# Acquisition of the Kronecker Sequence using Recursive Soft Sequential Estimation for DS-SS Systems in Multipath Channels

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***Abstract*— Recursive soft sequential estimation (RSSE) acquisition scheme is an improved sequential estimation method based on the principle of recursive soft-in/soft-out (SISO) decoding. The employment of the RSSE acquisition scheme in a DS-SS system is beneficial for the initial acquisition of long *m*-sequences in AWGN channel. The RSSE scheme is known to be faster than a serial search and with much less hardware complexity than a parallel search scheme. However, using RSSE scheme to acquire the phase of an *m*-sequence is infeasible in a multipath scenario. Hence, we suggest employing Kronecker sequence as a spreading sequence. Here the inner sequence is a Barker sequence and the outer sequence is an *m*-sequence. The Barker sequence can significantly suppress the multipath interference and improve the initial acquisition performance of DS-SS system using RSSE scheme.**

***Index Terms*— sequence acquisition, Kronecker sequence, recursive soft sequential estimation (RSSE)**

## I. Introduction

In DS-SS systems, code acquisition is important because data demodulation is possible only after code acquisition has been obtained [1]. The code acquisition is usually achieved in two steps: initial acquisition for coarse alignment and tracking for fine alignment, of which the former is addressed in this paper.

Ward [2] proposed a fast initial acquisition scheme based on the principle of sequential estimation by exploiting the inherent properties of *m*-sequence, i.e., for an *m*-sequence with length $2^S - 1$, the $S$ consecutive chips explicitly determine the next chip. Hence, if the consecutive chips are correctly received by the acquisition device and are loaded into the local *m*-sequence generator, successive shifts of the chips in the generator will generate chips that exactly match the received chips of the transmitted *m*-sequence. However, in the presence of noise, one or more of the consecutive chips might be in error, potentially resulting in erroneous loading. Thus, the merits of conventional sequential estimation technique decrease with decreasing SNR. In [3], Yang and Hanzo proposed an improved m-sequential acquisition scheme, namely, recursive soft sequential estimation (RSSE) acquisition scheme, which estimates consecutive chips of an *m*-sequence using a recursive soft-in/soft-out (SISO) decoder. The recursive SISO decoder receives soft information from the channel, and soft extrinsic information [4] from the previous chips, to generate the extrinsic information for the following decoding steps. The acquisition device is capable of observing the reliabilities of the consecutive chips by observing their log-likelihood ratio values, i.e., $L$-values [4]. In an AWGN channel, the reliabilities associated with $S$ consecutive chips can be accumulated by receiving more and more chips of an *m*-sequence, providing a sufficiently low erroneous loading probability even at a low SNR. The RSSE acquisition scheme has an algorithmic complexity similar to that of an *m*-sequence generator, and the acquisition time of the RSSE acquisition scheme is a linear function of the number of stages in the *m*-sequence generator.

Clearly, the most critical requirement for attaining the successful acquisition of *m*-sequence based on sequential estimation is that $S$ consecutive chips have to be correctly estimated. However, in a multipath scenario, the $S$ consecutive chips are contaminated by both AWGN and multipath interference. The reliabilities associated with $S$ consecutive chips can not be accumulated when the acquisition device keeps on receiving noise and interference-contaminated chips, resulting in incorrect loading of $S$ consecutive chips. Thus, using RSSE scheme to acquire the phase of *m*-sequence is infeasible in the multipath channels, as illustrated by the simulation results provided in this paper.

Hence, in the case of multipath, we suggest employing Kronecker concatenated sequence [5], [6], as a spreading sequence, where the inner sequence is a Barker sequence and the outer sequence is an *m*-sequence. In [7], the authors showed that the correlation properties and BER performance of Kronecker sequences are only slightly poorer than those of conventional spreading codes (e.g. *m*-, Gold, and Kasami sequences) if the inner and outer sequences are chosen properly. A small compromise on the BER performance allows us using the rapid RSSE acquisition scheme. The acquisition of Kronecker sequence can be done in two stages: acquisition of inner sequence and then of the outer sequence. In this paper, we assume the phase of the short inner sequence, i.e., Barker sequence, has already been successfully acquired. Then, we study the acquisition performance of the outer sequence by using RSSE scheme. Since the Barker sequence significantly suppresses the multipath interference, the RSSE scheme works well in the acquisition of the outer sequence.

The remainder of the paper is organized as follows. The definition of Kronecker sequences and the properties of the

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inner Barker sequence and the outer *m*-sequence are given in Section II. In Section III, we describe the RSSE acquisition scheme and discuss its *m*-sequence acquisition performance in the AWGN channel. In Section IV, we present a multipath channel model, followed by the acquisition of Kronecker sequence in multipath channels. We draw the concluding remarks in Section V.

## II. The Kronecker Sequence

We consider a sequence $\mathbf{a} = \{a_n\}$ of length $N = N_1 N_2$ which is the concatenation of an inner sequence $\mathbf{b}$ of length $N_1$ and an outer sequence $\mathbf{c}$ of length $N_2$,

$$a_{n_2 N_1 + n_1} = b_{n_1} c_{n_2}, \;\; 0 \leq n \leq N-1$$

This also can be seen as the Kronecker product of inner sequence $\mathbf{b}$ and outer sequence $\mathbf{c}$,

$$\mathbf{a} = \mathbf{b} \otimes \mathbf{c}$$

where $\otimes$ denotes the Kronecker product. Hence, sequence $\mathbf{a}$ is called a Kronecker sequence or a concatenated sequence.

### A. Barker Sequence

Let $\mathbf{b} = \{b_n\}$ be a complex-valued sequence with sequence length $N$, its aperiodic autocorrelation function (ACF) is given by,

$$A_{\mathbf{b}}(l) = \sum_{n=0}^{N-l-1} b_n b_{n+l}^*, \; 0 \leq l \leq N-1$$

where $b_n^*$ denotes the complex conjugate of element $b_n$ .

Barker considered the problem of synthesizing sequences with aperiodic ACF sidelobes bounded by

$$|A_{\mathbf{b}}(l)| \leq 1$$

for real binary sequences with $b_n = \pm 1$. Binary sequences satisfying this condition are called Barker sequences. To date, only 7 Barker sequences have been found, as listed in Table 1.

TABLE I

Binary Barker Sequences

| Length N | Binary Barker Sequence | Merit Factor |
|---|---|---|
| 2 | +1 +1 | 2.00 |
| 3 | +1 +1 -1 | 4.50 |
| 4 | +1 +1 +1 -1; +1 +1 -1 +1 | 4.00 |
| 5 | +1 +1 +1 -1 +1 | 6.25 |
| 7 | +1 +1 +1 -1 -1 +1 -1 | 8.17 |
| 11 | +1 +1 +1 -1 -1 -1 +1 -1 -1 +1 -1 | 12.10 |
| 13 | +1 +1 +1 +1 +1 -1 -1 +1 +1 -1 +1 -1 +1 | 14.08 |

### B. M-sequence Generator

The *m*-sequences are generated using linear feedback shift registers (LFSR) as shown in Fig. 1. In the *m*-sequence generator, $D$ represents a unity time-delay operation. Each of the coefficients, $h_i \in \{1, 0\}$, represents the presence of a connection if it is a one, or the absence of a connection if it is a zero. The above configuration corresponds to the generator polynomial

$$h(D) = 1 + D^{s_1} + D^{s_2} + ... + D^{s_M = S} \tag{1}$$

where $\{s_1, s_2, ..., s_M = S\}$ is an index set corresponding to the set of feedback taps, implying that in Fig. 1, we have $h_{s_1} = h_{s_2} = ... = h_{s_M = S} = 1$, while the remaining coefficients are zero.

In this paper, we assume that the *m*-sequence generator outputs duo-binary symbols $\{+1, -1\}$, representing a logical zero with +1. Consequently, the conventional modulo-2 addition defined over the field of $\{1, 0\}$ is now replaced by the modulo-2 multiplication operation defined in the field of $\{+1, -1\}$ as shown in Fig. 1. Let us denote the output binary sequence as $c_i \in \{+1, -1\}$, then the output symbols of the *m*-sequence generator can be expressed as,

$$c_i = \prod_{m=1}^{M} c_{i-s_m}, \;\; for \;\;\; i = ..., -1, 0, 1, ... \tag{2}$$

Eq. 2 implies that if the receiver has the knowledge of the *m*-sequence chip values $c_{i-1}, c_{i-2}, ..., c_{i-S}$, it can generate the forthcoming chips.

## III. Recursive Soft Sequential Estimation Acquisition

Recursive soft sequential estimation (RSSE) acquisition scheme is an improved sequential estimation method based on the principle of recursive soft-in/soft-out (SISO) decoding. In this section, we briefly introduce the RSSE scheme and its acquisition performance in AWGN channel. The details can be found in [3].

### A. Description of the RSSE

We assume that the DS-SS system transmits the phase-coded carrier signal without data modulation. The schematic diagram of the RSSE acquisition scheme is shown in Fig. 1, which includes four fundamental blocks, namely, an *m*-sequence generator, a soft-chip register, a SISO decoder, and a code phase-tracking loop. The soft-chip register has the same number of delay units, which we refer to as soft-chip delay units (SCDUs), and the same feedback branches as the *m*-sequence generator. The SCDUs store the instantaneous *L*-values of consecutive $S$ chips. By considering the intrinsic information associated with a given chip of the *m*-sequence received from the channel and its extrinsic information provided by the previous decoded *L*-values stored in the SCDUs, the SISO decoder outputs the *L*-value of the current chip. The *L*-value is shifted to the right-most position of the SCDUs in the

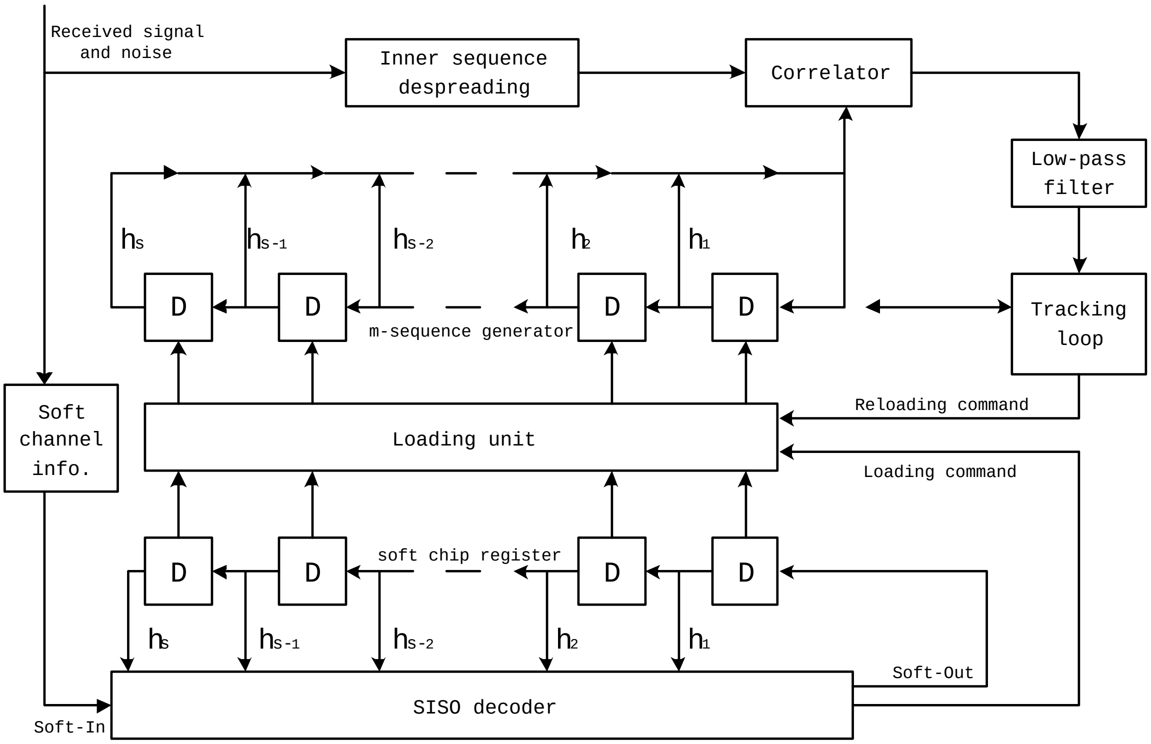


Fig. 1. Schematic diagram of the proposed soft recursive sequential estimator.

soft-chip register, while the soft value in the left-most SCDUs is shifted out and discarded. Following a number of recursions, when the amplitudes of the most recent $S$ soft outputs of the SISO decoder become sufficiently high for guaranteeing a sufficiently low erroneous loading probability, a "loading command" is activated. The $S$ consecutive chips are determined by employing hard decisions on their *L*-values and loaded into the corresponding delay units of the local *m*-sequence generator. Furthermore, if the tracking loop is incapable of tracking the phase, the code tracking loop activates a "reloading command" in order to load another group of $S$ consecutive chips into the delay units. The whole process can be repeated until successful code tracking is achieved.

### *B. RSSE Acquisition Algorithm*

As shown in Fig. 1, the SISO decoder requires both soft channel-output information and extrinsic information provided by the previous estimates of the SISO decoder in order to compute the soft output and update the contents in the soft-chip register.

Let $Z_i = \alpha_i c_i + n_i$ represent the received channel output corresponding to chip $c_i$, where $i = 0, 1, ....$ When communicating over a fading channel, $\alpha_i$ denotes the fading amplitude, whereas for an additive white Gaussian noise (AWGN) channel, we set $\alpha_i = 1$. Furthermore, $n_i$ denotes the AWGN having zero mean and a normalized variance of $N_0/2E_c$, where $N_0$ represents the single-sided power spectral density of the AWGN, $E_c$ represents the transmitted chip energy, and $E_c/N_0$ represents the signal-to-noise ratio (SNR) per chip. The soft channel-output information, in terms of $c_i$, is the *L*-value conditioned on the channel output $Z_i$, which is given by

$$L(c_i|Z_i) = L_c \cdot Z_i + L(c_i), \quad i = 0, 1, ... \tag{3}$$

where $L_c = 4\alpha_i \cdot E_c/N_0$, which is referred to as the reliability value of the channel. In (3), $L(c_i)$ is the *L*-value of a random variable $c_i$, which is computed as

$$L(c_i) = \log\left(\frac{P(c_i = +1)}{P(c_i = -1)}\right) \quad i = 0, 1, ... \tag{4}$$

where $L(c_i) = 0$ if we have no a priori information related to $c_i$. Let the previous $S$ number of soft outputs of the SISO decoder be $L(y_{i-1}), L(y_{i-2}), ..., L(y_{i-S})$. According to Eq. 2, the extrinsic information used for enhancing the correct decoding probability of $c_i$ can be approximately expressed as

$$\begin{aligned} L_e(c_i) &= \left[\prod_{m=1}^{M} \mathrm{sign}\{\mathrm{L(y_{i-s_m})}\}\right] \\ &\cdot \min\{|L(y_{i-s_1})|, |L(y_{i-s_1})|, ... \\ &\quad |L(y_{i-s_M})|\} \quad i = 0, 1, ... \end{aligned} \tag{5}$$

where we assume that $L_e(c_{-\infty}) = ... = L_e(c_{-2}) = L_e(c_{-1}) = 0$.

Finally, with the aid of the channel output information and the extrinsic information, the soft output of the SISO decoder associated with chip $c_i$ can be expressed as

$$L(y_i) \simeq L_c \cdot Z_i + \left[\prod_{m=1}^{M} \mathrm{sign}\{\mathrm{L(y_{i-s_m})}\}\right] \cdot \min\{|L(y_{i-s_1})|, |L(y_{i-s_1})|, ... |L(y_{i-s_M})|\} \quad i = 0, 1, ... \tag{6}$$

where again $L_e(c_{-\infty}) = ... = L_e(c_{-2}) = L_e(c_{-1}) = 0$.

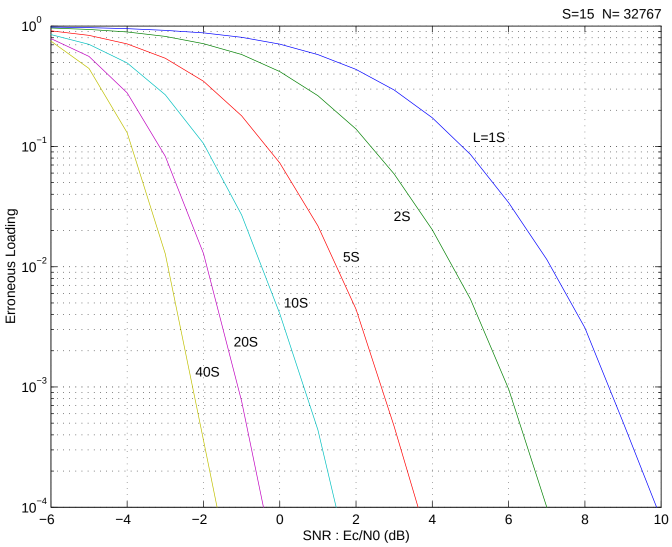


Fig. 2. Erroneous loading probability versus the SNR/chip $E_c/N_0$ performance for various numbers of chips invoked into the iterative SISO decoder, when transmitting the *m*-sequence generated using the generator polynomial of $h(D) = 1 + D^{14} + D^{15}$ over AWGN channels.

### C. RSSE Acquisition Performance in AWGN

Fig. 2 shows the acquisition performance of an *m*-sequence having a period of $N = 32767$ chips, which was generated by a 15-stage generator $(S = 15)$ using the generator polynomial of $h(D) = 1 + D^{14} + D^{15}$. As shown in Fig. 2, the *m*-sequence can be reliably acquired at an SNR per chip value of $E_c/N_0 = -0.5$dB by invoking about $L = 20S = 20 * 15 = 300$ chips into the recursive SISO decoder. By contrast using the conventional sequential estimation acquisition scheme [2], the PN code acquisition scheme has to operate at the SNR per chip value of $E_c/N_0 = 10$dB in order to achieve the erroneous loading probability of $10^{-4}$. Hence, the SNR per chip gain on the erroneous loading probability is about $10.5$dB, when $L = 300$ chips are invoked into the recursive SISO decoder.

However, in a multipath scenario, the $S$ consecutive chips are contaminated by both AWGN and multipath interference. The reliabilities associated with $S$ consecutive chips can not be accumulated when the acquisition device keeps on receiving more and more contaminated chips, resulting in incorrect loading $S$ consecutive chips into the local *m*-sequence generator. Thus, using RSSE scheme to acquire the phase of *m*-sequence is infeasible in the multipath channels. After describing the multipath channel model in the next section, we show how Kronecker sequence allows using the RSSE scheme.

## IV. Kronecker Sequence Acquisition in Multipath Channels

### A. The Channel Model

A widely accepted model for a frequency-selective multipath fading channel is a finite-length tapped delay line with a tap spacing of one chip [8]. Assuming that fading for each resolvable path is constant over the correlation interval, each tap is multiplied by an independent complex Gaussian random variable. The amplitude and phase of the fading for the $l$th resolvable path are respectively represented as $\alpha_l$ and $\theta_l$, where $\alpha_l$ is a Rayleigh random variable and $\theta_l$ is a uniform random variable over $[0, 2\pi)$. The multipath intensity profile (MIP) is assumed to be exponentially decaying with the decay rate $\mu$. When the total fading power in all of the resolvable paths is normalized to unity, the average fading power in each resolvable path is represented as,

$$E\{\alpha_l^2\} = \begin{cases} 1/L_p, & \mu = 0 \\ \frac{1-e^{-\mu}}{1-e^{-\mu L_p}} e^{-(l-1)\mu}, & \mu \neq 0 \end{cases} \tag{7}$$

where, $l = 1, 2, ..., L_p$ and $E\{\cdot\}$ denotes the statistical expectation.

### B. Simulation Results

As an example, we assume the multipath component is 10, and the decay rate $\mu$ is chosen such that the mean power of the last path is 30 dB decay to that of the first path. The inner sequence is the binary Barker sequence with length 11 and the outer *m*-sequence is derived from the generator polynomial $h(D) = 1 + D^{14} + D^{15}$. For the acquisition of Kronecker sequence, we assume the phase of inner sequence has been acquired and the correlator receiver has despreaded the inner sequence as a chip of the outer *m*-sequence. We study the performance of RSSE scheme on acquisition *m*-sequence in multipath channels.

Fig. 3 shows the reliability associated with correct loading $S$ consecutive chips of an *m*-sequence over multipath channels. The reliability of correct loading is defined as the sum of the absolute *L*-values of $S$ consecutive chips in the soft register. As more and more chips received by the acquisition device, the reliability of correct loading increase when we use Kronecker sequence. By contrast, for the DS-SS system uses the *m*-sequence, the decision reliability can not be accumulated into a high satisfactory value. Fig. 4 depicts the probability of erroneous loading versus the SNR/chip performance of RSSE scheme for various numbers of chips invoked into the iterative SISO decoder on acquisition of *m*-sequence in multipath channels. The high error floor claims the failure of using RSSE scheme.

Fig. 5 shows the erroneous loading probability versus the SNR/chip of DS-SS system employing Kronecker sequence

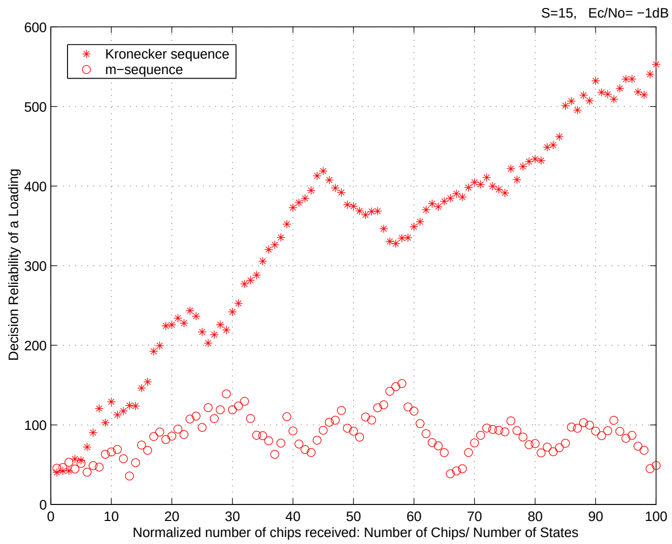


Fig. 3. Decision reliability versus the normalized number of received chips invoked in the recursive SISO decoding, when communicating over a multipath channel and using SNR per chip of -1dB for S=15.

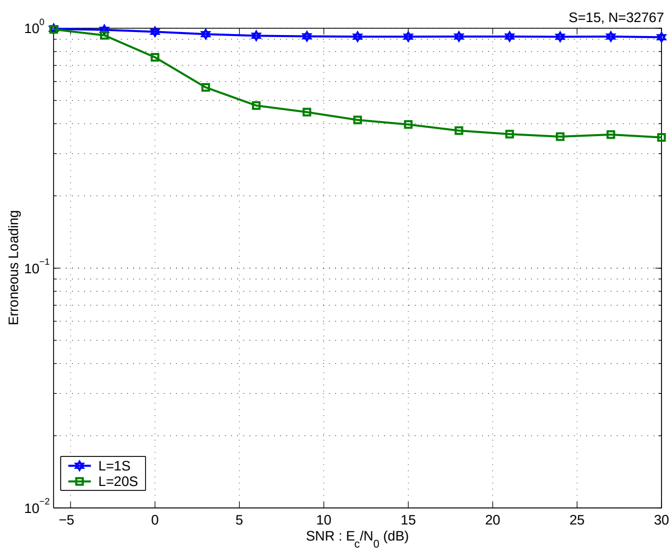


Fig. 4. Erroneous loading probability versus the SNR/chip $E_c/N_0$ performance for various numbers of chips invoked into the iterative SISO decoder, when transmitting the *m*-sequence generated using the generator polynomial of $h(D) = 1 + D^{14} + D^{15}$ over multipath channels.

instead of *m*-sequence. Since the inner Barker sequence significantly suppress the multipath interference, the reliability of correct loading can be accumulated sufficiently for guaranteeing a sufficiently low erroneous loading probability. Hence, it allows the RSSE scheme to acquire the phase of outer *m*-sequence.

## V. Conclusion

The RSSE acquisition scheme, as an improved sequential estimation method based on SISO decoding, is beneficial

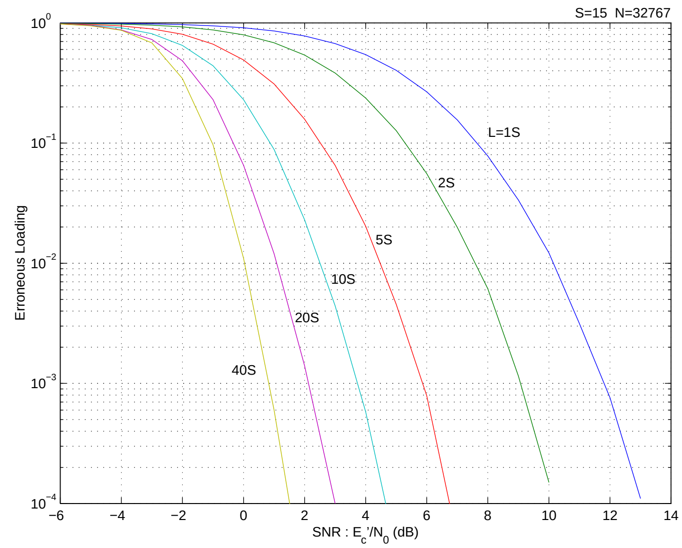


Fig. 5. Erroneous loading probability versus the SNR/chip $E_c/N_0$ performance for various numbers of chips invoked into the iterative SISO decoder, when transmitting the barker sequence and *m*-sequence generated using the generator polynomial of $h(D) = 1 + D^{14} + D^{15}$ over multipath channels.

for the initial acquisition of long *m*-sequences in AWGN channel. However, if we have no knowledge of the channel, the RSSE scheme fails to acquire the phase of an *m*-sequence in a multipath scenario. Thus, we suggest employing the Kronecker sequence as a spreading sequence, where the inner sequence is a Barker sequence and the outer sequence is an *m*-sequence. A small compromise on the BER performance allows us employing the rapid RSSE acquisition scheme. Since the Barker sequence significantly suppresses the multipath interference, the RSSE scheme works well in the acquisition of the Kronecker sequence.

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