation of spectrum prior to transmission is an important feature for next generation wireless networks. In this work, we develop and analyze a model for dynamic spectrum allocation, that is applicable for a broad class of practical systems. We consider multiple service providers (SPs), in the same geographic region, that share a fixed spectrum, on a non-interference basis. This spectrum is allocated to their customer end users for transmission to the SPs. Assuming that a user can obtain service from all the SPs, this work develops an efficient algorithm for spectrum allocation. The quality of service depends on system parameters such as number of users and SPs, the channel conditions between the users and SPs and the total transmit power of each user. The SPs have different efficiencies of reception. We adopt a user utility maximization framework to analyze this system. We develop the notion of spectrum price that enables a simple distributed spectrum allocation with minimal coordination among the SPs and users. Given the user utility functions and the system parameters, we characterize the spectrum price and the users' optimal bandwidth allocations. Our work provides theoretical bounds on performance limits of practical operator to user based dynamic spectrum allocation systems and also gives insights to actual system design.**

I. Introduction

We are witnessing a large growth in the scope of wireless communications services. In future new broadband applications like Worldwide Interoperability for Microwave Access (WiMAX) and Third Generation Partnership Project- Long Term Evolution (3GPP-LTE) will co-exist with traditional technologies like WLAN and 2G cellular. Spectrum allocation among different wireless systems is thus important for ensuring fairness and efficiency for end-to-end applications.

The traditional regulatory process for spectrum allocation has been largely non-responsive to application requirements. This has lead to an artificial scarcity of spectrum and reduced QoS for the users who are being serviced. This has motivated the development of dynamic spectrum allocation (DSA) techniques that take into account the application requirements, presence of other devices in the region and link gains between the transmit-receive pairs.

In 2004, the IEEE set up a working group to develop the 802.22 cognitive radio standard to employ the unused spectrum in the VHF and UHF TV bands to offer wireless broadband services [1]. It has been decided that fixed wireless access will be provided in these bands by professionally installed Wireless Regional Area Network (WRAN) base stations to WRAN user terminals. A service provider (SP) operating a base station will share the total spectrum with other SPs in the region and further allocate this spectrum to users efficiently.

Motivated by this SP-user model of DSA, we propose and analyze a dynamic spectrum allocation algorithm based on limited coordination among devices in this paper. We consider a two tiered spectrum allocation scheme as shown in Figure 1. There is some total spectrum C Hz available in a geographic area which is allocated to the users through the SPs. The users are permitted to obtain spectrum from all the SPs. We assume that the users obtain non-overlapping chunks of spectrum from the SPs to avoid interference. Assuming that each user application has an associated utility which is concave and increasing as a function of spectrum obtained, we adopt an utility maximization framework [2] to analyze the system. Given user utility functions, channel coefficients between users and SPs and user power constraints, our aim is to derive how much spectrum should a user obtain from a SP and how power should be subsequently allocated for sending information to the SPs. We assume that the spectrum utilized by a SP is the spectrum it has to allocate to the users and allow for simple SP interaction to share the spectrum C. We facilitate SPs sharing the spectrum C by a spectrum clearing house (SCH), akin to an FCC-controlled regional spectrum broker [3]. Based on our analysis, we develop the notion of a *spectrum price* and use it to propose a simple distributed allocation algorithm.

A. Related Work and Our Contribution

Our work lies in the domain of systems with non-strategic users who follow a common spectrum allocation protocol without greedily trying to maximize their objectives. Such systems could be centralized or distributed. Upcoming OFDMA based cellular systems such as 3GPP-LTE fall in the centralized category. User to subcarrier assignment and power allocation for OFDMA have been considered for the downlink [4] and for uplink [5]. These works mostly consider weighted sum rate maximization while we generalize this to concave utility functions of rates. Also prior work had considered a single SP with fixed frequency bins (OFDM tones) whereas in our work we allow for multiple SPs and treat spectrum as a continuous resource. Treating spectrum as continuous is justified for systems where the subcarrier spacing is small and the number of subcarriers is large. An example system is LTE which can operate with 15 KHz spacing and 2048

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subcarriers [6]. In addition, we also allow the different SPs to have different efficiencies which is defined as the fraction of Shannon capacity that the SP can reliably deliver.

In the distributed non-strategic regime, each user follows a distributed spectrum sharing algorithm. Reference [3] introduce the notion of Coordinated Access Bands of spectrum to achieve distributed coordination for spectrum sharing. Spectrum etiquette protocols that act as simple overlays over interfering devices such as bluetooth and 802.11a/b/g devices have been studied in [7]. While [3], [7] have mostly focused on the network architecture and protocol signaling, this paper introduces an analytical price based-distributed algorithm.

The assumption of non-strategic users is valid when the users are transmitter-receiver pairs of a single system or when different systems in a geographic region are jointly designed with a common objective [8]. This would require the users and SPs operating in a region to agree on simple spectrum etiquettes which precludes strategic user behavior. In passing, we note that strategic behavior in wireless systems has been studied for centralized mechanism design systems in [9] and for decentralized Nash power games in [8], [10].

We also assume that the SPs do not maximize profits and simply provide diversity to the users in their choice of channels for transmission. This is reasonable for many cases such as the spectrum being allocated by a single SP who sets up multiple access points in a region. In practice, SPs could engage in competition to attract users [11]. In such situations our work also provides a baseline case for understanding the performance loss and changes in pricing structure as a result of competition.

Our model precludes spectrum overlap as we assume that the various users are close to each other and to the SPs and thus spectral overlap would cause significant interference. As noted in [8], for regimes when the cross gains between transmit receive pairs are stronger than direct gains, only orthogonal spectrum allocation guarantees Pareto efficiency. Because of orthogonal allocation, our model has similarities with network flow control models [2], [12]. The notion of SP efficiency and the usage of the Shannon rate function (defined later) in the user utilities distinguishes our work. Practical transmitters might employ directional antennae to achieve frequency reuse but in this work, we limit ourselves to finding the fundamental limits on gain possible with only bandwidth allocation. The result will serve as a baseline case for understanding the additional benefits if multiple antennae are deployed.

II. System Model

The network topology is shown in Figure 1. There are N SPs and L end users and a central Spectrum Clearing House (SCH). Based on the demand for spectrum, Service Provider i provides X_i units to the L users or a subset of them. Let x_{ij} be the amount of spectrum obtained by user j from SP i. The users and SPs are assumed to be capable of transmitting and receiving over any spectrum band x_{ij} which lies within C. This could be achieved using non-contiguous OFDM technology [13]. Subsequently, user j transmits his data to SP i over spectrum x_{ij} at rate r_{ij} and with power p_{ij}.

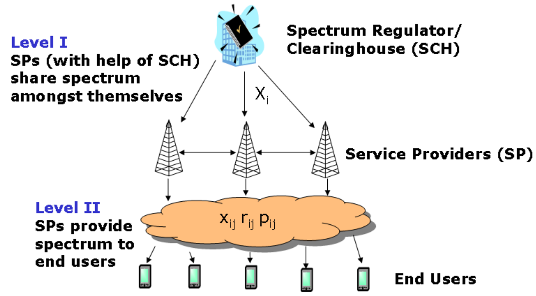

Fig. 1. The network topology

Each user has a total transmit power constraint. The channel between SP i and user j is characterized by the link gain coefficient h_{ij}, which remains constant during the period of spectrum allocation and subsequent transmission to the SP. We assume that h_{ij} is flat over frequency and hence is same no matter in which band x_{ij} lies. The coefficients h_{ij} are assumed to be known by both users and SPs. The background additive Gaussian noise is assumed of unit power spectral density. We first introduce some notations. A source transmitting with power p, over a flat channel of bandwidth x and link gain h has signal to noise ratio $\mathsf{snr}(x,p,h) = hp/x$ and achieves the rate

$$r(x,p,h) = x\log\left(1+\mathsf{snr}(x,p,h)\right). \tag{1}$$

In terms of $r(x,p,h)$ the rate r_{ij} is given by

$$r_{ij} = \eta_i r(x_{ij}, p_{ij}, h_{ij}), \tag{2}$$

where η_i is the fraction of the Shannon capacity that can be reliably guaranteed by SP i to a user. A possible example would be SP i, who has invested in a better decoder (a Turbo decoder with more iterations or better interleaver design) has a higher η_i than an SP with a conventional Viterbi decoder. Thus the total rate at which user j can transmit reliably is

$$R_j = R(\mathbf{x}_j, \mathbf{p}_j, \mathbf{h}_j) = \sum_{i=1}^{N} r_{ij}, \tag{3}$$

where $\mathbf{x}_j = [x_{1j}\cdots x_{Nj}]$, $\mathbf{h}_j = [h_{1j}\cdots h_{Nj}]$ and $\mathbf{p}_j = [p_{1j}\cdots p_{Nj}]$.

There is a utility function $U_j(R_j)$ associated with user j which is concave and increasing in R_j. The operating principle of the network is to maximize social welfare or the sum utility of the users. The optimization problem is

$$\max_{x_{ij}\geq 0, p_{ij}\geq 0, X_i\geq 0} \sum_{j=1}^{L} U_j\left(R_j\right) \tag{4a}$$

$$\text{s.t.} \sum_{j=1}^{L} x_{ij} \leq X_i,\ 1\leq i\leq N, \tag{4b}$$

$$\sum_{i=1}^{N} p_{ij} \leq P_j,\ 1\leq j\leq L, \tag{4c}$$

$$\sum_{i=1}^{N} X_i \leq C. \tag{4d}$$

As shown in (4b), X_i is the spectrum utilized by SP i which is equal to the spectrum it has to allocate to the users. User j transmits with power p_{ij} to SP i and as (4c) shows there is a constraint P_j on the total transmit power. The total amount of available spectrum is C. User j optimizes over x_{ij} and p_{ij}. In Appendix A, we show that the objective is concave in these variables and since the constraints are linear, the problem can be solved efficiently.

A. Distributed Solution and Pricing

In this section we give a distributed implementation of the spectrum allocation problem (4). First we relax the constraints (4b) and (4d) in the objective function to form the partial Lagrangian $\mathcal{L}$ [14]

$$\mathcal{L}(x_{ij}, p_{ij}, X_i, \boldsymbol{\lambda}, \mu) = \sum_{j=1}^{L} U_j(R_j) + \sum_{i=1}^{N} \lambda_i \left(X_i - \sum_{j=1}^{L} x_{ij} \right) + \mu \left(C - \sum_{i=1}^{N} X_i \right), \tag{5}$$

where $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_N]^T$. The stationarity conditions w.r.t. X_i can be expressed as,

$$\frac{\partial \mathcal{L}}{\partial X_i} = \lambda_i - \mu \leq 0, \tag{6}$$

with equality holding *iff* $X_i > 0$. Interpreting μ as the price the SCH charges to the SPs and λ_i as the price that SP i charges to its users [2] we see that each SP i that provides non-zero spectrum ($X_i > 0$) charges the same price $\lambda_i = \mu$. This is because the SPs have no objectives of their own.

Thus we form the Lagrangian

$$\mathcal{L}(x_{ij}, p_{ij}, \mu) = \sum_{j=1}^{L} U_j(R_j) - \mu \sum_{i=1}^{N} \sum_{j=1}^{L} x_{ij} + \mu C \tag{7}$$

for the new optimization problem and the dual

$$D(\mu) = \max_{x_{ij} \geq 0, p_{ij} \geq 0} \mathcal{L}(x_{ij}, p_{ij}, \mu) \quad \text{s.t.} \sum_{i=1}^{N} p_{ij} \leq P_j,\ 1 \leq j \leq L. \tag{8}$$

The spectrum price μ is set jointly by the SPs and the SCH by minimizing the dual

$$\min_{\mu > 0} D(\mu). \tag{9}$$

From (7) the optimization in (8) decomposes into separate optimization problems for the users [14]. The optimization subproblem for user j is

$$\mathcal{U}_j = \max_{x_{ij} \geq 0, p_{ij} \geq 0} U_j(R_j) - \mu \sum_{i=1}^{N} x_{ij} \tag{10a}$$

$$\text{s.t.} \sum_{i=1}^{N} p_{ij} \leq P_j. \tag{10b}$$

Quantity $\mathcal{U}_j$, called the *user surplus* in microeconomics [15, Chapter 14], is the residual utility of user j after paying the spectrum cost. In the context of a spectrum price, this is the payment in terms of the utility function that has to be given to user j to persuade him to give up his consumption of spectrum. The price μ is set by a distributed price update for (9)

$$\mu(t+1) = \left[\mu(t) - \alpha_\mu(t) \left(C - \sum_{i=1}^{N} \sum_{j=1}^{L} x_{ij}(\mu(t)) \right) \right]^+ \tag{11}$$

where $x_{ij}(\mu(t))$ is the spectrum obtained by user j from SP i, for a given value of $\mu(t)$ and $\alpha_\mu(t)$ is a positive step size. From (11) we see that if the spectrum is under-utilized, $C - \sum_{i=1}^{N} \sum_{j=1}^{L} x_{ij}(\mu(t))$ is positive and thus the price decreases to facilitate greater utilization of spectrum. Similarly if spectrum is over-utilized, the price increases. This is summarized in the following theorem

Theorem 1: The global spectrum price μ charged by all the SPs is set such that the entire spectrum is utilized.

The distributed spectrum allocation mechanism is given in Table I. From [16, Proposition 3.4], $\mu(t)$ given in Table I converges to the equilibrium price μ for proper choice of step size $\alpha_\mu(t)$.

Distributed Spectrum Allocation Mechanism
1) At time t, SPs broadcast price $\mu(t)$.
2) Each user j solves (10) and calculates $x_{ij}(\mu(t))$ and $p_{ij}(\mu(t))$ for all i SPs.
3) All users pass $x_{ij}(\mu(t))$ to each SP i.
4) The SPs calculate $\mu(t+1)$ from (11).

TABLE I
DISTRIBUTED UPDATE OF SPECTRUM AND POWER

III. CHARACTERIZING THE SPECTRUM ALLOCATION

We will denote the first and second derivatives of the utility function by

$$\dot{U}_j(R_j) \triangleq \frac{\partial U_j}{\partial R_j}, \qquad \ddot{U}_j(R_j) \triangleq \frac{\partial^2 U_j}{\partial R_j^2}. \tag{12}$$

The derivatives of the rate function $r(x, p, h)$, in (1), are

$$\Gamma_p(x, p, h) \triangleq \frac{\partial r}{\partial p} = \frac{hx}{x + hp} = \frac{h}{1 + \mathsf{snr}(x, p, h)} \tag{13a}$$

$$\Gamma_x(x, p, h) \triangleq \frac{\partial r}{\partial x} = \log\left(1 + \frac{hp}{x}\right) - \frac{hp}{x + hp} = \log(1 + \mathsf{snr}(x, p, h)) - \frac{\mathsf{snr}(x, p, h)}{1 + \mathsf{snr}(x, p, h)} \tag{13b}$$

It follows from (2) and (3) that the derivatives of R_j wrt x_{ij} and p_{ij} can be expressed as,

$$\frac{\partial R_j}{\partial p_{ij}} = \eta_i \Gamma_p(x_{ij}, p_{ij}, h_{ij}), \tag{14a}$$

$$\frac{\partial R_j}{\partial x_{ij}} = \eta_i \Gamma_x(x_{ij}, p_{ij}, h_{ij}). \tag{14b}$$

To arrive at the optimal solution for the user subproblem (10), we first write its Lagrangian

$$\mathcal{L}_j = U_j(R_j) - \sum_{i=1}^{N} \mu x_{ij} + \gamma_j \left(P_j - \sum_{i=1}^{N} p_{ij} \right), \tag{15}$$

where all Lagrange multipliers are positive. The stationarity conditions for the Lagrangian are

$$\frac{\partial \mathcal{L}_j}{\partial x_{ij}} \equiv \eta_i \dot{U}_j(R_j)\Gamma_x(x_{ij}, p_{ij}, h_{ij}) \leq \mu, \quad (16a)$$

$$\frac{\partial \mathcal{L}_j}{\partial p_{ij}} \equiv \eta_i \dot{U}_j(R_j)\Gamma_p(x_{ij}, p_{ij}, h_{ij}) \leq \gamma_j, \quad (16b)$$

with equality holding for users with $x_{ij} > 0$ and $p_{ij} > 0$ respectively.

Theorem 2: In the optimal solution of (10) only one SP is active per user almost surely.

Proof: Consider user j and SP i and assume $x_{ij} > 0$ and $p_{ij} > 0$. Thus (16a) and (16b) are satisfied with equality. Dividing (16a) by (16b) and after some manipulation we obtain,

$$\left(1 + \frac{h_{ij}p_{ij}}{x_{ij}}\right) \log\left(1 + \frac{h_{ij}p_{ij}}{x_{ij}}\right) - \frac{h_{ij}p_{ij}}{x_{ij}} = \kappa_j h_{ij}, \quad (17)$$

where $\kappa_j = \mu/\gamma_j$. Now consider the function $\Psi(\mathsf{snr}) = (1 + \mathsf{snr}) \log(1 + \mathsf{snr}) - \mathsf{snr}$, which can be shown to be one-to-one and increasing in snr. Substituting for $\mathsf{snr} = h_{ij}p_{ij}/x_{ij} = \Psi^{-1}(\kappa_j h_{ij})$ in (13a) and then substituting for $\Gamma_p(\cdot)$ in (16b) we obtain

$$\eta_i \dot{U}_j(R_j) \left[\frac{h_{ij}}{1 + \Psi^{-1}(\kappa_j h_{ij})}\right] = \gamma_j. \quad (18)$$

We prove the rest by contradiction. Let user j obtain spectrum from SPs i and k. From (18)

$$\frac{\eta_i h_{ij}}{1 + \Psi^{-1}(\kappa_j h_{ij})} = \frac{\eta_k h_{kj}}{1 + \Psi^{-1}(\kappa_j h_{kj})}. \quad (19)$$

Now since h_{ij} is a continuous random variable the probability of event (19) is zero. Thus each user obtains spectrum from one SP *almost surely*. ■

Various flavors of Theorem 2 are also observed in [17], [18]. If instead of a net spectrum constraint ((4b) and (4d) together) there were individual spectrum constraints at each SP (only (4b)), then the problem of *ties* would occur [4], [5].

Let the active SP of user j be denoted by i_j^*. Denote $x_{i_j^* j}$, $h_{i_j^* j}$ and $\eta_{i_j^*}$ by x_j^*, h_j^* and η_j^* respectively. The user optimization in (10) can be re-written by considering only $i = i_j^*$. The rate R_j given in (3) has contribution only from $r_{i_j^* j}$ and is denoted by $R_j^* = \eta_j^* x_j^* \log\left(1 + h_j^* P_j / x_j^*\right)$.

A. Insights to SP User Assignment

Since user j is attached to SP i_j^*, from (4), it means that if it were allocated the optimum spectrum x_j^* from any other SP $i \neq i_j^*$, it would have still obtained a lower utility. Since the utility $U_j(R_j)$ is an increasing function of R_j this implies that user j obtains the highest rate from SP i_j^*, for given spectrum x_j^*. Define the signal to noise ratio, $\mathsf{snr}_{ij} = h_{ij}P/x_j^*$. Thus

$$i_j^* = \arg\max_i \ \eta_i x_j^* \log(1 + \mathsf{snr}_{ij}) \quad (20a)$$

$$= \arg\max_i \ (1 + \mathsf{snr}_{ij})^{\eta_i}. \quad (20b)$$

Actually if user j were to be associated with SP $i \neq i_j^*$, the allocated spectrum would be different from x_j^*, but this does not affect our result.

Observation 1: The following observations can be made

a) *Low snr_{ij} regime:* Use $(1 + x)^n \simeq 1 + nx$ in (20b) to obtain $i_j^* = \arg\max_i \eta_i h_{ij}$.

b) *High snr_{ij} regime:* Use the approximation $(1 + x)^n \simeq x^n$ in (20b). For a better insight consider 2 SPs with SP 1 being more efficient. Thus $\eta = \eta_1/\eta_2 > 1$. The condition for which SP 1 is the active SP for user j turns out to be

$$x_j^* < P \left(\frac{h_{1j}^\eta}{h_{2j}}\right)^{1/(\eta-1)}. \quad (21)$$

Thus user j attaches to the more efficient SP when the optimal bandwidth allocation x_j^* is less than a threshold. That is, user j will use the more efficient SP when bandwidth becomes scarce.

Corollary 1: If all SPs have the same efficiency, then each user obtains spectrum from the SP to which it has the highest link gain.

Proof: Follows from condition (20b) with $\eta_i = \eta$. ■

Lemma 1: The following facts hold,

a) $\dot{U}(R)$ for $R = r(x, P, h)$, as defined in (1), is a decreasing function of x.

b) $\Gamma_x(x, P, h)$ is a strictly decreasing function of x and is positive for all values of $\mathbf{v} = [x, P]$ for fixed h.

Proof: a) Since $U(R)$ is concave, $\ddot{U}(R) < 0$. This means $\dot{U}(R)$ is decreasing in R. But R increases in x from (1). Combining we get the desired result.

b) It can be verified that $r(x, P, h)$ is concave and increasing in $\mathbf{v} = [x, P]$ for fixed h and thus concave and increasing in x for fixed P and h. From concavity of $r(x, P, h)$ wrt x, $\Gamma_x(x, P, h)$ is monotonic decreasing in x and since $R(x, P, h)$ is increasing, we conclude that $\Gamma_x(x, P, h) > 0$. ■

In the next Theorem, we verify that each user obtains a strictly positive spectrum allocation. Intuitively this makes sense as if a user is not allocated spectrum then the potential increase to the sum utility due to his transmit power is wasted. The proof appearing in the Appendix B shows that when a new user $L+1$ joins the system of L users, a new allocation in which each of the original L users forfeits spectrum ϵ and user L obtains spectrum $L\epsilon$ provides higher sum utility for small ϵ.

Theorem 3: In the optimal allocation each user j obtains spectrum $x_j^* > 0$.

B. Dependence on Marginal Utility and Received Power

Theorem 4: When two users have the same channel gains, transmit powers and active SP efficiencies, the optimal allocation of spectrum favors the user with a higher *marginal utility of spectrum* i.e. whose utility function has a higher rate of increase with spectrum.

Proof: Consider users j and k with utility functions satisfying $\dot{U}_j(R) > \dot{U}_k(R)$ for all R and for whom $h_k^* = h_j^* = h$, $P_k = P_j = P$ and $\eta_k^* = \eta_j^*$. Let the allocated spectrum for users j and k be x_j^* and x_k^* respectively. We have to show that $x_j^* > x_k^*$.

Assume the contrary i.e. $x_k^* \geq x_j^*$. Now consider (16a) for both users

$$\dot{U}_j(R_j^*)\Gamma_x(x_j^*, P, h) = \dot{U}_k(R_k^*)\Gamma_x(x_k^*, P, h) = \mu. \quad (22)$$

Consider $x_k^* \geq x_j^*$. Let $R_j^* = R_j(x_j^*, P, h)$ and $R_k^* = R_j(x_k^*, P, h)$. This implies

1) $\dot{U}_j(R_j^*) \overset{(a)}{>} \dot{U}_k(R_j^*) \overset{(b)}{\geq} \dot{U}_k(R_k^*)$ where (a) is given in the statement of the problem and (b) is true from Lemma 1(a)
2) $\Gamma_x(x_j^*, P, h) \geq \Gamma_x(x_k^*, P, h)$ from Lemma 1(b)

Thus $\dot{U}_j(R_j^*)\Gamma_x(x_j^*, P, h) > \dot{U}_k(R_k^*)\Gamma_x(x_k^*, P, h)$ from points 1) and 2), which contradicts (22). ■

This is because a unit of spectrum Δx yields a higher contribution to sum utility when allocated to user j than to user k. This has also been observed in [12] for a network flow control problem.

We can illustrate this phenomenon with the class of exponential utilities given by

$$U_j(R) = \Gamma_j \left(1 - e^{-R/\Gamma_j}\right), \tag{23}$$

where Γ_j is the *target rate* of user j. For example, $\Gamma_j = 10^6$ b/s might be appropriate for a file transfer while $\Gamma_j = 10^4$ b/s would be adequate for a voice application. Since $\dot{U}_j(R) = e^{-R/\Gamma_j}$ is increasing in Γ_j for all R, the high target rate users are allocated more spectrum than those with low ones. As $R \to \infty$, these utilities become flat, i.e. $U_j(R) \to \Gamma_j$.

Another class of utilities used to model elastic applications are α utilities [19], given by

$$U_\alpha(R) = \frac{1}{\alpha} R^\alpha, \;\; 0 < \alpha \leq 1, \;\; U_0(R) = \log(R). \tag{24}$$

$\alpha = 1$ gives rate as the utility and for lower values of α, the utility increases sub-linearly for rates above a threshold. Thus high α models applications with high rate requirements.

Lemma 2: For α utilities, $\dot{U}(R)\Gamma_x(x, P, h)$ is a strictly increasing function of P for fixed x.

Proof: Refer to Appendix C. ■

Note that $\dot{U}(R(x, P, h))$ is actually *decreasing* in P while $\Gamma_x(x, P, h)$ is increasing in P. For α utilities, we show, in Appendix C, that their product increases with P. This need not be true for any arbitrary increasing concave function, such as the exponential utilities in (23) as they flatten out at Γ_j.

Theorem 5: If all users have α utilities and the received power of one user increases and user to SP assignments remain the same or the user switches to a SP with same efficiency, then that user obtains more spectrum and the spectrum price increases.

Proof: Consider user j and let $k \neq j$ be any other user. Let the price be μ and users j and k obtain spectrum x_j^* and x_k^*. Let user j increases his power from P_j to $\tilde{P}_j > P_j$. Let the new allocations be $\tilde{x}_j^*$ and $\tilde{x}_k^*$ for users j and k. The spectrum price changes from μ to $\tilde{\mu}$ and the rates from R_j^* and R_k^* to $\tilde{R}_j^*$ and $\tilde{R}_k^*$ for users j and k. By Theorem 3, all spectrum allocations are strictly positive and relation (16a) holds with equality for the old and new allocations and

$$\eta_j^* \dot{U}_j(R_j^*)\Gamma_x(x_j^*, P_j, h_j) = \eta_k^* \dot{U}_k(R_k^*)\Gamma_x(x_k^*, P_k, h_k) = \mu. \tag{25a}$$

$$\eta_j^* \dot{U}_j(\tilde{R}_j^*)\Gamma_x(\tilde{x}_j^*, \tilde{P}_j, h_j) = \eta_k^* \dot{U}_k(\tilde{R}_k^*)\Gamma_x(\tilde{x}_k^*, P_k, h_k) = \tilde{\mu}. \tag{25b}$$

We have to show that $\tilde{x}_j^* > x_j^*$. Assume the contrary that the event $\mathcal{A} \equiv \tilde{x}_j^* \leq x_j^*$ holds. Since there is a sum spectrum constraint, $\mathcal{A} \Rightarrow \mathcal{B}$, where $\mathcal{B} \equiv \tilde{x}_k^* \geq x_k^*$ for some user $k \neq j$.

From (25a), the old allocation for user k satisfies $\eta_k^* \dot{U}_k(R_k^*)\Gamma_x(x_k^*, P_k, h_k) = \mu$. From $\mathcal{B}$, $\tilde{x}_k^* \geq x_k^*$ and applying Lemma 1 we obtain,

$$\eta_k^* \dot{U}_k(\tilde{R}_k^*)\Gamma_x(\tilde{x}_k^*, P_k, h_k) \leq \mu. \tag{26}$$

For user j, there are two changes: a decrease in allocated spectrum and an increase in transmit power. Let us see their effects in isolation. First keep transmit power unchanged. From $\mathcal{A}$, $\tilde{x}_j^* \leq x_j^*$ and using Lemma 1 we get

$$\underbrace{\eta_j^* \dot{U}_j(R_j(\tilde{x}_j^*, P_j, h_j))\Gamma_x(\tilde{x}_j^*, P_j, h_j)}_{\partial U_j/\partial \tilde{x}_j^*} \geq \mu. \tag{27}$$

Next we keep the spectrum fixed and consider the increase in transmit power. From Lemma 2

$$\underbrace{\eta_j^* \dot{U}_j(R_j(x_j^*, \tilde{P}_j, h_j))\Gamma_x(x_j^*, \tilde{P}_j, h_j)}_{\partial U_j/\partial \tilde{P}_j} > \mu. \tag{28}$$

Recall that $\tilde{R}_j^* = R_j(\tilde{x}_j^*, \tilde{P}_j, h_j)$. Since $U_j(\cdot)$ is jointly concave in x_j^* and P_j from Appendix A we conclude from (27) and (28) that,

$$\eta_j^* \dot{U}_j(\tilde{R}_j^*)\Gamma_x(\tilde{x}_j^*, \tilde{P}_j, h_j) > \mu, \tag{29}$$

But (26) and (29) taken together contradict (25b). Hence our original assumption, events $\mathcal{A}$ and $\mathcal{B}$ are wrong. Thus $\tilde{x}_j^* > x_j^*$ which implies $\tilde{x}_k^* < x_k^*$ for some user $k \neq j$. Hence,

$$\tilde{\mu} \overset{(a)}{=} \eta_k^* \dot{U}_k(\tilde{R}_k^*)\Gamma_x(\tilde{x}_k^*, P_k, h_k) \overset{(b)}{>} \mu, \tag{30}$$

where (a) follows from relation (25b) and (b) follows from Lemma 1. Hence proved. ■

Thus user j demands more spectrum as his transmit power increases. This leads to a higher price and all other users obtain less spectrum.

Corollary 2: The user with increased power derives a higher utility and surplus and the sum utility also increases.

Proof: The utility of user j, $U_j(R_j^*)$ increases as it is an increasing function of both P_j and x_j^*. In Appendix D we show that the surplus, $\mathcal{U}_j(x_j^*) = U_j(R_j^*) - \mu x_j^*$ increases with x_j^*. The increase in sum utility can be proved indirectly as follows: consider the suboptimal allocation where each user l is retained at x_l^*. Since the power of user j increases, this allocation will still increase the utility of user j and thus the sum utility. The optimal utility can not be worse. ■

C. Dependence on number of SPs and Users

Theorem 6: As more users are added to the system, the spectrum price increases.

Proof: Assume that the system is in equilibrium with L users who have been allocated spectrum and user $L+1$ user joins in with link gain h_{L+1}^* and transmit power P_{L+1}. From Theorem 3, in the new equilibrium, he is allocated non-zero spectrum. This will reduce the allocated spectrum for all other users j, $1 \leq j \leq L$. Since h_j^* and P_j stay the same, this means that the price of spectrum goes up from Lemma 1 and (16a) considered with equality at the new price. A new user increases the demand for spectrum thus raising the price. ∎

Theorem 7: If all SPs are equally efficient and users have α utilities then the addition of an SP either increases the spectrum price or keeps it unchanged.

Proof: Assume that the system is in equilibrium and SP $N+1$ joins in the system. If it offers no better channel to any of the users than their existing ones, i.e. if $h_j^* > h_{(N+1)j}$ for all j, then no user engages itself to the SP and the optimal solution (spectrum price, spectrum allocated etc) is the same as before.

However, if for user j, the new SP provides a better channel coefficient, i.e $h_j^* < h_{(N+1)j}$, then user j engages itself to SP $N+1$ and adjusts its engaged SP index to $i_j^* = N+1$ and channel coefficient to $h_j^* = h_{(N+1)j}$. Thus user j's channel condition to his active SP has improved and as per Theorem 5, the price goes up. ∎

As more SPs join the system, a subset of them offer better link gains to users resulting in *better access* to the spectrum. This increases demand for spectrum and hence the price increases. To understand this consider an analogy from beachfront property: There exist beach-houses (analogous to spectrum) and they are in demand from vacationers. If good roads are built so that these houses become easily *accessible* (analogous to improving link gains or transmit power) then their demand goes up and so do their prices.

IV. Linear Utility Functions, $U_j(R_j) = R_j$

This is the sum rate maximization problem and gives an indication of the capacity of the user-SP vector channel. We present the results and the reader is referred to our previous work [20] for the details.

Theorem 8: For given link gain h_j^*, power P_j and efficiency η_j^*, user j operates at a unique signal to noise ratio, snr_j^* which is given by the solution of

$$\Phi(\mathsf{snr}_j^*) = \log\left(1+\mathsf{snr}_j^*\right) - \frac{\mathsf{snr}_j^*}{1+\mathsf{snr}_j^*} = \frac{\mu}{\eta_j^*}. \qquad (31)$$

From (31) we can also interpret SP efficiency as a scaling factor of spectrum price μ, i.e. a SP with higher efficiency has a smaller *effective* price μ/η_j^*.

Corollary 3: If all SPs are equally efficient, allocated spectrum and user surplus are given by

$$x_j^* = \frac{h_j^* P_j}{\sum_{k=1}^{L} h_k^* P_k} C, \qquad (32a)$$

$$\mathcal{U}(x_j^*) = \frac{h_j^* P_j}{1 + \sum_{k=1}^{L} h_k^* P_k / C}. \qquad (32b)$$

It can be shown that (32b) is an increasing function of h_j^* thus validating Theorem 5. From (32a), the spectrum allocation is directly proportional to the received signal power and hence can be very unfair if the users have wide variations in link gains and transmit powers. The use of exponential and α utilities mentioned in Section III lead to more fair allocation of spectrum as the allocation now depend on the marginal utilities which have a lesser variation than the link gains. We will explore this in Section V via numerical experiments.

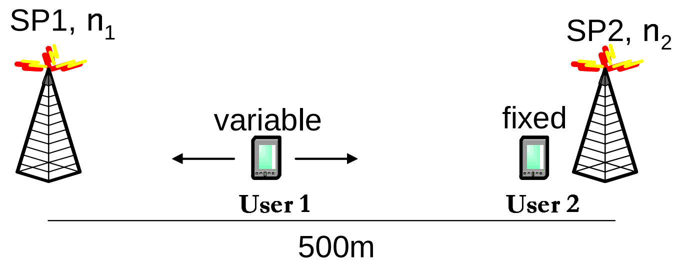

Fig. 2. The linear network with two SPs and two users.

V. Numerical Results

The spectrum allocation algorithm has the following basic steps

1) *SP selection by users*: The atomic setting is a network with one user and two SPs with different efficiencies.
2) *Spectrum allocation to users*: The atomic setting is a network with one SP and two users with different received powers.

We consider a network of two users and two SPs which incorporates both steps. We believe that insights from this network will be applicable to bigger networks as well. We consider two SPs in a linear cell with inter-base distance of 500 meters as shown in Figure 2. For path loss, we choose the COST-231 propagation model for outdoor WiMAX environments [21] at an operating frequency of 2.4 GHz. Let the noise power spectral density of $N_0 = -174$ dBm/Hz. Denote the SP i to user j distance by d_{ij} and the link gain, that incorporates N_0, by h_{ij},

$$h_{ij,dB} = P_{\text{loss}} - N_0 = -31.5 - 35\log(d_{ij}) - N_0. \qquad (33)$$

The distances are measured with SP 1 located at the origin. User 2 is fixed at a distance of $d_{22} = 100$ m from the SP 2 and the location of user 1 is varied from $d_{11} = 1$ m to $d_{11} = 499$ m from SP 1 in steps of 1 m. The total spectrum is 50 KHz. The following classes of utilities are considered based on the required rates of a user,

a) *low* required rate: For α utilities $U(R) = \log(R)$ and for exponential utilities $\Gamma = 1$ Kbps.
b) *high* required rate: For α utilities $U(R) = R$ and for exponential utilities $\Gamma = 1$ Mbps.

We first consider the spectrum allocation for users with exponential utilities. Let user 2 have a *high* required rate. SP efficiency ratios of $\eta_2/\eta_1 = 1$ and 10 are considered. Figure 3 shows the fraction of the spectrum allocated to user 1. It also indicates the active SP of user 1. The term *SP Switch* at distance $d = d_S$ means that for $d < d_S$ user 1 is attached to SP 1 and for $d > d_S$ it switches to SP 2. First consider that user 1 has a *high* required rate. Note that the switch to SP 2 occuers earlier when it is more efficient. The spectrum ratio is mostly increasing in the link gain to the active SP, h_1^*, as the rate function in (1) is increasing in h_1^* and spectrum x_1^* and if h_1^* improves then the rate achieved is increased even more by allocating more spectrum. Also an increase in R for low/medium R increases the utility $U(R)$. However x_1^* becomes constant in the region $\mathcal{V}$ defined by $\eta_2/\eta_1 = 10$ and

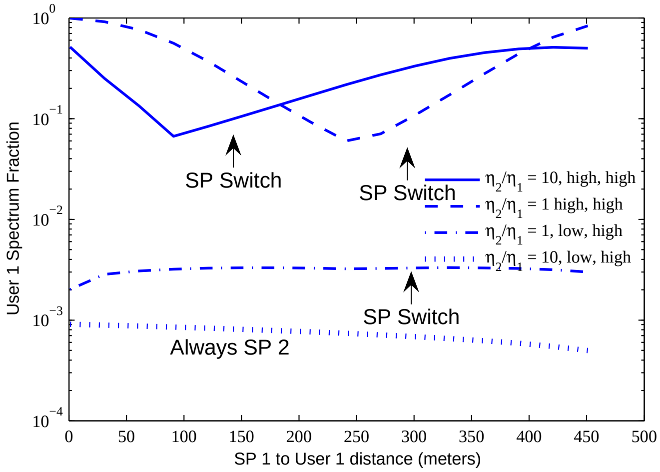

Fig. 3. Fraction of total spectrum allocated to user 1 as a function of distance for different target rates and SP efficiencies. Both users have exponential utilities and user 2 is fixed at 100m from SP 2

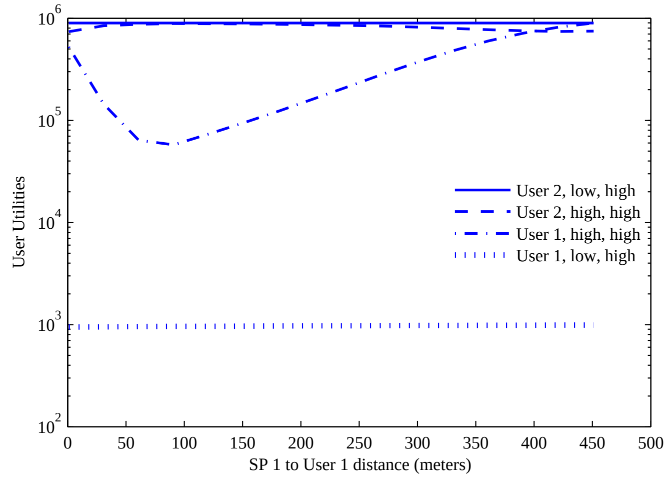

Fig. 5. The utilities for both users as a function of distance for different target rates. Both users have exponential utilities and user 2 is fixed at 100m from SP 2. The efficiency ratio is $\eta_2/\eta_1 = 10$

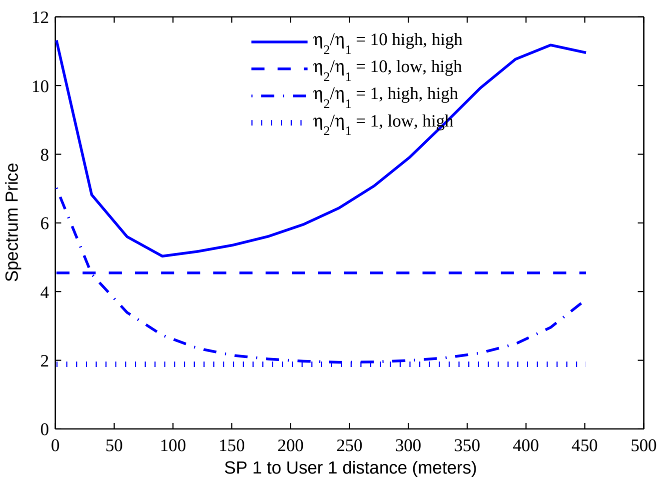

Fig. 4. The spectrum price μ as a function of user 1 distance from SP 2 for different target rates and SP efficiencies. Both users have exponential utilities and user 2 is fixed at 100m from SP 2

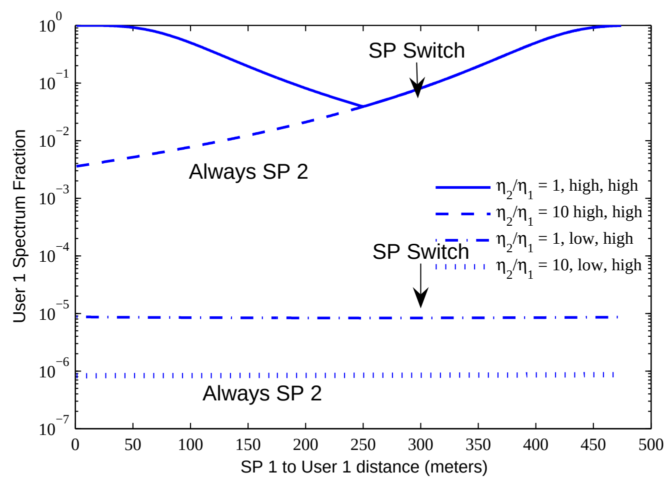

Fig. 6. Fraction of total spectrum allocated to user 1 as a function of distance for different target rates and SP efficiencies. Both users have α utilities and user 2 is fixed at 100m from SP 2

$d_{21} > 400$ m. This is because the exponential utility $U_1(R)$ flattens near the value of Γ_1 at high R. In region $\mathcal{V}$ user 1 has a very high h_1^* (to SP 2) and SP 2 is more efficient. So user 1 achieves a high rate and his utility is near Γ_1. This can be seen in Figure 5. Thus as user 1 gets closer to SP 2, any extra spectrum would increase its rate but not its utility. Another way to interpret this is to look at the prices in Figure 4. For region $\mathcal{V}$ both users are close to the flat regions of utilities and hence demand for additional spectrum is less. Consequently the prices are initially constant and then falls slightly.

Figures 3-5 also show results when user 1 has *low* required rate. Allocation x_1^* is much less as per Theorem 4. However x_1^* is enough to satisfy user 1's utility. Since x_1^* is less, user 1 always attaches to the more efficient SP as per observation 1(b). The prices are almost invariant to changes in d_{21}. This is because user 2 gets majority of the spectrum and thus sets the demand. Since it is stationary the prices change only with SP efficiencies.

The corresponding results when users have α utilities are shown in Figures 6-8. The same trends of exponential utility results are observed but the disparities between the users in terms of spectrum allocated and utilities are much more severe for dissimilar link gains. Comparing Figures 3 and 6 we see that when user 1 has *low* required rate, allocation x_1^* for α utilities is significantly less than x_1^* for the exponential utilities. The user j with a stronger h_j^* has a much larger impact on the prices for α utilities. From Figures 4 and 7 we see that when $h_2^* > h_1^*$, the α prices vary much less with d_{21} than the exponential prices. The unbounded nature of α utilities also mean that there is always demand for spectrum. Accordingly Figures 4 and 7 for the (high,high) case show that α prices in region $\mathcal{V}$ keeps on increasing unlike the exponential prices. Overall exponential utilities yield more equitable spectrum allocation than α utilities.

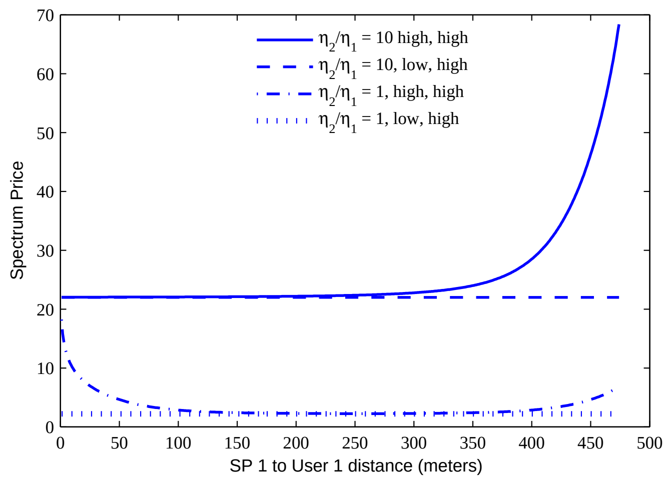

Fig. 7. The spectrum price μ as a function of user 1 distance from SP 2 for different target rates and SP efficiencies. Both users have α utilities and user 2 is fixed at 100m from SP 2

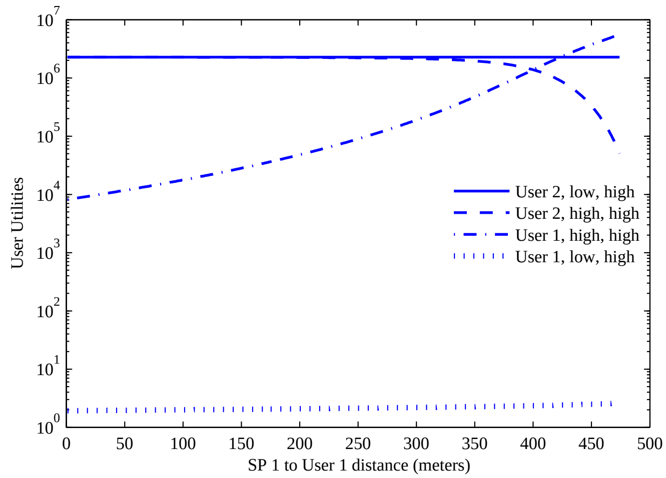

Fig. 8. The utilities for both users as a function of distance for different target rates. Both users have α utilities and user 2 is fixed at 100m from SP 2. The efficiency ratio is $\eta_2/\eta_1 = 10$

VI. Discussions and Conclusion

Dynamic spectrum allocation is important both for centralized broadband access networks and decentralized cognitive radio systems. Efficient networks are often designed for non-strategic behavior either by a central command and control plane or by adherence to a distributed protocol. In this work we have developed and analyzed a two tier allocation system for non-strategic users who obtain spectrum from multiple SPs. We model the system from user welfare maximization framework. We show that in the optimal policy each user obtains spectrum only from one service provider given by a function of the link gains and provider efficiency. Based on our analysis we develop the notion of a spectrum price to facilitate distributed allocation. For two general classes of concave utility functions namely exponential and α, we analytically characterize the spectrum allocation and price. We show that our results are consistent with basic economics principles. Our work provides theoretical bounds on performance limits of practical operator to user based dynamic spectrum allocation systems and also gives insights to actual system design.

We have assumed that system parameters such as h_{ij}, N and L stay constant during the optimization operation and the subsequent transmission. Whenever they change the optimization needs to be re-done. While we have not addressed such timescale issues, it is safe to say that proposed price based allocation is ideal for static outdoor settings with a strong Line-of-Sight component between users and SPs. For more mobile environments the average values of link gains can be used to derive reasonable allocations.

Appendix A
Properties of the Utility Function

Lemma 3: If $U(R)$ is an increasing and concave function in R then $U(\eta R)$ for $R = x\log(1 + hP/x)$ is an increasing and concave function of the vector $\mathbf{v} = [x, P]$

Proof: $U(R)$ is increasing and concave in R. It can be shown that $R(x, P, h)$ is concave in $\mathbf{v} = [x, P]$. The rest follows from [14, Section 3.2.4]. ∎

Theorem 9: If $U_j(R_j)$ is increasing and concave then $U_j\left(\sum_{i=1}^N \eta_i x_{ij} \log(1 + h_{ij}p_{ij}/x_{ij})\right)$ is increasing and concave in the vector $\mathbf{v^{(N)}} = [\mathbf{v_1}, \cdots, \mathbf{v_N}]$ where $\mathbf{v_i} = [x_{ij}, p_{ij}]$.

Proof: Let $r_{ij}(\mathbf{v}_i) = \eta_i x_{ij} \log(1 + h_{ij}p_{ij}/x_{ij})$ and $R_j\left(\mathbf{v^{(N)}}\right) = \sum_{i=1}^N r_{ij}(\mathbf{v}_i)$. Thus $\nabla^2 R_j = \text{diag}[D_1, \cdots, D_N]$ is the hessian. Consider any vector $\mathbf{z} \in \mathbf{R}^{2N}$. $\mathbf{z}^T(\nabla^2 R_j)\mathbf{z} = \sum_{i=1}^N \mathbf{z_i}^T D_i \mathbf{z_i}$ where $\mathbf{z_i} = [z_{2i-1}, z_{2i}]$. Since each D_i is negative definite from Lemma (3), the sum is negative. ∎

Appendix B
Proof of Theorem (3)

Define $\nabla U_j(x_j^*) = \partial U_j/\partial x_j^* = \dot{U}_j(R_j)\Gamma_x(x_j^*, P, h)$ and $M = \max_j \nabla U_j(x_j^* - \epsilon)$ for some $\epsilon > 0$. From Lemma 1, $\nabla U_j(x_j^*)$ is decreasing. The decrease in sum utility is

$$\Delta U_{dec} = \sum_{j=1}^{L} \int_{x_j^*-\epsilon}^{x_j^*} \nabla U_j(x)\, dx \leq \sum_{j=1}^{L} \nabla U_j(x_j^* - \epsilon)\epsilon < ML\epsilon. \tag{34}$$

However the utility of user $L + 1$ is

$$\Delta U_{inc} = \int_0^{L\epsilon} \nabla U_{L+1}(x)dx \geq \nabla U_{L+1}(L\epsilon)L\epsilon. \tag{35}$$

From (34) and (35), we have to show existence of $\epsilon > 0$ such that $\nabla U_{L+1}(L\epsilon) > M$. Now M is increasing in ϵ while $\nabla U_{L+1}(L\epsilon)$ is decreasing in ϵ. As $\epsilon \to 0$, $\nabla U_{L+1}(L\epsilon) \to \infty$ due to Γ_x while $M \to \max_j \nabla U_j(x_j^*)$. Thus at $\epsilon = 0$, the decreasing function is above the increasing function and so they are sure to intersect at some $x = x_s$. So for ϵ satisfying $0 < \epsilon < x_s$ there is a net increase in sum utility by allocating spectrum to user $L + 1$.

Appendix C
Proof of Lemma 2

We have to show that $\dot{U}(R(x, p, h))\Gamma_x(x, p, h)$ is a strictly increasing function of p for fixed x when $U(R) = R^\alpha/\alpha$ for

$0 \leq \alpha \leq 1$. Alternatively substituting $z = hp/x$ we have to show that the following is strictly increasing in z,

$$
\begin{aligned}
&(x \log(1+z))^{\alpha-1}\left[\log(1+z) - \frac{z}{1+z}\right] \\
=& x^{\alpha-1}(\log(1+z))^{\alpha}\left[1 - \frac{z/(1+z)}{\log(1+z)}\right]. \qquad (36)
\end{aligned}
$$

Since $(\log(1+z))^{\alpha}$ is strictly increasing in z, a sufficient condition is to show that $f(z) = (1+z)\log(1+z)/z$ is strictly increasing in z, which is proved by evaluating $\dot{f}(z)$ and using the fact that $z - \log(1+z) > 0$ for all $z > 0$.

Appendix D
User Surplus in Corollary 2

We have to show that $\mathcal{U} = U(R(x,p,h)) - \mu x$ for $\mu = \dot{U}(R(x,p,h))\Gamma_x(x,p,h)$ is increasing in p. A sufficient condition is to show that $\mathcal{U}(x,p,h)$ is increasing in both x and p for fixed h, since Theorem 5 proved that increasing p increases x. Define $R_{xx}(x,p,h) = \partial\Gamma_x/\partial x$. We can show

$$\frac{\partial \mathcal{U}}{\partial x} = -x\left[\dot{U}(R)R_{xx}(x,p,h) + \ddot{U}_j(R)\Gamma_x^2(x,p,h)\right]. \qquad (37)$$

Since $U(R)$ is increasing and concave, $\dot{U}(R) > 0$ and $\ddot{U}(R) < 0$. Since $R(x,p,h)$ is concave in x, $R_{xx}(x,p,h) < 0$. Using all these we can show that $\partial\mathcal{U}/\partial x > 0$. Differentiating

$$
\begin{aligned}
\frac{\partial \mathcal{U}}{\partial p} &= \dot{U}(R)\Gamma_p - x\left[\ddot{U}(R)\Gamma_p\Gamma_x + \dot{U}(R)\frac{\partial \Gamma_x}{\partial p}\right] \\
&= \dot{U}(R)\left[\Gamma_p - x\frac{\partial \Gamma_x}{\partial p}\right] - x\ddot{U}(R)\Gamma_p\Gamma_x.
\end{aligned}
$$

It can be shown that,

$$\Gamma_p - x\frac{\partial \Gamma_x}{\partial p} = \frac{hx^2}{(x+hP)^2} > 0. \qquad (38)$$

With this information we can also show that $\partial\mathcal{U}/\partial p$ is positive.

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