Ternary Complementary Sets for Orthogonal Pulse based UWB

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***Abstract*— Ternary complementary set based UWB signaling employing a set of orthogonal chip pulses is proposed. Each user transmits the same information bit over a set of parallel channels each characterized by an orthogonal pulse and a spreading sequence from a ternary complementary set. Ternary complementary sets assigned to two users are mutually uncorrelated in the complementary sense. Hence, the multipath interference as well as multiple access interference are mitigated. Furthermore, sequence length selection is significantly more flexible than for corresponding binary signaling techniques. Thus, the proposed signaling is in particular suitable for adaptive high rate systems. Construction methods for ternary mutually orthogonal complementary sets are also included.**

I. Introduction

Recently, there has been a considerable interest in applying pulsed multicarrier technique [1] to Ultra-wide band systems [2]. Some example systems have been proposed in [3] [4], in which a set of orthogonal or nearly orthogonal chip pulses were suggested. In [5], an approach of generating the orthogonal pulses based on modified Hermite polynomial functions has been proposed. A corresponding example UWB system can be found in [6]. The orthogonal pulses provide additional advantages that can be exploited to help with some of the difficult tradeoffs that one is faced with when designing high data rate impulse based UWB systems.

In this paper, we propose a DS-UWB system with multiple parallel channels based on the orthogonal chip pulses. The same information bit of the user is transmitted over a set of parallel channels characterized by orthogonal pulses and spreading sequences from a complementary set. The complementary sets assigned to any two users are uncorrelated in the complementary sense. Thus, both the multipath interference and multiple access interference are alleviated.

We study ternary direct sequence based UWB (TS-UWB) [7] [8] signaling which includes epochs of zero signal amplitude as a natural extension of binary antipodal signaling. With ternary signaling, it is possible to construct the spreading sequences or spreading sequence sets with good correlation properties for a larger number of lengths than for binary sequences. Constructions of ternary complementary set and ternary mutually orthogonal complementary sets are included in this paper.

The paper is organized as follows. In Section 2, the multiple parallel channels UWB signaling and the UWB channel model are introduced. In Section 3, we describe the constructions of ternary spreading sequences for a multiple channel UWB signaling. Simulation results are presented in Section 4. We conclude the paper in Section 5.

II. System Model

We transmit the same information bit over M parallel channels. Each bit is modulated by the channel pulse train. The corresponding transmitted signal for user k is given by

$$s^{(k)}(t) = \sum_r \sum_{m=1}^{M} b_r^{(k)} P_m^{(k)}(t - rT_p) \tag{1}$$

where the pulse train for user k and code channel m is

$$P_m^{(k)}(t) = \sum_{n=0}^{N-1} c_{m,n}^{(k)} \psi_m(t - nT_c) \tag{2}$$

r is the index of the information symbols and N is the length of the spreading sequence. The spreading sequence set $\{c_m\}_{m=1}^{M}$ for each user is a ternary complementary set. b_r are binary antipodal symbols transmitted over M parallel channels, T_c is the chip duration time and $T_p = NT_c$ is the symbol period. $\psi_m(t)$ is the unit energy signaling pulse chosen from the orthogonal set of pulses and assumed known to the receiver.

The impulse response of the UWB channel with L resolvable paths is

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l) \tag{3}$$

where α_l and τ_l denote the channel gain and the propagation delay of the l_{th} path, respectively.

When sufficient multipath resolution is available, small changes in the propagation time only affect the path delay and path component distortion can be neglected. Under these assumptions, path coefficients α_l can be modelled as independent real valued random variables whose sign is a function of the material properties and, generally, depends on the wave polarization, angle of incidence, and the frequency of the propagating wave [9]. We quantize the multipath delay into bins, i.e. $\tau_l = lT_c$.

For an asynchronous UWB system with K users, the corresponding received signal model is:

$$r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L-1} \alpha_l s^{(k)}(t - lT_c - \tau^{(k)}) + n(t) \tag{4}$$

where $\tau^{(k)}$ accounts for propagation delay and lack of synchronism between transmitters, $n(t)$ is a white Gaussian noise process. For the system with short sequences, the delay $\tau^{(k)}$ is assumed to be uniformly distributed in the interval $[0, NT_c)$, and in this paper, we quantize it into bins.

III. Ternary Spreading Sequences Design

Let θ_{a_i,a_i} denotes the aperiodic autocorrelation function of sequences a_i with length N. A set of M sequences $\{a_i\}_{i=1}^{M}$ is said to be a complementary set of sequences, if the sum of the aperiodic autocorrelation functions of the M sequences vanishes for every $l \neq 0$, i.e.

$$\sum_{i=1}^{M} \theta_{a_i,a_i}(l) = \sum_{i=1}^{M} \sum_{n=0}^{N-1-l} a_{i,n} a_{i,n+l} = 0, \forall l \neq 0. \tag{5}$$

where $a_{i,n}$ denotes the nth element in the sequence a_i.

A set of complementary sequences $\{b_i\}_{i=1}^{M}$ is a mate of the set $\{a_i\}_{i=1}^{M}$ if the length of b_i is equal to the length of a_i, for $1 \leq i \leq M$, and $\sum_{i=1}^{M} \theta_{a_i,b_i}(l) = 0$ for any l. Complementary sets of sequences are said to be mutually orthogonal (MO) complementary sets if any pair of them are mates.

In the following section, we describe approaches of generating ternary complementary and ternary MO complementary sets.

A. Ternary Complementary Set Design

Ternary complementary sets can be constructed from perfect ternary sequences [10]. Let c be a perfect ternary sequence [11] with length N, for which its periodic autocorrelation function $R_{c,c}$ satisfies

$$R_{c,c}(l) = \begin{cases} N_1 & \text{if } (l \bmod N) \equiv 0 \\ 0 & \text{if } (l \bmod N) \neq 0 \end{cases} \tag{6}$$

where N_1 is the number of non-zero elements of the ternary sequence. Then all its different cyclic time shifted versions form a complementary set.

Example 1: Perfect ternary sequence $[+00-0-+]$ and all its cyclic shifts form a ternary complementary set of sequences

$$\{c_m\}_{m=1}^{7} = \begin{bmatrix} + & 0 & 0 & - & 0 & - & + \\ + & + & 0 & 0 & - & 0 & - \\ - & + & + & 0 & 0 & - & 0 \\ 0 & - & + & + & 0 & 0 & - \\ - & 0 & - & + & + & 0 & 0 \\ 0 & - & 0 & - & + & + & 0 \\ 0 & 0 & - & 0 & - & + & + \end{bmatrix}$$

where $'+'$ denotes 1 and $'-'$ denotes -1.

B. Ternary MO Complementary Set Design

We describe two approaches for constructing of ternary MO complementary sets, namely recursive approach and combination approach.

Large family size MO complementary sets can be constructed from small family size MO complementary sets using a recursive procedure [12]. Let $\triangle^{(p)}$ be a matrix of sequences with $M^{(p)}$ rows, each row contains $M^{(p)}$ sequences with equal length $N^{(p)}$. The recursive procedure provides a larger matrix of sequences $\triangle^{(p+1)}$ with $2M^{(p)}$ rows, each row contains $2M^{(p)}$ sequences with length $2N^{(p)}$. That is,

$$\triangle^{(p+1)} = \begin{bmatrix} \triangle^{(p)} \otimes \triangle^{(p)} & -\triangle^{(p)} \otimes \triangle^{(p)} \\ -\triangle^{(p)} \otimes \triangle^{(p)} & \triangle^{(p)} \otimes \triangle^{(p)} \end{bmatrix} \tag{7}$$

where $-\triangle^{(p)}$ denotes the matrix whose ijth entry is the ijth entry negation of $\triangle^{(p)}$ and $'\otimes'$ denotes interleaving. Two sequences $a = \{a_1, a_2, a_3, ...\}$ and $b = \{b_1, b_2, b_3, ...\}$ are interleaved as $a \otimes b = \{a_1, b_1, a_2, b_2, ...\}$. The two matrices of sequences are interleaved by interleaving their corresponding sequences as described in Example 2.

The matrix $\triangle^{(p+1)}$ can be partitioned into MO complementary sets of twice the family size corresponding to $\triangle^{(p)}$.

Starting with a ternary complementary pair (TCP) $\{c_1, c_2\}$ [13], the seed matrix $\triangle^{(0)}$ is constructed as follows,

$$\triangle^{(0)} = \begin{bmatrix} c_1 & \overleftarrow{c_2} \\ c_2 & -\overleftarrow{c_1} \end{bmatrix} \tag{8}$$

where $\overleftarrow{c_2}$ denotes the reverse of the sequence c_2 and $-\overleftarrow{c_1}$ denotes the sequence whose ith element is the negation of ith element in sequence $\overleftarrow{c_1}$. The, $\triangle^{(0)}$ can be partitioned into two MO complementary sets $\{c_1, c_2\}$ and $\{\overleftarrow{c_2}, -\overleftarrow{c_1}\}$. Following (7), we can recursively construct larger MO complementary sets based on the seed matrix $\triangle^{(0)}$.

Example 2: Let $\{c_1, c_2\} = \{++-, +0+\}$. Based on (8), the seed matrix is given by

$$\triangle^{(0)} = \begin{bmatrix} ++- & +0+ \\ +0+ & +-- \end{bmatrix}$$

Note that $\triangle^{(0)}$ is comprised of two mutually orthogonal complementary sequence sets $\left(\begin{matrix} ++- \\ +0+ \end{matrix}\right)$ and $\left(\begin{matrix} +0+ \\ +-- \end{matrix}\right)$.

Based on the following recursive procedure,

$$\triangle^{(0)} \otimes \triangle^{(0)} = \begin{bmatrix} ++-\otimes++- & +0+\otimes+0+ \\ +0+\otimes+0+ & +--\otimes+-- \end{bmatrix}$$

and

$$-\triangle^{(0)} \otimes \triangle^{(0)} = \begin{bmatrix} --+\otimes++- & -0-\otimes+0+ \\ -0-\otimes+0+ & -++\otimes+-- \end{bmatrix}$$

From (7), we obtain the mutually orthogonal complementary sets $\triangle^{(1)}$ =

$$\begin{bmatrix} ++++-- & ++00++ & -+-++- & -+00-+ \\ ++00++ & ++---- & -+00-+ & -++-+- \\ -+-++- & -+00-+ & ++++-- & ++00++ \\ -+00-+ & -++-+- & ++00++ & ++---- \end{bmatrix}$$

Hence, from any starting TCP with sequence length $N^{(0)}$, we can construct the seed matrix $\triangle^{(0)}$. Repeating the recursive approach p times, we obtain 2^{p+1} MO complementary sets with equal sequence length $N^{(p)} = 2^p N^{(0)}$. Note that, the starting TCP sequence length $N^{(0)}$ can be any positive integer. But the sequence length of binary complementary pairs is given by $N = 2^a 10^b 26^c$, where a, b, c are nonnegative integers. Thus, the ternary signaling provides more flexibility in the spreading sequence design than binary signaling does.

The combination method [14] [15] MO complementary set construction is based on the following structure:

$$(a) \begin{bmatrix} +\Delta_1 & +\Delta_2 \\ +\Delta_2 & -\Delta_1 \end{bmatrix} \quad (b) \begin{bmatrix} +\Delta_1 & +\Delta_2 \\ -\Delta_2 & +\Delta_1 \end{bmatrix}$$

$$(c) \begin{bmatrix} -\Delta_1 & +\Delta_2 \\ +\Delta_2 & +\Delta_1 \end{bmatrix} \quad (d) \begin{bmatrix} +\Delta_1 & -\Delta_2 \\ +\Delta_2 & +\Delta_1 \end{bmatrix} \tag{9}$$

The next example demonstrates the combination approach.

Example3: Let the seed matrices be two uncorrelated complementary sets:

$$\Delta_{a1} = \begin{bmatrix} + & 0 \\ 0 & - \end{bmatrix} \quad \Delta_{a2} = \begin{bmatrix} + & 0 \\ 0 & + \end{bmatrix} \tag{10}$$

By employing all the structures $9(a) - 9(d)$, we generate four MO complementary sets with sequence length $N = 4$ and label them as $A_1 - A_4$.

$$(A_1) \begin{bmatrix} + & 0 & + & 0 \\ 0 & - & 0 & + \\ + & 0 & - & 0 \\ 0 & + & 0 & + \end{bmatrix} \quad (A_2) \begin{bmatrix} + & 0 & + & 0 \\ 0 & - & 0 & + \\ - & 0 & + & 0 \\ 0 & - & 0 & - \end{bmatrix}$$

$$(A_3) \begin{bmatrix} - & 0 & + & 0 \\ 0 & + & 0 & + \\ + & 0 & + & 0 \\ 0 & + & 0 & - \end{bmatrix} \quad (A_4) \begin{bmatrix} + & 0 & - & 0 \\ 0 & - & 0 & - \\ + & 0 & + & 0 \\ 0 & + & 0 & - \end{bmatrix} \tag{11}$$

Now taking another two uncorrelated complementary sets:

$$\Delta_{b1} = \begin{bmatrix} + & - \\ + & + \end{bmatrix} \quad \Delta_{b2} = \begin{bmatrix} - & - \\ - & + \end{bmatrix} \tag{12}$$

Repeating the structures $9(a) - 9(d)$, we generate another MO complementary sets comprised of four sets labelled as $B_1 - B_4$:

$$(B_1) \begin{bmatrix} + & - & - & - \\ + & + & - & + \\ - & - & - & + \\ - & + & - & - \end{bmatrix} \quad (B_2) \begin{bmatrix} + & - & - & - \\ + & + & - & + \\ + & + & + & - \\ + & - & + & + \end{bmatrix}$$

$$(B_3) \begin{bmatrix} - & + & - & - \\ - & - & - & + \\ - & - & + & - \\ - & + & + & + \end{bmatrix} \quad (B_4) \begin{bmatrix} + & - & + & + \\ + & + & + & - \\ - & - & + & - \\ - & + & + & + \end{bmatrix} \tag{13}$$

From the above eight sets with sequence length $N = 4$, by employing the following structures based on $9(a) - 9(d)$, we generate the MO complementary sets with sequence length $N = 8$:

$$(a) \begin{bmatrix} +A_1 & +B_4 \\ +B_4 & -A_1 \end{bmatrix} \quad (b) \begin{bmatrix} +A_1 & -B_4 \\ +B_4 & +A_1 \end{bmatrix}$$

$$(c) \begin{bmatrix} +A_4 & +B_1 \\ +B_1 & -A_4 \end{bmatrix} \quad (d) \begin{bmatrix} +A_4 & -B_1 \\ +B_1 & +A_4 \end{bmatrix}$$

$$(e) \begin{bmatrix} +A_2 & +B_3 \\ -B_3 & +A_2 \end{bmatrix} \quad (f) \begin{bmatrix} -A_2 & +B_3 \\ +B_3 & +A_2 \end{bmatrix}$$

$$(g) \begin{bmatrix} +A_3 & +B_2 \\ -B_2 & +A_3 \end{bmatrix} \quad (h) \begin{bmatrix} -A_3 & +B_2 \\ +B_2 & +A_3 \end{bmatrix} \tag{14}$$

All these eight complementary sequence sets are mates of each other. For example, set $14(a)$ is:

$$\{c_m\}_{m=1}^{8} = \begin{bmatrix} + & 0 & + & 0 & + & - & + & + \\ 0 & - & 0 & + & + & + & + & - \\ + & 0 & - & 0 & - & - & + & - \\ 0 & + & 0 & + & - & + & + & + \\ + & - & + & + & - & 0 & - & 0 \\ + & + & + & - & 0 & + & 0 & - \\ - & - & + & - & - & 0 & + & 0 \\ - & + & + & + & 0 & - & 0 & - \end{bmatrix}$$

IV. ANALYSIS

We study correlator receiver performance for ternary complementary pair, m-, and perfect ternary sequence in a single user system. For multiuser system, we compare the ternary complementary sets, Walsh-Hadamard codes, and preferred m-sequence [16]. Bit energy for all signaling schemes is normalized.

The mean power of multipath components are chosen to be equal to average value given in [17], which is based on the indoor line of sight (LOS) measurements performed in 23 homes. In [17], it is observed that the line of sight component and the first 10 paths account for 33% and 75% of the total power, respectively. The sign of the reflected path coefficient is modelled as a uniformly distributed random variable [18]. The path power is quantized into 0.4 nanosecond bins corresponding to a chip duration T_c. We assume that each bin contains exactly one multipath component (emulating a dense multipath environment) and the delay spread was restricted to be 4 nanosecond. The effects of interchip interference has been assumed negligible.

In Figure 1, a TCP with length 15 has been assigned to a single user for its two parallel channels, i.e. $[++-+++++-0-+-0+]$ and $[++-+++---0+-+0-]$. We compare it with single user single channel UWB system using the same length m-sequence $[---+--++-+-++++]$ and perfect ternary sequence $[+++++-+0+0-++-00+-0--]$. The results show that the UWB signaling employing ternary complementary pair suffers much less multipath interference than the signaling using m- and perfect sequences. At $SNR = 10^{-3}$, more than 1 and $2dB$ gain can be achieved by employing TCP over m- and perfect ternary sequences respectively.

Figure 2 demonstrates the BER performance of complementary sets and preferred m-sequence in a two user scenario. The MO complementary sets are constructed from 14(a,b) and assigned to two users. Based on the approach described in Section III. A, we derive another pair of complementary sets from arbitrary perfect ternary sequence $[+--0-00]$ and $[+00-0--]$ and assign them to the two users. Note that, by employing MO complementary sets, the multiple access interference is mitigated significantly.

In a dense multipath UWB environment, the spreading sequence with short length commonly suffers sever ISI, while with the multiple channel, the MO ternary complementary set of sequences with length 7 has better performance than the preferred m-sequences with length 31. We note that $2dB$ gain can be observed at $SNR = 10^{-3}$.

In Figure 3, we assign four ternary MO complementary sets 14 (a-d) to four users respectively. The ternary complementary set of sequences with length 8 can achieve the same BER performance with the Goldlike sequences with length 63. Hence, without any loss of the BER performance, in a multiple channel UWB system, we can increase the bit rate more than 7 times than the single channel UWB signals employing Goldlike sequences.

V. Conclusion

In this paper, we show that by simply using the correlator receiver, the orthogonal pulse based TS-UWB signaling with MO complementary sets can allow for efficient suppression of both the multipath interference and multiple access interference. Furthermore, ternary sequence sets with good correlation properties can be found for a larger number of sequence lengths than binary sequence based signaling, thus allowing for more flexibility in rate adaptive systems. The approach is also suitable for RAKE receivers to combine the multipath energy, whose analysis has not been addressed here.

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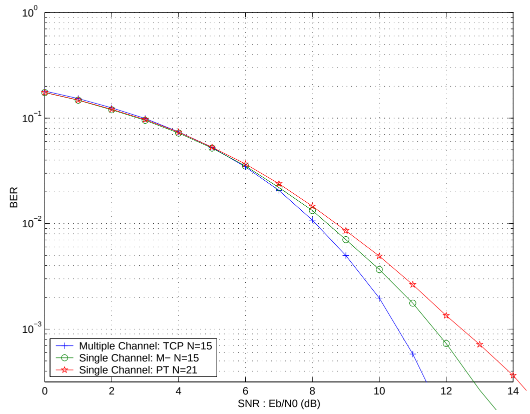

Fig. 1. Single User: Parallel channel UWB system with TCP versus Single channel UWB system with m- and perfect ternary sequence

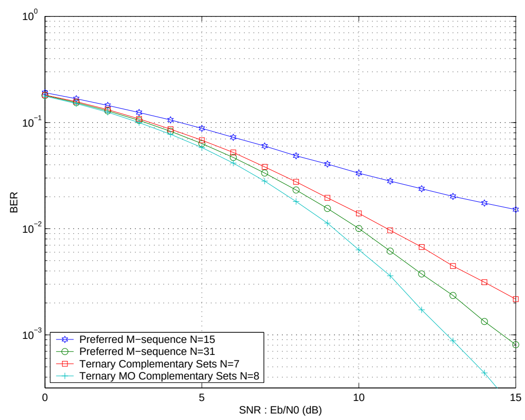

Fig. 2. Two Users: Parallel channel UWB system with ternary complementary sets versus Single channel UWB system with preferred m-sequences

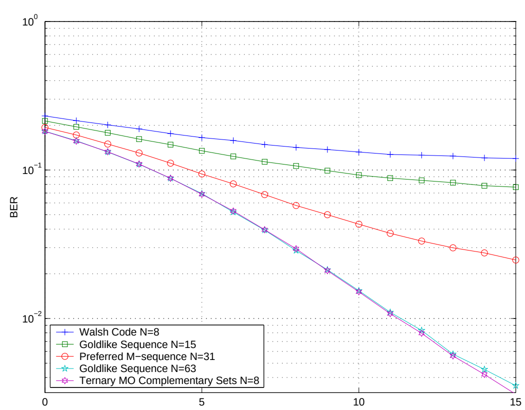

Fig. 3. Four Users: BER performance for MO ternary complementary sets, Walsh-Hadamard codes and preferred m-sequences