SPREAD SPECTRUM INTERFERENCE ISSUES IN THE 900 MHZ ISM BAND

BY RAJNIISH SINHA

A thesis submitted to the
Graduate School—New Brunswick
Rutgers, The State University of New Jersey
in partial fulfillment of the requirements
for the degree of
Master of Science
Graduate Program in Electrical Engineering

Written under the direction of
Professor Roy D. Yates
and approved by

________________________________________

________________________________________

________________________________________

________________________________________

New Brunswick, New Jersey
January, 1997
ABSTRACT OF THE THESIS

Spread Spectrum Interference Issues In The 900 MHz ISM Band

by Rajnish Sinha

Thesis Director: Professor Roy D. Yates

In this thesis we investigate the use of the 902-928 MHz unlicensed band for wireless communications. This band has been allocated by the FCC (Federal Communications Commission) for industrial, scientific and medical (ISM) applications. Although a license is not required to operate in this band, one does need to comply with a minimum set of rules laid down by the FCC.

Due to the high cost of licensing of radio spectrum, the unlicensed band offers a free and attractive alternative to service providers. As a consequence, this band is witnessing a rapid growth in both the number of systems as well as the diverse technologies being used by them. However, this latitude comes at a price in the form of susceptibility to excessive mutual interference. Since the systems operate independently and do not cooperate with each other, it becomes necessary for them to be able to sustain the mutual interference.

Among the various products operating in the ISM band, this thesis will focus on a particular class of products designated by the FCC as the ‘15.247 Devices.’ Part 15.247 devices include wireless LANs, Cordless phones and automatic meter readers.
(AMRs). These devices have proliferated the ISM band and their numbers continue to rise. Since the FCC requires that all 15.247 devices use spread spectrum communications, the systems use either DS-SS (Direct Sequence Spread Spectrum) or FH-SS (Frequency Hopping Spread Spectrum). Due to the difference between these communication technologies, the mutual interference experienced has different characteristics as well.

In this thesis, we evaluate the performance of DS-SS and FH-SS systems under the influence of mutual interference. We have developed simple yet general models that represent some of the DS and FH systems operating in this spectrum. Through the models, we achieve an understanding of the problems encountered by these systems due to their susceptibility to excessive mutual interference.
Acknowledgements

I would like to thank my advisor Dr. Roy D. Yates for his patience and encouragement during the work on this thesis. I would also like to thank Dr. Jack M. Holtzman and Dr. C. Rose for being on my thesis committee. Finally, I would like to thank all my relatives and friends for their constant love and support.
Dedication

To my parents
Capt. and Mrs. P. K. Sinha
# Table of Contents

Abstract ................................................................. ii

Acknowledgements ......................................................... iv

Dedication ................................................................... v

List of Figures ................................................................. viii

1. Introduction ................................................................. 1

   1.1. Previous work on Mutual Interference between DS and FH Spread Spectrum Systems .................................................... 1

   1.2. Comparison with Cellular and other Unlicensed bands ............. 2

   1.3. The ISM Band and the Part 15 Devices ................................. 3

   1.4. Rules for 15.247 FH & DS Devices .................................... 4

   1.5. Objectives and System Models ......................................... 4

2. System Models ............................................................... 6

   2.1. The FH Model ........................................................ 6

   2.2. The DS Model ......................................................... 12

      2.2.1. The Physical Layer Technology ............................... 12

      2.2.2. CSMA/CA - The MAC Layer Protocol ....................... 12

      2.2.3. The Model ...................................................... 14
2.2.4. Effect of the DS System’s Busy and Idle Periods on an FH Transmission ........................................ 15

3. Probability of Unacceptable DS Interference Experienced by an FH Message at the FH Base Station .......................................................... 20
   3.1. The Power Spectral Density of a DS-SS signal .......................................................... 20
   3.2. The Effect of DS Interference on the FH System ..................................................... 23
   3.3. The Effect of the Ratio of Interferer (DS) and Source (FH) Distances and their Transmitted Powers on the Fraction of FH Frequencies Blocked .................................................. 28

4. The Effect of Narrowband (FH) Interference on a Wideband (DS) System .......................................................... 37

5. Conclusions and Future Work .......................................................... 49
   5.1. FH as Source and DS as Interferer .......................................................... 49
   5.2. DS as Source and FH as Interferer .......................................................... 50
   5.3. Suggested Future Work .......................................................... 50

References .......................................................... 52
List of Figures

2.1. Message transmission process at an FH terminal ........................................ 8

2.2. Probability of at least one FH message being received OK vs. the total number of FH units activated. \( P\{\mathcal{G}\} \) is the probability of a non-colliding FH message being received correctly, and \( m \) is the number of message repetitions in an FH burst. .................................................. 11

2.3. Probability of at least one FH message being received OK vs. the total number of FH units activated \( K \) for different values of \( m \). \( P\{\mathcal{G}\} \) is fixed at 0.9. .......................................................... 11

2.4. \( P\{W\} \) versus \( K \) for different values of message repetitions and \( P\{\mathcal{G}\} \). \( m \) and \( n \) are the number of message repetitions for \( P\{\mathcal{G}\} = 0.5 \) and 0.9, respectively. ................................................................. 12

2.5. An M/M/1 model for the ‘compound’ DS transmitter .................................... 14

2.6. Message delivery process in the combined DS system ................................. 15

2.7. Comparison of FH message transmissions with the busy and idle periods of the DS system for different values of throughput \( \tau \) and mean frame lengths \( E[L] \) (in octets). .................................................. 17

2.8. Busy and Idle periods of the DS system for different mean values of frame lengths \( E[L] \) and throughput \( \tau \). ........................................................... 18

2.9. Probability that an FH message doesn’t collide with a DS transmission \( v/s \) DS throughput \( \tau \) for DS frame lengths \( E[L] = 250, 500, 750, \ldots, 2500 \) octets ........................................... 18
3.1. (a) System Model (b) A more descriptive version of the same illustrating the different parameters used in the analysis. The table lists the signal and distance variables for the DS/FH model ............................ 21

3.2. Plot of $|Z(f)|^2$ ................................................. 24

3.3. Plot of $\text{sinc}^2|fT_c||Z(f)|^2$ ................................. 24

3.4. Spectrum of a DS terminal's transmission ......................... 25

3.5. A superposition of DS and FH spectra to illustrate interfering frequencies at the FH receiver. .................................................. 27

3.6. SNR [dB] (FH signal $S$ to AWGN $N$) versus $d_w$ the FH terminal's distance from its base station. ................................. 34

3.7. Fraction $P\{\mathcal{G}\}$ of FH frequencies that are not blocked by the DS interference versus ratio of distances $(d_u/d_w)$ for different ratio of transmitter powers .................................................. 34

3.8. The lower and upper cutoff-ratio $(d_u/d_w)_a$ and $(d_u/d_w)_b$ as a function of $D/F$ the ratio of DS to FH transmitter powers. ......................... 35

3.9. (a) None of the FH hops are blocked since the DS interferer power is below the threshold (b) As the DS source comes closer to the FH base and exceeds the threshold, it suddenly blocks almost all the hops due to its 'broad-shouldered' PSD ........................... 36

4.1. A Simplified DS Receiver ........................................ 38

4.2. The received DS and FH signals (a) before despreading and (b) after despreading ......................................................... 39

4.3. The spatial distribution model for FH users (interferers). .................. 42

4.4. We assume that the DS receiver is located very close to the FH base station. This causes the maximum interference to the DS signals at their receiver and represents a worst-case scenario. ......................... 44
4.5. The SIR $\Gamma^*$ (limiting case) of the DS source as a function of its distance $d$ from its base station for different values of the FH terminal density $\rho$.
Each curve corresponds to a certain ratio of DS/FH transmitter power $D/F$. ................................. 48
Chapter 1

Introduction

The FCC has allocated 26 MHz of spectrum (902-928 MHz) known as the Unlicensed band for Industrial, Scientific and Medical (ISM) applications. Since a license is not required to operate in this band, this portion of the spectrum is witnessing a very rapid growth in the number of services being offered. Consequently, the devices operating in the ISM band often experience excessive mutual interference. In this work, we study the effects of interference of a particular system operating in the ISM band on a coexisting one. The details of our goal will become more apparent in the following sections.

1.1 Previous work on Mutual Interference between DS and FH Spread Spectrum Systems

Mutual interference between narrowband and wideband signals has been under investigation for quite some time now. Since earlier commercial systems used narrowband signals for communication, various wideband overlay techniques have been suggested to increase the bandwidth utilization [1, 2]. Spread spectrum communications, namely DS-SS, has been a popular choice due to its inherent resistance to narrowband interference. Researchers have long been studying the effects of overlaying a wideband CDMA system on an existing narrowband system [3]-[11]. Most of these works employ narrowband suppression and rejection techniques which limit the mutual interference between the co-existing systems. In this thesis, we assume that the devices operating in the ISM band are relatively simple in complexity and do not use any narrowband rejection or
suppression techniques. This is in fact the case with most of the systems currently operating in the ISM band. Also, since we are mainly interested in the worst-case mutual interference scenario, the above assumption is a reasonable one.

Research activity related to implementation of spread spectrum technologies in the ISM band is also gaining momentum and the focus recently has been on different design alternatives for new DS and FH techniques [13, 14] as well as some hybrid FH/DS systems [12]. We consider simple DS and FH systems that capture the essence of the problem at hand. Our derivations of the mutual interference are based upon the approach adopted in [15, 23]. To evaluate the effect of the interference however, we took into account the following factors which have been neglected in most of the references cited above.

One of the most important things that this thesis takes into consideration is the effect of the traffic characteristics of the DS and FH sources. This is crucial because the interference experienced at any instant is a function of the activity at the source and interferer terminals. Our interference analysis also takes into account the shape of the DS spectrum. The shape plays an important role because the DS interference caused to the FH system is non-uniformly distributed over the band of interest, and hence, different FH hops are susceptible to different levels of interference. To facilitate a better physical interpretation of the effects of mutual interference, we chose to evaluate certain key parameters such as the SIR and the number of frequency hops that are blocked as a function of the following parameters. The main variables in our analysis consist of the relative distances of the source and interferer from the receiver of interest, their relative transmitter powers, and their respective user densities.

1.2 Comparison with Cellular and other Unlicensed bands

Since there exists yet another unlicensed band called the U-PCS (Unlicensed Personal Communications Services) band (1910-1930 MHz), it is worthwhile to mention the differences between the U-PCS and the ISM band. The devices operating in the U-PCS
band follow a set of rules [19] called the “etiquette” which, by itself, is neither a specification nor a standard. The existence of the different systems offering services in this band is based on a “dynamic coordination” of spectrum usage, which follows three basic principles: (1) listen-before-transmit (LBT) protocol (2) limited transmitter power, and (3) limited time duration of transmission. In contrast, the ISM band advocates no explicit cooperation between the operating systems. The system engineers must design their systems to sustain the mutual interference. Table 1.2 illustrates the similarities and differences between the Cellular, ISM and the U-PCS bands.

<table>
<thead>
<tr>
<th>ISSUES</th>
<th>CELLULAR</th>
<th>ISM</th>
<th>U-PCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial Frequency Reuse</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Access Control</td>
<td>Centralized</td>
<td>None</td>
<td>Etiquette</td>
</tr>
<tr>
<td>Power Control</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Predictability of Interference</td>
<td>High</td>
<td>Low</td>
<td>Moderate</td>
</tr>
<tr>
<td>Control over New Services</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
</tbody>
</table>

Table 1.1: Cellular vs. ISM vs. U-PCS Bands

1.3 The ISM Band and the Part 15 Devices

In the hierarchy of products operating in the ISM band, there is a specific class known as the ‘Part 15 Devices.’ As shown in Table 1.2, these devices are further classified into 15.247 (Cordless phones, Wireless LANs, Automatic Meter Readers (AMR), etc.) and 15.249 (garage door openers, Cordless phones, inventory tags, AMR, etc.). The 15.247 devices have a restriction of 1 W on their peak transmitter output power. On the other hand, the 15.249 devices are relatively lower powered and the maximum field strength of the fundamental frequency at 3 meters can be 50 mV/meter which translates into 0.5 mW RF power on a dipole. Since the FCC requires that all 15.247 devices use spread spectrum techniques for communication, every 15.247 product either uses Direct Sequence (DS) or Frequency Hopping (FH) spread spectrum as shown under the 15.247 column of Table 1.2. We are going to focus primarily on the 15.247 devices.
The rules for 15.247 Frequency Hopping systems differ from those of 15.247 Direct Sequence systems. We will first look at the rules for the 15.247 FH systems. The hopping channel carrier frequencies have to be separated by at least 25 kHz or the 20 dB bandwidth of the hopping channel, whichever is greater. There have to be at least 50 pseudorandomly ordered hopping frequencies each of which must be used equally on average by each transmitter. The 20 dB bandwidth of each hopping channel should not exceed 500 kHz and an average frequency hop should have an occupancy time not greater than 0.4 seconds within any 20 second period. The rules for the 15.247 DS systems are as follows: A minimum 6 dB bandwidth of 500 kHz is required and the transmitted power density averaged over any one second interval should not exceed 8 dBm in any 3 kHz bandwidth. Also, the DS systems are required to have a Processing Gain of at least 10 dB.

1.5 Objectives and System Models

The goal of this project is to analyze and highlight some of the interference issues between two coexisting 15.247 systems, one using DS spread spectrum and the other using FH spread spectrum. We have developed simple yet general models for the DS and FH systems. Each model represents a whole class of products that use that particular technology in the ISM band.
As an example, we have looked at the effects of mutual interference between a Wireless Local Area Network (WLAN) and an Automatic Meter Reading (AMR) system. Our WLAN model uses Direct Sequence Spread Spectrum and is very similar to systems like WaveLAN. We will refer to it as the DS system from here onwards. The AMR model resembles an Itron like Frequency Hopping system and from now on we will refer to it as the FH system. The DS system operates on the entire 902-928 MHz ISM band whereas the FH system uses the 910-920 MHz sub-band.

We must mention that the analysis performed and the conclusions drawn in this thesis are based upon our understanding of the respective DS and FH systems. We have assumed values for model parameters wherever the information was unavailable. Therefore, the quantitative performance of the actual systems in the presence of interference might vary from that suggested in this thesis. However, even if the values assigned to the parameters are changed, the results will change numerically but not qualitatively, i.e., although the curves plotted might shift, they will still maintain their basic characteristics.

This work can be broadly classified into two parts. In the first, we treat the DS terminals as interferers and the FH system as the one of interest, while in the second we consider the reverse situation.
Chapter 2

System Models

2.1 The FH Model

In the FH system, AMR units are connected to electric power meters in offices or homes to facilitate remote access to the meter readings. Upon activation by a base station, these AMR units transmit their meter readings using FH techniques. The base station can be either mobile or fixed. A mobile base station might be a handheld device or a receiver located in a van. A fixed base station may be located on a lamp post or similar fixture. In a typical scenario, a van would drive down a street transmitting 'wake-up' signals to the AMR units located within a certain activation radius \( R \).

In order to develop a general model, we are going to define certain variables corresponding to the following events. We are going to assume that there are \( h_{\text{max}} \) hopping frequencies and that on receiving a wake-up signal from its base station, an FH meter transmits a burst of \( m \) identical messages each on a randomly chosen hop. A message transmission time is \( a \) msecs long. The delay between messages is represented by an exponential random variable \( V \) with mean \( s \). The protocol permits transmission of a new burst after \( b \) secs. However, we will focus on the transmission of a single burst. We also assume that there are \( K \) FH terminals present within the activation radius \( R \) which respond to the wake-up signal. Table 2.1 summarizes the parameters associated with the FH model.

For the FH system, we define a ‘collision’ as an event where at least two units transmit their messages on the same hop simultaneously. In the case of a collision, we assume that none of the colliding messages can be decoded correctly at the receiver.
<table>
<thead>
<tr>
<th>Event</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active Message Time</td>
<td>$a$</td>
<td>6 msec</td>
</tr>
<tr>
<td>Mean Silent Period between Messages</td>
<td>$s$</td>
<td>100 msec</td>
</tr>
<tr>
<td>Inter-Burst Duration</td>
<td>$b$</td>
<td>10 secs</td>
</tr>
<tr>
<td>Number of Hopping Frequencies</td>
<td>$h_{\text{max}}$</td>
<td>48</td>
</tr>
<tr>
<td>Number of FH Terminals</td>
<td>$K$</td>
<td>Variable</td>
</tr>
<tr>
<td>Number of Message Repetitions in a Burst</td>
<td>$m$</td>
<td>8</td>
</tr>
<tr>
<td>Prob of Non Colliding FH Message Recvd OK</td>
<td>$P{\mathcal{G}}$</td>
<td>Variable</td>
</tr>
</tbody>
</table>

Table 2.1: Parameters used in the FH Model

When other FH or DS systems coexist, we have to take into account the fact that even if an FH message doesn’t collide with any of its peers, there is a finite possibility of it facing ‘unacceptable’ interference from the other systems. For this purpose we define an FH message as a ‘non-colliding’ one if it does not suffer collision specifically from any of the other FH units.

A non-colliding FH message is assumed to have been received correctly if its SIR (Signal to Interference Ratio) at the FH receiver (base station) is greater than a required threshold. This event will be denoted by $\mathcal{G}$ and its probability will be denoted by $P\{\mathcal{G}\}$. Since $P\{\mathcal{G}\}$ represents the effect of all the ‘external’ interference from the non-FH type system(s), its importance cannot be over emphasized. A significant portion of the next chapter revolves around the evaluation of this important parameter.

A ‘wake-up’ is said to be ‘successful’ when at least one of the $m$ messages in the burst is received correctly. In this section we evaluate the probability $P\{W\}$ of a successful wake-up.

For the FH system, we assume that each AMR unit will either be in the ‘on’ or ‘off’ state depending on whether the AMR is transmitting a message. Hence, as shown in Figure 2.1, we can model the activity of each of them by an on/off process or an alternating renewal process [20].

We know from [20] that if we denote the durations of the ‘on’ and ‘off’ periods by the random variables $U$ and $V$ respectively, then the probability of a system being in an
Figure 2.1: Message transmission process at an FH terminal

‘on’ state at an arbitrary time is given by

\[ P_{on} = \frac{E[U]}{E[U] + E[V]} \]  

From Table 2.1, \( E[U] = a \) and \( E[V] = s \). Therefore,

\[ P_{on} = \frac{a}{a + s} \]  

The FH analysis proceeds as follows. Assuming that user 1 sends a message burst, the probability an arbitrary user \( i \) is idle is

\[ P\{ I \} = P\{ \text{user } i \text{ is idle at the beginning of the burst} \} \times P\{ \text{user } i \text{ stays silent during burst} \} \]

We are also going to assume that the source processes at the various FH terminals are independent of each other. Then, the probability of user \( i \) being idle at the beginning of the burst equals the probability of it being idle at a randomly chosen time, and for the two state model

\[ P\{ \text{user } i \text{ is idle at the beginning of the burst} \} = \frac{s}{a + s} \]  

Since the silent period \( V \) has been assumed to be exponential with mean \( s \), its distribution [20] is

\[ f_V(v) = \begin{cases} \frac{1}{s}e^{-v/s} & \text{if } v \geq 0 \\ 0 & \text{otherwise} \end{cases} \]  

Therefore,

\[ P\{ \text{user } i \text{ stays silent during burst} \} = P\{ V > a \} \]
\[ P(I) = P\{\text{user } i \text{ idle during burst}\} = \left( \frac{s}{a + s} \right) e^{-a/s} \]  

(2.6)

We are going to assume a uniform PMF (probability mass function) on the probability of occurrence of the hopping frequency \( f_H \), where the random variable \( H \) denotes the hop number. Therefore,

\[
P\{f_H = f_h\} = \begin{cases} 
\frac{1}{h_{\text{max}}} & h = 1, 2, \ldots, h_{\text{max}} \\
0 & \text{otherwise}
\end{cases}
\]  

(2.7)

We assume \( h_{\text{max}} = 48 \). We define \( P(C) \) as the probability that an arbitrary user \( i \) collides with user 1. Then, given a user \( i \) is ‘on’ during the burst of user 1, a collision occurs if both users happen to be on the same hop as well. The probability of this happening is

\[
P(C) = P\{\text{user } i \text{ is active during burst}\} \times P\{\text{user } i \text{ is also on that particular hop}\} \]

(2.8)

\[
= (1 - P(I)) \frac{1}{h_{\text{max}}} \]

(2.9)

Since there are \( K \) FH units that transmit independently,

\[
P\{\text{user 1 doesn’t collide with any of the other } K - 1 \text{ users}\} = (1 - P(C))^{K-1}
\]

Let \( P(S) \) denote the probability of user 1’s message being received successfully. As mentioned earlier in this section, we consider the simultaneous occurrence of the following events a success: The event that user 1 does not collide with any of the other \((K - 1)\) users, and the event that the non-colliding message is received at its base station with an SIR above a required threshold. The probability of the latter event was denoted by \( P(\mathcal{G}) \). Hence,

\[
P(S) = P(\mathcal{G})(1 - P(C))^{K-1} \]  

(2.10)
Since a ‘successful wake-up’ corresponds to a successful transmission of at least one of the \( m \) messages in a burst, the probability that a wake-up worked is
\[
P\{W\} = 1 - (1 - P\{S\})^m
\]
(2.11)

As before, let us assume that user 1 is the desired user and that the remaining \((K - 1)\) of them are the interferers. In Equation (2.11) above, we assume that the event where user 1’s FH message collides with an interfering one is independent of the outcome of the previous message transmission.

In Figure 2.2, we look at the effect of \( K \) the total number of FH units activated on \( P\{W\} \). Figure 2.3 plots \( P\{W\} \) against \( K \) for different values of \( m \) with \( P\{\mathcal{G}\} \) held constant at 0.9. This figure shows the variation in \( P\{W\} \) as the number of message repetitions \( m \) is changed. We observe that for a particular \( P\{W\} \), as \( m \) is increased, the corresponding system capacity \( K \) does not increase proportionately.

For large values of \( K \), \( P\{S\} \ll 1 \) and so Equation (2.11) can be approximated as
\[
P\{W\} \approx 1 - (1 - mP\{S\}) = mP\{S\} = mP\{\mathcal{G}\}(1 - P\{C\})^{K^{-1}}
\]
(2.12)

\( P\{C\} = 0.0023 \) for the FH model and therefore, for a fixed \( K \), \( P\{W\} \propto mP\{\mathcal{G}\} \). Figure 2.4 plots \( P\{W\} \) as a function of \( K \) (as expressed in Equation (2.11)) for different values of \( m \) and \( P\{\mathcal{G}\} \). We plotted curves for \( P\{\mathcal{G}\} = 0.5, 0.9 \) and for \( m = 2, 4, 8, 16, 32 \). To avoid confusion, we represent the number of message repetitions by \( m \) and \( n \) for \( P\{\mathcal{G}\} = 0.9 \) and 0.5, respectively. Note that the \( P\{\mathcal{G}\} = 0.9 \) and 0.5 curves overlap for roughly constant values of \( mP\{\mathcal{G}\} \). For example, the curve corresponding to \((m, P\{\mathcal{G}\}) = (4, 0.9)\) overlaps with the corresponding \((m, P\{\mathcal{G}\}) = (8, 0.5)\) curve. This implies that if \( P\{\mathcal{G}\} \) the probability that a non-colliding FH message is received correctly is high(low), then the FH transmitter can use lesser(higher) number of message repetitions in a burst and still achieve the same performance.

Note that as pointed out before, \( P\{\mathcal{G}\} \) is the factor which will incorporate the effects of background AWGN and the interference due to the DS system. It will be evaluated later on in Chapter 3.
Figure 2.2: *Probability of at least one FH message being received OK vs. the total number of FH units activated.* $P\{G\}$ is the probability of a non-colliding FH message being received correctly, and $m$ is the number of message repetitions in an FH burst.

Figure 2.3: *Probability of at least one FH message being received OK vs. the total number of FH units activated $K$ for different values of $m$. $P\{G\}$ is fixed at 0.9.*
Figure 2.4: $P\{W\}$ versus $K$ for different values of message repetitions and $P\{G\}$. $m$ and $n$ are the number of message repetitions for $P\{G\} = 0.5$ and 0.9, respectively.

2.2 The DS Model

2.2.1 The Physical Layer Technology

We assume that every DS mobile unit uses the same spreading chip sequence which is an eleven bit Barker code; see [25]. Hence, spread spectrum is used by the DS system simply to combat interference and not for any multiple access scheme. The data rate is assumed to be 1 Mbits/sec and the modulation technique is BPSK. The chip rate is 11 Mchips/sec and the carrier frequency is 915 MHz, which happens to be the center of the ISM band.

2.2.2 CSMA/CA - The MAC Layer Protocol

In the DS system, the base stations (also called access points or APs) are connected to the fixed network and the mobile terminals communicate with the APs using IEEE 802.11 CSMA/CA (Carrier Sense Multiple Access/Collision Avoidance) as the MAC layer protocol [26], [27] and [28]. CSMA/CA is an extension of the usual CSMA protocol, where a terminal tries to detect signal energy in the band to assess its availability.
In CSMA, if the channel is found to be idle, the terminal initiates transmission. Otherwise, it waits till the current transmission is over. Once the channel becomes free, the waiting terminal transmits with a probability $p$, and this algorithm is known as the ‘$p$-persistent’ algorithm.

It is interesting to note that a different protocol is used over the wired networks but before explaining why it is unsuitable for the wireless medium, we briefly discuss the protocol itself. The IEEE 802.3 standard ([21] and [22]) called CSMA/CD (collision detection) is used for bus/tree topologies in wired networks. It is a refinement of CSMA in that a transmitting terminal ‘listens’ for a collision while it is transmitting. If the propagation delay is small compared to the transmission time, then a collision can be detected faster and the medium can be utilized more efficiently than with pure CSMA. When a collision takes place on the cable, there is an increase in the voltage level of the echo or reflected wave and this is how a ‘listening’ terminal is able to detect a collision. Also, CSMA/CD uses a 1-persistent algorithm for accessing the channel. Since it uses properties of transmission lines to detect collisions, CSMA/CD cannot be used as an ‘over the air’ protocol and hence the need for the alternative CSMA/CA. One of the main distinguishing features between CSMA/CD and CSMA/CA is that the terminals in the former use a ‘listen while talk’ approach, whereas in the latter they employ a ‘listen before talk’ scheme.

In CSMA/CA however, if the channel is found to be busy at the first attempt [27], the terminal calculates a ‘random backoff’ period (in slots) which is drawn from a particular probability distribution. Once the channel becomes idle, the contending terminal defers its transmission by this period. This approach is adopted to reduce the probability of collision among stations contending for the medium at the end of the current transmission. Note that for any frame the terminal wishes to transmit, it calculates the ‘random backoff’ period only once (after the very first attempt). The terminal does not draw another random starting slot if another terminal starts transmission while it is waiting. Instead, the random number is treated like a timer and decremented every slot. When it reaches zero, transmission is attempted.
### 2.2.3 The Model

Since we are interested in the combined effect of all the DS terminals on the performance of the FH system, we chose to model the DS traffic as a single M/M/1 queue as shown in Figure 2.5. Essentially, we have modeled the effect of all the DS transmitters that could interfere with the FH system as a single transmitter. We call this single transmitter the ‘compound transmitter’. Further, the packet arrival process into the buffer of the compound transmitter has been assumed to be Poisson. We denote the arrival rate by $\lambda$ and the service rate by $\nu$. It will be assumed that $\lambda<\nu$.

As shown in Figure 2.6, the activity at the server of the M/M/1 queue can be modeled as an alternating renewal process with the ‘on/off’ period represented as the ‘busy/idle’ period. We assume that the compound transmitter is always ‘on’ as long as there is a backlog of packets. Let $P_n$ be the probability that there are $n$ packets in the buffer. From [20], we have:

$$ P_n = \left(\frac{\lambda}{\nu}\right)^n \left(1 - \frac{\lambda}{\nu}\right) $$

(2.13)

Therefore,

$$ P_0 = 1 - \frac{\lambda}{\nu} $$

(2.14)

If we denote idle and busy time random variables by $Y$ and $Z$ respectively, then in the limiting case as the observation interval becomes very large, $P_0$ equals the fraction of time the system is idle. Then, from Equation (2.1) and [20], the probability of the compound source being in an ‘off’ state is

$$ P_0 = \frac{E[Y]}{E[Y] + E[Z]} $$

(2.15)

$Y$ represents the time from when a packet is transmitted and leaves the buffer empty until the next arrival. From the M/M/1 queue model, $Y$ is exponential with rate $\lambda$ and
so

\[ E[Y] = \frac{1}{\lambda} \quad (2.16) \]

Therefore, on expressing Equation (2.15) in terms of \( E[Z] \), and after substituting \( 1/\lambda \) for \( E[Y] \),

\[ E[Z] = \frac{1 - P_0}{\lambda P_0} \quad (2.17) \]

Substituting the expression for \( P_0 \) from Equation (2.14) into the above equation,

\[ E[Z] = \frac{1}{\nu - \lambda} \quad (2.18) \]

Therefore, if \( P_{on} \) denotes the fraction of time the compound server is busy, then,

\[ P_{on} = 1 - P_0 = \frac{\lambda}{\nu} \quad (2.19) \]

### 2.2.4 Effect of the DS System’s Busy and Idle Periods on an FH Transmission

To understand how a DS busy period \( E[Z] \) affects an FH message, we have to compare their average lengths. We know from Table 2.1 that an FH message length is about 6 msecs with a mean silence of 100 msecs. So the question now is how long is a DS transmission? Is it comparable to an FH renewal period (message length + silence) or is it long enough to affect an entire FH burst? Also, is the idle period \( E[Y] \) for the DS system long enough to allow FH messages to squeeze through? These issues are critical for the successful operation of the FH system and we address them in the following analysis.
Typically, a DS frame (packet) can be anywhere from 34 to 2346 octets long [29]. Let us denote this frame length (in octets) by the random variable $L$ and the corresponding average length by $E[L]$. The CSMA/CA protocol allows only one DS terminal to transmit at any given time. In addition, a DS terminal transmits at 1 Mbps. Hence, the service rate of the DS system is $\nu = 1 \times 10^6 / 8E[L]$ frames/sec. By defining the throughput of the M/M/1 queue model to be $\tau$, we have

$$\tau = \frac{\lambda}{\nu} \quad (2.20)$$

Combining Equations (2.18) and (2.20), we get the following expression for the busy period of the DS transmitter

$$E[Z] = \frac{1}{\nu(1 - \tau)} = \frac{8E[L]}{1 \times 10^3 (1 - \tau)} \quad [\text{msec}] \quad (2.21)$$

Furthermore, the DS system’s mean idle period is

$$E[Y] = \frac{1}{\lambda} = \frac{1}{\nu \tau} = \frac{8E[L]}{1 \times 10^3 \tau} \quad [\text{msec}] \quad (2.22)$$

Figure 2.7 illustrates the effect of the DS throughput $\tau$, and the average DS frame length $E[L]$, on the relative lengths of $E[Z]$ and $E[Y]$ through a few examples. We can see from the different situations that an FH message transmission has a good chance of not colliding with a DS transmission if $\tau$ is very small and $E[L]$ is very large. However, if $\tau$ is very large (say 0.9) and $E[L]$ is very small (say 250 octets), then the DS idle period $E[Y]$ is not sufficiently long to allow an FH message to pass through without a collision.

In Figure 2.8 we plotted $E[Z]$ (solid lines) and $E[Y]$ (broken lines) against the throughput $\tau$ for $E[L] = 250$ and 2500 octets. Notice once again that for high values of $\tau$ and low values of $E[L]$, the DS idle period $E[Y]$ is not long enough to allow an FH message to pass through without collision. The probability $P\{A\}$ that an FH message does not collide with a DS transmission can be written as

$$P\{A\} = P\{\text{DS terminal is silent at start of FH msg}\} \times P\{\text{DS terminal remains silent for the duration of FH msg}\} \quad (2.23)$$
Figure 2.7: Comparison of FH message transmissions with the busy and idle periods of the DS system for different values of throughput $\tau$ and mean frame lengths $E[L]$ (in octets).
Figure 2.8: Busy and Idle periods of the DS system for different mean values of frame lengths $E[L]$ and throughput $\tau$

Figure 2.9: Probability that an FH message doesn’t collide with a DS transmission v/s DS throughput $\tau$ for DS frame lengths $E[L] = 250, 500, 750, \ldots, 2500$ octets
\[
E[Y]e^{-a\lambda} \quad (2.24)
\]
\[
= \frac{E[Y]}{E[Y] + E[Z]} 
\]
\[
= (1 - \tau)e^{-\alpha/E[Y]} \quad (2.25)
\]
\[
= (1 - \tau)e^{-125\alpha/E[L]} \quad (2.26)
\]

We can see from Figure 2.9 that for \( \tau > 0.7 \), \( P\{A\} < 0.25 \) for all values of \( E[L] \). This means that for high values of DS throughput \( \tau \), the FH message is very likely to collide with a DS transmission. This agrees with our observations in Figures 2.7 and 2.8. Since \( P\{A\} \) is very low for high values of \( \tau \) (possible in a practical system), we are going to assume from here on that \( \tau = 1 \). Therefore, we are assuming that an FH message is always going to collide with a DS transmission. However, if the DS interference is below a certain threshold and the FH message satisfies its SIR requirements at its receiver, then it is assumed that the FH message will be decoded correctly. In Chapter 3, we analyze some of the constraints within which an FH system must operate if it is to achieve the required SIR at its receiver.
Chapter 3

Probability of Unacceptable DS Interference Experienced by an FH Message at the FH Base Station

In the previous chapter we had introduced a parameter $P\{g\}$ which represented the effect of the ‘external’ interference on a non-colliding FH message. We had defined $P\{g\}$ as the ‘probability of a non-colliding FH message being received correctly.’ In our case $P\{g\}$ embodies the effect of the AWGN background noise as well as the interference due to the DS system. In this chapter, we are going to develop an expression for $P\{g\}$ in terms of the AWGN and various other parameters related to the FH and DS systems. Once again, we would like to emphasize that the model is a general one in that it can be applied to most DS and FH systems interfering with each other.

We are going to perform the interference analysis in the spectral domain and therefore we first need to find an expression for the Power Spectral Density (PSD) of the DS transmission. This will help us in calculating the amount of DS power that interferes with the FH transmission.

3.1 The Power Spectral Density of a DS-SS signal

The received signal in a BPSK DS system is of the form

$$r(t) = \sqrt{2R}u(t)m(t)\cos(2\pi f_c t)$$

(3.1)

where $m(t)$ and $u(t)$ are the message and chip sequences respectively [23, p. 59]. The bit and chip durations are $T_b$ and $T_c$ respectively. The number of chips per bit then is $\kappa = T_b/T_c$. The average received power of the signal is $R$. Let $M(f)$ and $U(f)$ denote
Figure 3.1: (a) System Model (b) A more descriptive version of the same illustrating the different parameters used in the analysis. The table lists the signal and distance variables for the DS/FH model.
the Fourier transforms of $m(t)$ and $u(t)$ respectively. Typically, $\kappa \gg 1$ and therefore $M(f)$ appears as a Dirac delta function to the relatively wideband $U(f)$ spectrum. For a single bit of duration $T_b$, the Fourier transform of $r(t)$ is given by

$$
R(f) = \sqrt{\frac{R}{2}} [U_{T_b}(f + f_c) + U_{T_b}(f - f_c)]
$$

(3.2)

$U_{T_b}(f)$ is the Fourier transform of the $\kappa$ chips in a single bit duration $T_b$. It is given by

$$
U_{T_b}(f) = T_c \text{sinc}(fT_c) \left[ \sum_{k=1}^{\kappa} u_k e^{-j\pi f(2k-1)/T_c} \right]
$$

(3.3)

where $u_k$ is $\pm 1$ and $\text{sinc}(x) = \sin(\pi x)/\pi x$. Also, let

$$
Z(f) = \sum_{k=1}^{\kappa} u_k e^{-j\pi f(2k-1)/T_c}
$$

(3.4)

Then,

$$
|U_{T_b}(f)|^2 = T_c^2 |Z(f)|^2 \text{sinc}^2(fT_c)
$$

(3.5)

It was mentioned before in Section 2.2.1 that in the case of the DS system the spreading gain is 11, and the same 11 chip Barker code is used for signal spreading in all the terminals. In fact, the chip sequence $u(t)$ has been hard-coded in every DS terminal. The Barker chip sequence is

$$
u(t): [1 -1 1 1 -1 1 1 1 -1 -1]
$$

Therefore, $|Z(f)|$ can be calculated exactly in our case. For this particular sequence, Figure 3.2 plots $|Z(f)|^2$ as a function of the frequency $f$.

From Figure 3.2, we see that $|Z(f)|^2$ has a constant value of 12 for all the frequencies except for the notch which occurs at exactly the same frequency as the first null of the associated $\text{sinc}^2(fT_c)$ term. To show the effect of the $|Z(f)|^2$ term on the $\text{sinc}^2(fT_c)$ term in Equation (3.5), Fig 3.3 plots the curves corresponding to $\text{sinc}^2(fT_c)$ as well as the $|Z(f)|^2\text{sinc}^2(fT_c)$ term on the same graph. The curve corresponding to the latter expression was scaled down by a factor of $l = 12$ so that its shape could be compared with that of the former. The result is interesting because the two curves
exactly coincide. Therefore we can safely conclude that the $|Z(f)|^2$ term merely scales the $\text{sinc}^2(\cdot)$ term by a constant factor of $l = 12$ over the frequency band of interest. Note that $l$ is a function of $\kappa$ the number of chips per bit as well as the particular chip sequence. Therefore its value varies with the choice of these parameters. Rewriting Equation (3.5) as a function of $l$, we have

$$|U_{Th}(f)|^2 = l T^2_c \text{sinc}^2(f T_c)$$  \hspace{1cm} (3.6)

In our case, since $f_c = 915$ MHz and the $U_{Th}(f)$ baseband bandwidth is 11 MHz, we can assume that negligible overlap exists between $U_{Th}(f + f_c)$ and $U_{Th}(f - f_c)$. The ESD (Energy Spectral Density) of $r(t)$ is then given by

$$\mathcal{E}(f) = |R(f)|^2 = \frac{R}{2} \left[ |U_{Th}(f + f_c)|^2 + |U_{Th}(f - f_c)|^2 \right]$$

$$= \frac{R T^2_c}{2} \left[ \text{sinc}^2((f + f_c)T_c) + \text{sinc}^2((f - f_c)T_c) \right]$$  \hspace{1cm} (3.7)

### 3.2 The Effect of DS Interference on the FH System

Let $W$ be the bandwidth of each frequency hop. Then, the amount of DS interference energy experienced in that bandwidth by a hopper located at frequency $f_h$ is given in [23] as

$$E_I(f_h) = 2 \int_{f_h-W/2}^{f_h+W/2} \frac{R T^2_c}{2} \text{sinc}^2((f - f_c)T_c) df \quad [\text{Watts/Hz}]$$  \hspace{1cm} (3.8)

If we further assume that the frequency hopper has a very narrow bandwidth compared to that of the spread DS signal, then we can assume the DS spectrum to be relatively constant over that small bandwidth $W$. Hence, the DS interference energy received at the FH base station can be rewritten as

$$E_I(f_h) = W R T^2_c \text{sinc}^2((f_h - f_c)T_c) \quad [\text{Watts/Hz}]$$  \hspace{1cm} (3.9)

Therefore, the interference power from the DS source at frequency $f_H$, in that small bandwidth $W$, can be expressed as

$$Q(f_H) = W^2 R T^2_c \text{sinc}^2((f_H - f_c)T_c) \quad [\text{Watts}]$$  \hspace{1cm} (3.10)
Figure 3.2: Plot of $|Z(f)|^2$

Figure 3.3: Plot of $\text{sinc}^2[fT_c]|Z(f)|^2$
Recall from Equation (2.10) in Section 2.1 that the probability of success of a particular FH user was given as:

\[ P(S) = P(\mathcal{G})(1 - P(C))^K \]

where \( P(\mathcal{G}) \) had been defined as the probability of a non-colliding FH message being received correctly.

The factor \( P(\mathcal{G}) \) models the effect of interference on an FH message that did not suffer collisions from any of its FH peers. In the following analysis, we will assume that the DS system and the background AWGN are the only sources of interference to the FH transmission. In this section we develop an expression for \( P(\mathcal{G}) \) as a function of \( Q(f_H) \) and other parameters.

In Section 2.2.3, we defined \( P_0 \) as the fraction of time the compound DS transmitter is idle; see Equation (2.15). Note that \( P_0 \) is the same as \( P_{\text{off}} \). Let \( \mathcal{B} \) denote the event that the FH message suffers a collision with a DS transmission, and let \( P(\mathcal{B}) \) be the corresponding probability. In Section 2.2.4 we had assumed the DS throughput \( \tau = 1 \) as a consequence of which the FH transmission would always experience interference from the DS system. Hence, \( P(\mathcal{B}) = 1 \).
Let \( S \) denote the average received FH power and \( N \) the thermal noise (AWGN) power equal to \( kT/\beta \) \cite{24}. \( k \) is the Boltzmann's constant equal to \( 1.37 \times 10^{-23} \) Joules/degree, \( T \) is the temperature in degrees Kelvin, and \( \beta \) is the bandwidth of interest (\( W \) in our case).

If a frequency hop experiences intolerable interference as a result of which it cannot be decoded correctly, we say that the frequency hop has been ‘blocked’. The probability that the SIR of the received FH signal is lower than the required threshold given that it has collided with a DS transmission can be written as

\[
P(\mathcal{L}) = P \left( \frac{S}{Q(f_H) + N} \right) < \Gamma_{th} = P \left( Q(f_H) > \frac{S}{\Gamma_{th}} - N \right) \quad (3.11)
\]

In Figure 3.5, for any frequency \( f_H \) such that \((f_c - \Delta f) \leq f_H \leq (f_c + \Delta f)\), \( Q(f_H) \) is greater than the threshold \((S/\Gamma_{th}) - N\). As a result, all FH frequencies between \((f_c - \Delta f)\) and \((f_c + \Delta f)\) will experience an unacceptable level of interference and will get blocked. Hence, the probability the SIR is below the threshold is equal to the probability that a particular hop \( f_H \) occupies the shaded region in Figure 3.5, i.e.

\[
P(\mathcal{L}) = P \left( Q(f_H) > \frac{S}{\Gamma_{th}} - N \right) = \frac{2\Delta f}{B} \quad (3.12)
\]

where \( B \) is the total bandwidth of the FH system. We will now calculate an expression for \( \Delta f \) in terms of the known parameters. At the frequency where the SIR is equal to \((S/\Gamma_{th}) - N\),

\[
Q(f_H) = Q(f_c \pm \Delta f) = \frac{S}{\Gamma_{th}} - N \quad (3.13)
\]

Then from Equation (3.10)

\[
Q(f_H) = lW^2T_c^2R\text{sinc}^2[(f_H - f_c)T_c]
\]

\[
= lW^2T_c^2R\text{sinc}^2[\Delta fT_c] \quad (3.14)
\]

Equations (3.13) and (3.14) together yield

\[
\frac{S}{\Gamma_{th}} - N = lW^2T_c^2R\text{sinc}^2[\Delta fT_c] \quad (3.15)
\]
The frequencies $f_i$ in the shaded region are the FH frequencies that will experience an interference level of $(S/ \Gamma_{th}) - N$ or higher.

Figure 3.5: A superposition of DS and FH spectra to illustrate interfering frequencies at the FH receiver.
Equivalently,
\[
\sin^2(\Delta f T_c) = \frac{1}{I W^2 T_c^2 R} \left( \frac{S}{\Gamma_{th}} - N \right) \tag{3.16}
\]

A sinc function does not have a unique inverse. However, we are only interested in the ‘main-lobe’ portion of the spectrum over which we can define an inverse. In particular, for frequencies \(0 \leq f \leq 1/T\), we define a function \(S_T(f)\) to be the ‘windowed sinc squared’ function
\[
S_T(f) \approx \sin^2(fT)[u(f) - u(f - \frac{1}{T})] \tag{3.17}
\]
where the unit step functions within the square parentheses represent a window of unit height centered at \(1/2T\) and occupying a bandwidth of \(1/T\) Hz. The null of \(S_T(f)\) occurs at \(1/T\). Rewriting Equation (3.16) in terms of \(S_T(\Delta f)\), we have,
\[
S_T(\Delta f) = \frac{1}{I W^2 T_c^2 R} \left( \frac{S}{\Gamma_{th}} - N \right) \tag{3.18}
\]
\(\sin^2(\cdot)\) is a monotonically decreasing function till the first null (region of our interest), and consequently so is the corresponding \(S(\cdot)\) function. Equation (3.18) then yields
\[
\Delta f = \frac{1}{T_c} S_T^{-1} \left( \frac{1}{I W^2 T_c^2 R} \left( \frac{S}{\Gamma_{th}} - N \right) \right) \tag{3.19}
\]
Therefore, from Equations (3.12) and (3.19),
\[
P\{L\} \approx \frac{2\Delta f}{B} \tag{3.20}
\]
\[
= \frac{2}{BT_c} S_T^{-1} \left( \frac{1}{I W^2 T_c^2 R} \left( \frac{S}{\Gamma_{th}} - N \right) \right) \tag{3.21}
\]

3.3 The Effect of the Ratio of Interferer (DS) and Source (FH) Distances and their Transmitted Powers on the Fraction of FH Frequencies Blocked

In the previous section we observed that the DS interference experienced by an FH source is a function of its location in the spectrum due to the non-uniform PSD of the DS interference.
The fraction of FH frequencies blocked $P\{\mathcal{L}\}$ is a function of the relative distances and transmitter powers of the FH and DS sources. There are other factors that can influence $P\{\mathcal{L}\}$ but we focus only on these particular ones. As the interferer (DS source) approaches the FH base station and its interference power crosses the SIR threshold, a certain number of FH hopping frequencies get blocked. Towards the end of this section, we express $P\{\mathcal{L}\}$ as a function of the ratio of DS to FH transmitter distances (from the FH base station) as well as the ratio of their transmitted powers. Table 3.1 lists some of the important parameters along with their definitions.

Since we have assumed $S$ and $R$ to be the average received powers of the FH and DS sources respectively,

$$S = \frac{k_w F}{d_w^\gamma}$$

$$R = \frac{k_u D}{d_u^\gamma}$$

where $k_u$ and $k_w$ are the propagation loss constants. We are distinguishing between them because they could be different due to the different antenna characteristics and device losses. Then, rewriting Equation (3.21) in terms of the transmitted powers and distances from the FH base station,

$$P\{\mathcal{L}\} = \frac{2}{BT_c} S_{T_c}^{-1} \left\{ \frac{1}{lW^2T_c^{2}\Gamma_{th}} \left( \frac{d_u}{d_w} \right)^\gamma F \frac{k_w}{k_u} \left( 1 - \frac{N\Gamma_{th}d_u^\gamma}{k_w F} \right) \right\}$$

Using the Egli propagation model [18], the transmission loss is

$$L_t(dB) = 85.9 + 20\log(f_c) + 10\gamma \log(d) - 20\log(h_b h_m)$$

where $\gamma$ is the path loss constant, $f_c$ is the carrier frequency (MHz), $d$ is the distance of transmitter from base (in km), and $h_b$ and $h_m$ are the antenna heights (in meters) of the base station and the mobile unit respectively. If we also include a device loss of 5 dB and a building penetration loss of 10 dB, then for $f_c = 915$ MHz,

$$L_t(dB) = 160 + 10\gamma \log(d) - 20\log(h_b h_x)$$

where the subscript $x$ could denote either a DS or an FH terminal. In our case, $h_b$ is the height of the FH base station at which signals from the DS and the FH terminals are
received. Since $L_d = -10 \log (k_x/d_x^2)$ dB, substituting it on the LHS of Equation (3.26) yields

$$10 \log (k_x) = -160 + 20 \log (h_b h_x) \quad (3.27)$$

If we consider the FH system and assume that both $h_b$ and $h_x = h_w$ are equal to 2 m, then $k_w = 10^{-14.8}$.

We also need to express the ratio $k_w/k_u$ in terms of known quantities. We assume that the characteristics of the antenna and the propagation medium are the same for the DS and FH systems. Therefore, if we further assume that on average the carrier frequency for the FH system is the same as that of the DS system, then from Equation (3.27)

$$10 \log (k_w) - 10 \log (k_u) = 20 \log (h_w) - 20 \log (h_u)$$

Note that in our scenario, the FH base station is common to both the source and the interferer and that is why its height does not appear in the expression above. Taking anti-log on both sides, we have

$$\frac{k_w}{k_u} = \left(\frac{h_w}{h_u}\right)^2 \quad (3.28)$$

Substituting $k_w/k_u = (h_w/h_u)^2$ in Equation (3.24), we get

$$P\{\mathcal{L}\} = \frac{2}{BT_c} S_{\mathcal{I}}^{-1} \left\{ \frac{1}{lW^2 T_c^2 \Gamma_{th}} \left( \frac{d_u}{d_w} \right)^\gamma \frac{F}{D} \left( \frac{h_w}{h_u} \right)^2 \left( 1 - \frac{N T_{th} d_w^2}{k_w F} \right) \right\} \quad (3.29)$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Assigned Variable</th>
<th>Value Assumed</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIR threshold for FH and DS systems</td>
<td>$\Gamma_{th}$</td>
<td>10 dB</td>
</tr>
<tr>
<td>Propagation loss exponent</td>
<td>$\gamma$</td>
<td>4.0</td>
</tr>
<tr>
<td>Number of hopping frequencies</td>
<td>$h_{\text{max}}$</td>
<td>48</td>
</tr>
<tr>
<td>Fraction of FH Frequencies Blocked</td>
<td>$P{\mathcal{L}}$</td>
<td>Variable</td>
</tr>
<tr>
<td>Total bandwidth of FH source</td>
<td>$B$</td>
<td>10 MHz</td>
</tr>
<tr>
<td>IF bandwidth of an FH transmission</td>
<td>$W$</td>
<td>200 kHz</td>
</tr>
<tr>
<td>Average transmitter power of the DS source</td>
<td>$D$</td>
<td>Variable</td>
</tr>
<tr>
<td>Average transmitter power of the FH source</td>
<td>$F$</td>
<td>Variable</td>
</tr>
<tr>
<td>Chip duration of DS source</td>
<td>$T_c$</td>
<td>1/11 MHz</td>
</tr>
<tr>
<td>Bit duration of the DS source</td>
<td>$T_b$</td>
<td>1/1 Mbps</td>
</tr>
<tr>
<td>Antenna height of FH source</td>
<td>$h_w$</td>
<td>2 m</td>
</tr>
<tr>
<td>Antenna height of DS source</td>
<td>$h_u$</td>
<td>2 m</td>
</tr>
</tbody>
</table>

Table 3.1: FH and DS System Parameters
Recall that since $P\{G\}$ had been defined as the probability of a non-colliding FH message being received correctly, it is equal to the probability of the non-colliding FH message being received above the required SIR. That is,

$$P\{G\} = 1 - P\{L\}$$  \hspace{1cm} (3.30)

Therefore,

$$P\{G\} = 1 - \frac{2}{BT_c S^{-1}_{T_u}} \left\{ \frac{1}{lW^2 T^2_c \Gamma_{th}} \left( \frac{d_u}{d_w} \right) \frac{\gamma F}{D} \left( \frac{h_w}{h_u} \right)^2 \left( 1 - \frac{N \Gamma_{th} d^\gamma_{w}}{k_w F} \right) \right\}$$  \hspace{1cm} (3.31)

Since $S = k_w F / d^\gamma_{w}$ (Equation (3.22)), the $N \Gamma_{th} d^\gamma_{w} / k_w F$ term in Equation (3.31) can be written as $N \Gamma_{th} / S$ or as $\Gamma_{th} / (S/N)$. Rewriting Equation (3.31) as a function of $S/N$, we have

$$P\{G\} = 1 - \frac{2}{BT_c S^{-1}_{T_u}} \left\{ \frac{1}{lW^2 T^2_c \Gamma_{th}} \left( \frac{d_u}{d_w} \right) \frac{\gamma F}{D} \left( \frac{h_w}{h_u} \right)^2 \left( 1 - \frac{\Gamma_{th}}{(S/N)} \right) \right\}$$  \hspace{1cm} (3.32)

We would like to know how large should the $S/N$ term be so that $(1 - \Gamma_{th} / (S/N))$ is approximately equal to 1. In other words, for what values of $S/N$ can we neglect the AWGN $N$ and consider only the interference due to the DS system. The AWGN noise floor is typically about $-174$ dBm/Hz at room temperature, and since the FH system’s IF filter bandwidth is 200 kHz, $N = -121$ dBm = $10^{-15}$ W. Figure 3.6 plots $S/N$ as a function of $d_w$, the FH terminal’s distance from its receiver. Notice that for $\Gamma_{th} / (S/N) > 1$ the $(1 - \Gamma_{th} / (S/N))$ term in Equation (3.32) becomes negative. Since $\text{sinc}^2$ only takes on positive values, $S^{-1}_{T_u}$ of a negative term is meaningless. Therefore we will only consider those values of $d_w$ for which an inverse does exist. The maximum value of $d_w$ for which $\Gamma_{th} / (S/N) < 1$ can be easily calculated by substituting $S = k_w F / d^\gamma_{w}$ in the inequality and then solving for $d_w$. For $F = 0.25$ mW, $k_w = 10^{-14.8}$, $N = 10^{-15}$ W, and $\Gamma_{th} = 10$ dB, the maximum value of $d_w$ is around 80 meters. From Figure 3.6, $d_w < 80$ m corresponds to $S/N > 25$ dB (which is quite high), and so we can neglect the background noise $N$.

If we neglect the AWGN,

$$P\{G\} = \frac{2}{BT_c S^{-1}_{T_u}} \left\{ \frac{1}{lW^2 T^2_c \Gamma_{th}} \left( \frac{d_u}{d_w} \right) \frac{\gamma F}{D} \left( \frac{h_w}{h_u} \right)^2 \right\}$$  \hspace{1cm} (3.33)
Figure 3.7 illustrates the effect of the ratio \((d_u/d_w)\) on the fraction of FH frequencies that are not blocked \((P\{g\})\). We have plotted a set of curves corresponding to different ratio of average transmitter powers. The curves rise sharply probably due to the high path loss exponent \(\gamma\). A high value of \(\gamma\) causes a large change in the received power for a relatively smaller change in distance. Another way of looking at it is to consider the influence of the shape of the DS spectrum on the FH frequencies being blocked. The DS spectrum appears to have a ‘wide-shoulder’ relative to the freedom in distance of the FH terminal. In other words, once the DS interferer is operating at a distance from where it can block certain FH frequencies, even a small decrease in \(d_u/d_w\) causes the DS power to increase substantially, enough to block most of the frequencies. Figure 3.9 illustrates that as the DS source approaches the FH base station and the DS power crosses the SIR threshold, there is a sharp transition from very few FH hops being blocked to suddenly almost all being blocked.

We can see from Figure 3.7 that for a particular value of \(D/F\), there is a range of values of \(d_u/d_w\) within which \(P\{g\}\) varies from 0 to 1. The maximum value of \(d_u/d_w\) at which \(P\{g\} = 0\) will be denoted by \((d_u/d_w)_a\) and the minimum value of \(d_u/d_w\) at which \(P\{g\} = 1\) will be denoted by \((d_u/d_w)_b\). \((d_u/d_w)_a\) and \((d_u/d_w)_b\) will be referred to as the lower and upper cutoff-ratio respectively. This implies that if \(d_u/d_w \leq (d_u/d_w)_a\) then all the frequency hops will experience intolerable interference and will get blocked.

On the other hand, if \(d_u/d_w \geq (d_u/d_w)_b\), then the DS interference will not be sufficient to block any of the FH frequency hops. \((d_u/d_w)_a\) and \((d_u/d_w)_b\) can be calculated from Equation (3.33) by substituting \(P\{g\} = 0\) and \(P\{g\} = 1\), respectively.

If we let \(\beta = h_w^2/lW^2T_c^2\Gamma th_uh_u^2\) and \(t = D/F\), then
\[
\frac{d_u}{d_w} = \left( \frac{t}{\beta} \right)^{\frac{1}{\gamma}} S_{T_c} \left[ \left( 1 - P\{g\} \right) \frac{BT_c}{2} \right]^{\frac{1}{\gamma}}
\]

Therefore, for \(P\{g\} = 0\),
\[
\left( \frac{d_u}{d_w} \right)_a = \left( \frac{t}{\beta} S_{T_c} \left[ \frac{BT_c}{2} \right]^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}}
\]

and for \(P\{g\} = 1\),
\[
\left( \frac{d_u}{d_w} \right)_b = \left( \frac{t}{\beta} \right)^{\frac{1}{\gamma}}
\]
In Figure 3.8, we plot \((d_u/d_w)_a\) and \((d_u/d_w)_b\) as a function of the ratio \(D/F\) of the DS/FH transmitter powers.

The relationships between most of the curves we have plotted till now can be demonstrated through an example. Before going through the example, let us mention that there can be many design objectives and various approaches to meet them. Here, we only consider a few for purposes of illustration. Let us assume that we are interested in implementing an FH system similar to the one we have chosen as our model. First, we would like to decide upon a minimum value of \(P\{W\}\) - the probability of a ‘wake-up’ being successful as our design criterion. Next, we would determine \(K\) - the number of FH terminals to be activated in the area. Then, from Figure 2.2 we will pick the minimum approximate value of \(P\{G\}\) which will satisfy our constraints on \(P\{W\}\) and \(K\) simultaneously. For example, if we choose \(P\{W\} = 0.92\) and \(K = 500\), then \(P\{G\}_{\text{min}} = 0.9\).

In the case of DS/FH systems, \(D/F = 1000\) and so from Figure 3.7 we would be able to conclude that \(d_u/d_w\) should be at least about 2.5 to satisfy our design objectives.

It is also possible to determine \(P\{G\}\) experimentally. To do this, we would have to turn off all the FH terminals except for the one of interest. After sending it various wake-up signals and measuring the number of successes, we would be able to determine \(P\{W\}\). Then on substituting \(K = 1\) in Equation (2.10) we would get \(P\{S\} = P\{G\}\). Subsequently, Equation (2.11) can be solved for \(P\{G\}\). If the \(P\{G\}\) is too low for the number of terminals we would like to support, then from Figure 3.7 we can determine the value of \(d_u/d_w\) that would give us the higher desired \(P\{G\}\) for a known value of \(D/F\).
Figure 3.6: SNR [dB] (FH signal S to AWGN N) versus $d_w$ the FH terminal’s distance from its base station.

Figure 3.7: Fraction $P\{G\}$ of FH frequencies that are not blocked by the DS interference versus ratio of distances ($d_u/d_w$) for different ratio of transmitter powers.
Figure 3.8: The lower and upper cutoff-ratio $(d_u/d_w)_a$ and $(d_u/d_w)_b$ as a function of $D/F$ the ratio of DS to FH transmitter powers.
Figure 3.9: (a) None of the FH hops are blocked since the DS interferer power is below the threshold (b) As the DS source comes closer to the FH base and exceeds the threshold, it suddenly blocks almost all the hops due to its ‘broad-shouldered’ PSD
Chapter 4

The Effect of Narrowband (FH) Interference on a Wideband (DS) System

In the previous sections we treated the FH signal as our signal of interest and considered the DS transmission as interference. In this section we look at the opposite scenario where the DS transmission is the one of interest and the FH transmission appears as narrowband interference to it.

Although the DS signal has a fixed carrier frequency $f_c$ about which it is spread, the FH transmission hops about different frequencies. We want to address the following question: If we have a frequency hopper at a frequency $f_H$, then how much of that energy appears as noise to the DS signal after the filtering is done by the conventional receiver as shown in Figure 4.1. The filter [23] is actually centered about an IF (intermediate frequency), but to keep matters simple we assume it is centered at $f_c$ itself.

The assumptions regarding the DS transmitted signal are going to be the same as those in Chapter 3. Table 4.1 summarizes some of the definitions and values used.

We assume that

$$\frac{1}{T_J} \ll \frac{1}{T_b} \ll \frac{1}{T_c}$$  \hspace{1cm} (4.1)

where $T_J$ is the time period of the FH signal. $T_b$ and $T_c$, as before, are the time periods of the DS bit and chip respectively. Equation (4.1) implies that the FH signal can be considered to be narrow-band compared to the DS signals as shown in Figure 4.2. Further, let the received FH signal be

$$c(t) = \sqrt{2S} \cos(2\pi f_H t)$$  \hspace{1cm} (4.2)
where $f_H$ is the hopping frequency.

The Fourier transform of the whitening sequence $u(t)$ (chip sequence) is given as (also see Equation (3.3))

\[
U(f) = T_c \text{sinc}(fT_c) \left[ \sum_{k=1}^{K} u_k e^{-j\pi f(2k-1)/T_c} \right] \tag{4.3}
\]

\[
= T_c Z(f) \text{sinc}(fT_c) \tag{4.4}
\]

where $Z(f)$ is the summation term in Equation (4.3). Therefore, if

\[
x(t) = c(t) u(t) \tag{4.5}
\]

then, Equations (4.2) and (4.5) imply

\[
X(f) = \frac{\sqrt{2S}}{2} [U(f + f_H) + U(f - f_H)] \tag{4.6}
\]

As in Equation (3.7), the ESD (Energy Spectral Density) of the whitened FH signal can be expressed as

\[
\mathcal{E}_X(f) = |X(f)|^2 = \frac{S}{2} \left[ |U(f - f_H)|^2 + |U(f + f_H)|^2 \right] \tag{4.7}
\]
Figure 4.2: The received DS and FH signals (a) before despreading and (b) after despreading.
### Table 4.1: Definitions and values of some of the parameters used.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Assigned Variable</th>
<th>Value Assumed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth of DS Message Signal ( m(t) )</td>
<td>( 1/T_b )</td>
<td>1MHz</td>
</tr>
<tr>
<td>Bandwidth of Chip Sequence ( u(t) )</td>
<td>( 1/T_c )</td>
<td>11MHz</td>
</tr>
<tr>
<td>Bandwidth of Narrowband FH Signal ( c(t) )</td>
<td>( 1/T_J )</td>
<td>16kHz</td>
</tr>
<tr>
<td>Total Bandwidth of FH System</td>
<td>( B )</td>
<td>10MHz</td>
</tr>
<tr>
<td>Average Transmitted Power of the FH Source</td>
<td>( F )</td>
<td>0.25mW</td>
</tr>
<tr>
<td>Average Transmitted Power of the DS Source</td>
<td>( D )</td>
<td>250mW</td>
</tr>
<tr>
<td>Average Received Power of the FH Source</td>
<td>( S )</td>
<td>Variable</td>
</tr>
<tr>
<td>Average Received Power of the DS Source</td>
<td>( R )</td>
<td>Variable</td>
</tr>
</tbody>
</table>

The DS receiver has a bandwidth \( 1/T_b \) Hz centered at \( f_c \) as shown in Figure 4.1. Therefore, the amount of FH energy \( E_B \) actually lying within the passband of the receiver can then be approximately given by

\[
E_B(f_H) \approx 2 \int_{f_c - 1/2T_b}^{f_c + 1/2T_b} S \frac{1}{2} |U(f - f_H)|^2 df \ \text{[Watts/Hz]} \tag{4.8}
\]

Since the receiver bandwidth is very small compared to that of the whitened FH signal \( X(f) \), we can assume that \( E_X(f) \) remains constant over this small ‘window.’ Hence, the FH energy lying in that band can be given as

\[
E_B(f_H) \approx \frac{S}{T_b} |U(f_c - f_H)|^2 \ \text{[Watts/Hz]} \tag{4.9}
\]

Therefore, the FH interference power that ‘leaks’ into the DS receiver of bandwidth \( 1/T_b \), is given by

\[
I(H) = \frac{S}{T_b^2} |U(f_c - f_H)|^2 \ \text{[Watts]} \tag{4.10}
\]

where the random variable \( H \) denotes the hop at which the FH unit transmits its current message.

Since the distance of the interferer also affects the received power, \( I(H) \) can actually take on a continuum of values. To see this more clearly, let us rewrite Equation (4.10) expressing \( I(H) \) as a function of the FH unit’s transmitted power \( F \) and its distance \( X \) from the DS base station. We are now going to assume that there are \( N \) FH interferers, all transmitting at the same average power \( F \). The average power received from the \( i^{th} \)
FH interferer will be denoted by $S_i$ and will depend on $X_i$, its distance from the DS base station. In the equations that follow, $I_i$ denotes the interference power of the $i^{th}$ FH interferer.

\[ I_i = \frac{S_i}{T_b^2} |U(f_c - f_{H_i})|^2 \]  
(4.11)

\[ = \left( \frac{kF}{X_i} \right) \frac{1}{T_b^2} |U(f_c - f_{H_i})|^2 \]  
(4.12)

Absorbing the constants $k_f, F$, and $1/T_b^2$ into a single one $k$, we have

\[ I_i = \frac{k}{X_i^2} |U(f_c - f_{H_i})|^2 \]  
(4.13)

Now, let

\[ y(H_i) = k |U(f_c - f_{H_i})|^2 \]  
(4.14)

Therefore,

\[ I_i = \frac{y(H_i)}{X_i^2} \]  
(4.15)

The total interference power $P$ caused by the $N$ FH units is then

\[ P = I_1 + I_2 + \ldots + I_N \]  
(4.16)

where $N$ is itself a random variable. We are going to assume that the interferers (FH users) are distributed according to a spatial Poisson distribution [16]. If $\rho$ is the average number of users per unit area, then the number of FH users $N$ in an area of size $A$ has the Poisson PMF

\[ P(N = n) = \begin{cases} \frac{e^{-\rho A} \rho^n}{n!} & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \]  
(4.17)

In the case of the FH system, the base station activates all the FH units which are within an activation radius $R$. We are also going to assume that the activated FH units are located in an annular ring around the FH base station as shown in Figure 4.3. Note that due to the annular structure, $A = \pi (R^2 - r_0^2)$.

If we assume that the $I_i \ i = 1, \ldots, N$ are i.i.d., then $P$ is a random sum of independent random variables. Since $N$ the number of FH terminals activated is independent of the
interference $I_i$ of each user $i$, we can write [20]

$$\mu_P = E[P] = E[I_i]E[N] = \rho AE \left[ \frac{y(H_i)}{X_i^2} \right]$$

(4.18)

The independence of $y(H_i)$ and $X_i$ further yields

$$\mu_P = \rho AE[y(H_i)] E \left[ \frac{1}{X_i^2} \right]$$

(4.19)

From Equations (4.4) and (4.14), we have

$$y(H_i) = k|U(f_c - f_{H_i})|^2$$

$$= kT_c^2 |\text{sinc}((f_c - f_{H_i})T_c)Z(f_c - f_{H_i})|^2$$

(4.20)

But we have seen before in Subsection (3.1) that $|Z(f)|^2$ only acts as a scaling factor (a factor of $l = 12$). Therefore,

$$y(H_i) = kT_c^2 \text{sinc}^2((f_c - f_{H_i})T_c)$$

(4.21)

Let $k_o = kT_c^2$ for convenience. Recall that $y(H_i)$ is the FH interference power (centered at hop $H_i$) that ‘leaks’ into the DS receiver bandwidth. Hence, $y(H_i)$ is a discrete random variable and its value depends on the location of the frequency hop. Since we have assumed the hops to be uniformly distributed over $H = 1, \ldots, h_{\text{max}}$, the values of
$y(H_i)$ are equiprobable as well. In our case $h_{\max} = 48$ and so $y(H_i)$ may take on any of the 48 values with equal probability. Note that due to the symmetry of the $\text{sinc}^2()$ function, $y(1) = y(48)$, $y(2) = y(47)$, $\ldots$, $y(24) = y(25)$. However, due to the shape of the $\text{sinc}^2()$ function, $y(1) < y(2) < \ldots < y(24)$.

Now,

$$E[y(H_i)] = \frac{1}{h_{\max}} \sum_{h=1}^{h_{\max}} y(h)$$

$$= \frac{2k_0}{h_{\max}} \sum_{h=1}^{h_{\max}/2} \text{sinc}^2[(f_c - f_h)T_c]$$

$$= \frac{2k_0}{h_{\max}} \sum_{h=1}^{h_{\max}/2} \text{sinc}^2(hwT)$$

where $w$ is the frequency spacing between hops (200 kHz for the FH system). Since the hops are located at integral multiples of $w$ from $f_c$, $(f_c - f_h) = hw$.

If we let

$$\alpha = \sum_{h=1}^{h_{\max}/2} \text{sinc}^2(hwT)$$

then

$$E[y(H_i)] = \frac{2k_0}{h_{\max}} \alpha$$

Further, let $E[y(H_i)]$ be denoted by $\mu_y$. Then,

$$\mu_P = \rho A \mu_y E\left[\frac{1}{X^\gamma}\right]$$

To calculate $E[1/X^\gamma]$, we are going to make yet another important assumption. Let us assume that the base stations of the DS and the FH systems are located very close to each other and are located at the center of the circle in Figure 4.4. This is the worst case scenario because in this case when the FH base activates the FH units around it, their responses are going to cause the maximum interference at the DS base station. Had the two base stations been far apart, only a fraction of this total interference power would have affected the DS receiver.
Figure 4.4: We assume that the DS receiver is located very close to the FH base station. This causes the maximum interference to the DS signals at their receiver and represents a worst-case scenario.

Proceeding under the assumptions made above, we have,

\[
E \left[ \frac{1}{X^\gamma} \right] = \int_{r_o}^R \frac{1}{x^\gamma} f_X(x) dx \tag{4.27}
\]

Since \( f_X(x) = dF_X(x)/dx \), let us find an expression for \( F_X(x) \). By definition,

\[
F_X(x) = \Pr \{ X \leq x \}
\]

\[
= \Pr \{ \text{being in the shaded region within the dotted circle} \}
\]

\[
= \left( \frac{\pi x^2 - \pi r_o^2}{\pi R^2 - \pi r_o^2} \right)
\]

\[
= \left( \frac{x^2 - r_o^2}{R^2 - r_o^2} \right) \tag{4.28}
\]

Then,

\[
f_X(x) = \frac{d}{dx} F_X(x) = \frac{2x}{R^2 - r_o^2} \tag{4.29}
\]

Consequently, on substituting the above expression for \( f_X(x) \) into Equation (4.27), we have

\[
E \left[ \frac{1}{X^\gamma} \right] = \int_{r_o}^R \frac{1}{x^\gamma} \left( \frac{2x}{R^2 - r_o^2} \right) dx
\]

\[
= \frac{2}{R^2 - r_o^2} \int_{r_o}^R \frac{1}{x^{\gamma-1}} dx
\]

\[
= \frac{2}{(R^2 - r_o^2)(\gamma - 2)} \left[ \frac{1}{r_o^{\gamma-2}} - \frac{1}{R^{\gamma-2}} \right] \tag{4.30}
\]
Then, substituting $E[1/X^\gamma]$ in Equation (4.26), we have

$$\mu_P = \rho A \mu_y \frac{\gamma}{R \gamma - 2} \left( \frac{1}{\gamma - 2} - \frac{1}{R \gamma - 2} \right)$$

(4.31)

Therefore the SIR at the DS receiver, after the despreading of the DS signal has taken place, can be written as

$$\text{SIR} = \Gamma = \frac{\text{Average Recvd DS signal power}}{\text{Avg FH interference}} = \frac{R}{\mu_P}$$

(4.32)

From Equation (3.23), $R = k_u D/d_0^2$ where $D$ is the DS unit’s transmitted power and $d$ is its distance from its base station. Rewriting the above expression in terms of all the other constants, we have

$$\Gamma = \left[ \frac{k_d D}{d^4} \right] \left[ \frac{1}{\rho A} \right] \left[ \frac{1}{\mu_y} \right] \left[ \frac{(\gamma - 2)(R^2 - r_0^2)(R r_o)^{\gamma - 2}}{2(R^\gamma - 2 - r_0^\gamma - 2)} \right]$$

(4.33)

From Equation (4.25), $\mu_y = E[Y] = 2k_o \alpha / h_{\text{max}}$. Recall that the constant $k_o$ embedded in $\mu_y$ represents the product $k_d T_c^2$ where $k = k_f F / T_b^2$. Rewriting Equation (4.33) in terms of these constants (and for $\gamma = 4$),

$$\Gamma = \left[ \frac{1}{\rho A} \right] \left[ \frac{k_d D}{d^4} \right] \left[ \frac{h_{\text{max}} T_b^2}{2k_f F T_c^2 \alpha} \right] \left[ (R r_o)^2 \right]$$

(4.34)

On rearranging the terms in the expression above, we obtain

$$\Gamma = \frac{1}{\rho A} \frac{k_d D}{k_f F} \left( \frac{T_b}{T_c} \right)^2 \frac{h_{\text{max}}}{2l \alpha} \left( \frac{R^2 r_o^2}{d^4} \right)$$

(4.35)

We had earlier defined $T_b/T_c = \kappa$ as the spreading gain for the DS system. As we have shown in the previous section, the ratio of the path loss constants $k_d/k_f$ is equal to the ratio of the squares of their respective antenna heights provided the other factors remain the same, i.e. $k_d/k_f = \frac{h_d^2}{h_f^2}$. Substituting these into Equation (4.35), we have

$$\Gamma = \frac{1}{\rho A} \left( \frac{h_d}{h_f} \right)^2 \left( \frac{D}{F} \right) \kappa^2 \left( \frac{h_{\text{max}}}{2l \alpha} \right) \left( \frac{R^2 r_o^2}{d^4} \right)$$

(4.36)
The variables in the above expression are $\rho$, $R$, $d$, and the ratio of the transmitted powers $- D/F$. Therefore let us parameterize $\Gamma$ in terms of these variables.

$$\Gamma = \frac{R^2}{\rho Ad^3} \left( \frac{D}{F} \right) \left[ \frac{2}{r_o} \left( \frac{h_d}{h_f} \right)^2 \kappa^2 \left( \frac{h_{\text{max}}}{2\lambda} \right) \right]$$  \hspace{1cm} (4.37)

Now let

$$\sigma = \left[ \frac{2}{r_o} \left( \frac{h_d}{h_f} \right)^2 \kappa^2 \left( \frac{h_{\text{max}}}{2\lambda} \right) \right]$$  \hspace{1cm} (4.38)

Then,

$$\Gamma = \sigma \left( \frac{D}{F} \right) \left[ \frac{R^2}{\rho Ad^3} \right]$$  \hspace{1cm} (4.39)

$$= \frac{\sigma D}{\pi \rho F d^4} \frac{1}{1 - (\frac{r_o}{R})^2}$$  \hspace{1cm} (4.40)

In the limiting case, as the activation radius $R$ becomes very large with respect to $r_o$, the DS SIR $\gamma$ approaches $\gamma^*$ which is independent of $R$. That is,

$$\gamma^* = \frac{\sigma D}{\pi \rho F d^4}$$  \hspace{1cm} (4.41)

A physical interpretation of $\gamma$ approaching $\gamma^*$ could be that the FH terminals closest to the DS base station dominate the interference experienced by the DS transmitter. Therefore, as we move away from the DS base station, the contribution of every additional FH unit decreases significantly due to the high path loss exponent.

In Figure 4.5 we plot $\gamma^*$ against $d$ the DS terminal’s distance from its base station. The curves correspond to different values of $D/F$ the ratio of DS/FH transmitter powers, and $\rho$ the FH user density. In this figure, we look at both an urban as well as a suburban model.

In the ‘urban’ or an office type environment, the density of the FH terminals has been assumed to be $\rho = 1$ FH terminal per 100 m$^2$. We plot two sets of curves, each corresponding to a different value of $D/F$. For $D/F = 1$, to achieve an SIR of $\gamma_{th} = 10$ dB, the DS transmitter has to be extremely close to its base station (Access Point). However, for $D/F = 1000$ which is the case for our DS/FH model, the same SIR can be
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
<td>SIR of a DS transmission (dB)</td>
</tr>
<tr>
<td>$\Gamma^*$</td>
<td>Limiting value of DS SIR $\Gamma$ as $R \to \infty$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>FH user density (users per unit area)</td>
</tr>
<tr>
<td>$A$</td>
<td>Activation area: $A = \pi(R^2 - r_o^2)$</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of FH units activated $(n)$</td>
</tr>
<tr>
<td>$R$</td>
<td>Activation radius (meters)</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance of DS source from its base (meters)</td>
</tr>
<tr>
<td>$F$</td>
<td>Average transmitted power of the FH source</td>
</tr>
<tr>
<td>$D$</td>
<td>Average transmitted power of the DS source</td>
</tr>
<tr>
<td>$r_o$</td>
<td>Inner radius: 1 meter</td>
</tr>
<tr>
<td>$h_d/h_f$</td>
<td>Ratio of antenna heights (assumed 1)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Spreading gain of DS system: 11</td>
</tr>
<tr>
<td>$h_{\text{max}}$</td>
<td>Number of frequency hops available: 48</td>
</tr>
</tbody>
</table>

Table 4.2: Summary of variables used

...achieved by the DS transmitter from a greater distance. Hence, under the assumptions made above, a DS transmitter should be able to survive the FH interference if it is within 15 m of its receiver.

In a typical ‘suburban’ type environment, the DS system could be in a house along with the FH unit, and the FH base station could be located in a van which drives through the neighborhood sending wake-up signals. We assume a density of $\rho = 1$ FH terminal per 1000 m$^2$. As expected, due to the sparse distribution of the FH terminals, the DS system doesn’t experience as high an interference as in the urban model. Here, even for $D/F = 1$, an SIR of $\Gamma_{th} = 10$ dB can be easily achieved from much further than in the urban case. Therefore, the DS system should be able to sustain the FH interference in a suburban environment as well.
Figure 4.5: The SIR $\Gamma^*$ (limiting case) of the DS source as a function of its distance $d$ from its base station for different values of the FH terminal density $\rho$. Each curve corresponds to a certain ratio of DS/FH transmitter power $D/F$. 

Solid Lines (Suburban Model): $\rho = 1$ FH terminal per $1000\, m^2$

Broken Lines (Urban Model): $\rho = 1$ FH terminal per $100\, m^2$
Chapter 5

Conclusions and Future Work

This thesis looked at some of the interference issues between a DS and an FH system operating in the unlicensed band (902-928 MHz). Since this band is unregulated, the systems do not cooperate with each other. Consequently, a system might become a victim of excessive interference and thus fail to operate successfully. The models we developed are general and can be extended to represent any DS or FH system. We first looked at the performance of the FH system under the influence of interference from the DS transmitters. Then, we considered the reverse case.

5.1 FH as Source and DS as Interferer

As shown in Figures 2.7, 2.8 and 2.9, for high values of DS throughput $\tau$, the probability of an FH message colliding with a DS transmission is very high. We studied a worst case scenario by assuming that the DS system is always operating at a high throughput rate of $\tau = 1$. The DS spectrum is centered at 915 MHz which is the center of the FH band as well as that of the unlicensed band. Due to the DS interference, FH messages sent on some of the frequency hops experience excessive interference. As a result, their SIR is below the required threshold and hence they cannot be decoded correctly at their receiver. In Figure 3.7, we plot the fraction $P\{\mathcal{G}\}$ of FH frequencies that have the SIR above the required threshold versus the ratio ($d_u/d_w$) of the DS to FH transmitter distances from the FH base station. We plotted curves for different ratio of DS to FH transmitter powers $D/F$. The curves rise sharply due to the flatness of the DS spectrum in the band of interest; see Figure 3.9. Figure 3.7 indicates that for a DS/FH
model similar to ours, where the ratio of transmitter powers is 1000, the DS transmitter can be twice as far as the FH transmitter from the FH base station and still block all its frequencies. The values of $d_u/d_w$ between which $P\{\mathcal{G}\}$ varies from 0 to 1 are called the lower and upper cutoff-ratio. They have been denoted by $(d_u/d_w)_a$ and $(d_u/d_w)_b$, respectively. In Figure 3.8, we plot $(d_u/d_w)_a$ and $(d_u/d_w)_b$ as a function of the ratio $D/F$.

5.2 DS as Source and FH as Interferer

In this case we considered a DS base station located at roughly the same spot as the FH base station. This is a worst case scenario because here, when the FH base station activates its transmitters within a certain activation radius $R$, their responses cause the maximum interference to the DS transmission received at the DS base station. We assumed that the interfering FH terminals are distributed in an annular ring in accordance with a spatial Poisson distribution. We observed that as $R$ becomes very large with respect to $r_o$, the internal radius of the annular ring, the DS SIR $\Gamma \to \Gamma^*$, a value independent of $R$. This may be due to the fact that the FH terminals closest to the DS base station contribute to most of the interference. Since the attenuation in power increases rapidly with distance, the FH terminals beyond a certain distance will have negligible effect on the DS system. Figure 4.5 plots $\Gamma^*$ as a function of $d$ the DS transmitter's distance from its receiver. Both urban and suburban type environments have been considered. For $D/F = 1000$, Figure 4.5 suggests that the DS transmitter should be able to achieve its required SIR of 10 dB (at its receiver) from a maximum distance of 15 m and 110 m approximately for the urban and suburban models, respectively.

5.3 Suggested Future Work

Since we have developed a general framework for analyzing DS and FH system performance under mutual interference, one can extend this model to study interference
between other DS/DS or FH/FH systems. It may be useful to study the interference due to other systems like LMS (Location and Monitoring Services) which also operate in the unlicensed band. This work can also be extended to incorporate the effects of channel fading and multipaths.
References


