Ternary Complementary Sets for Multiple Channel DS-UWB with Reduced Peak to Average Power Ratio

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***Abstract*— We study a multiple channel direct sequence ultra-wideband (DS-UWB) system which transmits the same information bit over a set of parallel channels corresponding to a set of orthogonal pulses. It is known that by employing mutually orthogonal (MO) ternary complementary set of spreading sequences, one can efficiently suppress both multipath and multiuser interference. Similar to a multicarrier DS-CDMA system, the multichannel DS-UWB system can have a high peak to average power ratio (PAPR) which may limit its application. In this paper, we analyze the PAPR of the multichannel UWB system by studying its upper bound. The bound illustrates how column sequences of MO complementary set matrices with small out-off-phase aperiodic autocorrelation functions (ACF) allow for lowering the PAPR. Hence, we develop an algorithm to construct the spreading sequence sets which may result in data sequences with reduced PAPR, while at the same time, preserving the complementarity and orthogonality which alleviate the multipath and multiuser interference.**

***Index Terms*— ultra-wideband (UWB), ternary sequence, complementary set, peak to average power ratio (PAPR)**

I. Introduction

We study ternary direct sequence based UWB (TS-UWB) [1]. Ternary signaling which includes epochs of zero signal amplitude constitutes a natural framework for studying impulse based UWB systems. As an extension of binary antipodal signaling, ternary signaling gives the designer much more flexibility in spreading sequence design.

For impulse based UWB systems, we have proposed a multichannel TS-UWB signaling which simultaneously transmits orthogonal pulses carrying the same information bit [2]. We demonstrated how, by employing ternary mutually orthogonal complementary set of sequences as the spreading sequences, the multipath interference and multiple access interference are efficiently suppressed.

Akin to the multicarrier direct-sequence code-division multiple access (MC-DS-CDMA) signaling [3], the transmitted signal of the multichannel UWB system is the sum of the signals from all parallel channels, so its envelope and transmitted power may vary significantly. Hence, such signaling potentially suffers a high peak to average power ratio (PAPR) which may limit its application. Great efforts have been devoted to PAPR analysis and reduction in multicarrier modulation systems (see, e.g., [3]–[5]). However, to our best knowledge, there is no PAPR analysis for impulse based multichannel UWB signaling in the literature, this being a promising technique for high bit-rate transmission, in particular, for indoor environments where multipath can be significant.

In this paper, we derive an upper bound on the PAPR for multichannel DS-UWB system using orthogonal pulses. The bound shows that data sequence [5] with small out-off-phase aperiodic autocorrelation functions (ACF) can lower the PAPR of the system. Therefore, we construct the spreading sequence sets which can result in data sequences with small aperiodic ACF. On the other hand, the spreading sequence sets preserve the complementarity and orthogonality to mitigate the multipath and multiuser interference.

The remainder of this paper is organized as follows. The multiple channel DS-UWB system is briefly described in Section II. In Section III, we analysis the PAPR of the multichannel DS-UWB system and describe the upper bound on its PAPR. The PAPR reduced ternary complementary sets are constructed in Section IV. Finally, the conclusion is given in Section V.

II. Multiple Channel UWB System

We transmit the same information bit over M parallel channels. The information bit is spread over a set of parallel channels each corresponding to one of a set of orthogonal pulses. The transmitted signal for user k is given by

$$S^{(k)}(t) = \sum_{r} \sum_{m=1}^{M} b_r^{(k)} P_m^{(k)}(t - rT_s) \tag{1}$$

where the pulse train for user k and code channel m is

$$P_m^{(k)}(t) = \sum_{n=0}^{N-1} c_{m,n}^{(k)} \psi_m(t - nT_c) \tag{2}$$

b_r is the binary antipodal symbol transmitted over M parallel channels and r is its index. The M spreading sequences $(c_{m,0}^{(k)}, c_{m,1}^{(k)}, \cdots, c_{m,N-1}^{(k)})$, $m = 1, 2, ...M$, assigned respectively to M parallel channels are ternary sequences in this paper. N is the sequence length. T_c is the chip duration time and $T_s = NT_c$ is the symbol period. $\psi_m(t)$ is the signaling pulse chosen from the orthogonal set of pulses and assumed known to the receiver.

The impulse response of the UWB channel with L resolvable paths is

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l) \tag{3}$$

where α_l and τ_l denote the channel gain and the propagation delay of the l_{th} path, respectively.

When sufficient multipath resolution is available, small changes in the propagation time only affect the path delay and path component distortion can be neglected. Under these assumptions, path coefficients α_l can be modelled as independent real valued random variables whose sign is a function of the material properties and, generally, depends on the wave polarization, angle of incidence, and the frequency of the propagating wave [6]. We quantize the multipath delay into bins, i.e. $\tau_l = lT_c$.

For an asynchronous UWB system with K users, the corresponding received signal model is:

$$r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L-1} \alpha_l s^{(k)}(t - lT_c - \tau^{(k)}) + n(t) \tag{4}$$

where $\tau^{(k)}$ accounts for propagation delay and lack of synchronism between transmitters, $n(t)$ is a white Gaussian noise process. For the system with short sequences, the delay $\tau^{(k)}$ is assumed to be uniformly distributed in the interval $[0, NT_c)$, and in this paper, we quantize it into bins.

We have demonstrated [2] that, when the spreading sequence set $\{(c_{m,0}^{(k)}, c_{m,1}^{(k)}, \cdots, c_{m,N-1}^{(k)})\}_{m=1}^{M}$ assigned to user k is a ternary complementary set and the ternary complementary sets between any two users are mutually orthogonal, the multipath interference as well as multiple access interference are efficiently suppressed by employing a correlator receiver.

Hence, the purpose of this paper is to solve the potential PAPR problem caused by multiple channels while at the same time maintaining the complementarity and orthogonality of the spreading sequence sets.

III. Peak to Average Power Ratio Analysis

The complementary set contains M complementary sequences with sequence length N is represented by matrix $\mathbf{C}$ as follow,

$$\mathbf{C} = \begin{bmatrix} c_{1,0} & c_{1,1} & c_{1,2} & \cdots & c_{1,N-1} \\ c_{2,0} & c_{2,1} & c_{2,2} & \cdots & c_{2,N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{M,0} & c_{M,1} & c_{M,2} & \cdots & c_{M,N-1} \end{bmatrix}_{M\times N}$$

where the sum of the aperiodic ACF of the M sequences, i.e. $(c_{m,0}, c_{m,1}, c_{m,2}, \cdots, c_{m,N-1})$, $m = 1, 2, ...M$, vanishes for every $l \neq 0$,

$$\sum_{i=1}^{M} \sum_{n=0}^{N-1-l} c_{i,n} c_{i,n+l} = 0, \forall l \neq 0$$

The complementary set matrix $\mathbf{C}$ can also be represented by its resulting data sequences $\mathbf{c}_0, \mathbf{c}_1, ...\mathbf{c}_{N-1}$, i.e. $\mathbf{C} = [\mathbf{c}_0 \mid \mathbf{c}_1 \mid \mathbf{c}_2 \mid ... \mid \mathbf{c}_{N-1}]_{M\times N}$. The N resulting data sequences $\mathbf{c}_n = (c_{1,n}, c_{2,n}, c_{3,n}, \cdots c_{M,n})$, $n = 0, 1, ...N-1$, affect the PAPR of the multichannel UWB system. Note that the information bits won't affect the instance power of the transmitted signal, since all the parallel channels carry the same information bit at the same time. Thus, in this section, we can analysis the PAPR and derive an upper bound on it by ignoring the information bits.

During a chip period T_c, the corresponding multichannel UWB signal of a single user is given by

$$S_{\mathbf{c}_n}(t) = \sum_{m=1}^{M} c_{m,n} \psi_m(t - nT_c), \quad for \quad n = 0, 1, ...N-1$$

Then, the corresponding instantaneous envelope power is,

$$\begin{aligned} P_{\mathbf{c}_n}(t) &= S_{\mathbf{c}_n}^2(t) \\ &= \left(\sum_{m=1}^{M} c_{m,n}\psi_m(t-nT_c)\right)\left(\sum_{l=1}^{M} c_{l,n}\psi_l(t-nT_c)\right) \\ &= 2\sum_{l=1}^{M-1}\sum_{m=1}^{M-l} c_{m,n}c_{m+l,n}\psi_m(t-nT_c)\psi_{m+l}(t-nT_c) \\ &\quad + \sum_{m=1}^{M} c_{m,n}^2\psi_m^2(t-nT_c) \quad for \quad n = 0, 1, ...N-1 \end{aligned}$$

We assume the orthogonal pulses have the zero amplitude out of the chip duration. Then, the instantaneous power over symbol period is given by,

$$P_{\mathbf{C}}(t) = \sum_{n=0}^{N-1} P_{\mathbf{c}_n}(t) U(t - nT_c)$$

where $U(t) = \begin{cases} 1 & t \in [\,0, T_c) \\ 0 & otherwise \end{cases}$.

The peak amplitude of all the orthogonal pulses may or may not be the same value, we define

$$\lambda = \max_m \left(\sup_{t\in[0,T_c]} |\psi_m(t)|\right) \quad for \quad m = 1, 2, ...M$$

where $\sup_{t\in[0,T_c]} |\psi_m(t)|$ denotes the peak value of pulse $|\psi_m(t)|$ over time period $[0, T_c]$.

Hence, the peak envelope power (PEP) corresponding with spreading sequence set $\mathbf{C}$ during the whole symbol period T_s is given by,

$$\begin{aligned} PEP(\mathbf{C}) &= \sup_{t\in[0,T_s]} P_{\mathbf{C}}(t) \\ &= \max_{\mathbf{c}_n}\left(\sup_{t\in[nT_c,(n+1)T_c]} P_{\mathbf{c}_n}(t)\right) \\ &\leq \lambda^2 \max_{\mathbf{c}_n}\left(\left|\sum_{m=1}^{M} c_{m,n}^2\right| + 2\left|\sum_{m=1}^{M-1}\sum_{l=m+1}^{M} c_{m,n}c_{l,n}\right|\right) \\ &\leq \lambda^2 \max_{\mathbf{c}_n}\left(\theta_{\mathbf{c}_n}(0) + 2\sum_{l\neq 0} |\theta_{\mathbf{c}_n}(l)|\right) \end{aligned} \tag{5}$$

where $\theta_{\mathbf{c}_n}$ denotes the aperiodic autocorrelation function (ACF) of sequence $\mathbf{c}_n = (c_{1,n}, c_{2,n}, c_{3,n}, \cdots c_{M,n})$, $n = 0, 1, ...N-1$ which is

$$\theta_{\mathbf{c}_n}(l) = \sum_{m=1}^{M-l} c_{m,n}c_{m+l,n} \quad for \quad l = 0, 1, ...M-1$$

We normalize the information symbol energy over M channels to be 1. Thus,

$$\int_0^{T_s} P_{\mathbf{C}}(t)dt = \sum_{m=1}^{M}\left(\int_0^{T_s}\sum_{n=0}^{N-1} c_{m,n}^2\psi_m^2(t-nT_c)dt\right) = 1$$

Then, the average power corresponding to the spreading sequence set $\mathbf{C}$ over a symbol period T_s is,

$$P_{av}(\mathbf{C}) = \frac{1}{T_s}\int_0^{T_s} P_{\mathbf{C}}(t)dt = \frac{1}{T_s} \quad (6)$$

From (5) and (6), the PAPR can be derived as,

$$\begin{aligned} PAR(\mathbf{C}) &= \frac{PEP(\mathbf{C})}{P_{av}(\mathbf{C})} \\ &\leq \frac{\lambda^2}{T_s}\max_{\mathbf{c}_n}\left(\theta_{\mathbf{c}_n}(0) + 2\sum_{l\neq 0}|\theta_{\mathbf{c}_n}(l)|\right) \quad (7) \end{aligned}$$

The above upper bound on the PAPR of multichannel UWB system associated with spreading sequence set $\mathbf{C}$ highlights the intimate relationship with the aperiodic ACF of $\mathbf{C}$'s resulting data sequences. In the next section, we construct the ternary complementary set $\mathbf{C}$ whose resulting data sequences have very small aperiodic ACF, thus lower the PAPR of the multichannel UWB system.

IV. Design of Ternary Complementary Sets with PAPR Reduction

Before we introduce the algorithm to construct the ternary complementary sets with PAPR reduction, let us go through some concepts and lemmas [7] first.

A set of M sequences $\{\mathbf{a}_i\}_{i=1}^M$ is said to be a complementary set of sequences, if the sum of the aperiodic ACF of the M sequences vanishes for every $l \neq 0$,

$$\sum_{i=1}^{M}\theta_{\mathbf{a}_i,\mathbf{a}_i}(l) = \sum_{i=1}^{M}\sum_{n=0}^{N-1-l} a_{i,n}a_{i,n+l} = 0, \forall l \neq 0$$

where $\theta_{\mathbf{a}_i,\mathbf{a}_i}$ denotes the aperiodic ACF of sequences $\mathbf{a}_i$ with length N and $a_{i,n}$ denotes the nth element in the sequence $\mathbf{a}_i$. When $M = 2$, then we said $\{\mathbf{a}_1, \mathbf{a}_2\}$ is a complementary pair.

A set of complementary sequences $\{\mathbf{b}_i\}_{i=1}^M$ is a mate of the set $\{\mathbf{a}_i\}_{i=1}^M$ if the length of $\mathbf{b}_i$ is equal to the length of $\mathbf{a}_i$, for $1 \leq i \leq M$, and $\sum_{i=1}^M \theta_{\mathbf{a}_i,\mathbf{b}_i}(l) = 0$ for any l. Complementary sets are said to be mutually orthogonal (MO) complementary sets if any pair of them are mates.

Lemma 1: Let $\{\mathbf{a}_1, \mathbf{b}_1\}$ be a complementary pair with sequence length N, then $\{\mathbf{a}_1\overleftarrow{\mathbf{b}_1}, \mathbf{b}_1(-\overleftarrow{\mathbf{a}_1})\}$ is a complementary pair with sequence length $2N$, where $\mathbf{a}_1\overleftarrow{\mathbf{b}_1}$ denotes the concatenation of two sequences $\mathbf{a}_1$ and $\overleftarrow{\mathbf{b}_1}$, $\overleftarrow{\mathbf{b}_1}$ denotes the reverse of the sequence $\mathbf{b}_1$ and $-\overleftarrow{\mathbf{a}_1}$ denotes the sequence whose ith element is the negation of the ith element in sequence $\overleftarrow{\mathbf{a}_1}$.

Lemma 2: Let $\{\mathbf{a}_i, \mathbf{b}_i\}$,$i = 1, 2, ...M$, respectively be M complementary pairs with the same sequence length. Then, all these $2M$ sequences form a complementary set $\{\mathbf{a}_1, \mathbf{b}_1, \mathbf{a}_2, \mathbf{b}_2, ..., \mathbf{a}_M, \mathbf{b}_M\}$.

A. Design Algorithm

Step by step, we construct ternary complementary sets whose resulting data sequences are all within ternary sequence set $\mathbf{T}(M, Z, S_{AP})$, where M is the length of resulting data sequence and is required to be an even number, Z is the number of the zero elements in the ternary sequence. For any ternary sequence $\mathbf{c} \in \mathbf{T}$, the sum of the absolute values of the aperiodic ACF of all the non-zero shifts, i.e. $\sum_{l=1}^{M-1}|\theta_{\mathbf{c},\mathbf{c}}(l)|$, is equal to S_{AP}. By choosing the S_{AP} as small as possible, we construct the ternary complementary set with PAPR reduction.

Step 1: In the ternary sequence set $\mathbf{T}(M, Z, S_{AP})$, we do the brute force search work to find seed sequence $\mathbf{c}_0 = (c_{1,0}, c_{2,0}, ..., c_{M,0})$, such that,

$$\begin{aligned} \mathbf{c}_1 &= (c_{1,1}, c_{2,1}, ..., c_{M,1}) \\ &= (c_{2,0}, -c_{1,0}, c_{4,0}, -c_{3,0}, ..., c_{M,0}, -c_{M-1,0}) \end{aligned}$$

where both $\mathbf{c}_0$ and $\mathbf{c}_1$ are within set $\mathbf{T}(M, Z, S_{AP})$.

Thus, we get the PAPR reduced ternary complementary set $\mathbf{C}^{(0)}$ as follow,

$$\mathbf{C}^{(0)} = \begin{bmatrix} c_{1,0} & c_{1,1} \\ c_{2,0} & c_{2,1} \\ \vdots & \vdots \\ c_{M-1,0} & c_{M-1,1} \\ c_{M,0} & c_{M,1} \end{bmatrix} = \begin{bmatrix} c_{1,0} & c_{2,0} \\ c_{2,0} & -c_{1,0} \\ \vdots & \vdots \\ c_{M-1,0} & c_{M,0} \\ c_{M,0} & -c_{M-1,0} \end{bmatrix}$$

Example 1: In set $\mathbf{T}(8, 1, 5)$, we can find seed sequence as $\mathbf{c}_0 = (+ - - + + + \, 0 +)$. We reorganize the elements of $\mathbf{c}_0$ and get the sequence

$$\begin{aligned} \mathbf{c}_1 &= (c_{1,1}, c_{2,1}, ..., c_{M,1}) \\ &= (c_{2,0}, -c_{1,0}, c_{4,0}, -c_{3,0}, ..., c_{M,0}, -c_{M-1,0}) \\ &= (- - + + + - + 0) \end{aligned}$$

It can be verified that both of them are within the set $\mathbf{T}(8, 1, 5)$. Thus, $\mathbf{C}^{(0)}$ is given by,

$$\mathbf{C}^{(0)} = \begin{bmatrix} c_{1,0} & c_{1,1} \\ c_{2,0} & c_{2,1} \\ \vdots & \vdots \\ c_{8,0} & c_{8,1} \end{bmatrix} = \begin{bmatrix} + & - \\ - & - \\ - & + \\ + & + \\ + & + \\ + & - \\ 0 & + \\ + & 0 \end{bmatrix}_{8\times 2}$$

Proposition 1: $\mathbf{C}^{(0)}$ is a ternary complementary set with PAPR reduction.

Proof: The complementarity of $\mathbf{C}^{(0)}$ can be proved by using *Lemma* 1 and *Lemma* 2. Since the sequence $\mathbf{c}_0$ and $\mathbf{c}_1$ satisfy the requirement that $\{(c_{2i-1,0}, c_{2i-1,1}), (c_{2i,0}, c_{2i,1})\}$, $i = 1, 2, ...M/2$, are respectively $M/2$ ternary complementary pairs. Based on Lemma 2, $\mathbf{C}^{(0)} = \{(c_{1,0}, c_{1,1}), (c_{2,0}, c_{2,1}), \cdots, (c_{M,0}, c_{M,1})\}$ is a ternary complementary set. On the other hand, based on the properties of seed sequence $\mathbf{c}_0$, both sequence $\mathbf{c}_0$ and $\mathbf{c}_1$ are within small aperiodic ACF sequence set $\mathbf{T}(M, Z, S_{AP})$.

Step 2: We can recursively extend $\mathbf{C}^{(0)}$ to be the PAPR reduced ternary complementary set with longer spreading sequence length.

TABLE I

THE SPREADING SEQUENCE LENGTH $N^{(p)}$

p	0	1	2	3	4	5	6	$\cdots$
$N^{(p)}$	2	4	8	16	32	64	128	$\cdots$

Let $\mathbf{C}^{(p)}$ denotes ternary complementary set consisting $\frac{M}{2}$ complementary pairs with sequence length $N^{(p)}$. By applying the procedure in $Lemma$ 1, we obtain $\frac{M}{2}$ complementary pairs with sequence length $N^{(p+1)} = 2N^{(p)}$, according to $Lemma$ 2, all these $\frac{M}{2}$ complementary pairs form a ternary complementary set $\mathbf{C}^{(p+1)}$. The spreading sequence lengths N of each step are shown in TABLE I.

Example 2: Based on $Lemma$ 1, we construct ternary complementary set $\mathbf{C}^{(1)}$ and $\mathbf{C}^{(2)}$ with PAPR reduction from ternary complementary set $\mathbf{C}^{(0)}$ constructed in $Example$ 1.

$$\mathbf{C}^{(0)} = \begin{bmatrix} + & - \\ - & - \\ - & + \\ + & + \\ + & + \\ + & - \\ 0 & + \\ + & 0 \end{bmatrix}_{8\times 2} \Longrightarrow \mathbf{C}^{(1)} = \begin{bmatrix} + & - & - & - \\ - & - & + & - \\ - & + & + & + \\ + & + & - & + \\ + & + & - & + \\ + & - & - & - \\ 0 & + & 0 & + \\ + & 0 & - & 0 \end{bmatrix}_{8\times 4}$$

Further, we can get

$$\mathbf{C}^{(2)} = \begin{bmatrix} + & - & - & - & - & + & - & - \\ - & - & + & - & + & + & + & - \\ - & + & + & + & + & - & + & + \\ + & + & - & + & - & - & - & + \\ + & + & - & + & - & - & - & + \\ + & - & - & - & - & + & - & - \\ 0 & + & 0 & + & 0 & - & 0 & + \\ + & 0 & - & 0 & - & 0 & - & 0 \end{bmatrix}_{8\times 8}$$

Proposition 2: The extended sequence sets $\mathbf{C}^{(p)}$, $p = 0, 1, 2, 3...$, are ternary complementary sets with PAPR reduction.

Proof: The complementarity of the sequence set $\mathbf{C}^{(p)}$ can be verified by using $Lemma$ 2 and $Lemma$ 3. The PAPR reduction can be proved by observing that all the resulting data sequences $\mathbf{c}_0^{(p+1)}$, $\mathbf{c}_1^{(p+1)}$,... $\mathbf{c}_{N^{(p+1)}-1}^{(p+1)}$, can be expressed in terms of $\mathbf{c}_0^{(p)}$, $\mathbf{c}_1^{(p)}$,... $\mathbf{c}_{N^{(p)}-1}^{(p)}$, that is

$$\mathbf{c}_i^{(p+1)} = \begin{cases} \mathbf{c}_i^{(p)} & for\ 0 \le i \le N^{(p)} - 1 \\ -\mathbf{c}_{i-N^{(p)}}^{(p)} & for\ N^{(p)} \le i \le \frac{3}{2}N^{(p)} - 1 \\ \mathbf{c}_{i-N^{(p)}}^{(p)} & for\ \frac{3}{2}N^{(p)} \le i \le 2N^{(p)} - 1 \end{cases}$$

Note that $\mathbf{c}_0^{(0)}$ and $\mathbf{c}_1^{(0)}$, the resulting data sequences of $\mathbf{C}^{(0)}$, are within set $\mathbf{T}(M, Z, S_{AP})$. Thus, all resulting data sequences of $\mathbf{C}^{(p)}$, i.e. $\mathbf{c}_0^{(p)}$, $\mathbf{c}_1^{(p)}$,... $\mathbf{c}_{N^{(p)}-1}^{(p)}$, $p = 0, 1, 2, ...$, are within the same ternary sequence set $\mathbf{T}(M, Z, S_{AP})$ as well.

Step 3: Based on a single PAPR reduced ternary complementary set $\mathbf{C} = [\mathbf{c}_0, \mathbf{c}_1, \cdots \mathbf{c}_{M-1}]_{M\times M}$ constructed above, we design the mutually orthogonal complementary sets with PAPR reduction.

Let's start the construction by introducing the Hadamard matrix $\mathbf{H}$ which can be generated from the following iteration:

$$\begin{aligned} \mathbf{H}^{(0)} &= [+] \\ \mathbf{H}^{(p+1)} &= \begin{bmatrix} \mathbf{H}^{(p)} & \mathbf{H}^{(p)} \\ \mathbf{H}^{(p)} & -\mathbf{H}^{(p)} \end{bmatrix}, \quad p = 0, 1, 2... \end{aligned}$$

Represent $\mathbf{H}_{M\times M}$ in details as:

$$\mathbf{H}_{M\times M} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_M \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,M} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,1} & h_{M,2} & \cdots & h_{M,M} \end{bmatrix}_{M\times M}$$

We denote the Kronecker product as $\otimes$. By using Kronecker product between $\mathbf{H}_{M\times M}$ and $\mathbf{C}_{M\times M}$, we get the M^2 by M^2 matrix $\mathbf{D}$,

$$\begin{aligned} \mathbf{D} &= \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \\ \vdots \\ \mathbf{D}_M \end{bmatrix} = \mathbf{H}_{M\times M} \otimes \mathbf{C}_{M\times M} \\ &= \begin{bmatrix} h_{1,1}\mathbf{C} & h_{1,2}\mathbf{C} & \cdots & h_{1,M}\mathbf{C} \\ h_{2,1}\mathbf{C} & h_{2,2}\mathbf{C} & \cdots & h_{2,M}\mathbf{C} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,1}\mathbf{C} & h_{M,2}\mathbf{C} & \cdots & h_{M,M}\mathbf{C} \end{bmatrix}_{M^2\times M^2} \end{aligned}$$

The matrix $\mathbf{D}_i$, $i = 1, 2, ...M$, with the dimension $M \times M^2$ are shown as follows,

$$\begin{aligned} \mathbf{D}_i &= \begin{bmatrix} h_{i,1}\mathbf{C} & h_{i,2}\mathbf{C} & \cdots & h_{i,M}\mathbf{C} \end{bmatrix} \\ &= \begin{bmatrix} d_{1,1}^i & d_{1,2}^i & \cdots & d_{1,M^2}^i \\ d_{2,1}^i & d_{2,2}^i & \cdots & d_{2,M^2}^i \\ \vdots & \vdots & \ddots & \vdots \\ d_{M,1}^i & d_{M,2}^i & \cdots & d_{M,M^2}^i \end{bmatrix}_{M\times M^2} \end{aligned}$$

Reshape $\mathbf{H}_{M\times M}$ as $\mathbf{H}'_{1\times M^2}$ below,

$$\begin{aligned} \mathbf{H}'_{1\times M^2} &= (\mathbf{h}_1\mathbf{h}_2 \cdots \mathbf{h}_M) \\ &= (h_{1,1}h_{1,2} \cdots h_{1,M}h_{2,1}h_{2,2} \cdots h_{M,M})_{1\times M^2} \end{aligned}$$

Let $\widetilde{\mathbf{H}}$ denotes the following matrix,

$$\widetilde{\mathbf{H}}_{M\times M^2} = \begin{bmatrix} \mathbf{H}' \\ \mathbf{H}' \\ \vdots \\ \mathbf{H}' \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{M,M} \\ h_{1,1} & h_{1,2} & \cdots & h_{M,M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1,1} & h_{1,2} & \cdots & h_{M,M} \end{bmatrix}_{M\times M^2}$$

By using dot product $\odot$ between matrix $\mathbf{D}_i$ and matrix $\widetilde{\mathbf{H}}$, we obtain the complementary set Δ^i, $i = 1, 2..., M$, as shown in (8). All together, we can generate M MO complementary sets and assigned them to M users.

Proposition 3: Δ^i, $i = 1, 2, ...M$, are MO complementary sets, which can be verified by following the similar procedures in [8].

Proposition 4: Sequence sets Δ^i, $i = 1, 2, ...M$, are ternary MO complementary sets with PAPR reduction.

$$\Delta^i = \widetilde{\mathbf{H}} \odot \mathbf{D}_i = \begin{bmatrix} h_{1,1}h_{i,1}\mathbf{c}_0 & h_{1,2}h_{i,1}\mathbf{c}_1 & \cdots & h_{1,M}h_{i,1}\mathbf{c}_{M-1} & h_{2,1}h_{i,2}\mathbf{c}_0 & h_{2,2}h_{i,2}\mathbf{c}_1 & \cdots & h_{M,M}h_{i,M}\mathbf{c}_{M-1} \end{bmatrix}_{M\times M^2}$$

$$= \begin{bmatrix} h_{1,1}d^i_{1,1} & h_{1,2}d^i_{1,2} & \cdots & h_{1,M}d^i_{1,M^2} & h_{2,1}d^i_{1,M+1} & h_{2,2}d^i_{1,M+2} & \cdots & h_{M,M}d^i_{1,M^2} \\ h_{1,1}d^i_{2,1} & h_{1,2}d^i_{2,2} & \cdots & h_{1,M}d^i_{2,M^2} & h_{2,1}d^i_{2,M+1} & h_{2,2}d^i_{2,M+2} & \cdots & h_{M,M}d^i_{2,M^2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{1,1}d^i_{M,1} & h_{1,2}d^i_{M,2} & \cdots & h_{1,M}d^i_{M,M^2} & h_{2,1}d^i_{M,M+1} & h_{2,2}d^i_{M,M+2} & \cdots & h_{M,M}d^i_{M,M^2} \end{bmatrix}_{M\times M^2} \quad (8)$$

Proof: The resulting data sequence of Δ^i, $i = 1, 2, ...M$ can be expressed $h_{m,n}h_{i,k}\mathbf{c}_j$, where $m, n, k \in [1, M]$ and $j \in [0, M-1]$. Since $h_{m,n}$, $m, n \in [1, M]$, are elements of Hadamard matrix $\mathbf{H}$, then the resulting sequence of Δ^i is the corresponding resulting data sequence of $\mathbf{C}$, (i.e. $\mathbf{c}_0, \mathbf{c}_1, \cdots \mathbf{c}_{M-1}$) multiplied by either "+1" or "-1". From $Proposition$ 2, we know that all resulting data sequences of $\mathbf{C}$ are within the same ternary sequence set $\mathbf{T}(M, Z, S_{AP})$. Hence, all resulting data sequences of MO complementary sets Δ^i, $i = 1, 2, ...M$, are within the set $\mathbf{T}(M, Z, S_{AP})$.

In Fig. 1, we normalize the PAPR upper bound of proposed 8 by 8 ternary complementary set $\mathbf{C}^{(2)}$ in example 2 as 1. Then, we randomly generate complementary sets with the same dimension. Fig. 1 demonstrates that the proposed ternary complementary set may lower the PAPR upper bound by 4.5 dB with that of randomly generated complementary sets.

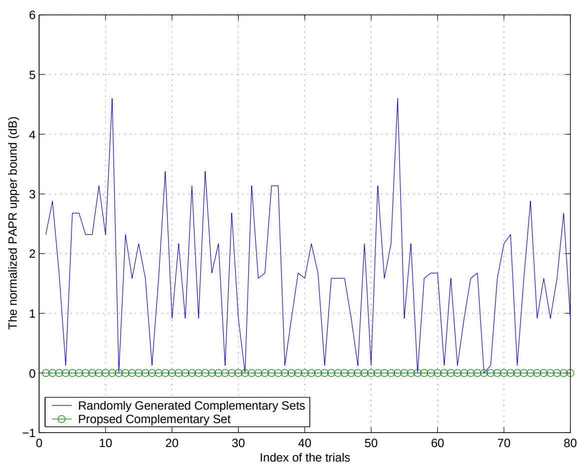

Fig. 1. Normalized PAPR upper bound of randomly generated complementary sets Versus that of proposed PAPR reduced ternary complementary set

V. Conclusion

In this paper, we analyze the PAPR of the multiple channel UWB system based on its upper bound. Based on the upper bound analysis, we further develop an algorithm to construct the spreading sequence sets which may result in data sequences with small aperiodic ACF, thus lower the PAPR of the signal. On the other hand, the spreading sequence sets preserve the complementarity and orthogonality to alleviate the multipath and multiple access interference.

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