Timing Acquisition for Non Contiguous OFDM based Dynamic Spectrum Access

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***Abstract*— Most current and upcoming communication systems like 802.11x, WiMAX etc. deploy some variant of Orthogonal Frequency Division Multiplexing as their physical layer technology. Symbol timing acquisition is the first operation performed at the receiver after which other signal processing, such as orthogonalizing the received data into parallel streams using Fast Fourier Transform (FFT), can take place. To ensure reliable communication, extensive work has been done in designing robust algorithms that estimate the symbol timing with high accuracy. Most of these works assume that there is some total bandwidth which is utilized by a single user. However in future cognitive radio systems, the spectrum access will be dynamic and multiple devices in a geographical region will sense a common pool of spectrum for the presence of vacant frequency bands to transmit in. In the OFDM context, this means that a device may transmit in non contiguous tones (termed as Non-Contiguous OFDM or NC-OFDM). It is not clear how the existing symbol timing acquisition algorithms will perform in this situation. The current research around cognitive radios is mostly focused on the sensing and resource allocation aspects but to our knowledge the symbol timing acquisition issues have not yet been studied. In this work we study the performance of cyclic prefix correlation based symbol timing acquisition algorithms for NC-OFDM transmission. We first derive the ML estimator when the channel is frequency non-selective and show that it has high computational complexity. Consequently we study the performance of low complexity, sub-optimal approaches both for frequency non-selective and frequency selective channels. Our simulations indicate that in some likely situations such as the users occupying multiple discontiguous sub-bands and having large differences in the timing offsets between their transmitters and receivers, cyclic prefix based timing acquisition algorithms can perform quite poorly. This points to the need for better algorithms of reasonable complexity, or entirely different approaches to symbol timing acquisition, for example based on the periodic transmission of known sequences.**

I. Introduction

Spectrum is an important wireless resource. In recent years, there has been increased research and policy-making to move away from a static *command-and-control* access of spectrum to a more dynamic and flexible access [1], [2], [3]. In the future, we are likely to see devices, possibly belonging to different technologies, share a common pool of spectrum in a given geographic region. More specifically, cellular operators could acquire spectrum to deploy in-home base stations called Femtocells [4] for improving coverage, which provides an opportunity for spectrum sharing across operators. Each operator could potentially use spectrum licensed out to competing operators through a sub-lease arrangement. Thus some sort of coordination and dynamic sharing amongst operators is assumed. However there could be other applications where the spectrum sharing is not coordinated by a common protocol. There could be a primary licensee of spectrum such as TV broadcasters in the 54 to 862 MHz band and secondary systems such as 802.22 based cognitive radio WRANs [5] could operate in the vacant bands of this spectrum to provide broadband access in rural areas.

A variety of technical challenges have to be addressed before such systems become a working reality. These include signal processing techniques for sensing vacant spectrum [6], wideband radio RF capabilities to operate over this spectrum, bonding of this discontiguous spectrum to achieve higher data rates and allocation of this spectrum amongst multiple cognitive radio terminals [7]. One possible physical layer that is well suited for such a discontiguous band transmission is OFDM, where a device transmits only in the tones corresponding to the vacant spectrum. This allows for tighter usage of spectrum compared with traditional FDM and is more efficient. The spectrum corresponding to the other tones could be used by other devices. Such a transmission scheme is called NC-OFDM in [8], [9] and differs from conventional OFDMA systems as the devices are uncoordinated.

For such a transmission scheme, it is important to study the symbol timing acquisition performance at the receiver. This can be explained as follows: assume that a NC-OFDM symbol is $N + L$ samples long with the first L samples, called the cyclic prefix (CP), being the same as the last L samples. Let the transmitted NC-OFDM samples be $s(k)$. The received samples $r(k)$, in presence of timing offset θ between transmitter and receiver, is given by

$$r(k) = \sum_{l=0}^{N_l} h(l)s(k-\theta-l) + s_I(k) + n(k), \tag{1}$$

where $h(l)$ represents a N_l tap frequency selective channel and $s_I(k)$ the signals from other users that interfere at the receiver. Timing acquisition is about estimating the OFDM symbol boundary by estimating θ at the receiver.

For NC-OFDM, the signals $s(k)$ and $s_I(k)$ will occupy non-overlapping set of tones. However, since the practical pulse-shaping filters are not ideally band-limited, part of the symbol energies will spill over to the adjacent bands causing interference. Hence, the performance of acquisition algorithms will improve with wider guard bands between the signals of the different users. For systems where the spectrum is licensed to a primary user and secondary users opportunistically use it, presence of wide guard bands may be assumed to protect the primary users from interference. However if the spectrum is unlicensed and a group of uncoordinated devices attempt to access it, then the spectrum can become tightly packed to maximize its usage leading to loss in timing acquisition performance.

A. Related Work

Acquisition performance is well understood for a single user system, i.e. $s_I(k) = 0$ in (1) and when the transmitter occupies the entire spectrum [10], [11]. Even in the presence of multiple users, single-user algorithms are often used as acquisition is the first step at the receiver and at this stage there are usually no signal processing methods to *distinguish* the signal from the interferer. The single user OFDM acquisition algorithms can be broadly classified as

1) Cyclic Prefix Correlation: The optimal ML estimator of symbol timing for a frequency non-selective channel in derived in [10]. The CP introduces correlations in the OFDM samples and that is used to perform a sliding window correlation between two L length sample blocks, placed N samples apart.

2) Pilot Symbol Correlation: The authors in [11], [12] postulate the transmission of two specially designed OFDM symbols to achieve symbol timing synchronization. The idea is to introduce known correlations in the samples of the OFDM symbol which could be tracked by the receiver.

3) Joint Cyclic Prefix and Pilot Symbol Correlation: Symbol timing recovery by transmitting pilot symbols and extending the CP correlation based approach of [10] is proposed in [13].

4) Blind acquisition Methods: Such methods do not rely on cyclic prefix or pilot symbol correlation. In [14], a method for achieving symbol acquisition is proposed by constructing certain autocorrelation matrices from the received signal and minimizing their rank. This method is shown to perform well even in frequency selective channels.

The presence of multiple users affects the acquisition performance. If the users are coordinated, as in the uplink of an OFDMA system, joint timing acquisition for all users can be performed [15], [16]. For such *cellular based* systems, the uplink performance is also helped by the fact that the users are already synchronized to a common system timing during initialization using the downlink signal. This phenomenon will be discussed in Section IV.

If there is no coordination amongst the interfering users, such as in an ad-hoc network, then the acquisition performance deteriorates. The interfering users could be OFDM transmitters themselves with different delays from the intended user. For example, the term $s_I(t)$ in (1) could be another OFDM sample stream $s'(k - \theta')$ with $\theta' \neq \theta$. The receiver can incorrectly estimate θ' as the timing instead of the correct instant θ. The presence of a narrowband interferer is studied in [17], [18] where one user occupies the entire bandwidth and uses a pilot based acquisition algorithm as in [11]. Distributed timing acquisition amongst different interfering devices can also be realized in the MAC layer if all the devices are assumed to follow a common MAC protocol for example 802.11 in ad-hoc mode [19].

B. Our Contribution

In this work, we consider a different scenario where a cognitive transmitter, employing NC-OFDM, only transmits in tones corresponding to the vacant spectrum and the receiver has to acquire the timing of the delayed signal. In fact depending on what fraction of the spectrum is vacant, the user could be narrowband instead of the interferer. The transmit power of the interfering users could be higher than that of the desired user. The user data could be in discontiguous tones and it is not clear upfront as to how this would affect the acquisition performance. Also the different devices participating in Dynamic Spectrum Access could come from different networks and employing different technologies to transmit over the spectrum and so the case of their following a common MAC protocol is improbable.

As a starting point, we consider the applicibality of single user OFDM acquistion algorithms of Section I-A for NC-OFDM. For pilot based correlation approaches there has to be some initial signaling for the receiver to know the pilot sequences or the receiver and transmitter should follow some pre-decided link level protocol. This can be ruled out for Dynamic Spectrum Access applications. The pilot based scheme in [11] requires a user to transmit in all tones in order to generate a symbol with symmetric samples after the IFFT which is ruled out for NC-OFDM. Thus we focus on CP based correlation, which only assumes that the transmission structure is OFDM based. This can be easily implemented at a receiver. Blind acquisition methods lead to wastage of subcarriers which could have been used for data transmission, besides when the presence of the CP guarantees a correlation in the OFDM samples, it seems natural to exploit it for purposes of synchronization. CP correlation based acquisition algorithms have been implemented in many practical systems and they yield satisfactory performance even in channels for which they are not optimal, for example in frequency selective channels [10]. This further motivates us to study performances of CP based algorithms. Thus in this paper we try to answer the following question,

Do CP based acquisition algorithms by themselves or with realizable enhancements, suffice to yield satisfactory timing acquisition performance for a NC-OFDM transmission?

To answer this question, we first derive the ML estimator for CP correlation based acquisition for NC-OFDM transmission in a frequency non-selective channel and show that it has high computational complexity. Consequently we consider the performance of low complexity, sub-optimal approaches such as using the ML estimator of frequency non-selective OFDM transmission [10] for NC-OFDM and also introducing a band pass filter at the receiver before the acquisition phase to filter out the interference from the other users. As a result of our simulations, we have been able to identify situations in the CP correlation based acquisition algorithms deliver satisfactory results and situations in which they do not.

The rest of the paper is organized as follows: In Section II we study the performance of CP based timing acquisition in a channel impaired with Gaussian noise and also derive the optimal ML algorithm. In Section III we introduce frequency selective fading and in Section IV we also consider the presence of an interfering user for studying the performance of CP based acquisition algorithms. In this work we will not investigate other forms of acquisition such as carrier frequency offset correction or frame synchronization.

II. CP BASED TIMING ACQUISITION FOR FREQUENCY NON-SELECTIVE CHANNELS

We reproduce the main result of [10] for OFDM symbol timing acquisition in channels impaired with Gaussian noise when one user occupies the entire bandwidth, i.e. $h(k) = \delta(k)$ ($N_l = 1$) and $s_I(k) = 0$ in (1). The optimal estimate that minimizes the mean square error is the ML estimate when the Tx signal can be modeled as a white Gaussian sequence. This is a reasonable assumption for full spectral occupancy, since the number of guard tones typically used is small relative to the FFT length. It is shown in [10] that the optimal ML estimate of θ is given by

$$\mathsf{CPCorr}: \quad \hat{\theta}_{\text{ML}} = \arg\max_{\theta} \{\text{Re}(\gamma(\theta)) - \rho\Phi(\theta)\} \tag{2a}$$

$$\gamma(m) = \sum_{k=m}^{m-L+1} r(k) r^*(k+N), \tag{2b}$$

$$\Phi(m) = \frac{1}{2} \sum_{k=m}^{m-L+1} |r(k)|^2 + |r(k+N)|^2, \tag{2c}$$

where $E[s^2(k)] = \sigma_s^2, E[n(k)^2] = \sigma_n^2$ and $\rho = \sigma_s^2/(\sigma_s^2 + \sigma_n^2)$. Define $\mathsf{snr} = \sigma_s^2/\sigma_n^2$. The quantity θ is modeled as deterministic but unknown and thus the mean square error in estimating θ is given by

$$\mathsf{mse}(L, \mathsf{snr}) = E\left(\theta - \hat{\theta}_{\text{ML}}\right)^2, \tag{3}$$

where the expectation is over the statistics of the estimate. We can interpret $\gamma(m)$ as the operator which captures the correlation energy between two L sample blocks separated N samples apart with the first sample of first block taken at time $k = m$. We term this algorithm as CPCorr. Note that for CPCorr to work, the receiver needs to know ρ apriori before the acquisition phase. Though this is not practical, we can assume that the transmit power and the receiver noise characteristics stay constant over the transmission interval and the receiver can obtain a good estimate of ρ based on past history. Also for moderate/high snr regimes, $\rho \sim 1$ irrespective of the actual value of snr.

The following situation could arise in NC-OFDM,

Scenario 1: Consider a system with available bandwidth W Hz and the user of interest transmits in some parts of the entire band, and the remaining parts of the band are not occupied by other users. The channel is frequency non-selective and is impaired only by Gaussian noise. This could correspond to a channel with a strong line of sight component.

A. The Optimal ML Algorithm

For Scenario 1, we derive the optimal ML estimator. Consider that the FFT length is N and the set of tones in which the desired user transmits be $\mathcal{T}$. Let the information symbol vector be $\mathbf{x} = [x(1), \cdots, x(N)]$ such that $x(j) = 0$ if $j \notin \mathcal{T}$. The transmitted symbol vector $\mathbf{s} = [s(1), \cdots, s(N)]$ is generated through IFFT of information vector $\mathbf{x}$,

$$\mathbf{s} = \mathbf{Q}\mathbf{x}, \tag{4}$$

where $\mathbf{Q} = [\mathbf{q}_1, \cdots, \mathbf{q}_N]$ is the IFFT matrix. We show that the transmitted symbols of $\mathbf{s}$, at two different time instants j and k are correlated even if the vector $\mathbf{x}$ has uncorrelated entries. The correlation between OFDM samples j and k is

$$\begin{aligned} E[s(j)s(k)] &= E[\mathbf{q}_j^H \mathbf{x}\mathbf{x}^H \mathbf{q}_k] = \mathbf{q}_j^H E[\mathbf{x}\mathbf{x}^H]\mathbf{q}_k \\ &= \mathbf{q}_j^H \mathbf{W} \mathbf{q}_k = \rho^{jk}, \end{aligned} \tag{5}$$

where $\mathbf{W}$ has 1's in diagonal positions given by $\mathcal{T}$ and zeros elsewhere. Thus the correlation is non-zero. We now use (5) to calculate the correlation matrix of the received signal vector. The receiver collects a $2N + L$ sample block $\mathbf{r}$, as this is sure to contain a single complete $(N + L)$ sample OFDM symbol which starts after θ samples. For a generic user, the received OFDM symbol is given by $\mathbf{r} = \mathbf{s} + \mathbf{n}$ with $\mathbf{s} = [\mathbf{u}|\mathbf{v}|\mathbf{u}]$. Samples $\mathbf{u} = [s(\theta), \cdots, s(\theta + L - 1)]$ are the prefix symbols and $\mathbf{v} = [s(\theta + L), \cdots, s(\theta + N - 1)]$ are the data symbols. We define the following matrices,

$$\mathbf{X} = E[\mathbf{u}\mathbf{u}^H],\ \mathbf{Y} = E[\mathbf{u}\mathbf{v}^H],\ \mathbf{Z} = E[\mathbf{v}\mathbf{v}^H]. \tag{6}$$

After some algebra, it can be shown that the correlation matrix of $\mathbf{r}$, $\tilde{\mathbf{C}} = E[\mathbf{r}\mathbf{r}^H]$ is given by,

$$\tilde{\mathbf{C}} = \begin{pmatrix} \sigma_n^2 \mathbf{I}_L + \mathbf{X} & \mathbf{Y} & \mathbf{X} \\ \mathbf{Y}^H & \sigma_n^2 \mathbf{I}_{N-L} + \mathbf{Z} & \mathbf{Y}^H \\ \mathbf{X} & \mathbf{Y} & \sigma_n^2 \mathbf{I}_L + \mathbf{X} \end{pmatrix}. \tag{7}$$

Let $\mathbf{C}_\theta$ be the actual correlation matrix of the received $2N+L$ sample window. It is given by,

$$\mathbf{C}_\theta = \text{diag}\left[(P + \sigma^2)\mathbf{I}_\theta, \tilde{\mathbf{C}}, (P + \sigma^2)\mathbf{I}_{N-\theta}\right]. \tag{8}$$

The optimal ML estimate of θ is thus given by

$$\arg\min_{\theta} \log\left(|\mathbf{C}_\theta|\right) + \frac{1}{2}\mathbf{r}^H \mathbf{C}_\theta^{-1} \mathbf{r} \tag{9}$$

To compare the performance of the algorithm CPCorr and the ML algorithm, we simulate the MSE performance when the total spectrum is W, the desired user transmits in bands $[0, f_a] \cup [f_b, W]$ with $0 < f_a < f_b < W$ and does not transmit in band $[f_a, f_b]$. We call this situation as *partial spectral occupancy* and the spectral occupancy is $1 - (f_b - f_a)/W$. CPCorr might not perform well in this situation as it assumes IID signal samples, whereas partial bandwidth occupancy

FFT Size, N	256
snr (dB)	4, 10, 16
$[f_a, f_b]$ (MHz) (bands not occupied)	$[0.25W, 0.35W]$ 90% occupancy $[0.15W, 0.6W]$ 55% occupancy

TABLE I

SIMULATION PARAMETERS FOR TESTING THE PERFORMANCE OF CPCORR AND ML ALGORITHM FOR A SINGLE USER WITH PARTIAL SPECTRAL OCCUPANCY IN A FREQUENCY NON-SELECTIVE CHANNEL

causes significant correlations (increasing with the fraction of unoccupied bandwidth). The simulation parameters are shown in Table I. Note that CPCorr metrics could be calculated for each of several OFDM symbols, added and then the sum be used for finding the best delay. In fact the the higher snr values like 16 dB can be regarded as an approximation of what would happen if we accumulated across OFDM symbols as mentioned.

An appropriate metric to study the acquisition performance is normalized mean square error,

$$\mathsf{nmse}(L, \mathsf{snr}) = \frac{\mathsf{mse}(L, \mathsf{snr})}{L^2}, \tag{10}$$

where mse is defined in (3). This is because symbol timing errors up to L do not result in intersymbol interference. Normalizing this way allows us to compare performance at different values of L. Thus for frequency non-selective channels as long as $\mathsf{nmse}(L, \mathsf{snr}) < 1$, the acquisition performance is satisfactory. For frequency selective channels, since the CP also has to provide immunity against the delay spread of the channel, we will consider a lower threshold of $\mathsf{nmse}(L, \mathsf{snr})$ than unity.

Observation 1: The simulation results for 90% and 55% spectral occupancy are shown in Figures 1 and 2. The following observations can be made

1) The ML algorithm yields a lower nmse(L,snr) value than CPCorr for all values of L and snr.
2) The relative loss of performance of CPCorr over the ML algorithm is more for lower spectral occupancies as CPCorr is optimal for full spectral occupancy.
3) With respect to the criterion $\mathsf{nmse}(L, \mathsf{snr}) < 1$, CPCorr performs satisfactorily for moderate/high values of snr.

Conclusion 1: Algorithm CPCorr satisfactorily acquires symbol timing for a single user transmission with partial spectral occupancy in a frequency non-selective channel.

B. A note about Sample Correlation for Partial Spectral Occupancies

The reason behind correlations in the OFDM samples when the user did not occupy all the tones is that the bandwidth of the signal was less than W but it was being oversampled at W. This can be avoided by carefully sampling the analog NC-OFDM signal at the correct rate depending on its bandwidth. However this depends on what is the bandwidth of the NC-OFDM signal which is a dynamic quantity. Also the transmitted signal might be in multiple discontiguous sub-bands

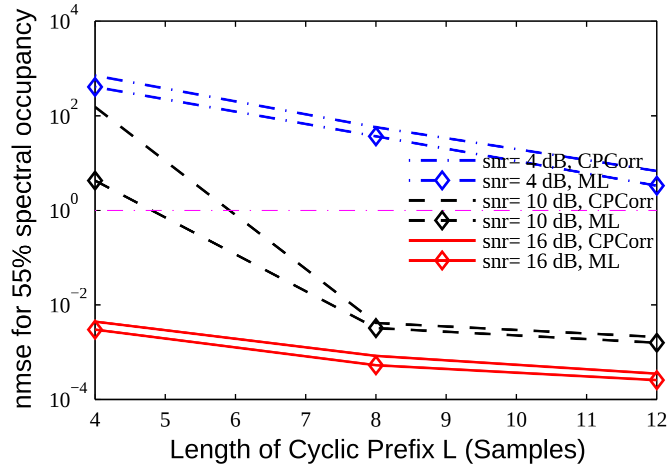

Fig. 1. Performance of CPCorr and ML algorithm for a single user frequency non-selective channel with partial spectral occupancy of 90% of total bandwidth W, with no transmission in $[f_a, f_b] = [0.25W, 0.35W]$ MHz for different values of snr

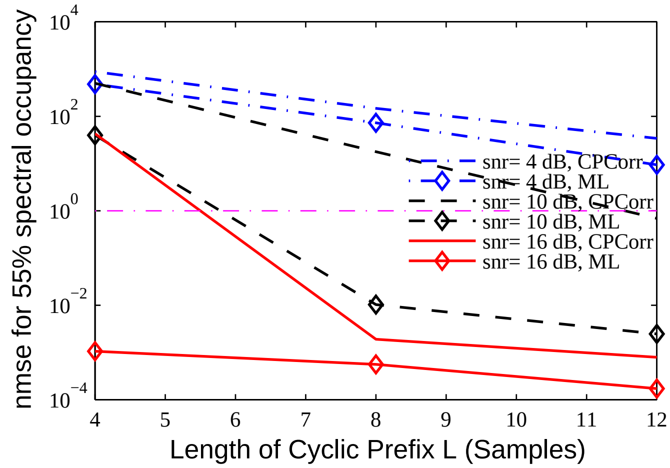

Fig. 2. Performance of CPCorr and ML algorithm for a single user frequency non-selective channel with partial spectral occupancy of 55% of total bandwidth W, with no transmission in $[f_a, f_b] = [0.15W, 0.6W]$ MHz for different values of snr

(instead of one contiguous sub-band as considered in Figures 1 and 2) and in that case, careful sampling over multiple sub-bands is needed. Implementing a fixed sampling rate of W is the simplest working algorithm.

Note that even in standard OFDM, there is partial spectral occupancy (and hence correlations) due to the guard tones at the ends of the band, but these are usually slight, i.e. the oversampling factor is typically small to cause noticeable degradation in acquisition performance.

III. TIMING ACQUISITION FOR FREQUENCY SELECTIVE CHANNELS

Even in the case of regular OFDM, deriving the optimal ML algorithm is difficult because of the lack of channel statistics during the acquisition step. Also as noted in (9), the optimal

Delay (μs)	0	0.31	0.71	1.09	1.73	2.51
Power (dB)	0	-1	-9	-10	-15	-20

TABLE II
THE POWER DELAY PROFILE OF THE VEHICULAR A CHANNEL MODEL

Bandwidth, W (MHz)	5
FFT size, N	512
CP, L	30, 40, 51, 76

TABLE III
OFDM PARAMETERS FOR SIMULATION. THE CP LENGTHS L CORRESPONDS TO $6\%, 8\%, 10\%$ AND 15% OF FFT LENGTH N

ML is computationally intense as the FFT size N grows. Since we want low comlexity acquisition algorithms, we'll focus only on the performance of CPCorr from now on.

Example 1: To illustrate how frequency selectivity affects CPCorr, consider a channel with $N_l = 2$ in (1) and $s_I(k) = 0$. From (1), we substitute for $r(k)$ in the expression for $\gamma(m)$ in (2b) and look at the correct timing instant $m = \theta$ to understand how it is affected in the presence of multiple paths. Define $k' = k - \theta$, so that $s(k'+i) = s(k'+i+N)$ for $i = 0, \cdots, L-1$. The following components are present in $\gamma(\theta)$

$$\begin{aligned}\gamma(\theta) \sim & \underbrace{|h(0)|^2 \sum_{k'=0}^{L-1} s(k')s^*(k'+N)}_{\mathcal{S}_1} \\ & + \underbrace{|h(1)|^2 \sum_{k'=0}^{L-1} s(k'-1)s^*(k'+N-1)}_{\mathcal{S}_2} \\ & + \underbrace{h(0)h^*(1) \sum_{k'=0}^{L-1} s(k')s^*(k'+N-1)}_{\mathcal{I}_1^S} \\ & + \underbrace{h(1)h^*(0) \sum_{k'=0}^{L-1} s(k'-1)s^*(k'+N)}_{\mathcal{I}_2^S}.\end{aligned} \quad (11)$$

The following observations can be made

a) Components $\mathcal{S}_1$ and $\mathcal{S}_2$ capture the correlation in the received signal. In $\mathcal{S}_1$ all L terms in the summation contribute toward the correct correlation but for $\mathcal{S}_2$ the first term is the product of two uncorrelated variables $s(-1)$ and $s(N-1)$. We'll call $s(-1)s^*(N-1)$ as *self interference*. For $N_l > 2$ the subsequent $\mathcal{S}_j$ terms where $2 < j \leq N_l$ have more self interference components in them but since they are weighted by $|h_j|^2$ which is usually decreasing in magnitude, their effect is less significant.
b) In the components $\mathcal{I}_1^S$ and $\mathcal{I}_2^S$, all terms act as self interference as all the products are amongst uncorrelated variables. The quality of the estimate deteriorates as the self interference increases.

Note that this method only gives an indication of the effects of frequency selective fading; for a complete analysis, we would have to investigate how it affects the timing instants other than the true value at $m = \theta$. However for those values, most terms would be products of uncorrelated variables with or without fading and thus looking only at $\gamma(\theta)$ is sufficient for qualitative purposes.

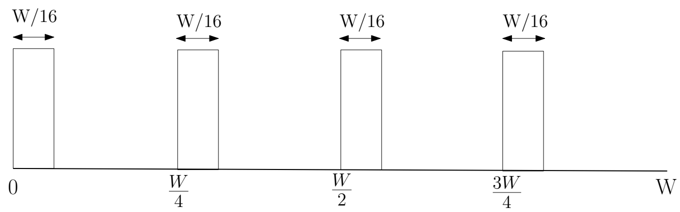

Fig. 3. Four sub-band *discontiguous* spectral occupancy of the first user in the band $[0, W]$

Scenario 2: For simulating a frequency selective channel, we consider the Vehicular A model which is given in Table II. The OFDM parameters are shown in Table III. Note that the length of the CP in samples is less than 15% of the FFT size to minimize the spectrum and power overhead. To test the robustness of CPCorr, we chose an unfavourable (but possible) scenario where the user has a 25% spectral occupancy and the vacant bands are split into four subbands as shown in Figure 3.

Observation 2: Figure 4 shows the performance of CPCorr for a frequency selective channel characterized by a Vehicular A power delay profile. We see that there is a penalty when there is frequency selectivity but by increasing snr to snr = 16 dB, and/or L, the performance can be made satisfactory. A threshold of nmse(L, snr) < 0.7 has been shown in Figure 4.

Conclusion 2: Algorithm CPCorr satisfactorily acquires symbol timing for a single user transmission with partial spectral occupancy in a frequency selective channel.

IV. TIMING ACQUISITION IN PRESENCE OF A SECOND USER

In this section we consider how the presence of a second user affects the acquisition performance. We assume that the second user also transmits OFDM signals with the same symbol duration. One important observation that we will make is that, if the timing delays of both users are similar, then the performance of the timing acquisition is enhanced as the signals from the two users reinforce each other and appear a single high power signal to the receiver. But if the timing delays of the two users are far apart, the receiver of user one might end up acquiring the timing of user two. We formalize this in the following example,

Example 2: Consider a two user system with the timing delays given by θ_1 and θ_2 respectively. Thus the signal of the desired user is $s_1(k-\theta_1)$ and $s_I(k) = s_2(k-\theta_2)$ in (1). Assume without loss of generality that $\theta_1 < \theta_2$ and let

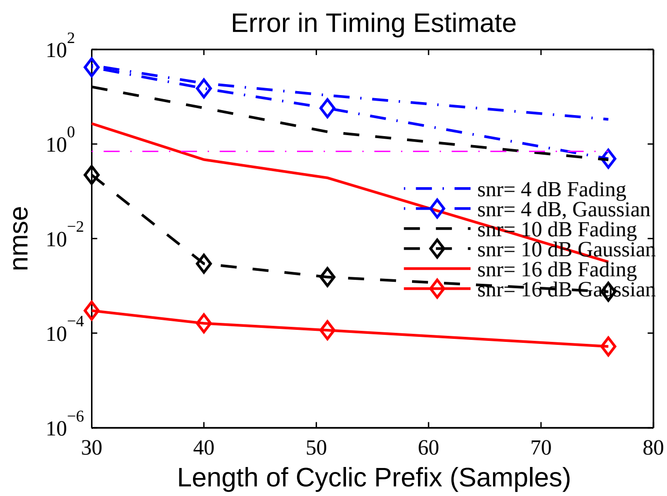

Fig. 4. Performance of CPCorr in a single user frequency selective channel with partial spectral occupancy of 25% as given in Fig 3 for different values of snr

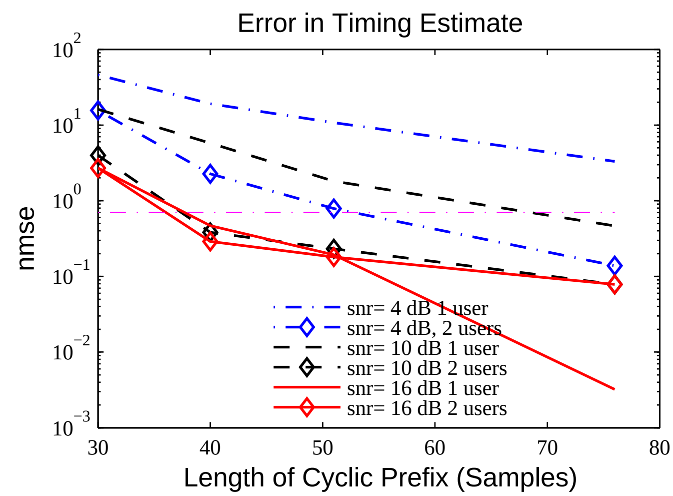

Fig. 5. Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 3 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case a) of Scenario 3 ($\tilde{\theta} < L$) for different values of snr

$\tilde{\theta} = \theta_2 - \theta_1$. For simplification, consider that the channel is impaired only with Gaussian noise. Thus $h(k) = \delta(k)$ ($N_l = 1$) in (1). From (1), we substitute for $r(k)$ in the expression for $\gamma(m)$ in (2b) and look at the correct timing instant $m = \theta_1$ to understand how it is affected in the presence of the second user. Define $k' = k - \theta_1$, such that $s_1(k'+i) = s_1(k'+i+N)$ for $i = 0, \cdots, L-1$. The following components are present in $\gamma(\theta)$

$$\gamma(\theta) \sim \underbrace{\sum_{k'=0}^{L-1} s_1(k')s_1^*(k'+N)}_{\mathcal{S}} + \underbrace{\sum_{k'=0}^{L-1} s_2(k'+\tilde{\theta})s_2^*(k'+\tilde{\theta}+N)}_{\mathcal{I}^M} \quad (12)$$

The following observations can be made,

1) Component $\mathcal{S}$ yield the sum of the correlation energies of the first user.
2) If $\tilde{\theta} < L$, then $\gamma(\theta)$ contains $L - \tilde{\theta}$ terms of the second user's signal that are repeated, i.e. the terms $s_2(k'+\tilde{\theta}) = s_2(k'+\tilde{\theta}+N)$ for $k' = 0, \cdots, L-\tilde{\theta}-1$. Thus these terms add to the correlation energy. However, the last $\tilde{\theta}$ terms of the second user's signal contribute as *multiple access interference (MAI)* as the terms $s_2(k'+\tilde{\theta}) \neq s_2(k'+\tilde{\theta}+N)$ for $k' = L-\tilde{\theta}, \cdots, L$.
3) If $\tilde{\theta} > L$, then the entire correlation energy of the second user constitutes as MAI.

Scenario 3: Consider a frequency selective channel with a Vehicular A power delay profile as given in Table II. Let there be two users with orthogonal spectral occupancies in $[0, W]$. Let the occupancy of the first user be given in Figure 3. Let the transmit power and thus transmit snr and the CP lengths of the two users be same. The OFDM parameters for both users are given in Table III. We will consider the following cases for simulation,

a) $\theta_1 = 25, \theta_2 = 51$ and thus $\tilde{\theta} < L$ for all of L. Note that the probability of $\tilde{\theta} < L$ is roughly L/N.
b) $\theta_1 = 10, \theta_2 = 150$ and thus $\tilde{\theta} > L$ for all L. Note that the probability of $\tilde{\theta} > L$ is roughly $1 - L/N$. Thus this is more probable than event a).

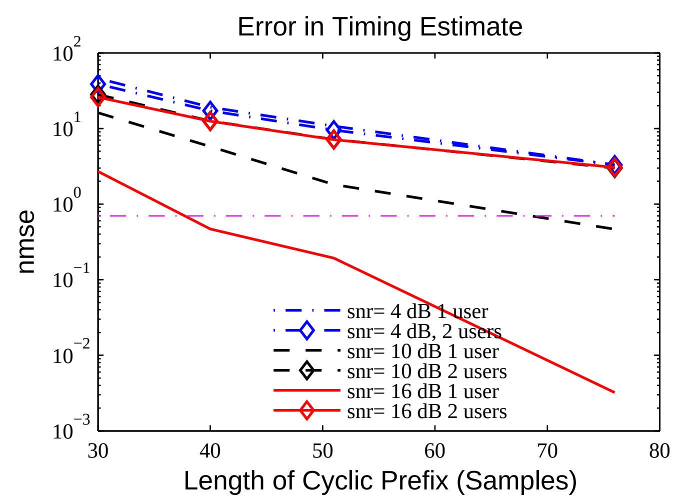

Fig. 6. Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 3 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case b) of Scenario 3 ($\tilde{\theta} > L$) for different values of snr

Observation 3: Figures 5 and 6 show the performance of CPCorr for cases 3a) and 3b) respectively. It is seen that

1) In case 3a), the contribution of useful correlation dominates over MAI and thus the presence of the second user helps. In case 3b), the presence of the second user degrades the performance for all values of L and snr.
2) Since in our model, the second user has the same received signal power as the desired user, accumulating

energy from multiple symbols to increase received snr does not help in enhancing the performance, as the received power of the second user is also increased. This effect is more acute for 3b), as the entire signal of the second user is MAI.

Conclusion 3: In presence of another user, algorithm CPCorr satisfactorily acquires symbol timing for the desired user, with a partial spectral occupancy in a frequency selective channel, only when the differential timing delay between the two users is within the length of the cyclic prefix. If the differential delay is much larger than the cyclic prefix, the performance is not satisfactory irrespective of snr. These assume that both users always transmit at the same power.

A. A Note about Timing Acquisition in Cellular

To put our work in proper context, we examine the timing acquisition in OFDM based cellular systems such as WiMAX where the same scenario of multiple transmissions during the acquisition phase is prevalent. Based on Conclusion 3, our main claim is that algorithms based on CPCorr, will give satisfactory performance for cellular systems. To see this we examine the downlink and uplink separately,

1) Downlink: Here the problem is users synchronizing to the BS. When a mobile is first turned on, it will receive multiple transmissions from interfering BSs and will end up associating with, and acquiring the timing of, the strongest BS. This is different from Scenario 3 where for a given transmitter, the receiver was fixed and there was an equal power interferer (which can arise for ad-hoc networks engaging in DSA). For cellular networks, the mobile to BS association process ensures that signals from interfering BSs have significantly lesser power than the associated BS and thus CPCorr will work.

2) Uplink: Here the problem is the BS synchronizing to the transmissions from different mobiles, who may be simultaneously transmitting. All those mobiles would have already been synchronized to the timing of the BS when they had first turned on. Due to differential path lengths from mobiles to the BS, there might be some difference in the timings of the signals from these mobiles when they reach the BS, but they will be close. Thus this is similar to the case considered in Scenario 3a) and by Conclusion 3, algorithms like CPCorr will yield satisfactory performance.

B. Effect of Filtering

The main reason for Conclusion 3 is that CPCorr performs energy capture from correlations and can't distinguish between the signal of the desired user from that of others. If we assume that the receiver of the first user knows the spectral occupancy of the transmitted signal, one possible way is to filter out the second user's signal before performing CPCorr. In this section, we explore this possibility. Let a band-pass filter $d(k)$ be applied to filter out the second user's signal. The received signals without filtering, $r(k)$ and with filtering, $r_f(k)$ are respectively given by,

Filter	A_{stop}(dB)	A_{pass}(dB)	δ(MHz)	Filter Lengths
A	15	5	0.2	[25, 15, 15, 15]
B	15	5	0.1	[49, 27, 27, 27]
C	20	1	0.1	[61, 61, 61, 61]

TABLE IV

PARAMETERS FOR BAND PASS FILTERS USED IN FIGURE 7

$$\begin{aligned} r(k) &= h_1(k) \otimes s_1(k-\theta_1) + h_2(k) \otimes s_2(k-\theta_2) + z(k) \\ r_f(k) &= \underbrace{d(k) \otimes h_1(k)}_{\text{more correlation}} \otimes s_1(k-\theta_1) \\ &\quad + \underbrace{d(k) \otimes h_2(k) \otimes s_2(k-\theta_2)}_{\text{suppression of } s_2(k)} + z(k). \end{aligned} \tag{13}$$

As seen in (13), filtering suppresses the signal of the second user but makes the desired user's signal pass through the effective channel, $d(k) \otimes h_1(k)$ which has a longer *effective delay spread*. This increases the self interference as noted in Example 1a). Thus roughly speaking, the introduction of a filter introduces a trade-off between suppressing multi-user interference and suppressing self interference. It would be insightful to analytically characterize this trade-off, for given channel parameters and snr. In this work, we however perform extensive simulations that enables us to identify some of the trends in the trade-off and decide if use of the filter makes the subsequent use of CPCorr satisfactory.

Scenario 4: Consider the system mentioned in Scenario 3 and assume that the receiver of user one knows its spectral occupancy. The receiver extracts the signal of the first user using band-pass filters, whose impulse responses are generated by Kaiser windowing technique of FIR filter generation [20, Chapter 10]. This allows to specify the stop-band attenuation, A_{stop} which corresponds to the bands occupied by other users, maximum allowable pass-band ripples, A_{pass} and the transition width between stop band and pass band, δ. The parameters used to generate the three such filters are shown in Table IV and their magnitude responses are given in Figure 7. Filter A has the least stringent specifications for MAI suppression but also the smallest length filters leading to least self-interference, while the opposite is true for Filter C.

Observation 4: Figures 8 and 9 compare the performance of CPCorr with and without filtering for $\tilde{\theta} < L$ and $\tilde{\theta} > L$ respectively. In general, as the length of CP increased, there is a cross-over point beyond which the performance with filtering becomes better than without filtering. Specifically when $\tilde{\theta} < L$, from Figure 8 we see that

a) For high value of L, the effective extra delay spread introduced due to the filter (self-interference) is less than L and hence having a filter is better due to MAI suppression.
b) For low L, the effective extra delay spread introduced due to the filter is significant compared to L and hence hence having a filter leads to worse performance.

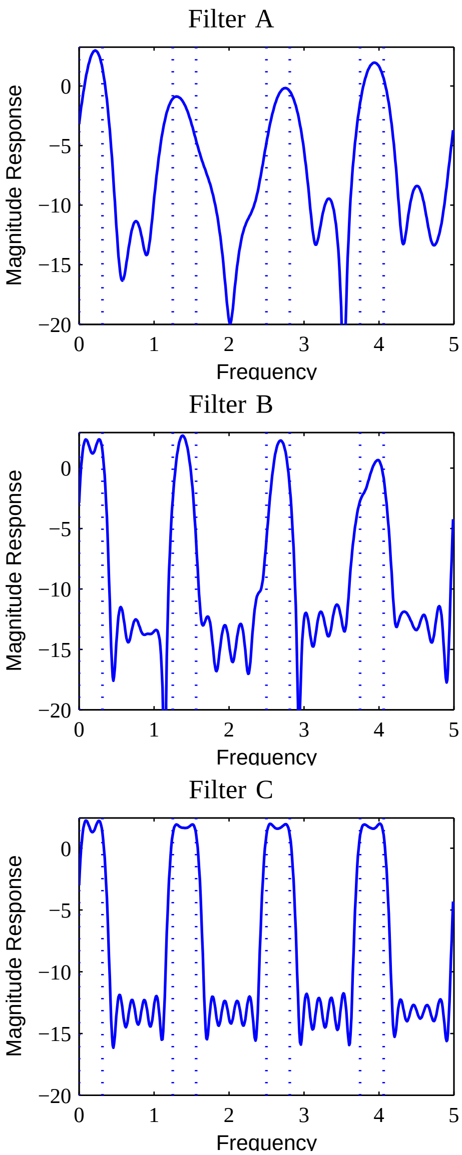

Fig. 7. Magnitude response of band pass filters used to filter out the second user's signal with spectral occupancy of the first user given in Figure 3. The corresponding parameters are given in Table IV

When $\tilde{\theta} > L$, all three filters fail to restore the performance of CPCorr to acceptable levels.

As a note, we conducted simulations with a variety of other channel power delay profiles and filters and the general trends in Observation 4 continue to hold.

C. Single Sun-band Spectral Occupancy

Filtering did not help to improve performance in the four sub-band spectral occupancy case as suppression of narrow bands required longer filters which increased the self-interference. However, if the spectral occupancies were not divided into such narrow bands as in Figure 3, then MAI could be reduced with shorter filters which would cause much less self-interference. Intuitively, the performance of CPCorr should improve.

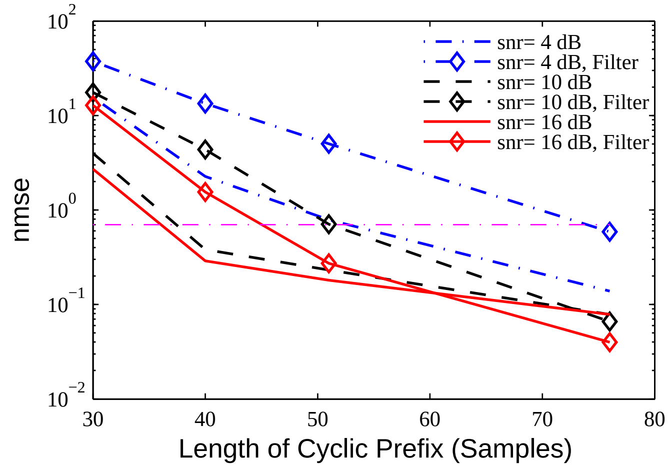

Fig. 8. Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 3 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case a) of Scenario 3 ($\tilde{\theta} < L$) for different values of snr. Band-pass filter B shown in Figure 7 has been used for filtering out the signal of user one prior to CPCorr.

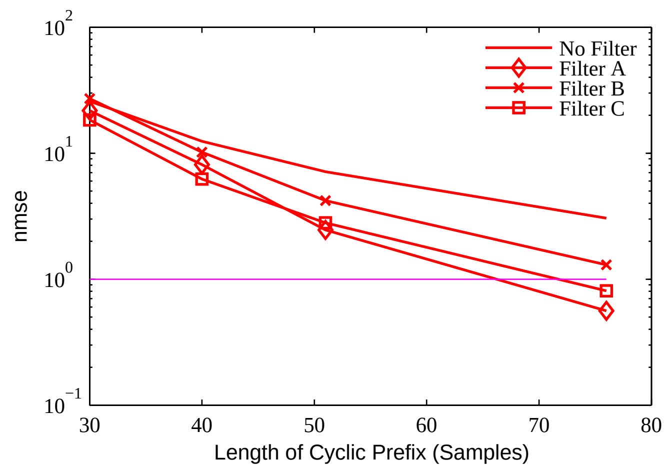

Fig. 9. Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 3 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case b) of Scenario 3 ($\tilde{\theta} > L$). The snr is fixed at 16 dB. All three band-pass filters shown in Figure 7 have been used for filtering out the signal of user one prior to CPCorr.

Scenario 5: Consider a frequency selective channel with a Vehicular A power delay profile as given in Table II. Let there be two users with orthogonal spectral occupancies in $[0, W]$. Let the occupancy of the first user be 25% and the occupancy be in a single contiguous subband as shown in Figure 10. Let $W_0 = 2$ MHz. The rest of the parameters are same as in Scenario 3.

For the uniform occupancy case we'll only consider the adverse situation of $\tilde{\theta} > L$, as our aim is to establish the shortcomings of CPCorr (if any) and for $\tilde{\theta} < L$, CPCorr performed satisfactorily, even for the discontiguous spectral

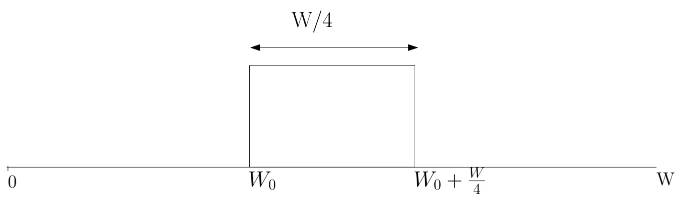

Fig. 10. Single sub-band *uniform* spectral occupancy of the first user in the band $[0, W]$

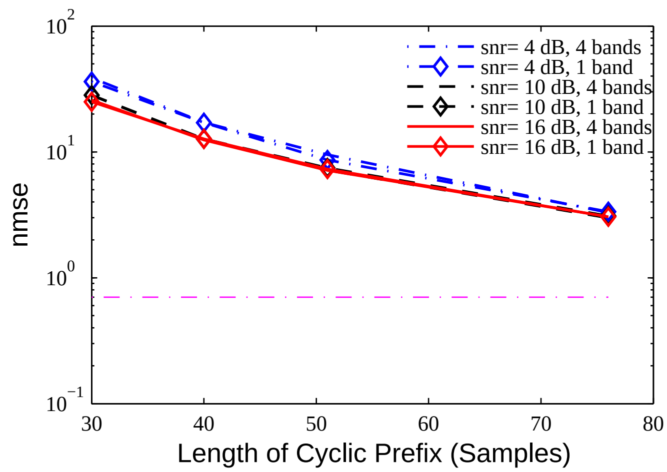

Fig. 11. Performance of CPCorr for a two user system with the first user having a 25% spectral occupancy. The single band vs four sub-band spectral occupancy has been compared.

occupancy case as noted in Conclusion 3. First we compare the performance of CPCorr, in absence of filtering for the four sub-bands vs the single sub-band spectral occupancy. This is shown in Figure 11. The performances are almost same which means that the exact nature of the spectral occupancy is not important for CPCorr. Finally we consider the effects of filtering for uniform spectral occupancy. The filters used have parameters as shown in Table V and their magnitude responses are plotted in Figure 12. The performance of CPCorr, when $\tilde{\theta} > L$ is shown in Figure 13.

Observation 5: The following observations can be made,

a) Filtering satisfactorily restores the performance of algorithm CPCorr for all filters. Performance of filter B is almost similar to that of filter A and is not shown in Figure 12 for purposes of clarity.
b) Since all filter lengths are less than the length of the prefix, choosing the longest filter (Filter C) gives best performance due to MAI suppression.

Conclusion 4: From Observations 4 and 5, we conclude that for a two user transmission when the differential delay is much larger than the length of the cyclic prefix, then for a given fraction of occupied bandwidth, filtering becomes less helpful as that occupied bandwidth is split between more and more contiguous parts.

The reason behind Conclusion 4 is that to suppress the interferer's signal, a smaller length filter is required in the case of contiguous spectrum occupancy. This also implies that, even in the case of discontiguous occupancy, if there are guard bands in between different users, then the requirements on filter are relaxed leading to smaller length filters and the acquisition performance can be improved. Presence of guard bands can be assumed if the spectrum is licensed to a primary user and secondary users opportunistically access it but must leave guard bands to minimize interference to the primary user.

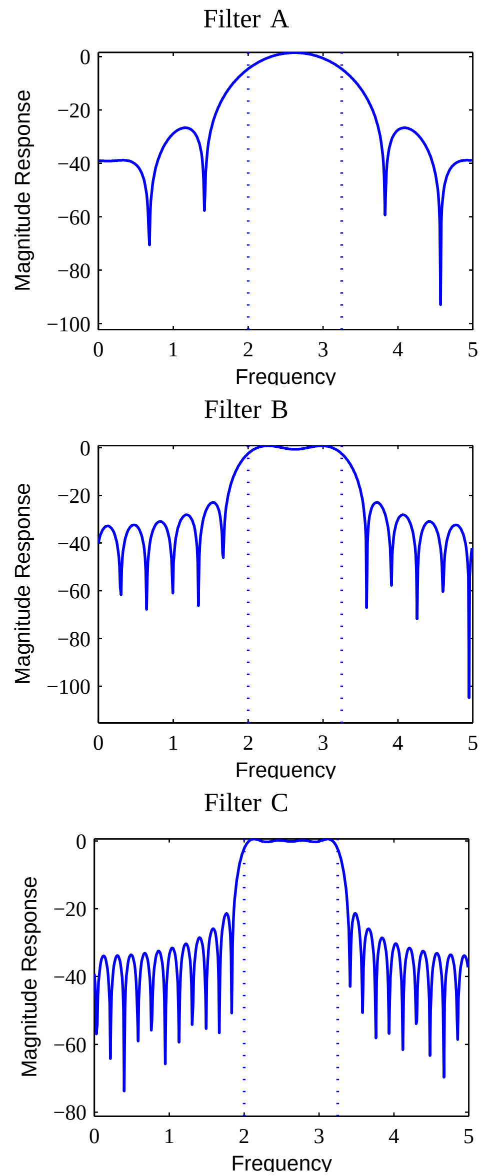

Fig. 12. Magnitude response of band pass filters used to filter out the first user's signal with spectral occupancy as given in Figure 10. The corresponding parameters are given in Table V

V. Conclusion

In this work we considered a scenario where co-located cognitive systems would dynamically share a given spectrum by transmitting in non-overlapping and possibly non-

Filter	A_{stop}(dB)	A_{pass}(dB)	δ(MHz)	Filter Lengths
A	10	5	0.2	7
B	15	5	0.2	15
C	15	5	0.1	27

TABLE V

PARAMETERS FOR BAND PASS FILTERS USED IN FIGURE 12

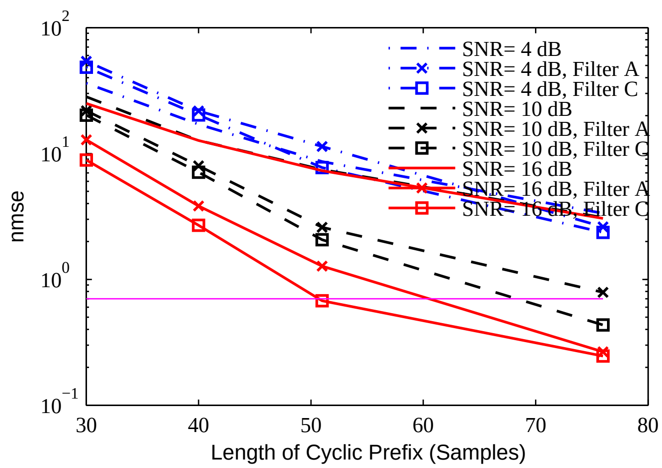

Fig. 13. Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 10 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case b) of Scenario 3 ($\tilde{\theta} > L$). Performances of two band-pass filters shown in Figure 12 have been used for filtering out the signal of user one prior to CPCorr.

contiguous bands and studied the performance of a practically implementable, cyclic prefix correlation based algorithm for OFDM symbol timing acquisition. Since the algorithm is optimal only for a single user transmission in a frequency non-selective channel, we developed a concept of self and multiple access interference to derive important insights into the working of the algorithm for multi-user transmissions for general frequency selective channels. We found that for a two user system, when the differential delays of the two users are much larger than the cyclic prefix, the performance of the algorithm, even with filtering, deteriorates as the occupied bandwidth is split into several pieces, and in some realistic cases becomes quite poor. Assuming that the receiver of the desired user is aware of the bands in which the transmitted signal lies, we showed that it could filter its intended signal and partially restore the performance. The single-user results point to the limits on acquisition performance imposed by occupying a small fraction of the band (e.g., cognitive radio over 100 MHz, with each user occupying only a few MHz), esp. when the occupied bandwidth is split into multiple pieces.

We feel that this work provides a systematic study of non-contiguous OFDM timing acquisition and has important implications in the design of future dynamic spectrum access based systems. There is ample scope for future work, notably in determining acquisition algorithms when the user transmits in multiple narrow sub-bands and also in investigating practical methods by which the receiver may infer the spectral occupancy of its transmitter during the acquisition phase.

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