A price based dynamic spectrum allocation scheme

Joydeep Acharya, Roy D. Yates
Wireless Information Network Laboratory (WINLAB)
Rutgers University, New Jersey

***Abstract*— We develop a system model where multiple service providers (SPs), in the same geographic region, share a fixed spectrum, on a non-interference basis. This spectrum is allocated to end-users for transmission to the SPs. We assume that a user can obtain service from all the SPs. The quality of service depends on system parameters such as number of users and SPs, the channel conditions between the users and SPs and the total transmit power of each user. We adopt a user utility maximization framework to analyze this system. We introduce a spectrum price that enables a simple distributed spectrum allocation with minimal coordination among the SPs and users. Given the user utility functions and the system parameters, we characterize the spectrum price and the optimal bandwidth that the users should obtain.**

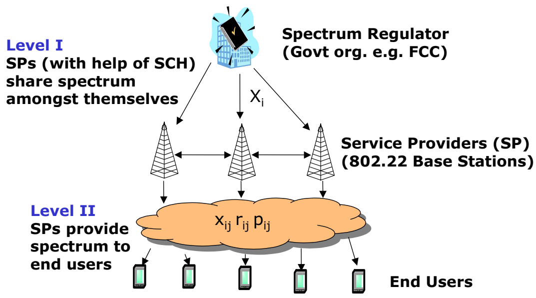

Fig. 1. The network topology

I. Introduction

In the future, we are likely to experience multiple wireless devices belonging to different technologies in the same geographic region. Spectrum allocation among different transmit-receive device pairs is thus important for ensuring fairness and efficiency for end-to-end applications. The traditional regulatory process for spectrum allocation has been largely non responsive to the application requirements. This has motivated the development of dynamic spectrum allocation (DSA) techniques that take into account the application requirements, presence of other devices in the region and link gains between the transmit-receive pairs.

In 2004, the IEEE set up a working group to develop the 802.22 cognitive radio standard to employ the unused spectrum in the VHF and UHF TV bands to offer wireless broadband services [1]. It has been decided that fixed wireless access will be provided in these bands by professionally installed Wireless Regional Area Network (WRAN) base stations to WRAN user terminals. A service provider (SP) operating a base station will sense unused spectrum and allocate it to users dynamically depending on their applications.

Motivated by the SP-user model of 802.22, in this paper we propose and analyze a dynamic spectrum allocation algorithm based on a *spectrum price* and *limited coordination among devices*. We consider a two tiered spectrum allocation scheme as shown in Figure 1. There is some total spectrum C Hz available in a geographic area which is allocated to the users through the SPs. The users are permitted to obtain spectrum from all the SPs. We assume that the users obtain non-overlapping chunks of spectrum from the SPs to avoid interference. Assuming that each user application has an associated utility which is concave and increasing as a function of spectrum obtained, we adopt an utility maximization framework [2] to analyze the system. Given user utility functions, channel coefficients between users and SPs and user power constraints, our aim is to derive how much spectrum should a user obtain from a SP and what power should be allocated for sending information to the SPs. We allow for simple SP coordination to share the spectrum C where the spectrum utilized by a SP depends on how much spectrum it has to allocate to the users. This is facilitated by a spectrum clearing house (SCH), akin to an FCC-controlled regional spectrum broker [3]. Based on our analysis, we also propose a simple distributed allocation algorithm based on the notion of *spectrum price*.

II. Related Work

Our work lies in the domain of systems with non-strategic users who follow a common spectrum allocation protocol without greedily trying to maximize their objectives. Such systems could be centralized or distributed. Upcoming OFDMA based cellular systems like WiMAX, 3GPP-LTE and 3GPP-2 UMB fall in the centralized category. User to subcarrier assignment and power allocation for OFDMA systems have been considered for the downlink [4], [5] and for uplink [6], [7]. These systems mostly consider one SP with fixed frequency bins (OFDM tones) wheras in our work we allow for multiple SPs and treat spectrum as a continuous resource. Treating spectrum as continuous is justified for systems where the subcarrier spacing is small and the number of subcarriers is large. An example system is LTE which can operate with 15 KHz spacing and 2048 subcarriers [8]. In addition we also allow the different SPs to have different efficiencies which is defined as the fraction of Shannon capacity that the SP can reliably deliver.

In the distributed regime each user follows a distributed spectrum sharing algorithm like CSMA/CA in 802.11a/b/g systems. Reference [3] introduce the notion of Coordinated Access Bands of spectrum to achieve limited coordination for spectrum sharing in a distributed way. Spectrum etiquette protocols that act as simple overlays over interfering devices such as bluetooth and 802.11a/b/g devices have been studied in [9]. While [3], [9] have mostly focused on the network architecture and protocol signaling, this paper studies the spectrum allocation problem from an analytical standpoint.

III. System Model

The network topology is shown in Figure 1. There are N SPs and L end users and a central Spectrum Clearing House (SCH). Service Provider i provides X_i units to the users. Let x_{ij} be the amount of spectrum obtained by user j from SP i. Subsequently, user j transmits his data to SP i over this spectrum at rate r_{ij} and with power p_{ij}. Each user has a total transmit power constraint. The channel between SP i and user j is characterized by the flat fading link gain coefficient h_{ij}. The coefficients h_{ij} are assumed to be known at both the users and the SPs. We also assume that the background additive Gaussian noise is of unit power spectral density. A source transmitting with power p, over a flat channel of bandwidth x and link gain h has signal to noise ration $\mathsf{snr}(x,p,h) = hp/x$ and achieves the rate

$$r(x,p,h) = x\log\left(1+\mathsf{snr}(x,p,h)\right). \tag{1}$$

In terms of $r(x,p,h)$ the rate r_{ij} is given by

$$r_{ij} = \eta_i r(x_{ij}, p_{ij}, h_{ij}), \tag{2}$$

where η_i which is the fraction of the Shannon capacity that can be reliably guaranteed by SP i to a user. A possible example would be SP i, who has invested in a better decoder (a Turbo decoder with more iterations or better interleaver design) that has a higher η_i than an SP with a conventional Viterbi decoder. Thus the total rate at which user j can transmit reliably is

$$R_j = R(\mathbf{x}_j, \mathbf{p}_j, \mathbf{h}_j) = \sum_{i=1}^{N} r_{ij}, \tag{3}$$

where $\mathbf{x}_j = [x_{1j}\cdots x_{Nj}]$, $\mathbf{h}_j = [h_{1j}\cdots h_{Nj}]$ and $\mathbf{p}_j = [p_{1j}\cdots p_{Nj}]$. We assume that there is a utility function $U_j(R_j)$ associated with user j which is concave and increasing in R_j. We assume that the operating principle of the network (consisting of the SCH, SPs and users) is to maximize social welfare of the users which corresponds to maximizing their sum utility. The optimization problem is

$$\max_{x_{ij}\geq 0, p_{ij}\geq 0, X_i \geq 0} \sum_{j=1}^{L} U_j\left(R_j\right) \tag{4a}$$

$$\text{s.t. } \sum_{j=1}^{L} x_{ij} \leq X_i,\ 1\leq i\leq N, \tag{4b}$$

$$\sum_{i=1}^{N} p_{ij} \leq P_j,\ 1\leq j\leq L, \tag{4c}$$

$$\sum_{i=1}^{N} X_i \leq C. \tag{4d}$$

As shown in (4b), X_i is the spectrum utilized by SP i which is equal to the spectrum it has to allocate to the users. User j transmits with power p_{ij} to SP i and as (4c) shows there is a constraint P_j on the total transmit power. The total amount of available spectrum is C. User j optimizes over x_{ij} and p_{ij} and the SP i optimizes over X_i. It is shown in [10] that this is a convex optimization problem and thus can be solved efficiently.

A. Distributed Solution and Pricing

Form the partial Lagrangian $\mathcal{L}$ [11] by relaxing the constraints (4b) and (4d) in the objective function to obtain

$$\mathcal{L}(x_{ij}, p_{ij}, X_i, \boldsymbol{\lambda}, \mu) = \sum_{j=1}^{L} U_j\left(R_j\right) + \sum_{i=1}^{N}\lambda_i\left(X_i - \sum_{j=1}^{L} x_{ij}\right) + \mu\left(C - \sum_{i=1}^{N} X_i\right) \tag{5}$$

where $\boldsymbol{\lambda} = [\lambda_1,\cdots,\lambda_N]^T$. The stationarity conditions w.r.t. X_i can be expressed as

$$\frac{\partial\mathcal{L}}{\partial X_i} = \lambda_i - \mu \begin{cases} = 0 & \text{if } X_i > 0. \\ < 0 & \text{if } X_i = 0. \end{cases} \tag{6}$$

Thus each SP i charges the same price $\lambda_i = \mu$ to the users. This is because the SPs have no objectives of their own, for example profit, to maximize and their role is to provide the users with a set of channel coefficients to choose from while transmitting. For $\lambda_i = \mu$, (5) and its dual becomes

$$\mathcal{L}(x_{ij}, p_{ij}, \mu) = \sum_{j=1}^{L} U_j\left(R_j\right) - \mu\sum_{i=1}^{N}\sum_{j=1}^{L} x_{ij} + \mu C. \tag{7}$$

$$D(\mu) = \max_{x_{ij}\geq 0, p_{ij}\geq 0} \mathcal{L}(x_{ij}, p_{ij}, \mu)$$
$$\text{s.t. } \sum_{i=1}^{N} p_{ij} \leq P_j,\ 1\leq j\leq L. \tag{8}$$

The spectrum price μ is set by the SPs by minimizing the dual

$$\min_{\mu > 0} D(\mu). \tag{9}$$

From $D(\mu)$ the optimization in (8) decomposes into separate optimization problems for the users [11]. The optimization subproblem for user j is

$$\max_{x_{ij}\geq 0, p_{ij}\geq 0} U_j\left(R_j\right) - \mu\sum_{i=1}^{N} x_{ij} \tag{10a}$$

$$\text{s.t. } \sum_{i=1}^{N} p_{ij} \leq P_j. \tag{10b}$$

Thus each user maximizes his utility minus the payment for spectrum. The prices are set by the distributed price update equation for (9)

$$\mu(t+1) = \left[\mu(t) - \alpha(t)\left(C - \sum_{i=1}^{N}\sum_{j=1}^{L} x_{ij}(\mu(t))\right)\right]^{+} \tag{11}$$

where $x_{ij}(\mu)$ is the spectrum obtained by user j from SP i, for a given value of μ and $\alpha(t)$ is a positive step size. From (11) we see that if the spectrum is underutilized,

Distributed Spectrum Allocation Mechanism
1) At time t, SPs broadcast price $\mu(t)$.
2) User j solves (10) for $x_{ij}(\mu(t))$ and $p_{ij}(\mu(t))$.
3) All users pass $x_{ij}(\mu(t))$ to each SP i.
4) The SPs calculate $\mu(t+1)$ from (11).

TABLE I

DISTRIBUTED UPDATE ALGORITHM

$C - \sum_{i=1}^{N}\sum_{j=1}^{L} x_{ij}(\mu(t))$ is positive and thus the price decreases to facilitate greater utilization of spectrum. Similarly if spectrum is over utilized, the price increases.

The distributed allocation algorithm as shown in Table I

IV. CHARACTERIZING THE SPECTRUM ALLOCATION

We introduce notations that will be used in the rest of the paper. The first and second derivatives of the utility function will be denoted by

$$\dot{U}_j(R_j) \triangleq \frac{\partial U_j}{\partial R_j}, \qquad \ddot{U}_j(R_j) \triangleq \frac{\partial^2 U_j}{\partial R_j^2}. \tag{12}$$

The derivatives of the rate function $r(x,p,h)$, defined in (1), are

$$\Gamma_p(x,p,h) \triangleq \frac{\partial r}{\partial p} = \frac{hx}{x+hp} = \frac{h}{1+\mathsf{snr}(x,p,h)} \tag{13a}$$

$$\Gamma_x(x,p,h) \triangleq \frac{\partial r}{\partial x} = \log\left(1+\frac{hp}{x}\right) - \frac{hp}{x+hp} = \log\left(1+\mathsf{snr}(x,p,h)\right) - \frac{\mathsf{snr}(x,p,h)}{1+\mathsf{snr}(x,p,h)}. \tag{13b}$$

It follows from (2) and (3) that the derivatives of R_j wrt x_{ij} and p_{ij} can be expressed as,

$$\frac{\partial R_j}{\partial p_{ij}} = \eta_i \Gamma_p(x_{ij}, p_{ij}, h_{ij}), \tag{14a}$$

$$\frac{\partial R_j}{\partial x_{ij}} = \eta_i \Gamma_x(x_{ij}, p_{ij}, h_{ij}). \tag{14b}$$

To arrive at the optimal solution for the user subproblem (10), we first write its Lagrangian,

$$\mathcal{L}_j = U_j\left(R_j\right) - \sum_{i=1}^{N} \mu x_{ij} + \gamma_j\left(P_j - \sum_{i=1}^{N} p_{ij}\right), \tag{15}$$

where all Lagrange multipliers are positive. The stationarity conditions for the Lagrangian are,

$$\frac{\partial \mathcal{L}_j}{\partial x_{ij}} = \eta_i \dot{U}_j(R_j)\Gamma_x(x_{ij}, h_{ij}p_{ij}) \le \mu, \tag{16a}$$

$$\frac{\partial \mathcal{L}_j}{\partial p_{ij}} = \eta_i \dot{U}_j(R_j)\Gamma_p(x_{ij}, h_{ij}p_{ij}) \le \gamma_j, \tag{16b}$$

with equality holding for users with $x_{ij} > 0$ and $p_{ij} > 0$ respectively.

We now state and derive some properties of the resulting allocation,

Lemma 1: In the optimal solution of (10) only one SP is active per user almost surely.

Proof: Consider user j and SP i and assume $x_{ij} > 0$ and $p_{ij} > 0$. Thus (16a) and (16b) are satisfied with equality. Dividing (16a) by (16b) and after some manipulation we obtain,

$$\left(1+\frac{h_{ij}p_{ij}}{x_{ij}}\right)\log\left(1+\frac{h_{ij}p_{ij}}{x_{ij}}\right) - \frac{h_{ij}p_{ij}}{x_{ij}} = \kappa_j h_{ij}, \tag{17}$$

where $\kappa_j = \mu/\gamma_j$. Now consider the function

$$\Psi(\mathsf{snr}) = (1+\mathsf{snr})\log(1+\mathsf{snr}) - \mathsf{snr}. \tag{18}$$

It can be shown that this function is one-to-one and increasing in snr. Substituting for $\mathsf{snr} = h_{ij}p_{ij}/x_{ij} = \Psi^{-1}(\kappa_j h_{ij})$ in (13a) and then substituting for $\Gamma_p(\cdot)$ in (16b) we obtain

$$\eta_i \dot{U}_j(R_j)\left[\frac{h_{ij}}{1+\Psi^{-1}(\kappa_j h_{ij})}\right] = \gamma_j. \tag{19}$$

We will now prove the rest by contradiction. Let user j obtain spectrum from two SPs i and i'. Because $\dot{U}_j(R_j)$ does not vary across i for a given user j it follows from (19) that

$$\frac{\eta_i h_{ij}}{1+\Psi^{-1}(\kappa_j h_{ij})} = \frac{\eta_{i'} h_{i'j}}{1+\Psi^{-1}(\kappa_j h_{i'j})}. \tag{20}$$

Now since h_{ij} is a continuous random variable the probability of event (20) is zero. Thus each user obtains spectrum from one SP *almost surely*. ■

Various flavors of Lemma 1 are also observed in [6], [12].

Let the SP to which user j transmits to be denoted by i_j^*. To simplify notation we denote $x_{i_j^*j}$, $h_{i_j^*j}$ and $\eta_{i_j^*}$ by x_j^*, h_j^* and η_j^* respectively. Thus the user optimization subproblem in (10) can be re-written by considering only $i = i_j^*$. The rate R_j as given in (3) has contribution only from $r_{i_j^*j}$ and we denote it by R_j^*. The user optimization is given by

$$\mathcal{U}_j(x_j^*) = \max_{x_j^* \ge 0} U_j\left(R_j^*\right) - \mu x_j^* \tag{21a}$$

$$R_j^* = \eta_j^* x_j^* \log\left(1+\frac{h_j^* P_j}{x_j^*}\right) \tag{21b}$$

We call $\mathcal{U}_j(x_j^*)$ the *user surplus* in conformance to the microeconomics literature [13, Chapter 14].

A. A Two SP case

It is instructive to look at the SP association problem for two SPs. Assume WLOG that SP 1 is the more efficient SP. Consider user j and let x_j^* be the allocated spectrum from the active SP. We will find out the conditions for which SP 1 is the active SP. Since user 1 chooses SP 1 over SP 2, he obtains a higher utility when associated with SP 1 assuming that the spectrum is x_j^* in both cases. Since the utility $U_j(R_j)$ is an increasing function of R_j this implies that user 1 obtains a higher rate from SP 1 for allocated spectrum x_j^*, i.e.

$$\eta_1 x_j^* \log\left(1+\frac{h_{1j}P}{x_j^*}\right) > \eta_2 x_j^* \log\left(1+\frac{h_{2j}P}{x_j^*}\right) \tag{22a}$$

$$\left(1+\frac{h_{1j}P}{x_j^*}\right)^{\eta_1} > \left(1+\frac{h_{2j}P}{x_j^*}\right)^{\eta_2}. \tag{22b}$$

Observation 1: The following observations can be made based on relative values of x_j^* and the signal to noise ratio, $\mathsf{snr}_{ij} = h_{ij}P/x_j^*$.

a) For a low snr_{ij} regime (high x_j^* and/or low $h_{ij}P$ value), we can use the approximation $(1+x)^n \simeq 1+nx$ in (22b) to obtain $\eta_1 h_{1j} > \eta_2 h_{2j}$.

b) For a high SNR and moderate spectrum range (high $h_{ij}P$ value), we can use the approximation $(1+x)^n \simeq x^n$ in (22b). We define $\eta = \eta_1/\eta_2 > 1$ as SP 1 is more efficient and obtain after some manipulations

$$x_j^* < P\left(\frac{h_{1j}^{\eta}}{h_{2j}}\right)^{1/(\eta-1)}. \tag{23}$$

Thus it is interesting to see that the optimal value of x_j^* is *less than a threshold* when the user attaches itself to the *more efficient SP*.

Lemma 2: If all SPs have the same efficiency, i.e. if $\eta_i = \eta$ for all i, then each user obtains spectrum from the SP to which it has the highest link gain.

Proof: This can be seen by substituting $\eta_1 = \eta_2$ in (22b). ■

Lemma 3: In the optimal allocation each user j obtains spectrum $x_j^* > 0$.

Proof: The proof is by induction. Consider that there are L users in the system all of whom have been allocated non zero spectrum. This is true for $L = 1$. We now show that if user $L+1$ joins the spectrum, he is allocated non zero spectrum in the new allocation. We indirectly prove this, by showing that a new allocation in which each of the original L users forfeit spectrum x_ϵ and user $L+1$ obtains spectrum Lx_ϵ provides higher sum utility for some small x_ϵ. Though this allocation need not be optimal, it implies that in the optimal allocation all users are allocated spectrum.

For proving that sum utility increases, we have to show that there exists choices of x_ϵ for which the utility of user $L+1$ due to this allocated spectrum is greater than the decrease in sum utility of all the L users due to reduction in allocated spectrum. We first define the quantities, $\nabla U_j = \partial U_j/\partial x_j^* = \dot{U}_j(R_j)\Gamma_x$ and $M = \max_j \nabla U_j(x_j^*)$.

From concavity the decrease in sum utility is,

$$\Delta U_{dec} \le \nabla U_1(x_1^*)x_\epsilon + \cdots + \nabla U_L(x_L^*)x_\epsilon < MLx_\epsilon, \tag{24}$$

However the utility of user $L+1$ is,

$$\Delta U_{inc} = \int_0^{Lx_\epsilon} \nabla U_{L+1}(x)dx \ge \nabla U_{L+1}(Lx_\epsilon)Lx_\epsilon \tag{25}$$

So for $x_\epsilon \neq 0$ we have to prove that we can choose x_ϵ such that $\nabla U_{L+1}(Lx_\epsilon) > \max_j \nabla U_j(x_j^*)$. This is true as the function in LHS is a decreasing convex function of x_ϵ and that in RHS is an increasing convex function of x_ϵ. As $x_\epsilon \to 0$ the value of RHS is bounded by $\max_j \nabla U_j(x_j^*)$ while that of LHS tends to infinity due to the Γ_x term. Thus at origin the decreasing convex function is above the increasing one and thus they are sure to intersect. If they intersect at $x_\epsilon = x_s$ then for $0 < x_\epsilon < x_s$ there is a net increase in sum utility by allocating spectrum to user $L+1$. ■

Intuitively this makes sense as if user $L+1$ is not allocated spectrum then the potential increase to the sum utility due to his transmit power is wasted.

Lemma 4: The following facts hold,

a) $\dot{U}_j(R_j)$ for $R_j = R_j(x_j, P, h)$ is a decreasing function of x_j.

b) $\Gamma_x(x, P, h)$ is a strictly decreasing function of x and is positive for all values of $\mathbf{v} = [x, P]$.

Proof:

a) Since $U_j(R_j)$ is concave, $\dot{U}_j(R_j)$ is decreasing in R_j and R_j increases in x_j^*.

b) The function $r(x, P, h)$ is concave and increasing in $\mathbf{v} = [x, P]$ and thus it is concave and increasing in x for fixed P and h. From concavity of $r(x, P, h)$ wrt x, $\Gamma_x(x, P, h)$ is monotonic decreasing in x and from increasing property of $R(x, P, h)$, we conclude that $\Gamma_x(x, P, h)$ is positive.

■

Lemma 5: When two users have the same channel gains, transmit powers and active SP efficiences, the optimal allocation of spectrum favors the user with a higher *marginal utility of spectrum* i.e. whose utility function has a higher rate of increase with spectrum.

Proof: Consider users j and k with utility functions satisfying $\dot{U}_j(R) > \dot{U}_k(R)$ for all R and for whom $h_k^* = h_j^* = h$, $P_k = P_j = P$ and $\eta_k^* = \eta_j^* = \eta$. Let the allocated spectrum for users j and k be x_j^* and x_k^* respectively. We show by contradiction that $x_j^* > x_k^*$.

Assume the contrary i.e. $x_k^* \ge x_j^*$. Now consider (16a) for both users,

$$\dot{U}_j(R_j^*)\Gamma_x(x_j^*, P, h) = \dot{U}_k(R_k^*)\Gamma_x(x_k^*, P, h) = \mu. \tag{26}$$

Consider $x_k^* \ge x_j^*$. Let $R_j^* = R_j(x_j^*, P, h)$ and $R_k^* = R_j(x_k^*, P, h)$. Then the following facts hold,

1) $\dot{U}_j(R_j^*) \overset{(a)}{>} \dot{U}_k(R_j^*) \overset{(b)}{\ge} \dot{U}_k(R_k^*)$ where (a) is given in the statement of the problem and (b) is true from Lemma 4(a)

2) $\Gamma_x(x_j^*, P, h) \ge \Gamma_x(x_k^*, P, h)$ from Lemma 4(b)

Now consider points 1 and 2. Together they yield

$$\dot{U}_j(R_j^*)\Gamma_x(x_j^*, P, h) > \dot{U}_k(R_k^*)\Gamma_x(x_k^*, P, h), \tag{27}$$

which is a contradiction. ■

This is because a unit of spectrum Δx yields a higher contribution to sum utility, when allocated to user j than to user k. This has also been observed by the authors of [14] for a network flow control problem.

We can illustrate this phenomenon with the class of exponential utilities given by

$$U_j(R) = \Gamma_j\left(1 - e^{-R/\Gamma_j}\right), \tag{28}$$

where Γ_j is the target rate of an user, which could be 1 Kbps for low data rates and 1 Mbps for file transfer. Since $\dot{U}_j(R) = e^{-R/\Gamma_j}$ is an increasing function of Γ_j, the high target rate users are allocated more spectrum than those with low target rates. Note that $U_j(R) \to \Gamma_j$, i.e. these utilities become flat at high rates.

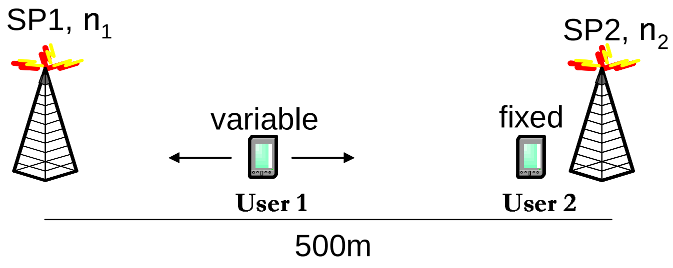

Fig. 2. The linear network with two SPs and two users.

V. Numerical Results

Consider two SPs in a linear cell with inter base distance of 500 meters as shown in Figure 2. The path loss is based on the COST-231 propagation model for outdoor WiMAX environments [15]. The noise power spectral density $N_0 = -174$ dBm/Hz. The SP i to user j distance is d_{ij} and the link gain (assuming unit noise spectral density) is h_{ij}. Thus

$$P_{\text{loss}} = -31.5 - 35\log(d_{ij}), \quad (29a)$$

$$h_{ij,dB} = P_{\text{loss}} - N_0. \quad (29b)$$

The distances are measured with SP 1 location as origin. User 2 is fixed at a distance of 100 m from the SP 2 and the location of user 1 is varied from 1m to 499m from SP 1. The total spectrum is 50 KHz. The users have exponential utilities as defined in (28) with parameters $\Gamma_1 = \Gamma_2 = 1$ Mbps.

Figure 3 shows the spectrum allocation. Subfigure 1 shows the SP that user 1 attaches to (*active* SP) for the cases $\eta_2/\eta_1 = 1$ and 10. User 1 is initially associated with SP 1 and then switches to SP 2 and the switch is sooner when SP 2 is more efficient. Subfigure 2 shows the ratio of the spectrum allocated to user 1. For most parts, it is increasing in the link gain to the active SP. This is because the rate function in (1) is increasing in link gain and spectrum and if link gain improves then the rate achieved is increased even more by allocating more spectrum. Further such an increase in R for low/medium R leads to increase in the utility $U(R)$. However the allocated spectrum becomes constant in the region $\mathcal{V}$ defined by $\eta_2/\eta_1 = 10$ and $d_{21} > 400$ m. This is because the exponential utility $U_1(R)$ flattens near the value of Γ_1 at high R. In region $\mathcal{V}$ user 1 has a very high link gain to SP 2 (its active SP) and SP 2 is more efficient. So user 1 achieves a high rate and his utility is near Γ_1. This can be seen in Subfigure 4. Thus as user 1 gets closer to SP 2, any extra spectrum would increase its rate but not its utility. Another way to interprete this is to look at the prices in subfigure 3. For region $\mathcal{V}$ both users are close to the flat regions of utilities and hence demand for additional spectrum is less. Consequently the prices are constant.

VI. Discussions and Conclusion

In this work we have analyzed a two tier allocation system for non strategic users who obtain spectrum from multiple SPs. We model the system from user welfare maximization framework. We show that in the optimal policy each user obtains spectrum only from one service provider. Based on our analysis we propose a spectrum price to facilitate distributed allocation. For two general classes of exponential concave utility functions, we analytically characterize the spectrum allocation and price. Extra insights are obtained from our simulations which are based in realistic channel parameters.

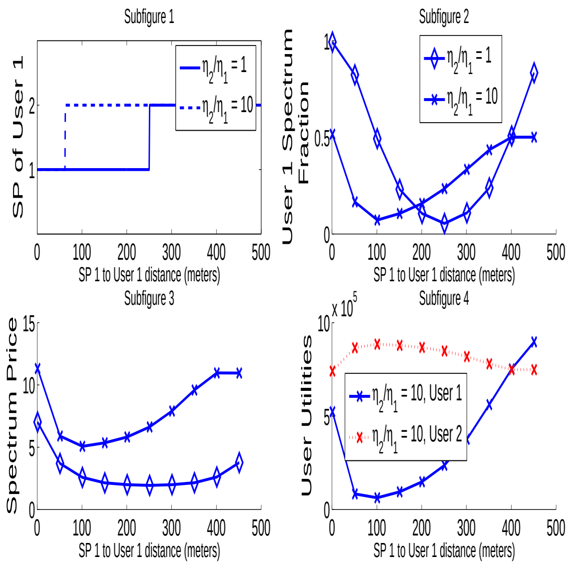

Fig. 3. Various graphs showing details of alloction when both users have exponential utilities with high target rates of 1 Mbps

References

[1] "IEEE 802.22 WRAN WG website," *http://ieee802.org/22/*.

[2] F. Kelly, A. Maulloo, and D. Tan, "Rate control in communication networks: shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, vol. 49, pp. 237–252, 1998.

[3] M. Buddhikot, P. Kolodzy, S. Miller, K. Ryan, and J. Evans, "DIMSUMnet: New directions in wireless networking using coordinated dynamic spectrum access," *IEEE WoWMoM*, pp. 78–85, Jun 2005.

[4] J. Huang, V. Subramanian, R. Agrawal, and R. Berry, "Downlink scheduling and resource allocation for OFDM systems," *In Proc. of IEEE Conf. on Information Sciences and Systems (CISS)*, March 2006.

[5] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," *In Proc. of IEEE Vehicular Technology Conf. (VTC)*, May 2000.

[6] W. Yu and J. M. Cioffi, "FDMA capacity of gaussian multiple access channels with ISI," *IEEE Trans. Communications*, vol. 50, January 2002.

[7] J. Huang, V. Subramanian, R. Berry, and R. Agrawal, "Joint scheduling and resource allocation OFDM systems: Algorithms and performance for the uplink," *In Proc. of Asilomar Conf. on Signals, Systems and Computers*, November 2007.

[8] "Long Term Evolution(LTE): Overview of LTE Air Interface, Technical White Paper," http://www.motorola.com/mot/doc/6/6993_MotDoc.pdf.

[9] X. Jing and D. Raychaudhuri, "Spectrum co-existence of IEEE 802.11b and 802.16a networks using reactive and proactive etiquette policies," *In Proc. of ACM SIGCOMM*, vol. 11, no. 4, pp. 539–554, 2006.

[10] J. Acharya and R. D. Yates, "Dynamic spectrum allocation for uplink users with heterogeneous utilities," *manuscript under preparation*.

[11] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[12] R. Knopp and P. A. Humblet, "Multiple–accessing over frequency–selective fading channels," *In Proc. of IEEE Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, vol. 3, September 1995.

[13] H. R. Varian, *Intermediate Microeconomics*. W. W. Norton, 1999.

[14] S. Low and D. E. Lapsley, "Optimization flow control, I: Basic algorithm and convergence," *IEEE/ACM Trans. Networking*, vol. 7, pp. 861–874, December 1999.

[15] "Mobile WiMAX part I: A technical overview and performance evaluation, WiMAX forum, http://www.wimaxforum.org/technology/downloads."