Subspace Based Estimation of the Signal to Interference Ratio for TDMA Cellular Systems

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Abstract

The Signal-to-Interference ratio (SIR) has been highlighted in the literature to be a most efficient criterion for several methods aiming at reducing the effects of cochannel interference, e.g. diversity reception, dynamic channel allocation and power control. In this paper we address the problem on how to obtain fast and accurate measurements of this parameter in a practical context. We develop a general SIR estimation technique for narrow-band cellular systems, that is based on a signal subspace approach using the sample covariance matrix of the received signal. Simulation results for a GSM like system show that the SIR can be estimated to within an error of 0.3 dB after only 200 ms, or within an error of 0.1 dB after only 0.6 seconds.

1 Introduction

Several methods and techniques have been developed to combat the effects of cochannel interference, to increase system capacity and improve communication quality in cellular radio systems. Many such techniques have used the *Signal to Interference Ratio (SIR)* to assess signal quality, and where it has been assumed that the receivers (base stations in the uplink and mobile stations in the downlink) can measure this parameter in real time during operation.

For instance, in the optimum ratio combining of signals from a diversity antenna, the signals are weighted with their corresponding SIR value in each branch. Further, in [13], it has been shown that an SIR-balancing power control is optimum in the sense that it maximizes the minimum SIR for a set of transmitters using a narrow-band channel. Based on this SIR-balancing property, several distributed power control algorithms have been derived, see e.g. [11]. The SIR parameter can also be used for handoff and dynamic channel allocation [2].

However, in contrast to many studies, in which the SIR criterion is used and assumed to be easily available, little attention has been devoted to the problem on how to obtain fast and accurate measurements of this parameter in a practical context.

The SIR estimation problem for analog (e.g. AMPS) cellular systems has been studied in [7,12]. In [7], it has been shown that by separating the received signal into two components at different frequencies, where the two components are known by the receiver, it is possible to get an estimate of the SIR.

The study in [12] is confined to systems employing QPSK modulation schemes. It has been demonstrated by numerical examples, that the average absolute difference between the in-phase and the quadrature envelopes are correlated to the SIR, and could, therefore, be used as an SIR estimator.

The methods above have focused on analog cellular systems, where the delay spread usually is negligible compared to the symbol duration. The second generation of cellular systems, e.g. D-AMPS (IS-54) and GSM, employ high bit rate TDMA schemes, resulting in time dispersive fading channels. SIR estimators for this type of systems have recently been studied in [1, 3, 4].

The study in [3] presents an estimation method, named Signal to Variation Power (SVR) estimator, which is based on the observation that on a short time scale, the signal from the desired transmitter will have a constant envelope, whilst the joint interference and noise signal produce oscillations. The SVR estimator has been applied to a DECT system in [4]. Numerical results reveal that the estimator suffers from a large bias for interesting values of the SIR.

Another SIR estimation method for TDMA cellular systems, has been developed in [1]. The algorithm is applied during the reception of the training sequence in each TDMA slot, in which case the receiver knows the transmitted sequence, providing an unbiased estimate of the SIR. The study in [1] has demonstrated by numerical examples, that the SIR can be estimated to within 2 dB in less than a second, when using the training and color code sequences in a D-AMPS cellular system. The method requires, though, some knowledge of the channel.

In this paper we develop a general SIR estimation technique for narrow-band cellular systems, that is based on a signal subspace method, using the sample covariance matrix of the received signal. The method requires essentially no information about the channel.

In Section 2, we introduce the model and describe our system. In Section 3, we outline some theory from linear algebra and derive the subspace based SIR estimator method. In Section 4, we evaluate the performance of the proposed estimator in an hexagonal cellular system using the GSM frame structure. Finally, we present our conclusions in Section 5

2 System Model

Consider a TDMA cellular radio system and let us focus on a generic time slot. The transmit filter, the channel, the matched filter and the sampler in a TDMA system can be represented by a discrete time transversal filter with a channel tap spacing equal to the symbol duration [9]. The number of channel taps, which determines the delay spread, is environment dependent. The delay spread, due to multipath propagation, is more dominant in urban areas where more objects causing reflections are present, compared to suburban areas and open terrain. Assume that the effective number of channel taps is equal to M.

Let $\{a_j\}$ denote the sequence of transmitted symbols. The j^{th} received symbol can now be expressed as,

$$r_j = \sum_{l=1}^{M} h_l(j) a_{j-l+1} \ , \tag{1}$$

where the index j is used to describe the channel tap coefficient, $h_l(j)$, that may vary in time. We further assume that we employ a modulation scheme, e.g. a PSK modulation method, with the property that $E[a_i] = 0$ where a_i is a data symbol.

Note that FDMA cellular systems can also be incorporated into our model. In such systems the delay spread usually is negligible compared to the symbol duration, which corresponds to a transversal filter with $M = 1$.

If the channel does not vary significantly over $L + M - 1$ consecutive symbols, we can form an *observation vector*, $\boldsymbol{y}$, of length L as follows,

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{h} \ , \tag{2}$$

where the channel taps are represented by the vector $\boldsymbol{h} = [h_1, \ldots, h_M]^T$, and where $\boldsymbol{x}^T$ denotes the transpose of $\boldsymbol{x}$.

The matrix $\boldsymbol{A}$ is a $L \times M$ Toeplitz matrix formed by any subsequence of consecutive transmitted symbols. For sake of illustration, let the subsequence be $\{a_1, a_2, \cdots, a_{M+L-1}\}$. Then the matrix $\boldsymbol{A}$ has the following form,

$$\boldsymbol{A} = [a_{ij}] = \begin{pmatrix} a_M & \ldots & a_2 & a_1 \\ a_{M+1} & \ldots & a_3 & a_2 \\ \vdots & \ddots & \vdots & \vdots \\ a_{M+L-1} & \ldots & a_{L+1} & a_L \end{pmatrix} , \tag{3}$$

where $\boldsymbol{y} = [r_M \cdots r_{M+L-1}]^\top$. Since we also will work with a sequence of observation vectors, let $\boldsymbol{y}(k)$ denote the k^{th} observation vector formed according to (2), i.e.,

$$\boldsymbol{y}(k) = \boldsymbol{A}(k)\boldsymbol{h}(k) \ . \tag{4}$$

Let us now expand the model to also include in the received signal, the signals from N_I number of interferers, and the receiver noise. The received signal from each of the interfering transmitters can be expressed as in (4). Hence, we obtain,

$$\boldsymbol{y}(k) = \boldsymbol{A}_0(k)\boldsymbol{h}_0(k) + \sum_{n=1}^{N_I} \boldsymbol{A}_n(k)\boldsymbol{h}_n(k) + \boldsymbol{n}(k) \ , \tag{5}$$

where the index 0 is associated with the desired transmitter. The receiver noise $\boldsymbol{n}(k)$ is modeled as an independent, zero-mean, complex Gaussian random process with second-order moments

$$E[\boldsymbol{n}(k)\boldsymbol{n}^H(l)] = \sigma_N^2 \boldsymbol{I}\delta_{kl} \quad ; \qquad E[\boldsymbol{n}^\top(k)\boldsymbol{n}(l)] = \mathbf{0} \ , \tag{6}$$

where $\boldsymbol{x}^H$ denotes the Hermitian transpose of $\boldsymbol{x}$, σ_N^2 is the noise power, δ_{kl} represents the Kronecker delta function, and $\boldsymbol{I}$ is the identity matrix.

We confine our framework to the case when $\boldsymbol{A}_0(k)$ consists of symbols that are known by the receiver. In TDMA cellular systems, every burst of symbols within a time slot is equipped with a training sequence to be used for purposes such as base station identification, symbol synchronization and estimation of channel characteristics. The training sequence consists of a known pattern of symbols, and is therefore used in this study to form the matrix $\boldsymbol{A}_0$. From now on, let the index k in (5) denote the k^{th} observation vector formed by the symbols obtained during the reception of the training sequence in the k^{th} time slot. The training sequence is identical for all time slots, which yields $\boldsymbol{A}_0(k) = \boldsymbol{A}_0 \ \forall k$.

We further assume that the received signals from the interferers are uncorrelated and mutually uncorrelated. This assumption is motivated by the following reason. In the GSM and D-AMPS cellular systems, the base stations are not synchronized. This implies that the time slots from any of the cochannel interferers may be received anywhere within the time slot received from the desired transmitter. Further, the training sequence is short compared to the entire length of a time slot. Therefore, it is reasonable to assume that the received symbols from the interferers, during the reception of the training sequence from the desired transmitter, originate from data symbols. From the assumption that the data symbols , excluding the symbols in the training sequence, for all transmitters are uncorrelated and mutually uncorrelated, the assumption stated above follows directly.

From (5), (6) and the assumptions above, the SIR, γ, can be written as follows. For notational convenience, let $h_j^n(k)$ denote the the j^{th} channel tap for the n^{th} interferer in the k^{th} observation vector. Then, we have

$$\gamma \ = \ \frac{E[\boldsymbol{h}_0^H(k)\boldsymbol{A}_0^H(k)\boldsymbol{A}_0(k)\boldsymbol{h}_0(k)]}{E\left[\sum_{n=1}^{N_I}\boldsymbol{h}_n^H(k)\boldsymbol{A}_n^H(k)\sum_{m=1}^{N_I}\boldsymbol{A}_m(k)\boldsymbol{h}_m(k)\right] + \sigma_N^2}$$

$$= \frac{E[\boldsymbol{h}_0^H(k)\boldsymbol{A}_0^H(k)\boldsymbol{A}_0(k)\boldsymbol{h}_0(k)]}{\sum_{n=1}^{N_I}\sigma_a^2\sum_{j=1}^{M}E\left[|h_j^n(k)|^2\right]+\sigma_N^2} = \frac{\sigma_S^2}{\sigma_{I+N}^2} \tag{7}$$

where σ_a^2 is the power of the transmitted symbols, σ_S^2 is the power of the desired signal and σ_{I+N}^2 denotes the interference plus noise power.

Study now the covariance matrix, $\boldsymbol{R}$, of the observation vectors, that will be used in the next section to derive the subspace based SIR estimator. The matrix $\boldsymbol{R}$ can be expressed as,

$$\begin{aligned}
\boldsymbol{R} &= E[\boldsymbol{y}(k)\boldsymbol{y}^H(k)] = \boldsymbol{A}_0 E[\boldsymbol{h}_0\boldsymbol{h}_0^H]\boldsymbol{A}_0^H + \\
&\quad + \sum_{n=1}^{N_I} E[\boldsymbol{A}_n(k)\boldsymbol{h}_n(k)\boldsymbol{h}_n^H(k)\boldsymbol{A}_n^H(k)] + \sigma_N^2\boldsymbol{I} \\
&= \boldsymbol{A}_0\boldsymbol{H}_0\boldsymbol{A}_0^H + \sigma_{I+N}^2\boldsymbol{I}\ ,
\end{aligned} \tag{8}$$

where we have assumed that the channel taps are uncorrelated, i.e., $E[h_i^n h_j^n] = 0,\ \forall\, i \neq j$.

3 Subspace Based SIR estimator

Consider the situation where each observation vector, $\boldsymbol{y}(k)$, is of length L, such that $L > M$. The main problem that we are confronted with is to separate the desired signal and the interference plus noise signal from the observation vector in (5). We solve this problem by making an eigenvector decomposition of the covariance matrix in (8).

Let the channel tap covariance matrix $\boldsymbol{H}_0$ in (8) have rank d. The rank d may be less than the number of channel taps, M, if the channel taps are coherent, i.e., if they are identical up to amplitude scaling and phase shift. This, however, will not affect our SIR estimation method. Since for all practical cases the matrix $\boldsymbol{A}_0$ has full rank, the covariance matrix of the desired signal, $\boldsymbol{A}_0\boldsymbol{H}_0\boldsymbol{A}_0^H$, is positive semi-definite and has rank d, where $d \leq M < L$. This implies that the observation space, of dimension L and spanned by the columns of the matrix $\boldsymbol{R}$, can be partitioned into a *signal subspace* spanned by the columns of $\boldsymbol{A}_0$, and into an *interference plus noise subspace* where only the power of the interference plus noise is found. This forms the basis of all signal subspace techniques.

Hence, the covariance matrix $\boldsymbol{R}$ can be expressed in terms of its eigenvector decomposition,

$$\boldsymbol{R} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{U}^H \ , \tag{9}$$

where $\boldsymbol{U} = [\boldsymbol{e}_1, \ldots, \boldsymbol{e}_L]$ consists of the orthonormal eigenvectors of $\boldsymbol{R}$. The diagonal matrix, $\boldsymbol{\Sigma}$ =diag(λ_i), contains the corresponding eigenvalues, where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L$. The eigenvalues of $\boldsymbol{R}$ have the following structure,

$$\lambda_i = \begin{cases} \sigma_{S_i}^2 + \sigma_{I+N}^2 \ , & \text{if } \ i = 1, \ldots, d \\ \\ \sigma_{I+N}^2 \ , & \text{otherwise} \end{cases} \ , \tag{10}$$

where $\sigma_{S_i}^2$ is the power (variance) of the desired signal along the i^{th} eigenvector. From (10), we realize that the d largest eigenvalues in $\boldsymbol{\Sigma}$ correspond to the signal subspace. Thus, if we know the dimension d, then the signal subspace can be identified. With this at hand and (10), we can obtain the powers of the desired signal and the interference plus noise, respectively. This is the basic idea with our estimation technique. How to find the dimension of the signal subspace is described below.

The true covariance matrix $\boldsymbol{R}$ is of course not known and has to be estimated. Since we want to track time variations of the SIR, we form a *moving average* or *Bartlett* estimate of the covariance matrix from the K most recent observation vectors. Let n denote the n^{th} observation vector, after which we would like to obtain an estimate of the SIR. Then we have,

$$\hat{\boldsymbol{R}}(n) = \frac{1}{K} \sum_{k=n-K+1}^{n} \boldsymbol{y}(k)\boldsymbol{y}^H(k) \ . \tag{11}$$

It has been shown, e.g. [8], that the eigenvalues of the sample covariance matrix in (11) is a maximum likelihood estimate of the eigenvalues of the true covariance matrix $\boldsymbol{R}$.

Almost all existing approaches to the determination of the dimension of the signal subspace are based on the observation that the smallest eigenvalue of the covariance matrix has multiplicity $L - d$. We have tested two information theoretic approaches, both proposed in [10], the An Information Criterion (AIC), and the Minimum Descriptive Length (MDL) principle. It turned out that the MDL criterion obtained by far the best performance, and was, consequently, chosen for this task. Strong consistency of the MDL method has been proved in [14].

To describe the MDL estimation method, let $\hat{\lambda}_1 \geq \ldots \geq \hat{\lambda}_L$ denote the eigenvalues of the sample covariance matrix in (11). Further, define the sphericity test function, $T_{sph}(m)$, as

$$T_{sph}(m) = \frac{1}{L-m} \frac{\sum_{i=m+1}^{L} \hat{\lambda}_i}{\left(\prod_{i=m+1}^{L} \hat{\lambda}_i\right)^{\frac{1}{L-m}}} , \tag{12}$$

and the MDL objective function, $F_{\mathrm{MDL}}(m)$, as

$$F_{\mathrm{MDL}}(m) = K(L-m)\log[T_{sph}(m)] + \frac{1}{2}m(2L-m)log(K) . \tag{13}$$

The dimension of the signal subspace is estimated as [10]

$$\hat{d} = \arg\min_{m} F_{\mathrm{MDL}}(m) . \tag{14}$$

From (10),(11) and (14), we now obtain our Subspace Based (SB) SIR estimator, $\hat{\gamma}$.

Subspace Based SIR Estimator

(1) Make an eigenvector decomposition of the sample covariance matrix $\hat{\boldsymbol{R}}$,

$$\hat{\boldsymbol{R}} = \hat{\boldsymbol{U}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{U}}^H ,$$

where $\hat{\boldsymbol{\Sigma}} = \mathrm{diag}(\hat{\lambda}_i)$, with $\hat{\lambda}_1 \geq \ldots \geq \hat{\lambda}_L$.

(2) Estimate the current dimension of the signal subspace, $\hat{d}$, using (12),(13) and (14).

(3) Estimate the interference power according to,

$$\hat{\sigma}_{I+N}^2 = \frac{1}{L-\hat{d}} \sum_{j=\hat{d}+1}^{L} \hat{\lambda}_j ,$$

and the signal power according to,

$$\hat{\sigma}_S^2 = (\sum_{j=1}^{\hat{d}} \hat{\lambda}_j) - \hat{d}\,\hat{\sigma}_{I+N}^2 .$$

(4) The estimate of the SIR is then obtained as

$$\hat{\gamma} = \frac{1}{L}\frac{\hat{\sigma}_S^2}{\hat{\sigma}_{I+N}^2} , \tag{15}$$

where the factor $1/L$ accounts for the fact that we have L samples (symbols) of the received signal within each observation vector.

In the next section we evaluate the performance of the signal subspace SIR estimator in (15). We compare its performance against the performance of the *Interference Projection* (IP) SIR estimator proposed in [1]. The basic idea with the IP estimator is to divide the observation vector in (5) into short observation vectors of length $2M$. The power of the interference plus noise is then estimated by projecting the observation vectors into the null spaces of the submatrices, that are formed from the partitioning of $\boldsymbol{A}_0$. Since the total signal power easily can be estimated, this method provides an estimate of the SIR. It is important to note that the interference projection estimator, in contrast to the estimator in (15), requires the knowledge of the current number of channel taps M.

4 Numerical Examples

The SIR estimators are evaluated numerically in an hexagonal cellular system, using the TDMA frame structure in the GSM specification. A TDMA frame for one carrier in GSM consists of eight time slots, where each time slot, in turn, contains a burst of 148 symbols (bits). In the middle of the burst, the training sequence consisting of 26 symbols (bits) is included. The frame rate is 216.69 frames/s. We employ a BPSK modulation scheme with transmitted symbols $a_i \in \{-1, 1\}$. The carrier frequency is the same as in GSM, $f_c = 900\ MHz$.

We confine the simulations to the downlink and consider the mobile receiver located in the center cell in the hexagonal cellular system, where the radius of a cell is $1\ km$. We use a fixed channel allocation scheme with reuse distance 3. The starting position of the mobile receiver is uniformly distributed over the cell area. The mobile moves with a constant speed in a direction uniformly distributed in $[0, 2\pi]$.

The channel tap, $h_i(k)$, is modeled as,

$$h_i(k) = \sqrt{l(kT_f)s(kT_f)}f_i(kT_f)e^{-j2\pi f_c(k-1)T_s}\ , \tag{16}$$

where $T_f = 4.615\ ms$ is the frame duration and $T_s = 3.69\ \mu s$ is the symbol time. The distance dependent path loss in (16), $l(t)$, is modeled as, $l(t) = d(t)^{-4}$, where $d(t)$ is the distance between the transmitter and the receiver. The shadow fading factor, $s(t)$, is assumed to be log-normally distributed with a mean of 0 dB, and a log-standard deviation of $\sigma_f = 8\ dB$. The shadow fading is further assumed to have the time correlation function proposed in [5], which has been derived based on field experimental data. If $z(t) = (10/\sigma_f)\log_{10} s(t)$, then

$$E[z(t+\tau)z(t)] = e^{-v\tau/X} \ , \tag{17}$$

where v is the velocity of the mobile user. The parameter X is the effective correlation distance of the shadow fading, and is assumed to be 43 m.

The Rayleigh fading factor, $f_i(t)$, is obtained from a ray-tracing propagation package, which provides a time correlated Rayleigh fading process as described in [6]. For simulation purposes, the number of channel taps is taken to be $M = 2$.

The received powers are determined as follows. The receiver noise is assumed to be $10^{-15}\ W$ ($-150\ dB$). All interferers use the same transmitter power. The received interference level is set to be 20 dB above the noise floor, and the signal power is adjusted so that a pre-determined SIR level is obtained in the receiver.

To evaluate the performance of the two estimators we use, as performance measure, the expected absolute error, $E[|\gamma - \hat{\gamma}|]$, where γ is the true SIR and $\hat{\gamma}$ is the estimated SIR value. Under every estimation algorithm we take 2000 independent trials.

Figure 1 depicts the expected absolute error as a function of the number of observation vectors, K (the averaging length), expressed in seconds. $K = 1$ is equivalent to 4.615 ms. Figure 1 shows the performance for different mobile speeds when one interferer is active. Similarly, Figure 2 shows the performance when six interferers are active.

We find that, with the use of the subspace based estimator, the SIR can be estimated to within an error of 0.3 dB after only 200 ms, or alternatively, within an error of 0.1 dB after 0.6 s.

We also observe that the SB estimator is robust against variations in mobile speeds and number of interferers, in contrast to the IP estimator.

Further, the SB estimator outperforms the IP estimator for almost all lengths of averaging. It is only when few observation vectors are available that the IP method provides better estimates. This is due to the fact that the IP technique obtains more than one vector sample within each received training sequence.

We have also studied the performance for different values of the SIR. Both estimators were found to be insensitive to SIR variations.

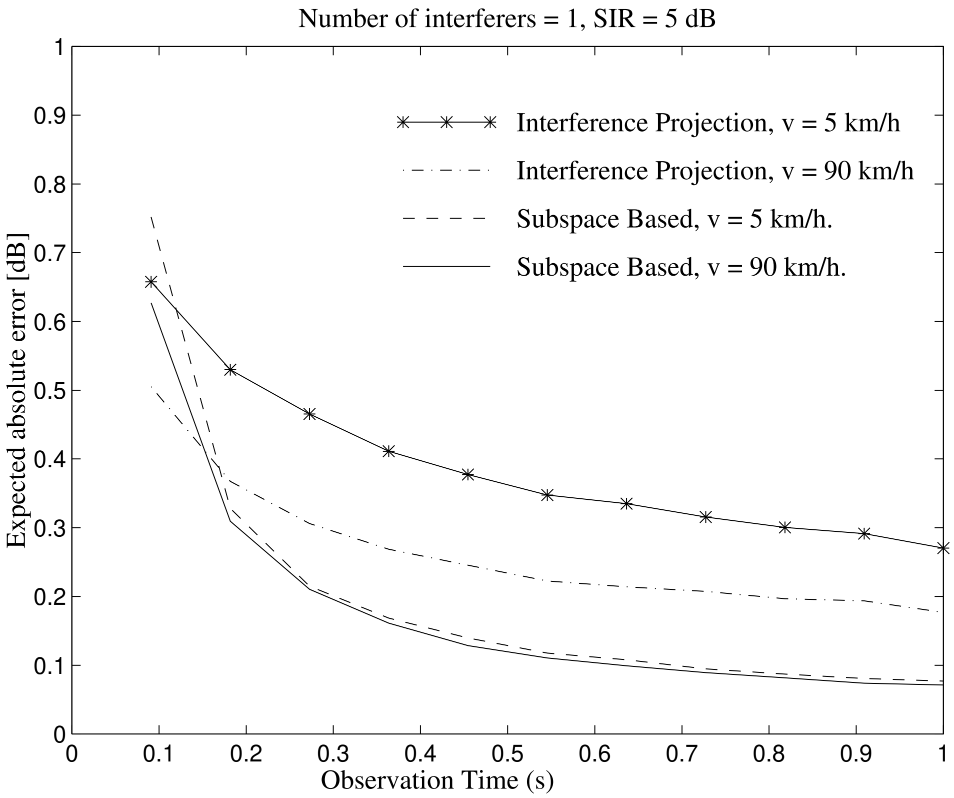

Figure 1: The expected absolute error in dB as a function of the observation interval. The true SIR $= 5\ dB$ and the number of interferers $N_I = 1$.

Note that we have depicted the performance when the sample covariance matrix is formed starting from "scratch", i.e. by starting with no observation vectors available. As soon as we have obtained a first estimate of the SIR, we can start tracking the time variations of the SIR according to (11). This implies that, during the connection of call (neglecting an initial phase of approximately half a second), we will be able to follow the true SIR within an error of less than 0.1 dB, when the SB estimator is used.

5 Conclusions

The SIR has been found to be a most efficient criterion for several radio resource management algorithms, that are designed to combat the effects of cochannel interference. In this paper we have studied the practical problem of obtaining accurate real time measurements of this parameter for narrow-band cellular systems. We have derived an SIR estimation method, that is based on an eigenvector decomposition of the sample covariance matrix of the received signal.

It has been shown by numerical examples that the subspace based SIR estimator is able to estimate the true SIR within an error of 0.3 dB after only 200 ms, or within an error of 0.1 dB after only 0.6 seconds. Further, the estimator has been demonstrated to outperform the interference projection estimator, previously proposed in [1].

Since the subspace based method has the ability to track variations in the channel characteristics, in terms of the actual number of channel taps, this makes it to an attractive SIR estimation technique for time dispersive fading channels.

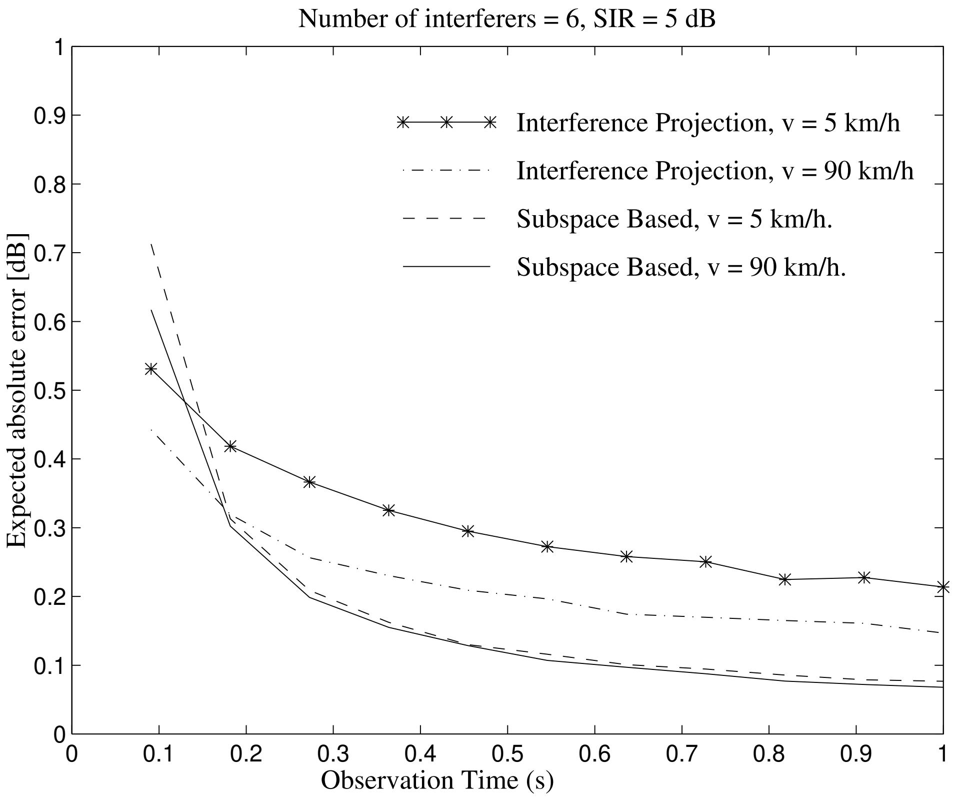

Figure 2: The expected absolute error in dB as a function of the observation interval. The true SIR $= 5\ dB$ and the number of interferers $N_I = 6$.

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