Can a Packet Walk Straight Through a Field of Randomly Dying Location-Unaware Wireless Nodes?

Silvija Kokalj-Filipović, Predrag Spasojević* and Roy D. Yates
WINLAB, ECE Department, Rutgers University
{skokalj,spasojev,ryates}@winlab.rutgers.edu

ABSTRACT

A protocol, dubbed BeSpoken, steers data transmissions along a straight path called a spoke through a wireless sensor network with many *location-unaware* nodes. BeSpoken implements a simple, spatially recursive communication process, where a set of control packets and a data packet are exchanged among daisy-chained relays that constitute the spoke. It directs data transmissions by randomly selecting relays from crescent-shaped areas along the spoke axis created by intersecting transmission ranges of control and data packets. To specify design rules for protocol parameters that minimize energy consumption while ensuring that spokes propagate far enough and have a limited wobble with respect to the spoke axis, our model of the spoke propagation matches the protocol parameters to the density of network nodes, assuming that nodes are spatially distributed as a Poisson point process of *known uniform* intensity. To avoid this requirement, we propose and characterize an adaptive mechanism that ensures desired spoke propagation in a network of *arbitrary density*. This necessitates a qualitatively new protocol model used to evaluate the spoke propagation under both the basic and the adaptive protocol. The introduced adaptive mechanism repairs the spoke when the crescent-shaped area is empty which may occur in the case of network thinning and as a result of random or arbitrary sensor death. Our analytical and simulation results demonstrate that the adaptive BeSpoken creates longer spokes both in networks with uniform distribution of nodes and in networks with holes. In addition, the adaptive protocol is significantly less sensitive to changes in the distribution of network nodes and their density.

Categories and Subject Descriptors: C.2.1 [Computer Networks]: Network Architecture and Design: Wireless communication

General Terms: Design, Performance, Theory.

Keywords: adaptive, directional propagation, location-unaware nodes, network thinning, random walk, stochastic model, wireless.

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1. INTRODUCTION

In randomly deployed *Wireless Sensor Networks (WSNs)* composed of location unaware nodes, data sources are frequently unaware of which data sinks have interest in their observations. An example is a network of cheap, battery-operated sensor nodes scattered over an area, used for environmental monitoring and expected to efficiently deliver gathered information to a data collector (sink) located at an arbitrary (and frequently random) position at the network boundary. Given scarce resources and limited processing power of WSN nodes, the unknown position of a data sink makes the task of delivering data especially challenging. Several new communication paradigms, like geocasting, data dissemination and data search, emerged from this problem [5]. In the geocasting problem [10] data needs to be routed to a geographic region instead of a destination node specified by an address.

Unrestricted flooding as a dissemination method and a trivial form of geocasting leads to a "broadcast storm" of redundant transmissions [11] and consumes more resources than necessary [4]. Two dissemination techniques that use flooding selectively are briefly described next. In a *push* approach [3], a publishing process plants pointers in the network that can be used by the interested sinks to establish a path to the correct source. Publishing mechanisms are largely based on flooding and consequent path reinforcement. Alternatively, in [6] the authors introduce a data-centric *pull* mechanism called *directed diffusion* in which interest requests (queries) are flooded into the network leaving gradient paths back to the sink. With location-unaware nodes, a more efficient alternative to flooding is to use landmark-based routing protocols [2] to store state information in selected nodes (possibly along a path) to direct the search toward the correct source [12].

Disseminating data along straight trajectories, studied here, is conceptually closest to geographic greedy forwarding schemes [7, 14] used for routing to known destinations, with an important distinction that, instead of greedily approaching the sink, in our approach the data is greedily directed away from the source. In the greedy geographic forwarding scheme a packet is forwarded to a one-hop neighbor which is closer to the destination than the current node. The similarity is only conceptual, since the assumptions are orthogonal to ours: a source node knows the location of the destination node, and network nodes are location-aware. Adhering to the previously introduced scenario where the data collector is located somewhere at the network boundary, we propose a push-pull model of data dissemination, based on BeSpoken, a protocol introduced in our earlier work [8,9]. BeSpoken greedily propagates data away from the source through a wireless sensor network with many location-unaware nodes and, ultimately, to the network perimeter. It does so by steering data transmissions along a straight path called a spoke. Here a source disseminates data advertisements along the source spokes, and a data collector (sink) sends a query along its own spokes that may intersect the source spokes. Each intersection represents a successful search.

BeSpoken implements a simple, spatially recursive communication process, where a basic set of control packets and a data packet are exchanged repeatedly among daisy-chained relays that constitute the spoke. It directs data transmissions by randomly selecting relays to retransmit data packets from crescent-shaped areas along the spoke axis, created by intersecting transmission ranges of control and data packets. The resulting random walk of the spoke hop sequence may be modeled as a two dimensional Markov process. As a result of an analysis of this model, in [8, 9] we proposed design rules for protocol parameters (such as transmission range of data and control packets) that minimize energy consumption while ensuring that spokes propagate far enough and have a limited wobble with respect to the spoke axis. The parameters were optimized for network nodes scattered as a planar Poisson point process of a known uniform intensity. It was essential to match the protocol parameters to the density of network nodes. However, randomly deployed WSNs can frequently exhibit local phenomena such as small areas scarcely covered by nodes, or communication voids. In this paper, we aim to demonstrate how the proposed protocol reacts to the variations in node density, and to propose and characterize mechanisms that mitigate these problems.

Any geographic forwarding suffers from this local minimum phenomenon, when all neighbors of the current packet recipient are farther away from the destination than the node itself. To help packets get out of the local minimum, Karp and Kung [7], and independently [1], proposed the idea of combining the greedy forwarding and the perimeter routing on a planar graph which describes the connectivity of the original network. In the context of Bespoken, the local minimum phenomenon arises when the absence of forwarding nodes in a local area results in premature termination of a spoke. To mitigate this effect, we here propose an adaptive mechanism for Bespoken that repairs the spoke when it encounters an empty crescent-shaped area along the spoke axis by seeking a replacement for the last forwarding node. We create a model for the BeSpoken enhanced with the proposed mechanism, and emulate its behavior in terms of decreased percentage of prematurely dying spokes (with respect to what we here refer to as basic BeSpoken), which is also confirmed by the protocol simulation.

A byproduct of the adaptive Bespoken mechanism is a reduced sensitivity of the protocol to variations in node density. The presented results clearly illustrate that the adaptive BeSpoken performs better than the basic one in cases where the applied protocol parameters have been underdesigned for the current network density. This is an important observation as any WSN can become scarcely populated over time due to random node dying. We also illustrate that the adaptive BeSpoken performs well in the presence of network holes, i.e. when sensors are systematically destroyed by hazardous events.

2. SYSTEM MODEL AND PROBLEM STATEMENT

The *basic BeSpoken* protocol organizes a sequence of fixed-power *relay* transmissions that propagate the source message hop-by-hop, without positional or directional information in a dense wireless network. The hop relays form a *spoke* which may deviate from the radial *spoke axis*. Each spoke hop is organized using a sequence of two control message transmissions followed by the hop data transmission. We define the transmission range as the maximum distance from the source at which nodes can reliably receive a packet. We assume that the physical layer modulation and coding are designed to compensate for short-scale fading effects and, thus, our transmit power requirements depend only on distance-dependent propagation path loss. Assuming radially symmetric attenuation (isotropic propagation), the area in which the transmitted packet is reliably received is a disk of a given radius. We use the same transmission power for both data and control packets, but different coding rate and/or modulation format, so that the communication rate for control messages is lower and translates to a longer range.

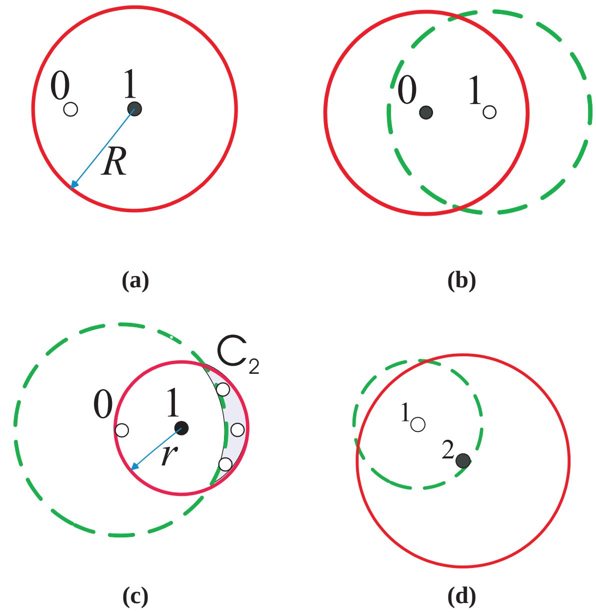

Figure 1: BeSpoken Protocol: **At each protocol stage, the current transmission range is denoted with the full circle while the previous range is denoted with a dashed circle.**

In this section we first give the basic BeSpoken protocol, then introduce protocol design parameters and relate them to two basic requirements that a spoke should satisfy: its propagation distance, alternatively referred to as Outage Constraint, and its deviation from the spoke axis, also referred to as Wobbliness Constraint. Next, we introduce the spoke evolution model and its quantized version. This model allows for quantifying the outage and the wobbliness constraints.

2.1 Basic BeSpoken Protocol

The BeSpoken protocol implements a recursive process illustrated in Figure 1 in the following way:

- **(a)** The leading relay (node 1) sends an RTS (request to send) control packet with range $R = rq$ where $q = 2 - \epsilon$, for small ϵ.
- **(b)** The pivot (node 0) sends a BTS (block to send) control packet with range R.
- **(c)** The leading relay transmits the data packet with range r and becomes the new pivot. The region in which nodes receive this data packet but do not receive the preceding BTS packet forms the *1-st hop crescent* C_2.
- **(d)** A random node from the crescent C_2 becomes the new leading relay by transmitting a new RTS. The process returns to (a) with node 1 as the pivot and node 2 as the leading relay.

This recursive process is initialized by assigning the role of the pivot to the source node which transmits the data packet with a range r. The first node which receives the data packet and gets access to the medium becomes the first leading relay. The underlying ALOHA-type Carrier Sense Multiple Access protocol would resolve any collisions; hence, after a possible additional delay, only one random node from the crescent would transmit the RTS packet.

2.2 Design Parameters and Requirements

The vector from node 0 to node 1 in Figure 1 defines the spoke axis. The crescent subtending angle determines how much the spoke may deviate from the spoke axis direction. The parameter $q = R/r$ determines the maximum crescent subtending angle. A large subtending angle fosters wobbliness, yet it implies a larger crescent, which increases chances that a relay will be found to retransmit data. Fixing q to a small value that limits wobbliness requires increasing r to generate a large enough crescent and decrease the outage probability. Note that the energy per hop grows as r^α, where $\alpha \geq 2$ is the propagation loss coefficient, so that the total energy per spoke grows as $dr^{\alpha-1}$. Hence, minimizing the transmission range r corresponds to a minimum energy objective.

These competing tendencies illustrate the importance of the protocol parameters design. In [8,9], we characterize the spoke behavior as a function of parameters r and q, with respect to the requirements:

- **Propagation Distance/Outage:** the probability that a spoke dies before reaching a distance d is small,

- **Wobbliness:** the deviation of the instantaneous spoke direction with respect to the spoke axis is within defined limits.

Next, we first provide a general BeSpoken model and also summarize the results on the spoke outage probability for the non-adaptive BeSpoken protocol and state the general wobbliness results. In the following section we model the adaptive BeSpoken protocol, which allows for spoke backward repair when it encounters an empty crescent, and quantify its outage probability. In the concluding section, we compare the performance of the two protocol versions in terms of their Outage behavior.

2.3 Markov Process/Chain Model

For the outage constraint, the Markov process of the hop-length L_k, where index k denotes the time step, is formally defined as a fictitious process $\{L_k\}$ that never encounters an empty crescent.

Observe that the region between the radius R control circle and the radius ρ arc defines an *interior crescent*, shown as the shaded area in Figure 2. From geometric arguments, it can be verified that the area of this interior crescent is

$$S_{\text{IC}}(l,\rho) = 2\rho^2\beta(l,\rho) - 2R^2\alpha(l,\rho) + Rl\sin\alpha(l,\rho) \qquad (1)$$

where $\alpha(l,\rho)$ is found from the law of cosines to satisfy $\cos\alpha(l,\rho) = (R^2-\rho^2+l^2)/(2lR)$. Note that L_{k+1} can vary from a minimum value of $R - L_k$ to a maximum value of r. The crescent C_{k+1} induced by the transmission of relay k (such as in Figure 1) has an area $S_c(L_k) = S_{\text{IC}}(L_k, r)$. We note that C_{k+1} is the set of all possible positions of the node $k+1$. Under the fictitious process model, the position of node $k+1$ will be uniformly distributed over the crescent C_{k+1}.

From Figure 2 we see that, given the current hop length $L_k = l_k$, the arc of radius ρ has length $2\rho\beta(l_k,\rho)$. The conditional probability that we find node $k+1$ in the annular segment of width $d\rho$ along the arc of radius ρ is $2\rho\beta(l_k,\rho)d\rho/S_c(l_k)$. It follows that the

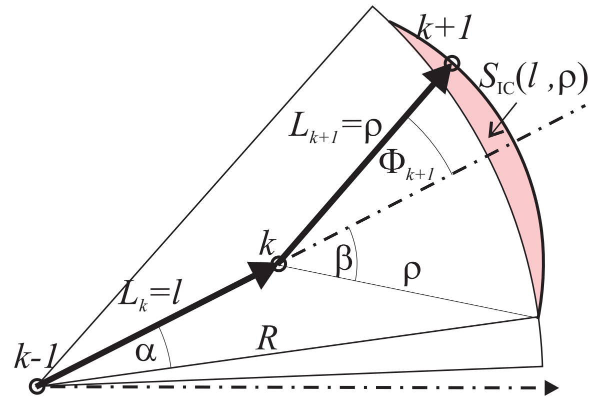

Figure 2: Given $L_k = l$ and $L_{k+1} = \rho$, the angular hop displacement Φ_{k+1} is constrained to the interval $-\beta \leq \Phi_{k+1} \leq \beta$ where the maximum angular displacement at hop $k+1$ is $\beta = \beta(l,\rho)$. The shaded area denotes the interior crescent of area $S_{\text{IC}}(l,\rho)$.

conditional pdf of the next hop length L_{k+1} given $L_k = l_k$ is

$$f_{L_{k+1}|L_k}(\rho|l_k) \;=\; \tfrac{2\rho\beta(l_k,\rho)}{S_c(l_k)} \quad R - l_k \leq \rho \leq r, \qquad (2)$$

and zero otherwise. We note that (2) provides a complete characterization of the fictitious process $\{L_k\}$.

Furthermore, we develop a Markov Chain model that approximates $\{L_k\}$. We start by quantizing the L_k process, yielding the m-state Markov chain $\hat{L}_k$. For each L_k the induced crescent C_{k+1}, which has an area $S_c(L_k) = 2r^2\beta(L_k,r) - 2R^2\alpha(L_k,r) + RL_k\sin\alpha(L_k,r)$, is also approximated by an area $S_c(\hat{L}_k)$, and quantized (see Figure 3). Here, a quantization interval $\mathcal{I}_{ij}$ corresponds to the strip of area

$$d_{ij} \;=\; \begin{cases} \int_{R-h_i}^{h_j} 2\rho\beta(h_i,\rho)\,d\rho, & j = j^*(i), \\ \int_{h_j-\Delta}^{h_j} 2\rho\beta(h_i,\rho)\,d\rho, & j > j^*(i), \end{cases} \qquad (3)$$

(and zero otherwise), and of width $|\mathcal{I}_{ij}|$ within the crescent $\hat{C}_k$ of area $c_i = \sum_j d_{ij}$. Here $j^*(i) = \min\{j : h_j > R - h_i\}$ is the index of the leftmost non-empty quantization interval within I_i. As shown in Figure 3, $c_{ij} = S_{\text{IC}}(h_i, h_j)$ is the *quantized interior crescent area* formed by the control circle (of radius R) centered at the kth hop relay and a circle of radius h_j centered at node $k+1$ at distance $\hat{L}_k = h_i$.

In the m-state uniform-quantization model, the hop-length states $\{h_i\}$ uniformly quantize the process state space $[R-r, r]$ so that $h_i = R - r + i\Delta$, where $\Delta = (2r-R)/m$ is the quantization interval. Furthermore, $j^*(i) = m - i$ so that the next-hop quantization intervals $\mathcal{I}_{ij}$ satisfy $\mathcal{I}_{ij} = (h_j - \Delta, h_j]$ for $j > m - i$ and are empty for $j \leq m - i$. The transition probabilities are now

$$P_{ij} \;=\; \frac{c_{ij} - c_{i(j-1)}}{c_i}, \quad i + j > m, \qquad (4)$$

and $P_{ij} = 0$ whenever $i + j \leq m$ follows since, in that case, $(h_{j-1}, h_j]$ and $\mathcal{I}_i = [R - h_i, r]$ intersect in at most one point.

Additional quantization details can be found in [8].

2.4 BeSpoken Outage Constraint

For analytical tractability, instead of requiring the spoke to travel distance d with high probability, we require it to travel η hops with

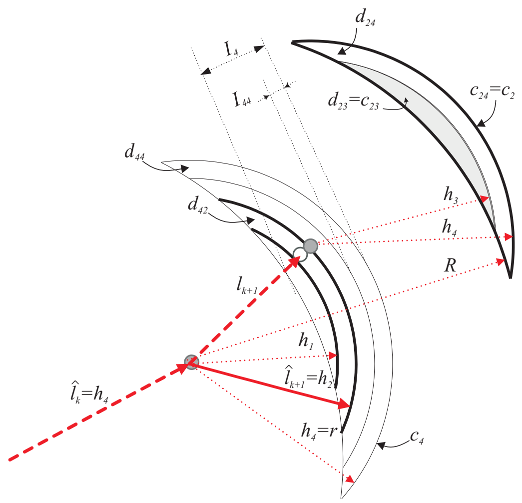

Figure 3: Ergodic Finite State Markov Chain: quantization example for a four-state chain ($m = 4$): $\hat{L}_k = h_4 = r$ results in the first crescent $\hat{C}_k$ of area c_4 partitioned into four strips of total area $c_4 = d_{41} + d_{42} + d_{43} + d_{44}$; L_{k+1}, quantized to $\hat{L}_{k+1} = h_2$, is followed by a crescent $\hat{C}_{k+1}$ of area c_2 and a hop span $I_2 = [R - h_2, r]$ which is (uniformly) quantized into a crescent of area $d_{23} = c_{23}$ (shaded region) and a crescent strip $d_{24} = c_2 - c_{23}$ (the unshaded area).

high probability. In particular, we define $\eta = \lceil d/r \rceil$ as the number of hops corresponding to an idealized straight-line spoke extending to the distance d.

For design purposes we assume that the spatial distribution of network nodes is a planar Poisson point process [13] of intensity $\lambda = 1$. Thus, a current crescent C_{k+1} forms a candidate set for node $k + 1$ with cardinality Z_k that is, conditionally, a Poisson random variable with conditional expected value

$$E[Z_k | L_k = l_k] = S_c(l_k). \tag{5}$$

For non-adaptive BeSpoken protocol, a spoke stops at hop k when the crescent C_{k+1} is empty, i.e., $Z_k = 0$. Since the nodes obey a planar Poisson process, it follows from (5) that the conditional probability the crescent C_{k+1} is empty is

$$\Pr\{Z_k = 0 | L_k = l_k\} = e^{-S_c(l_k)}. \tag{6}$$

In order to formalize the Outage constraint for non-adaptive BeSpoken protocol, we define

$$D^{NA} = \min\{n : Z_n = 0\} \tag{7}$$

as the first time the process encounters an empty crescent.

For the BeSpoken with adaptive mechanisms, a spoke does not stop at hop k when the crescent C_{k+1} is empty, unless the adaptive mechanism initiated at that point fails to repair the spoke. We first present the Outage model for the simpler non-adaptive case, and describe the adaptive Outage model in 3, only after properly introducing and formalizing the adaptive BeSpoken protocol.

2.4.1 Non-adaptive Outage Constraint

The non-adaptive outage constraint can be formalized as

$$\Pr\left\{D^{NA} \leq \eta\right\} \quad \leq \quad p. \tag{8}$$

The fact that a spoke stops at stage k when the crescent C_{k+1} is empty means that the spoke generation is a transient process. The Markov Chain model based on the fictitious hop-length process is appropriately expanded to include the outage event. For the m-state Markov chain, let us denote the event that the first η crescents $\hat{C}_k$, $k = 1, \cdots, \eta$, are not empty as $A_\eta^{NA} = \{\min_{k \leq \eta} Z_k > 0\}$. The probability that the crescents $\hat{C}_1, \ldots, \hat{C}_\eta$ are not empty, and that the system is in state j at time η is denoted $\kappa_j^{(\eta)} = \Pr\left\{\hat{L}_\eta = h_j, A_\eta^{NA}\right\}$. Using Markovity of $\hat{L}_k$ and conditional independence of Z_k given $\hat{L}_k$, it is straightforward to show that

$$\kappa_j^{(\eta)} \quad = \quad \sum_{i=1}^{m} \overline{e_j} P_{ij} \kappa_i^{(\eta-1)} \tag{9}$$

where $\overline{e_j} = 1 - \exp(-\lambda c_j)$ is the probability of a non-empty crescent while in state j. Using a recursive proof, in [8] we show that, given the initial state m, $\kappa_i^{(1)} = 0$ for $i < m$ and $\kappa_m^{(1)} = \overline{e_m}$. Furthermore, as $\Pr\left\{A_\eta^{NA}\right\} = \sum_{i=1}^{m} \kappa_i^{(\eta)}$, the probability that the spoke will stop at or before hop η (assuming that the chain always starts in state h_m) becomes

$$\begin{aligned} \Pr\left\{D^{NA} \leq \eta\right\} &= 1 - \Pr\left\{A_\eta^{NA}\right\} \\ &= 1 - [0 \cdots \overline{e_m}]\, \breve{\mathbf{P}}^{(\eta-1)} [1 \cdots 1]^T, \end{aligned} \tag{10}$$

where

$$\breve{P}_{ij} = P_{ij} \overline{e_j}. \tag{11}$$

The analysis of the outage constraint based on this Markov chain results in the following design rule for the data transmission range: for a spoke to reach η hops with probability p, given q, the range is required to be

$$r \geq 1 / \sqrt{\exp(1) \left(1 - (1-p)^{\frac{1}{\eta-1}}\right) f(q)}, \tag{12}$$

where $f(q) = S_c(r)/r^2$, and $S_c(r)$ is the maximum crescent area for transmission range r. Note that $f(q)$ is the crescent area for unit r (the ratio $S_c(r)/r^2$ does not depend on r).

2.5 BeSpoken Wobbliness Constraint

The evolution of the spoke's current angle is modeled as a zero mean Markov Modulated Random Walk *(MMRW)* [9]. The wobbliness constraint requires that the expected time $E[T_{\varphi_o}]$ until the spoke (i.e. its current angle) hits an angle threshold φ_o (or $-\varphi_o$) is bounded. The expected random-walk stopping time $E[T_{\varphi_o}]$ is obtained through an extension of Wald Identity based on a martingale transform of the MMRW. The above analysis results in a range of q-s that are then evaluated jointly with r, according to the following algorithm [8].

Given the desired distance d, and the angle threshold φ_o
$n = 1, \ldots \infty$
(a) Calculate q^* assuming $n = E[T_{\varphi_o}]$
(b) Given $q = q^*$ from (a), calculate r^* from (12)
(c) If $d/r^* < n$, goto (a) else BREAK.

3. ADAPTIVE BESPOKEN: A STEP TOWARD REAL NETWORKS

In sensor network deployments, spatial distributions of sensors are usually far from being uniform. Such networks often contain regions without enough sensor nodes, which we refer to as holes.

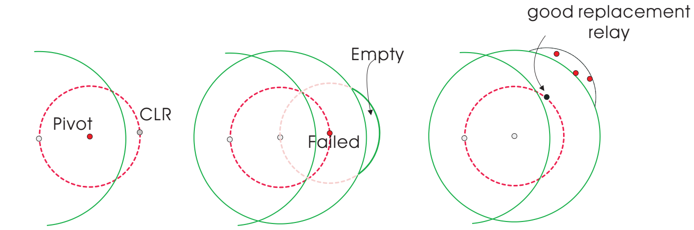

Figure 4: The triptych represents a single transition of the Markov process modeling the adaptive spoke: in the *first* step a new relay is selected by the pivot that becomes the Current Leading Relay (CLR); in the *second* step the CLR becomes aware that its induced crescent is empty, when it does not receive any RTS in due time; in the *third* step the CLR solicits its own replacement by sending the request intended for all the nodes in its own crescent - we show one such peer node that replaces the CLR and, having a non-empty induced crescent, repairs the spoke.

In routing, holes are communication voids that cause greedy forwarding to fail, and non-adaptive BeSpoken suffers from the same vulnerability. We propose to extend basic BeSpoken protocol with adaptive mechanisms to alleviate problems with small-scale network discontinuities (network thinning caused by random node dying, holes, voids). In this section, an adaptive BeSpoken protocol is proposed and its model is analyzed in order to establish a quantitative measure of the protocol performance. Its improvement over the non-adaptive version is evaluated in terms of the gain in the likelihood to achieve a given propagation distance when employing the same protocol parameters.

3.1 BeSpoken Backward Repair Mechanisms

In the following, the crescent to which the relay belongs is *the own crescent*, and the crescent which is formed as a result of relay's transmission is referred to as *the induced crescent*. The *Current Leading Relay (CLR)* activates the BeSpoken adaptive mechanism, since it has the ability to detect an empty candidate set by observing the absence of the RTS request within a time-out period. Upon encountering an empty crescent $S_c(l_k)$ without candidate relays, CLR node k can solicit another pivot from a previous crescent, while keeping its leading-relay status, which would effectively change the current spoke angle. Another *backward-repair* technique requires the CRL to solicit its own replacement from its own crescent $S_c(l_{k-1})$, i.e., among the peer candidate relays. The latter approach is termed the *one-step backward repair* protocol. The example shown in Figure 4 illustrates one possible scenario for the recovery attempt. Here, a replacement relay has been found that creates a large non-empty crescent, hence repairing the spoke. We can envision that even if another replacement relay was selected with an induced crescent smaller than the failed one, but at a sufficient distance from the empty crescent to result in a large enough disjoint area, the chances of repairing the spoke are worth performing this step back.

There is a whole spectrum of adaptive algorithms, nevertheless, for the sake of simplicity and without loss of generality, we only consider the one-step backward repair protocol.

3.2 One-Step Backward Repair Model

We here analyze the adaptive mechanism introduced in 3.1, where the current leading relay (kth node) runs into an empty set of relay candidate nodes, and solicits a single own replacement from its own crescent. The spoke stops when the replacement results in a non-empty relay candidate set. Extensions to the cases with multiple replacement trials are straightforward.

To model the adaptive mechanism, we use the uniform quantization model illustrated in Figure 3, with only two quantization levels, again, for simplicity and without loss of generalization. Hence, the BeSpoken hop-length evolution is modeled by a two-state Markov Chain $\hat{L}_k$. Following the notation from Section II.C, the states 1 and 2 correspond to quantized hop lengths $h_1 = R-r+\Delta = R/2$ and $h_2 = r$, where $\Delta = r - R/2$. The corresponding quantized areas are $c_1 = S_c(R/2)$ and $c_2 = S_c(r)$. To make a better distinction between crescents pertaining to different states, in the present work we will use the notation $s_L = S_c(h_2)$ and $s_S = S_c(h_1)$, to denote the large and the small crescent areas, respectively.

To establish a unifying model for both the hops that are completed without utilizing the adaptive mechanism, and those that are completed in the second attempt, through the adaptive algorithm, we redefine the conditions under which the underlying Markov Chain transitions to a new state. Now the chain does not transition into another state always when the next relay is found, but, as an additional condition, the induced crescent of the selected relay must not be empty. The new definition incorporates a lookahead element with each "hop-length" state to make sure that at least one subsequent hop is possible. This is necessary to seamlessly integrate the adaptive mechanism into the two-state model. This effectively adds a third trapping state T when the subsequent hop is not possible. The transition probabilities that (partially) characterize this redefined Markov Chain form the reduced matrix

$$\mathbf{P_{ad}} = \begin{bmatrix} 0 & P_{12}^A \\ P_{21}^A & P_{22}^A \end{bmatrix}, \tag{13}$$

since, as described in Section II, the chain can not transition to the same state from the state corresponding to the small crescent. The reduced matrix $\mathbf{P_{ad}}$ does not include the transitions to trapping state T. In order to formally define transition probabilities P_{ij}^A, we require additional notation. The data propagation will stop if both the induced crescent of the CLR and of its replacement are empty. Trivially, if there are no replacements in the current crescent, it is sufficient that the first induced crescent is empty. Let us denote the cardinality of the union of the two candidate relay sets (in these two induced crescents) with Z_k^2. Now, it follows that the transition probability (with lookahead) can be expressed as

$$P_{ij}^A = \Pr\left\{L_k = h_j, Z_k^2 > 0 | L_{k-1} = h_i, Z_{k-1} > 0\right\} \tag{14}$$

Note that $\mathbf{P_{ad}}$ is the adaptive counterpart (for the *One-Step Backward Repair BeSpoken Protocol*) of the transition matrix $\hat{\mathbf{P}}$ whose elements are given in (11).

The envelope of all possible replacing crescents created from state $p = 1$ is shown in Figure 10. The envelopes for the large current crescent are presented in Figures 11 and 12. Figure 11 fixes the replacement relay's state to state 1, and illustrates two general cases of the relative position of the failed CLR and its replacement. Their relative position determines the intersection of the induced crescents, where the support set of the replacement crescent is approximated with so-called small envelope. Figure 12 fixes the replacement relay's state to state 2, which brings forth so-called large envelope. In the following we evaluate $\mathbf{P_{ad}}$ for a Poisson node distribution and using area linearization when approximations are necessary.

The probability of repair for the two-state Markov chain model, given that the failed leading relay was in state $n \in [1, 2]$, and that the pivot was in the state $p \in [1, 2]$, depends on the position of the

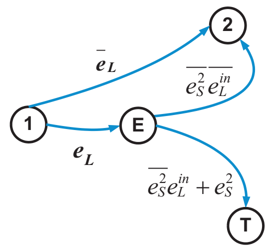

Figure 5: Transitions of the adaptive mechanism contributing to the transition probability P_{12}^{A}; here E denotes the empty crescent in state 2, and T denotes the trapping state, when the repair attempt fails.

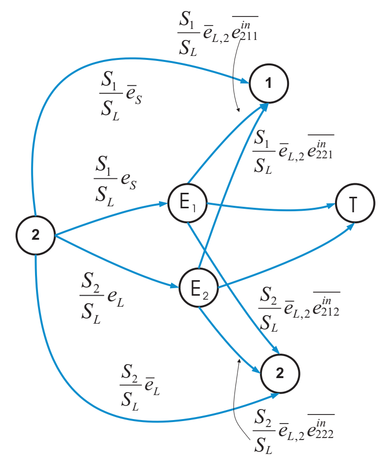

Figure 6: Transitions of the adaptive mechanism contributing to the transition probabilities P_{21}^{A} and P_{22}^{A}; here E_1 denotes the empty crescent in state 1, E_2 denotes the empty crescent in state 2, and T denotes the trapping state, when the repair attempt fails.

replacement relay, more precisely, on its quantization level, and on how much of its induced crescent area is disjoint from the crescent formed by the failed relay. We refer to this induced disjoint areas as the *innovation area*. The average innovation area, denoted with $\overline{S_{in}^{p,n}}$, is calculated as the average difference

$$\Delta^{n,m} = \left(S_c^n \bigcup S_c^m\right) - S_c^n,$$

between the first (failed) crescent area $S_c^n(k+1)$ and the crescent area $S_c^m(k+1)$ formed with the replacement relay. Here, m denotes the quantization state of the replacement relay.Hence,

$$\overline{S_{in}^{p,n}} = \sum_m P_{pm} E\left[\Delta^{n,m}\right]_{\Xi(pm)},$$

where the average is taken first over the envelope of possible crescents induced by replacement relays in state m, denoted with $\Xi\,(pm)$.

The average probability of finding a non-empty set of candidate relays for the next hop is

$$\overline{e_{p,m,n}^{in}} = E\left[\left(1 - e^{-S_{in}^{m,n}(k+1)}\right)\right]_{\Xi(pm)},$$

given that the pivot was at the state p, the failed relay was at the state n, and that a replacement relay was found corresponding to the MC transition from state p to some state m. Here, the averaging was done over the possible relative positions of the crescents (of respective areas) $S_c^m(k+1)$ and $S_c^n(k+1)$, the relative position being a random variable whose support set is determined by n and p, and ultimately by $\Xi\,(pm)$. In the appendix we calculate an approximation for $\overline{e_{p,m,n}^{in}}$ by linearizing both the envelope area and the area of the crescent.

Let $e_L = e^{-s_L}$ and $e_S = e^{-s_S}$ denote the probabilities of large and small crescent being empty, respectively; $\overline{e_L} = \left(1 - e^{-s_L}\right)$ and $\overline{e_S} = \left(1 - e^{-s_S}\right)$ denote the probabilities of large and small crescent having at least one node; $\overline{e_{L,2}} = \left(1 - e^{-s_L}\left(1 + s_L\right)\right)$ and $\overline{e_{S,2}} = \left(1 - e^{-s_S}\left(1 + s_S\right)\right)$, denote the probabilities that at least two nodes will be found within the crescent of area s_L and s_S.

The introduced notation is used to represent probabilities of different repair paths contributing to a particular adaptive BeSpoken transition, as illustrated in Figures 5 and 6, when pivot is in state 1 and 2, respectively. For example, Figure 5 states that to transition from state 1 to state 2 one can transition directly if CLR's induced crescent is not empty. If it is empty, its replacement exists, and its replacement's induced crescent is not empty, it will transition to state 2 in the second attempt. Otherwise, it will transition to state T and the spoke will stop. Similarly for Figure 6 which describes the repair paths for transitions from state 2. The edges of state diagrams are denoted with probabilities for each corresponding transition event. Hence, by summing the probabilities of the contributing transition repair paths, we obtain the elements P_{ij}^{A} of the matrix $\mathbf{P_{ad}}$ as follows:

$$\begin{aligned}
P_{11}^{A} &= 0 && (15)\\
P_{12}^{A} &= \overline{e_L} + e_L\overline{e_{S,2}}\overline{e_{122}^{in}} && (16)\\
P_{21}^{A} &= \frac{s_1}{s_L}\left[\overline{e_S} + \overline{e_{L,2}}\left(\frac{s_1}{s_L}e_S\overline{e_{211}^{in}} + \frac{s_2}{s_L}e_L\overline{e_{221}^{in}}\right)\right] && (17)\\
P_{22}^{A} &= \frac{s_2}{s_L}\left[\overline{e_L} + \overline{e_{L,2}}\left(\frac{s_1}{s_L}e_S\overline{e_{212}^{in}} + \frac{s_2}{s_L}e_L\overline{e_{222}^{in}}\right)\right]. && (18)
\end{aligned}$$

The derivation details are given in the appendix.

3.2.1 Adaptive Outage Constraint

In order to formalize the Outage constraint for the One-Step Backward Repair protocol, we now define

$$D^A = \min\left\{n : Z_n^2 = 0\right\} \tag{19}$$

as the first time the process fails to repair the spoke if it encounters an empty crescent. Hence, the adaptive outage constraint can be expressed as

$$\Pr\left\{D^A \leq \eta\right\} \quad \leq \quad p. \tag{20}$$

Let us denote the event that the spoke does not stop in the first η hops as

$$A_\eta^A = \left\{\min_{k\leq\eta} Z_k^2 > 0\right\}.$$

The probability that the spoke does not stop in the first η hops, and that the system is in state j at time η is

$$\mu_j^{(\eta)} = \Pr\left\{\hat{L}_\eta = h_j, A_\eta^A\right\}.$$

Using Markovity of $\hat{L}_k$ and conditional independence of Z_k^2 given $\hat{L}_{k-1}$, it is straightforward to show that for the adaptive BeSpoken

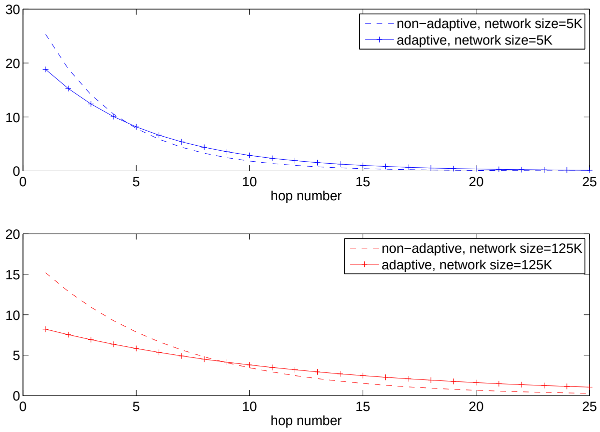

Figure 7: Percentage of spokes dying at each hop based on two-state uniformly quantized Markov Chain model: adaptive mechanism decreases the probability of spokes dying prematurely

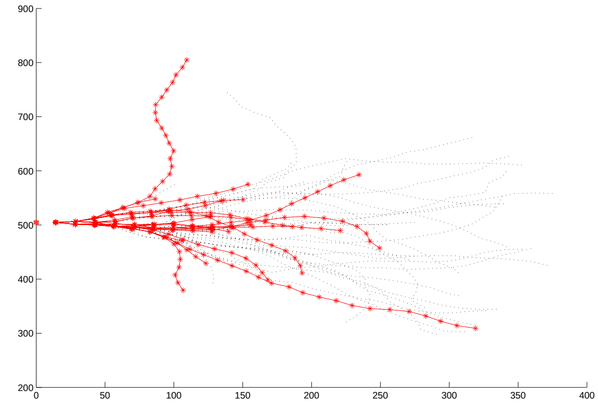

Figure 8: Two samples of spokes directed eastward in a *thinning* network where the density of nodes is decreased to one half of the initial density due to random node dying: thin-line spokes are created by the adaptive BeSpoken and asterix-marked spokes created by the basic BeSpoken; One-Step Backward adaptive protocol demonstrates better performance than the basic BeSpoken i.e. more spokes survive in the thin-line cloud that in the cloud of asterix-marked spokes.

the expression (9) becomes

$$\mu_j^{(\eta)} = \sum_{i=1}^{m} P_{ij}^{A} \mu_i^{(\eta-1)}. \tag{21}$$

By defining the vector $\mu^{(\eta)} = [\mu_1^{(\eta)}, \cdots, \mu_m^{(\eta)}]$, (21) becomes $\mu^{(\eta)} = \mu^{(\eta-1)}\mathbf{P_{ad}}$. Recursively, we obtain $\mu^{(\eta)} = \mu^{(1)}\left(\mathbf{P_{ad}}\right)^{\eta-1}$. Given the initial state m, and $\mu_i^{(1)} = 0$ for $i < m$, $\mu_m^{(1)} = \overline{e_m}$, we obtain $\mu^{(\eta)} = [0 \ \cdots \ \overline{e_m}]\left(\mathbf{P_{ad}}\right)^{\eta-1}$.

As $\Pr\left\{A_\eta^A\right\} = \sum_{i=1,\cdots,m} \mu_i^{(\eta)} = \mu^{(\eta)}\left[1 \ \cdots \ 1\right]^T$, the probability that the spoke will stop at or before hop η (assuming that the chain always starts in state h_m) becomes

$$\begin{aligned}\Pr\left\{D^A \leq \eta\right\} &= 1 - \Pr\left\{A_\eta^A\right\} \\ &= 1 - [0 \cdots \overline{e_m}]\left(\mathbf{P_{ad}}\right)^{(\eta-1)}[1 \cdots 1]^T \end{aligned} \tag{22}$$

4. CONCLUSION: NUMERICAL ANALYSIS AND SIMULATION RESULTS

We have numerically evaluated the outage probability based on the two-state uniformly quantized Markov Chain model, both for the basic BeSpoken (10) and its adaptive version (22). Due to a small number of quantization levels an error is introduced, but the evaluation is comparative, and we expect that the error is unbiased. In both cases Markov Chain transition probabilities were calculated using the same design parameters (based on (12) and 2.5). The results presented in Figure 7 for two different network sizes show that the adaptive algorithm works better. We also support our analysis with simulation results. For the same random network instances, we ran simulations of basic and adaptive versions of the protocol whose design parameters are based on the guidelines in [8] ((12) and the algorithm in 2.5), given a network of uniformly distributed nodes with unit density and known size.

The first experiment was designed to compare the performance of the two protocols for a thinned network whose node density is lower than the one used for the protocol parameter (r and q) design. Hence, the network nodes were deployed in a uniform manner over a square region ensuring a half-unit density. Spoke traces are generated as by extending a large number of spokes to follow the same direction. The adaptive mechanism used here is the analyzed One-Step Backward adaptive protocol. The overlapping traces of both protocol variants are shown in Figure 8. The presented result clearly illustrates that the adaptive BeSpoken performs better in cases where the applied protocol parameters have been underdesigned for the current network density. This is an important observation as any WSN can become scarcely populated due to random node dying.

The second simulation experiment was designed to evaluate the expected better performance of the adaptive BeSpoken for a network with a hole (small unpopulated network area). For established design parameters (r and q), we simulated a stationary network of unit density, with uniformly distributed nodes deployed over a square region, where all nodes in the bounded region highlighted in Figure 9, have been removed. Again, spoke traces are generated as we extended a large number of spokes to follow the same direction. The overlapping traces of both protocol variants are shown in Figure 9. The adaptive mechanism used here is Two-Step Backward adaptive protocol, as we expected that once the hole is encountered, the spoke needs to significantly change its direction in order to avoid the void. The presented result illustrates that this adaptive BeSpoken performs better in the presence of holes.

We suggest that each WSN application can be associated with higher probability of irregularities of a particular type. Hence, the appropriate adaptive protocol can be selected according to the application. For example, a WSN deployed for environmental monitoring over long time periods is likelier to suffer from network thinning, while a WSN deployed in disaster areas has higher chances to experience network holes as sensors can be systematically destroyed by hazardous events.

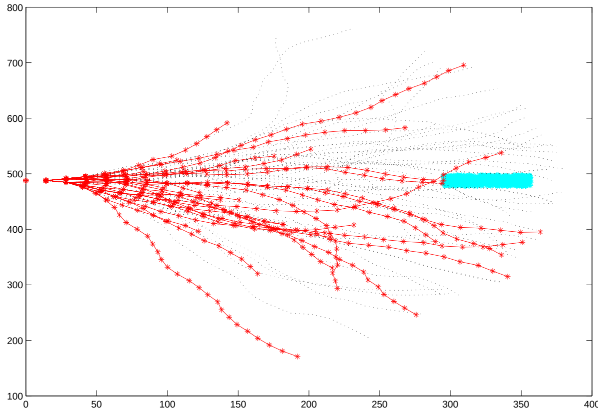

Figure 9: **Two samples of spokes directed eastward in a network where a *hole* has been created due to node destruction: thin-line spokes are created by the adaptive BeSpoken and asterix-marked spokes created by the basic BeSpoken; Two-Step Backward adaptive protocol demonstrates better performance than the basic BeSpoken.**

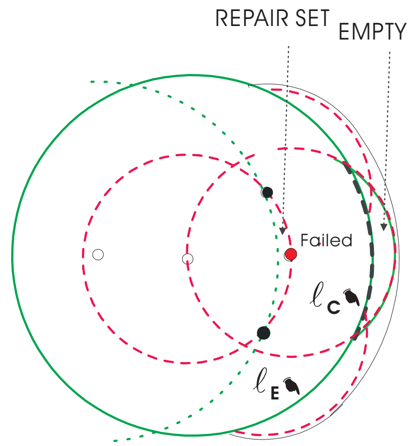

Figure 10: Adaptive Envelopes for Pivot State 1: Large envelope, large failed crescent (replacement must stay in state 2, failed CLR in state 2). The pointers indicate the length of the envelope ℓ_E and the length of the failed crescent ℓ_c.

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APPENDIX

A. ADAPTIVE MECHANISM DERIVATIONS

When calculating the average probability of a hop in One-Step Backward Repair Model, we approximate the crescent areas with the appropriate rectangular areas. The area of the envelope over which the average is taken, is calculated as the length of its lower boundary times its width, which equals $(r - R/2)$ for $m = 1$, and $(2r - R)$ for $m = 2$.

The same stands for the "‘inserted"’ (failed) crescent: the area is the product of its lower boundary and the width. We distinguish between two distinct areas, $s_S \approx l_{cs}\,(r - R/2)$, and $s_L \approx l_{cl}\,(2r - R)$, approximating the small and the large crescent area, respectively. Here, l_{cs} and l_{cl} are the lower boundaries of the respective crescents.

From Figure 10, for $p = 1$, $m = 2$, $n = 2$, we observe that the envelope length is $l_E = R(\cos^{-1}\left(R^2 - 2r^2\right)/\left(2r^2\right) + 2\cos^{-1}\left(3R^2 - 4r^2\right)/\left(4rR\right))$ and the crescent length is $l_{cl} = R\cos^{-1}\left(R^2 - 2r^2\right)/\left(2r^2\right)$.

In order to provide a universal representation of all the different cases of envelopes and relative positions of the failed relay, we now introduce the following notation

$$l_E = l_c\,(2 + \xi)\,, \tag{23}$$

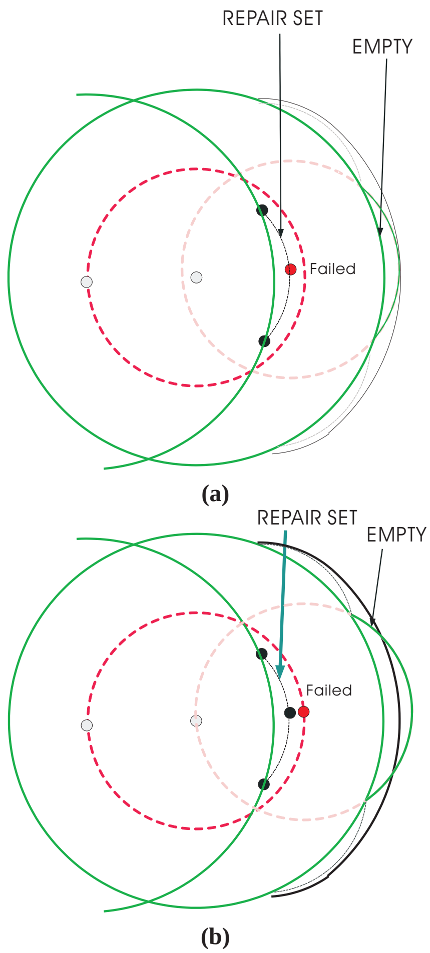

Figure 11: Small Adaptive Envelopes for Pivot State 2 (Replacement Relay in State 1): (a)Small envelope, small crescent (failed CLR also in state 1). (b) Small envelope, large crescent (failed CLR in state 2).

where, for $p = 1, m = 2$ and $n = 2$, $\xi = 2\frac{\cos^{-1}\frac{3R^2-4r^2}{4rR}}{\cos^{-1}\left(R^2-2r^2\right)/\left(2r^2\right)} - 1$ and $l_c = l_{cl}$.

Note the pointers in Figure 10 showing the envelope length l_E as the length of the large circular segment at the bottom of the envelope, and l_c as the length of the smaller circular segment. For other envelope cases, l_E and l_c refer to the analogous circular segments.

Next, in Figure 11 (a), for $p = 2$, $m = 1$, $n = 1$, the envelope length is

$$l_E = R(2\cos^{-1}\frac{5R^2-4r^2}{4R^2} + 2\cos^{-1}\frac{3R^2-4r^2}{4rR})$$

and the crescent length is $l_{cs} = 2R\cos^{-1}\left(5R^2-4r^2\right)/\left(4R^2\right)$; hence,

$$l_E = l_c\left(2+\xi\right), \tag{24}$$

where now

$$\xi = \frac{\cos^{-1}\frac{3R^2-4r^2}{4rR} - \cos^{-1}\frac{5R^2-4r^2}{4R^2}}{\cos^{-1}\frac{5R^2-4r^2}{4R^2}} \tag{25}$$

and $l_c = l_{cs}$.

In Figure 12 (a), for $p = 2$, $m = 2$, $n = 2$, the envelope length is

$$l_E = 3R\cos^{-1}\left(R^2-2r^2\right)/\left(2r^2\right) = 3l_c$$

and the crescent length is

$$l_{cl} = R\cos^{-1}\left(R^2-2r^2\right)/\left(2r^2\right)$$

.

Hence, we have

$$l_E = l_c\left(2+\xi\right), \qquad \xi = 1 \text{ and } l_c = l_{cl}. \tag{26}$$

Figure 11 (b) and Figure 12 (b) respectively illustrate cases when the failed relay was from state $n = 2$, and the relay attempting to repair is from state $m = 1$, and vice versa.

The probability of the repair success, for the case presented in Figure 11 (b), can be aproximated with the probability of success when both relays are from state $m = n = 1$ (Figure 11 (a)).

For the case presented in Figure 12 (b), i.e. $n = 1$, $m = 2$, note that the probability of repair success is larger than the probability of success when both relays (i.e. m and n) are in state 2 (Figure 12 (a)), as the innovation area is larger by a constant factor $\tilde{s} = s_L - s_S$.

Having introduced the universal notation, we calculate the average probability of repair success according to the following formula

$$\begin{aligned}
\overline{e^{in}_{p,m,n}} &= E\left[\left(1-e^{-S^{m,n}_{in}(k+1)}\right)\right]_{\Xi(pm)} \\
&= \frac{1}{(1+\xi)^2{l_c}^2} \\
&\quad \left(\int_0^{l_c}\int_0^{y}\left(1-e^{-\left(\frac{y-x}{l_c}s_1+\tilde{s}\right)}\right)d\,x\,d\,y\right. \\
&\quad + \int_{l_c}^{(1+\xi)l_c}\int_{y-l_c}^{y}\left(1-e^{-\left(\frac{y-x}{l_c}s_1+\tilde{s}\right)}\right)d\,x\,d\,y \\
&\quad + \int_{l_c}^{(1+\xi)l_c}\int_0^{y-l_c}\left(1-e^{-s_2}\right)d\,x\,d\,y \\
&\quad \int_0^{\xi l_c}\int_y^{y+l_c}\left(1-e^{-\left(\frac{x-y}{l_c}s_1+\tilde{s}\right)}\right)d\,x\,d\,y \\
&\quad + \int_{\xi l_c}^{(1+\xi)l_c}\int_y^{(1+\xi)l_c}\left(1-e^{-\left(\frac{x-y}{l_c}s_1+\tilde{s}\right)}\right)d\,x\,d\,y \\
&\quad \left.\int_0^{\xi l_c}\int_{y+l_c}^{(1+\xi)l_c}\left(1-e^{-s_2}\right)d\,x\,d\,y\right),
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
&\xi = 2\frac{\varphi^{12}}{\varphi^{22}} - 1 \text{ for } p = 1, m = 2 \\
&\xi = \frac{\varphi^{12} - \cos^{-1}\left(\frac{5R^2-4r^2}{4R^2}\right)}{\cos^{-1}\left(\frac{5R^2-4r^2}{4R^2}\right)} \text{ for } p = 2, m = 1 \\
&\xi = 1 \text{ for } p = 2, m = 2 \\
&s_1 = s_L \text{ and } s_2 = s_L \text{ for } p = 1, m = 2 \\
&s_1 = s_S \text{ and } s_2 = s_S \text{ for } p = 2, m = 1 \\
&s_1 = s_L \text{ and } s_2 = s_L \text{ for } p = 2, m = 2, n = 2 \\
&s_1 = s_S \text{ and } s_2 = s_L \text{ for } p = 2, m = 2, n = 1.
\end{aligned}$$

Here, $\varphi^{ij} = \beta(h_i, h_j)$, denotes the angular displacement associated with transition from state i to state j (when the previous hop length is h_i, and the current length is h_j); Also, $\tilde{s} = s_2 - s_1$.

Solving the integrals in (27) results in a closed-form expression for the probability of repair

$$\overline{e^{in}_{p,m,n}} = \frac{\xi^2}{(1+\xi)^2}\left(1-e^{-s_2}\right) + \frac{2}{(1+\xi)^2}\left(0.5 - \frac{e^{-\tilde{s}}}{s_1} + \frac{e^{-\tilde{s}}}{s_1^2}\left(1-e^{-s_1}\right) + \xi - \xi\frac{e^{-\tilde{s}}}{s_1}(1-e^{-s_1})\right). \quad (28)$$

The following algorithm calculates the transition probabilities of the Markov Chain that models the BeSpoken enhanced with One-Step Backward Repair mechanism.

$P^A_{pm} = P^B_{pm}$

for $n = (1,2)$

{

$P^A_{pm} += S_T(p,n)e^{-S_c(2,n)}(1-e^{-(S_c(2,p)-1)})S_T(p,m)\overline{e^{in}_{p,m,n}}$

} (29)

$P^A_{11} = 0,$ (30)

where $P^B_{pm} = \check{P}_{pm}$ are the following transition probabilities, related to the first attempt to find the next relay, i.e. without utilizing the adaptive mechanism:

$$\begin{aligned} P^B_{11} &= 0 \\ P^B_{12} &= \overline{e_L} \\ P^B_{21} &= \frac{s_1}{s_L}\overline{e_S} \\ P^B_{22} &= \frac{s_2}{s_L}\overline{e_L}, \end{aligned} \quad (31)$$

and coefficients $S_T(p,n)$ and $S_c(p,n)$ are the elements of two matrices associated with the two-state adaptive Markov Chain where the two-level uniform quantization approximation is applied, in that $s_1 = s_S$, and $s_2 = s_L - s_S$

$$\mathbf{S_C} = \begin{bmatrix} 0 & s_L - s_S \\ s_S & s_L - s_S \end{bmatrix}, \quad (32)$$

$$\mathbf{S_T} = \begin{bmatrix} 1 & 1 \\ s_S/s_L & 1 - s_S/s_L \end{bmatrix}. \quad (33)$$

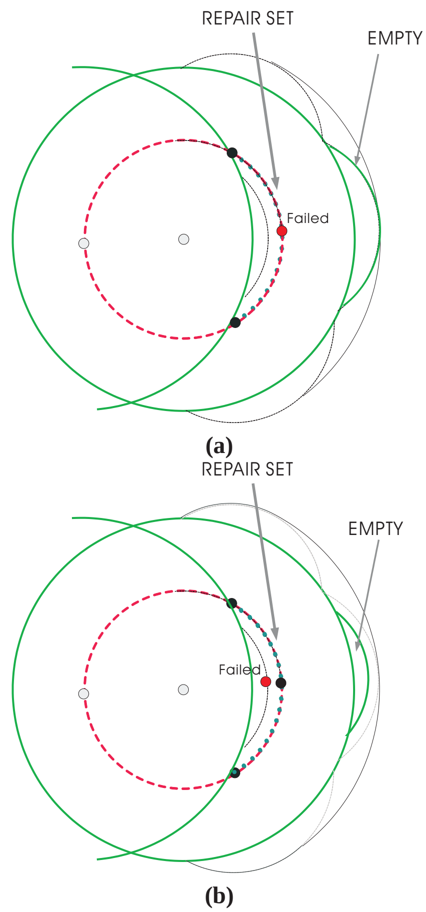

Figure 12: Large Adaptive Envelopes for Pivot State 2 (Replacement Relay in State 2): (a) Large envelope, large failed crescent (failed CLR in state 2). (b) Large envelope, small failed crescent (failed CLR in state 1).