Integrated Power Control and Base Station Assignment

Roy D. Yates and Ching-Yao Huang

Abstract— **In cellular wireless communication systems, transmitted power is regulated to provide each user an acceptable connection while limiting the interference seen by other users. Previous work has focused on maximizing the minimum carrier to interference ratio (CIR) or attaining a common CIR over all radio links. However, previous work has assumed the assignment of mobiles to base stations is known and fixed. In this work, we integrate power control and base station assignment. In the context of a CDMA system, we consider the minimization of the total transmitted uplink power subject to maintaining an individual target CIR for each mobile. This minimization occurs over the set of power vectors and base station assignments. We show that this problem has special structure and identify synchronous and asynchronous distributed algorithms that find the optimal power vector and base station assignment.**

Keywords— **Power control, Cellular radio, CDMA, Handoff, Base station assignment**

I. Introduction

There has been significant work on power control in mobile wireless communications systems. In this problem, there is a set of mobile users that wish to communicate with a set of base stations. These users share a common channel and one user's transmission causes interference for all other users. As a result, transmitted power is regulated both to provide each user an acceptable connection and to limit the interference seen by other users. Previous work [1–4] has focused on maximizing the minimum carrier to interference ratio (CIR) or attaining a common CIR over all radio links. When a base station is designated to receive the signal of user, we say that the user has been assigned to that base station. Previous work has assumed the assignment of users to base stations is fixed or specified by outside means. In this work, we wish to determine which base station should be responsible for receiving a user's signal. In particular, we consider the combined problem of regulating transmitter powers and assigning users to base stations.

The goal of power control and base station assignment is to provide each user an acceptable connection that meets each individual CIR requirement. As a result, we will formulate the Minimum Transmitted Power (MTP) problem in which the total transmitted uplink power is minimized subject to maintaining an individual target CIR for each mobile. This minimization occurs over the set of power vectors and base station assignments.

The authors are with the Wireless Information Networks Laboratory and the Department of Electrical and Computer Engineering, Rutgers University and can be reached at ECE Dept., P.O. Box 909, Piscataway, NJ 08855. This work was supported by National Science Foundation grant NSF-NCRI-92-06148

We emphasize that the objective of power minimization is in fact secondary to that of finding a feasible solution to the MTP problem. The benefit of an algorithm that solves the MTP problem is that it always will find a power vector and base station assignment that provides acceptable connections for all users as long as such a feasible solution exists. Of course, among all feasible solutions, the solution that minimizes total transmitted power would be desirable, particularly when users have limited battery power.

We will describe synchronous and asynchronous versions of a simple algorithm called Minimum Power Assignment (MPA) that iteratively solves the MTP problem. At each iteration of the MPA algorithm, each user selects a base station and transmitter power for which minimum power is needed to maintain an acceptable CIR under the assumption that all other users keep their powers fixed. The intuition behind MPA is simple. A user that reduces its transmitted power while maintaining an acceptable connection does all other users a favor. Similarly, a user that increases its transmitted power so that its CIR exceeds its target causes all other users unnecessary interference.

We emphasize that by iteratively assigning users to base stations, the MPA algorithm is a handoff algorithm that adapts to the number of users and to the location of each user in the system. Alternatively, one can view the base station reassignments specified by the MPA algorithm as defining cell boundaries that adapt to the location and signal attenuation of each user. In this sense, the MPA algorithm represents an implementation of Hanly's concept of expanding and contracting cells in [5].

In this work, we will assume there exists a feasible solution to the MTP problem. When a feasible solution does not exist, the primary issue becomes how to maximize the number of acceptable connections, a very different optimization problem. In future work, we hope to use the framework of the MTP problem to consider this problem.

II. The MTP Problem

We assume N users and M base stations and a common radio channel. Let p_i denote the transmitted power of user i so that $\boldsymbol{p} = [p_1, \ldots, p_N]^\top$ denotes the power vector of the system. The corresponding received signal power of user i at base k is $h_{ki}p_i$ where h_{ki} denotes the gain for user i to base k. The interference seen by user i at base k is $\sum_{j\neq i} h_{kj}p_j$. We define $a_i = k$ if user i is assigned to base k. A base station assignment is a vector $[a_1, \ldots, a_N]$ that specifies an assigned base for each mobile. We wish to solve the Minimum Transmitted Power (MTP) problem:

$$\begin{array}{lll} \text{minimize} & \sum_i p_i & \\ \text{subject to} & & \\ h_{a_i i} p_i \geq \gamma_i \left(\sum_{j \neq i} h_{a_i j} p_j + \sigma_{a_i} \right) & 1 \leq i \leq N & (1) \\ p_i \geq 0 & 1 \leq i \leq N & \\ a_i \in \{1, \ldots, M\} & 1 \leq i \leq N & \end{array}$$

The first set of constraints specifies that each user i has an acceptable connection that attains an individual target CIR γ_i. The second constraint simply demands nonnegativity of the power vector. The quantity σ_{a_i} describes the receiver noise at the assigned base of user i. Finally, we also specify that the assignment vector must take on integer values. In short, our problem is to minimize the total transmitted power subject to maintaining an acceptable connection for each user.

When user i is assigned to base k, we can write the CIR constraint for user i as

$$p_i \geq \boldsymbol{H}_i^{(k)} \boldsymbol{p} + \delta_i^{(k)} \qquad (2)$$

where $\delta_i^{(k)} = \gamma_i \sigma_k / h_{ki}$ and $\boldsymbol{H}_i^{(k)}$ is a normalized row vector with jth element

$$H_{ij}^{(k)} = \begin{cases} \dfrac{\gamma_i h_{kj}}{h_{ki}} & j \neq i \\ 0 & j = i \end{cases} \qquad (3)$$

Since each assignment is specified by $[a_1, \ldots, a_N]$, the number of possible assignment vectors is M^N. We can number the assignments $l = 1, 2, \ldots$ and associate assignment l with the assignment vector $[a_1(l), \ldots, a_N(l)]^\top$. We define $\boldsymbol{G}^{(l)}$, the normalized interference matrix under assignment l, as having i,jth element $G_{ij}^{(l)} = H_{ij}^{(a_i(l))}$. Similarly, the normalized noise vector $\boldsymbol{\sigma}^{(l)}$ is defined to have ith component $\sigma_i^{(l)} = \delta_i^{(a_i(l))}$. We say that a power vector $\boldsymbol{p}$ is feasible under assignment l, if for each user i, $p_i \geq \boldsymbol{G}_i^{(l)} p + \boldsymbol{\sigma}_i^{(l)}$, where $\boldsymbol{G}_i^{(l)}$ denotes the ith row of $\boldsymbol{G}^{(l)}$. That is, the set of feasible power vectors under assignment l is

$$P^{(l)} = \left\{ \boldsymbol{p} \geq 0 | \boldsymbol{p} \geq \boldsymbol{G}^{(l)} \boldsymbol{p} + \boldsymbol{\sigma}^{(l)} \right\} \qquad (4)$$

The set $P^{(l)}$ describes a cone of feasible powers in that if $\boldsymbol{p} \in P^{(l)}$ then $\alpha \boldsymbol{p} \in P^{(l)}$ for all $\alpha \geq 1$. Each cone $P^{(l)}$ is specified by a normalized interference matrix $\boldsymbol{G}^{(l)}$ and normalized noise vector $\boldsymbol{\sigma}^{(l)}$. The nonnegative noise vector $\boldsymbol{\sigma}^{(l)}$ displaces these cones from the origin. A depiction of these cones is provided in Figure 1. Finally, we mention that we could view the MTP problem as the minimization of total transmitted power over the set of feasible power vectors $\cup_l P^{(l)}$. However, as depicted in Figure 1, $\cup_l P^{(l)}$ is typically not a convex set so that the standard approaches to minimizing a linear function over a convex set are not directly applicable.

For a fixed assignment, [1–4, 6, 7] have examined forms of the following maxmin CIR problem [1]:

$$\begin{array}{ll} \text{maximize} & \gamma \\ \text{subject to} & \boldsymbol{p} \geq \gamma (\boldsymbol{G}\boldsymbol{p} + \boldsymbol{\sigma}) \\ & \boldsymbol{p} \geq 0 \end{array} \qquad (5)$$

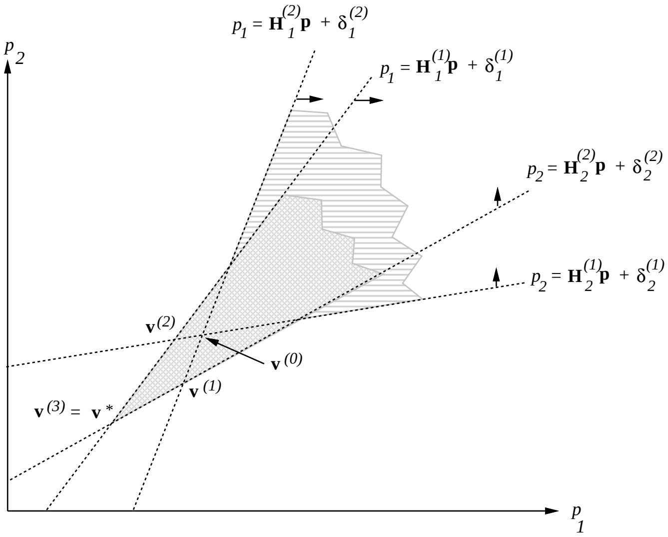
For fixed γ, the constraint set of the maxmin CIR problem defines a cone of feasible power vectors. In [7], Zander addresses the interference limited case of $\boldsymbol{\sigma} = 0$. Grandhi [8] and Foschini [9] have identified iterative algorithms that solve the subproblem of finding a feasible power vector $\boldsymbol{p}$ given a fixed assignment and common CIR target γ. In [10], Mitra proves geometric convergence for an asynchronous version of Foschini's algorithm. These methods find the power vector $\boldsymbol{p} = \boldsymbol{v}$, the vertex of the cone of feasible powers, satisfying $\boldsymbol{v} = \gamma (\boldsymbol{G}\boldsymbol{v} + \boldsymbol{\sigma})$.

In the context of the MTP problem, we embed the set of CIR targets $\{\gamma_i\}$ in the interference matrix $\boldsymbol{G}^{(l)}$ so that the vertex of $P^{(l)}$ is a power vector $\boldsymbol{v}^{(l)}$ satisfying

$$\boldsymbol{v}^{(l)} = \boldsymbol{G}^{(l)} \boldsymbol{v}^{(l)} + \boldsymbol{\sigma}^{(l)} \qquad (6)$$

We will assume that $\boldsymbol{G}^{(l)}$ is nondegenerate, implying $\boldsymbol{v}^{(l)}$ is the unique solution to (6). The vertex $\boldsymbol{v}^{(l)}$ has the following special property.

Lemma 1: For all $\boldsymbol{p} \in P^{(l)}$, $\boldsymbol{p} \geq \boldsymbol{v}^{(l)}$.



This figure depicts the set of feasible power vectors for a system of two users and two base stations. Rays labeled $p_i = \boldsymbol{H}_i^{(k)} \boldsymbol{p} + \delta_i^{(k)}$ describe the minimum power user i needs to communicate with base k as a function of the transmitted power of the other user. Four base station assignments are possible corresponding to the four cones with labeled vertices $\boldsymbol{v}^{(0)}, \ldots, \boldsymbol{v}^{(3)} = \boldsymbol{v}^*$. The union of shaded regions depicts the nonconvex set $\cup_l P^{(l)}$.

Fig. 1. The feasible region of an instance of MTP

[1] Note that $\boldsymbol{G}$ in (5) uses a somewhat different indexing scheme than that used in this work. In particular, G_{kj} is the normalized uplink gain from mobile j to the assigned base station of mobile k.

The proof can be found in Appendix A. We see that for a fixed assignment l, total transmitted power is minimized by choosing $\boldsymbol{p} = \boldsymbol{v}^{(l)}$. Moreover, the minimization of total power over all possible assignments can be reduced to finding the minimum power vertex $\boldsymbol{v}^{(l)}$ among all vertices.

Theorem 1: There exists a vertex $\boldsymbol{v}^*$ such that $\boldsymbol{v}^* \leq \boldsymbol{v}^{(l)}$ for all l.

Proof: Among all feasible assignments l, let l_i denote the assignment that minimizes $v_i^{(l)}$, the ith component of vertex $\boldsymbol{v}^{(l)}$. By definition, $v_i^{(l_i)} \leq v_i^{(l)}$ for all l and

$$v_i^{(l_i)} = \boldsymbol{G}_i^{(l_i)} \boldsymbol{v}^{(l_i)} + \sigma_i^{(l_i)} \tag{7}$$

where $\boldsymbol{G}_i^{(l_i)}$ denotes the ith row of $\boldsymbol{G}^{(l_i)}$. Consider $\boldsymbol{p} = [v_1^{(l_1)} \cdots v_N^{(l_N)}]^\top$. We observe that $\boldsymbol{p} \leq \boldsymbol{v}^{(l_i)}$ for all i, and that

$$\begin{aligned} p_i = v_i^{(l_i)} &= \boldsymbol{G}_i^{(l_i)} \boldsymbol{v}^{(l_i)} + \sigma_i^{(l_i)} && (8) \\ &\geq \boldsymbol{G}_i^{(l_i)} \boldsymbol{p} + \sigma_i^{(l_i)} && (9) \end{aligned}$$

That is, $\boldsymbol{p}$ is feasible with respect to the ith constraint of cone $P^{(l_i)}$. Let $\boldsymbol{G}^*$ denote the interference matrix with ith row $\boldsymbol{G}_i^* = \boldsymbol{G}_i^{(l_i)}$. Let $\boldsymbol{\sigma}^*$ denote the normalized noise vector with ith element $\sigma_i^* = \sigma_i^{(l_i)}$. The pair $\boldsymbol{G}^*$, $\boldsymbol{\sigma}^*$ describes the feasible cone P^* with a vertex $\boldsymbol{v}^*$. We have observed that $\boldsymbol{p} \in P^*$, so Lemma 1 implies $\boldsymbol{p} \geq \boldsymbol{v}^*$. By construction $\boldsymbol{p} \leq \boldsymbol{v}^{(l)}$ for all l. Hence $\boldsymbol{v}^* = \boldsymbol{p} \leq \boldsymbol{v}^{(l)}$ for all l. ■

Theorem 2: Over all feasible power vectors $\boldsymbol{p}$ and base assignments l, vertex $\boldsymbol{v}^*$ and its corresponding base station assignment solves the MTP problem.

Proof: Consider any feasible power vector $\boldsymbol{p}$. Since $\boldsymbol{p} \in P^{(l)}$ for some l, Lemma 1 and Theorem 1 imply $\boldsymbol{p} \geq \boldsymbol{v}^{(l)} \geq \boldsymbol{v}^*$. Hence, $\sum_i p_i^{(l)} \geq \sum_i v_i^*$. ■

Note that the power vector $\boldsymbol{v}^*$ is unique; however, $\boldsymbol{v}^*$ may be the vertex of more than one cone. In this case, there would be multiple base station assignments for which $\boldsymbol{v}^*$ is the vertex power vector. A trivial example occurs when two base stations share the same location in space.

In addition, we observe that if the objective function of the MTP problem were not the total transmitted power but rather $f(\boldsymbol{p})$, a nondecreasing function[2] of the power vector $\boldsymbol{p}$, then $\boldsymbol{v}^*$ would still be the optimal solution. However, we have not resolved how to solve for $\boldsymbol{v}^*$ nor how to find a corresponding optimal assignment for arbitrary $f(\boldsymbol{p})$.

III. Iterative Solutions to MTP

We now describe the iterative MPA (Minimum Power Assignment) algorithm that converges to the optimal assignment and power vector $\boldsymbol{v}^*$. To simplify the description of MPA, we define

$$\begin{aligned} M_i^{(k)}(\boldsymbol{p}) &= \boldsymbol{H}_i^{(k)} \boldsymbol{p} + \delta_i^{(k)} && (10) \\ M_i(\boldsymbol{p}) &= \min_k M_i^{(k)}(\boldsymbol{p}) && (11) \\ A_i(\boldsymbol{p}) &= \arg\min_k M_i^{(k)}(\boldsymbol{p}) && (12) \end{aligned}$$

[2] $f(\boldsymbol{p})$ is nondecreasing in the sense that $\boldsymbol{p} \leq \boldsymbol{p}'$ implies $f(\boldsymbol{p}) \leq f(\boldsymbol{p}')$.

From Equation (2), $M_i^{(k)}(\boldsymbol{p})$ equals the minimum power needed by user i to transmit to base station k with CIR γ_i if each user $j \neq i$ keeps its power p_j constant. Hence, $M_i(\boldsymbol{p})$ equals the minimum power user i needs to transmit to a base station with CIR γ_i and $A_i(\boldsymbol{p})$ would be the corresponding base station. Given a power vector $\boldsymbol{p}(n)$ and assignment $\boldsymbol{k}(n)$ at time n, an MPA iteration finds a new power vector $\boldsymbol{p}(n+1)$, and assignment vector $\boldsymbol{k}(n+1)$ described by

$$\begin{aligned} \boldsymbol{p}(n+1) &= \boldsymbol{T}(\boldsymbol{p}(n)) && (13) \\ \boldsymbol{k}(n+1) &= \boldsymbol{A}(\boldsymbol{p}(n)) && (14) \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{T}(\boldsymbol{p}) &= [M_1(\boldsymbol{p}), \ldots, M_N(\boldsymbol{p})]^\top && (15) \\ \boldsymbol{A}(\boldsymbol{p}) &= [A_1(\boldsymbol{p}), \cdots, A_N(\boldsymbol{p})]^\top && (16) \end{aligned}$$

Starting from an initial power vector $\boldsymbol{p}$, n iterations of the MPA algorithm produce the power vector $\boldsymbol{T}^n(\boldsymbol{p})$ and the base station assignment $\boldsymbol{k}(n) = \boldsymbol{A}(\boldsymbol{p}(n-1))$.

In the enumeration of base station assignments, there exists an assignment l_n such that $a_i(l_n) = k_i(n)$ for all i. Effectively, at step n, the users' base station choices define a new assignment l_n and the new power vector can be written

$$\boldsymbol{p}(n) = \boldsymbol{G}^{(l_n)} \boldsymbol{p}(n-1) + \boldsymbol{\sigma}^{(l_n)} \tag{17}$$

General proof of the convergence of MPA relies on proving convergence in two special cases, as shown in the following sequence of lemmas. Proofs of these lemmas can be found in Appendix A.

Lemma 2: For power vectors $\boldsymbol{p}$ and $\boldsymbol{p}'$, if $\boldsymbol{p} \leq \boldsymbol{p}'$, then $\boldsymbol{T}(\boldsymbol{p}) \leq \boldsymbol{T}(\boldsymbol{p}')$.

Lemma 3: If $\boldsymbol{p}$ is a feasible power vector, then $\boldsymbol{p} \geq \boldsymbol{T}(\boldsymbol{p})$ and $\boldsymbol{T}(\boldsymbol{p})$ is a feasible power vector.

Lemma 4: The power vector $\boldsymbol{v}^*$ is the unique fixed point of the synchronous MPA algorithm.

Lemma 5: Starting from a feasible power vector $\boldsymbol{p}$, the MPA algorithm produces $\boldsymbol{T}^n(\boldsymbol{p})$, a decreasing sequence of feasible power vectors that converges to $\boldsymbol{v}^*$.

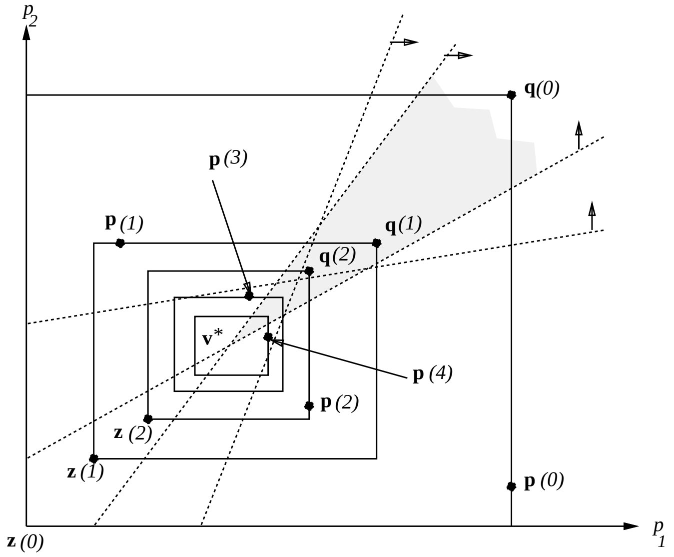
Lemma 6: Starting from $\boldsymbol{z}$, the all zero vector, $\boldsymbol{T}^n(\boldsymbol{z})$ is an increasing sequence of power vectors that converges to $\boldsymbol{v}^*$.

Theorem 3: From any initial power vector $\boldsymbol{p}$, the MPA algorithm produces a sequences of power vectors $\boldsymbol{T}^n(\boldsymbol{p})$ that converges to $\boldsymbol{v}^*$.

Proof: Since $\sigma_i^{(l)} > 0$ for all l, $v_i^* > 0$ for all i. Hence, there exists $\beta > 0$ such that $\boldsymbol{q} = \beta \boldsymbol{v}^* \geq \boldsymbol{p}$. Let $\boldsymbol{z}$ denote the all zero vector so that $\boldsymbol{z} \leq \boldsymbol{p} \leq \boldsymbol{q}$. Lemma 2 implies

$$\boldsymbol{T}^n(\boldsymbol{z}) \leq \boldsymbol{T}^n(\boldsymbol{p}) \leq \boldsymbol{T}^n(\boldsymbol{q}) \tag{18}$$

Since $\boldsymbol{q}$ and $\boldsymbol{z}$ meet the requirements of Lemmas 5 and 6 respectively for initial power vectors, $\lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{z}) = \lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{q}) = \boldsymbol{v}^*$. By Equation (18), we must have $\lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{p}) = \boldsymbol{v}^*$. ■



In this example, there are two base stations and two mobiles with powers p_1 and p_2. Each pair of rays, one marked by $\rightarrow$ and one by $\uparrow$, forms the cone corresponding to a base station assignment. The cone with vertex $\boldsymbol{v}^*$ is shaded. With respect to a particular assignment, rays marked by $\rightarrow$ indicate constraints that p_1 must exceed while rays marked by $\uparrow$ indicate constraints that p_2 must exceed. The initial power vector is $\boldsymbol{p}(0)$. Subsequent power vectors produced by MPA are denoted $\boldsymbol{p}(n)$. Following the proof of Theorem 3, $\boldsymbol{z}(n) = \boldsymbol{T}^n(\boldsymbol{z})$ and $\boldsymbol{q}(n) = \boldsymbol{T}^n(\boldsymbol{q})$ constrain $\boldsymbol{p}(n)$ to the box $\boldsymbol{z}(n) \leq \boldsymbol{p}(n) \leq \boldsymbol{q}(n)$.

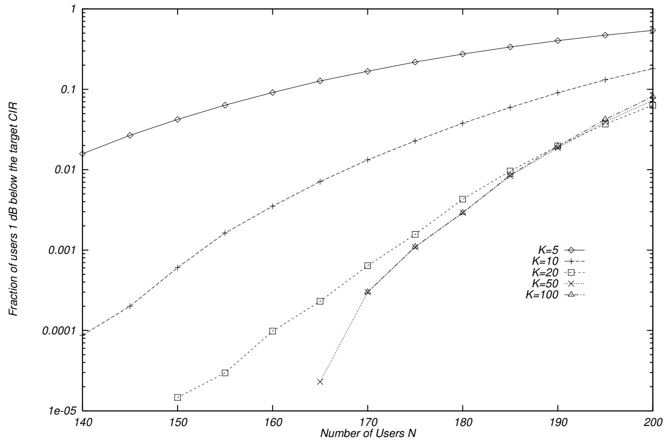
Fig. 2. Convergence of the MPA Algorithm

MPA is fairly practical in that the iteration step can be written as

$$p_i(n) = \min_k \frac{\gamma_i \left(R_k(n-1) - h_{ki} p_i(n-1)\right)}{h_{ki}} \qquad (19)$$

where $R_k(n-1) = \sum_j h_{kj} p_j(n-1) + \sigma_k$ is the total received power (signal plus noise) at base k using power vector $\boldsymbol{p}(n-1)$. The base station assignment $k_i(n)$ is simply the minimizing argument k of Equation (19). Although the minimization in Equation (19) occurs over all base stations k, in practice it would be necessary only to consider nearby base stations as a result of signal attenuation. Effectively, a user needs only to know the total received power and its uplink gain h_{ki} at each nearby base station. If the uplink and downlink face the same attenuation, then user i can deduce the uplink gain h_{ki} from a pilot tone from base station k. Furthermore, it would also be possible for the downlink communication channel to provide each user $R_k(n)$ for all nearby base stations k.

The convergence of the MPA algorithm to the optimal power vector $\boldsymbol{v}^*$ is graphically depicted in Figure 2. We observe that for an initial power vector $\boldsymbol{p} \geq \boldsymbol{v}^*$, the convergence occurs at least as quickly as the convergence of $\boldsymbol{T}^n(\boldsymbol{q})$. Starting from an initial power vector $\boldsymbol{q}(0) = \beta \boldsymbol{v}^*$, it is readily shown that the resulting sequence of power



As a function of the number of iterations K and the number of mobiles N, this plot records the fraction of stationary mobiles with CIR 1 dB below the target CIR over many repeated trials.

Fig. 3. Convergence of the MPA algorithm

vectors $\boldsymbol{q}(n)$ satisfies $\boldsymbol{q}(n+1) \leq \boldsymbol{G}^* \boldsymbol{q}(n) + \boldsymbol{\sigma}^*$. From the Perron-Frobenius Theorem [11, 12], for a nonnegative matrix $\boldsymbol{G}$ the iteration

$$\boldsymbol{p}(n+1) = \boldsymbol{G}\boldsymbol{p}(n) + \boldsymbol{\sigma} \qquad (20)$$

converges to a unique fixed point iff the largest eigenvalue of $\boldsymbol{G}$ is less than unity. In this case, the iteration (20) is a contraction mapping that converges at a geometric rate. Hence, if there exists a feasible power vector $\boldsymbol{p}$ for the MTP problem, then the interference matrix $\boldsymbol{G}^*$ must have largest eigenvalue less than unity. This implies that starting from a feasible solution $\boldsymbol{p}$, geometric convergence to $\boldsymbol{v}^*$ does occur.

From other starting points, however, the rate of convergence is not as well understood. In particular, one can construct examples in which from an infeasible power vector $\boldsymbol{p}$, the MPA algorithm chooses an assignment l_n such that the iteration at step n does not constitute a contraction mapping, thus complicating rate of convergence issues.

To provide some indication of the MPA rate of convergence, a simulation experiment was performed. We considered a system of ten base stations uniformly spaced every 2000 meters on a ring. For each base station, the uplink gain included an order $\alpha = 4$ propagation loss and a position dependent log normal shadow fading component. That is, expressed in dB, the uplink gain h_{ki} of user i to base k at a distance d_{ki} is

$$10 \log h_{ki} = -10\alpha \log d_{ki} + S_k(d_{ki}) \qquad (21)$$

The shadow fading component $S_k(d_{ki})$ was taken to be a normal random variable with mean 0 and standard deviation $\eta = 8$ dB. The shadow fading processes $S_k(d)$ and $S_{k'}(d')$ for distinct base stations k and k' were assumed to be independent. The correlation over distance of the shadow fading was described by the autocorrelation function

$$\rho(d) = E[S_j(d')S_j(d'+d)] = \eta^2 e^{-|d|/d_0} \tag{22}$$

with a correlation distance of $d_0 = 45$ meters.

Each mobile was required to achieve a CIR target of $\gamma = 1/20$ that reflected the processing gain associated with a CDMA system. Note that with $\gamma = 1/20$, a single base station in isolation would be able to support 20 mobiles. For the 10 base station system, we considered a number of mobiles N ranging from 140 to 200. For each N, we repeatedly placed N mobiles randomly on the ring. We defined a mobile to have unacceptable CIR if it was not within 1 dB of its target CIR. For each placement of the mobile users, we started the MPA algorithm with initial power zero and recorded the number of mobiles with unacceptable CIR as a function of K, the number of synchronous MPA iterations. In Figure 3, we graph the fraction of mobiles with unacceptable CIR over a large number of trials for $K = 5, 10, 20, 50, 100$. We see that 20 iterations are nearly as effective as 100 iterations and that for small N, 10 or fewer iterations is often sufficient.

Careful inspection of Figure 3 shows that for $N > 190$, 20 iterations leads to fewer users with unacceptable CIR than 50 or 100 iterations. This arises when the common CIR target γ is infeasible. In this case, the MPA iteration converges to a base station assignment l and for large n, the corresponding power control iteration becomes

$$\boldsymbol{p}(n) = \boldsymbol{G}^{(l)}\boldsymbol{p}(n-1) + \boldsymbol{\sigma}^{(l)} \tag{23}$$

Moreover, the power vector $\boldsymbol{p}(n)$ increases without bound while each user's CIR converges to the maximum attainable CIR, as noted in [13]. When the maximum attainable CIR is more than 1 dB below the target CIR, all users eventually end up with unacceptable CIR. However, in this situation, we have observed that starting from zero power, a modest number of iterations, such as $K = 20$, often leaves many of the users within 1 dB of the target CIR. Hence, when the number of users N is large and it becomes more likely that the maximum attainable CIR is more than 1 dB below the target CIR, a modest number of iterations tends to provide more users an acceptable CIR.

We note that this experiment assumed that all mobiles had perfect information about radio channel. That is, each mobile i knew at every step n both the uplink gains h_{ki} and the total received powers $R_k(n)$ at all base stations in order to execute the MPA using (19). This suggests that the limiting factor in the effectiveness of the MPA algorithm will be the ability of the mobile to collect timely, accurate information about the channel conditions.

IV. The Asynchronous MPA (AMPA) Algorithm

In this section, we examine an asynchronous version of the MPA algorithm using the framework for totally asynchronous algorithms of Bertsekas and Tsitsiklis [11]. The AMPA algorithm allows some users to perform power adjustments faster and execute more iterations than others. In addition, the AMPA algorithm allows users to perform these updates while using outdated information on the interference caused by other users. Let $p_i(t)$ denote the transmitted power of user i at time t so that the power vector at time t is $\boldsymbol{p}(t) = [p_1(t), \ldots, p_N(t)]^\top$.

We assume that user i may not have access to the most recent values of the components of $\boldsymbol{p}(t)$. This occurs when user i has outdated information about the received power at certain bases. At time t, let $\tau_j^i(t)$ denote the most recent time for which p_j is known to user i. Note that $0 \leq \tau_j^i(t) \leq t$. For user i at time t, $p_j(\tau_j^i(t))$ denotes the most recently known value of $p_j(t)$. Thus, at time t, user i may update its transmitter power and base station assignment using the power vector

$$\boldsymbol{p}(\tau^i(t)) = \left[p_1(\tau_1^i(t)), p_2(\tau_2^i(t)), ..., p_N(\tau_N^i(t))\right]^\top \tag{24}$$

We assume a set of times $T = \{0, 1, 2, ...\}$ at which one or more components $p_i(t)$ of $\boldsymbol{p}(t)$ are updated. Let T^i be the set of times at which $p_i(t)$ is updated. At times $t \notin T^i$, $p_i(t)$ is left unchanged. Hence, the AMPA algorithm can be written as

$$p_i(t+1) = \begin{cases} M_i(\boldsymbol{p}(\tau^i(t))) & t \in T^i \\ p_i(t) & t \notin T^i \end{cases} \tag{25}$$

$$k_i(t+1) = \begin{cases} A_i(\boldsymbol{p}(\tau^i(t))) & t \in T^i \\ k_i(t) & t \notin T^i \end{cases} \tag{26}$$

We assume the sets T^i are infinite and given any time t_0, there exists t_1 such that $\tau_j^i(t) \geq t_0$ for all $t \geq t_1$. Convergence of the AMPA algorithm will be proven by the Asynchronous Convergence Theorem as taken from [11]. We note that x and $f(x)$ in the statement of Theorem 4 represent the power vector $\boldsymbol{p}$ and iteration function $\boldsymbol{T}(\boldsymbol{p})$ in the context of this work.

Theorem 4: (Asynchronous Convergence Theorem) If there is a sequence of nonempty sets $\{X(n)\}$ with $X(n+1) \subset X(n)$ for all n satisfying the following two conditions:

1. Synchronous Convergence Condition: For all n and $x \in X(n)$, $f(x) \in X(n+1)$. If $\{y^n\}$ is a sequence such that $y^n \in X(n)$ for all n, then every limit point of $\{y^n\}$ is a fixed point of f.
2. Box Condition: For every n, there exists sets $X_i(n) \in X_i$ such that $X(n) = X_1(n) \times X_2(n) \times \cdots \times X_N(n)$.

and the initial solution estimate $x(0)$ belongs to the set $X(0)$, then every limit point of $\{x(t)\}$ is a fixed point of f.

Theorem 5: From any initial power vector $\boldsymbol{p}$, the AMPA algorithm converges to $\boldsymbol{v}^*$.

Proof: Given an initial power vector $\boldsymbol{p}$, we can choose $\beta > 0$ so that $\boldsymbol{q} = \beta\boldsymbol{v}^* \geq \boldsymbol{p}$. Using $\boldsymbol{z}$ to denote the all zero vector, we define

$$X(n) = \{\boldsymbol{p} | \boldsymbol{T}^n(\boldsymbol{z}) \leq \boldsymbol{p} \leq \boldsymbol{T}^n(\boldsymbol{q})\} \tag{27}$$

For all n, the set $X(n)$ satisfies the box condition. By Lemmas 5 and 6, $X(n+1) \subset X(n)$ for all n. From the proof of Theorem 3, $\lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{z}) = \lim_{n\to\infty} \boldsymbol{T}^n(\boldsymbol{q}) = \boldsymbol{v}^*$. Hence any sequence $\{y^n\}$ such that $y^n \in X(n)$ for all

n must converge to $\boldsymbol{v}^*$. Since the initial power vector $\boldsymbol{p}$ satisfies $\boldsymbol{p} \in X(0)$, the asynchronous convergence theorem implies the AMPA algorithm will converge to $\boldsymbol{v}^*$. ■

The implementation of AMPA defined by Equations (25) and (26) is not the most practical since it requires each user to notify every other user of its transmitted power. Just as in Equation (19) for MPA, the AMPA iteration can be expressed in terms of the total received power (signal plus noise) at each base. In this approach, each base station k uses the downlink channel to inform the users of $R_k(t)$, the total received power at base k at time t. Of course, $R_k(t)$ would not be continuously updated and users may use outdated values of $R_k(t)$ to perform updates. These delays would permit a user i at time t to know $R_k(\hat{\tau}_k^i(t))$, the received power at base k most recently communicated to user i. At time $t \in T^i$, updates of AMPA can be written

$$p_i(t+1) = \min_k \frac{\gamma_i \left(R_k(\hat{\tau}_k^i(t)) - h_{ki}p_i(t)\right)}{h_{ki}} \tag{28}$$

where

$$R_k(\hat{\tau}_k^i(t)) = \sum_i h_{ki}p_i(\hat{\tau}_k^i(t)) + \sigma_k \tag{29}$$

However, at base k, $R_k(\hat{\tau}_k^i(t))$ is calculated using $p_j(\hat{\tau}_k^i(t))$ while at base k', $R_{k'}(\hat{\tau}_{k'}^i(t))$ is calculated using $p_j(\hat{\tau}_{k'}^i(t))$. In the iteration of Equation (28), user i effectively uses p_j at time $\hat{\tau}_k^i(t)$ from base k while also using p_j at time $\hat{\tau}_{k'}^i(t)$ from base k'. The framework of the asynchronous convergence theorem assumes that updates by user i are based on $p_j(\tau_j^i(t))$, a single delayed value of $p_j(t)$. However, we know that the synchronous version of Equation (28) is equivalent to the MPA algorithm. An extension to the asynchronous convergence theorem[3] will verify that the asynchronous iteration defined by Equation (28) converges.

V. Discussion

We believe that MPA can appropriately handle new calls and call completions. In particular, upon a call completion, a user powers down. If all users had acceptable connections before the call completion, then the remaining users will continue to have acceptable connections since the call completion reduces interference. That is, the power vector associated with the remaining users stays feasible. Continued execution of MPA by the remaining users leads to subsequent power reductions and perhaps reassignments. A new call arrival can be initialized with zero power. During call setup, the new user participates in the MPA algorithm. If the call can be carried, MPA will find the optimal power vector and a corresponding base station assignment. However, if it is not possible to provide all users an acceptable CIR, then measures external to the MPA algorithm must be taken to block the new call. This subject merits further attention.

The AMPA algorithm is a flexible totally asynchronous version of MPA. Each user is permitted to adjust its transmitted power and base station assignment using possibly outdated estimates of the interference power at the base stations without waiting for all other users to perform updates. In addition, we note that a user update that includes a base station reassignment will require coordination between the user and the base stations and would take significantly longer than a simple power adjustment. Consequently, the asynchronous algorithm permits users to continue to adjust powers while other users perform more time consuming base station reassignments. Just as for the synchronous case, the rate of convergence of the AMPA algorithm needs further investigation.

We believe thus work provides an appropriate framework for the integration of power control and handoff. However, the interactions between user mobility, channel fading and the MPA algorithm must be investigated.

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After initial submission of this work, we have learned that Stephen Hanly has independently and concurrently proven the synchronous convergence of an algorithm essentially the same as the MPA algorithm in this paper. In [14], Hanly's Algorithm 2 differs from the MPA algorithm only in that it permits the possibility of restricting the base stations that a mobile can use.

Appendix

I. Additional Proofs

Proof: Lemma 1 Given $\boldsymbol{p} \in P^{(l)}$, let $\hat{\boldsymbol{p}} = \boldsymbol{v}^{(l)} + \alpha(\boldsymbol{p} - \boldsymbol{v}^{(l)})$. Since $\boldsymbol{v}^{(l)} = \boldsymbol{G}^{(l)}\boldsymbol{v}^{(l)} + \boldsymbol{\sigma}^{(l)}$ and $\boldsymbol{p} \geq \boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l)}$,

$$\begin{aligned} \hat{\boldsymbol{p}} - \left(\boldsymbol{G}^{(l)}\hat{\boldsymbol{p}} + \boldsymbol{\sigma}^{(l)}\right) &= \alpha\left(\boldsymbol{p} - \boldsymbol{G}^{(l)}\boldsymbol{p} - \boldsymbol{\sigma}^{(l)}\right) && (30) \\ &\geq 0 && (31) \end{aligned}$$

Hence, $\hat{\boldsymbol{p}} \in P^{(l)}$ for all nonnegative α. Now suppose that $p_i < \boldsymbol{v}_i^{(l)}$ for some i. In this case, we can choose α such that for some i, $\hat{p}_i = 0$ and $\hat{p}_j \geq 0$ for all $j \neq i$. For this choice of α,

$$0 = \hat{p}_i < \boldsymbol{G}_i^{(l)}\hat{\boldsymbol{p}} + \boldsymbol{\sigma}^{(l)} \tag{32}$$

which contradicts $\hat{\boldsymbol{p}} \in P^{(l)}$. ■

Proof: Lemma 2 Equation (10) implies that for all i and k, $M_i^{(k)}(\boldsymbol{p}) \leq M_i^{(k)}(\boldsymbol{p}')$. Hence, from Equation (11), $M_i(\boldsymbol{p})$, the ith component of $\boldsymbol{T}(\boldsymbol{p})$, satisfies $M_i(\boldsymbol{p}) \leq M_i(\boldsymbol{p}')$. ■

Proof: Lemma 3 Let $\boldsymbol{p}' = \boldsymbol{T}(\boldsymbol{p})$. Since $\boldsymbol{p}$ is feasible, $\boldsymbol{p} \in P^{(l)}$ for some l. Hence, for all i,

$$\begin{aligned} p_i' &= M_i(\boldsymbol{p}) && (33) \\ &\leq M_i^{(a_i(l))}(\boldsymbol{p}) && (34) \\ &\leq p_i && (35) \end{aligned}$$

Hence $\boldsymbol{p}' \leq \boldsymbol{p}$. To verify feasibility of $\boldsymbol{p}'$, let l' denote the assignment corresponding to $\boldsymbol{A}(\boldsymbol{p})$. In this case, from Equation (17)

$$\begin{aligned} p_i' &= \boldsymbol{G}_i^{(l')}\boldsymbol{p} + \boldsymbol{\sigma}^{(l')} && (36) \\ &\geq \boldsymbol{G}^{(l')}\boldsymbol{p}' + \boldsymbol{\sigma}^{(l')} && (37) \end{aligned}$$

[3] See Problem 2.2 in Chapter 6 of [11].

Hence, $\boldsymbol{p}' \in P^{(l')}$ and $\boldsymbol{T}(\boldsymbol{p})$ is feasible. ■

Proof: Lemma 4 By Lemma 3, since $\boldsymbol{v}^*$ is feasible, $\boldsymbol{T}(\boldsymbol{v}^*) \leq \boldsymbol{v}^*$ and $\boldsymbol{T}(\boldsymbol{v}^*)$ is also feasible. Since $\boldsymbol{T}(\boldsymbol{v}^*)$ is feasible, Lemma 1 and Theorem 1 imply $\boldsymbol{T}(\boldsymbol{v}^*) \geq \boldsymbol{v}^*$. Hence, $\boldsymbol{v}^* = \boldsymbol{T}(\boldsymbol{v}^*)$. To verify uniqueness of the fixed point $\boldsymbol{v}^*$, suppose there exists some other fixed point $\hat{\boldsymbol{v}} \neq \boldsymbol{v}^*$. Since $\hat{\boldsymbol{v}}$ is feasible, $\hat{\boldsymbol{v}} \geq \boldsymbol{v}^*$. Let $\alpha = \max_i(\hat{v}_i/v_i^*)$. Since $\hat{\boldsymbol{v}}$ and $\boldsymbol{v}^*$ are distinct, $\alpha > 1$, $\alpha\boldsymbol{v}^* \geq \hat{\boldsymbol{v}}$, and for some j, $\alpha v_j^* = \hat{v}_j$. In this case, since $\hat{\boldsymbol{v}}$ is a fixed point,

$$\hat{v}_j = \min_k \boldsymbol{H}_j^{(k)}\hat{\boldsymbol{v}} + \delta_j^{(k)} \tag{38}$$

$$\leq \min_k \boldsymbol{H}_j^{(k)}\alpha\boldsymbol{v}^* + \delta_j^{(k)} \tag{39}$$

$$< \alpha\left(\min_k \boldsymbol{H}_j^{(k)}\boldsymbol{v}^* + \delta_j^{(k)}\right) \tag{40}$$

$$= \alpha v_j^* \tag{41}$$

which is a contradiction. ■

Proof: Lemma 5 Let $\boldsymbol{p}(n) = \boldsymbol{T}^n(\boldsymbol{p})$. Since $\boldsymbol{p}(0)$ is feasible, Lemma 3 implies that $\boldsymbol{p}(0) \geq \boldsymbol{p}(1)$ and that $\boldsymbol{p}(1)$ is feasible. Now suppose $\boldsymbol{p}(0) \geq \boldsymbol{p}(1) \geq \cdots \geq \boldsymbol{p}(n)$. In this case, Lemma 3 implies $\boldsymbol{p}(n+1) \leq \boldsymbol{p}(n)$ and $\boldsymbol{p}(n+1)$ is feasible. Hence the sequence of $\boldsymbol{p}(n)$ is decreasing and always feasible. By Lemma 1 and Theorem 1, $\boldsymbol{p}(n) \geq \boldsymbol{v}^*$ for all n. Hence, the sequence of $\boldsymbol{p}(n)$ is bounded below by $\boldsymbol{v}^*$ and so must converge to the unique fixed point $\boldsymbol{v}^*$. ■

Proof: Lemma 6 Let $\boldsymbol{z}(n) = \boldsymbol{T}^n(\boldsymbol{z})$. First, we note that $\boldsymbol{z}(1) = \boldsymbol{T}(\boldsymbol{z}) \geq \boldsymbol{z}$. Assuming $\boldsymbol{z}(0) \leq \boldsymbol{z}(1) \leq \cdots \leq \boldsymbol{z}(n)$, Lemma 2 implies

$$\boldsymbol{z}(n+1) = \boldsymbol{T}(\boldsymbol{z}(n)) \geq \boldsymbol{T}(\boldsymbol{z}(n-1)) = \boldsymbol{z}(n) \tag{42}$$

Hence, the sequence of power vectors is nondecreasing. Second, since $\boldsymbol{z} \leq \boldsymbol{v}^*$, assuming $\boldsymbol{z}(n) \leq \boldsymbol{v}^*$ implies

$$\boldsymbol{z}(n+1) = \boldsymbol{T}(\boldsymbol{z}(n)) \leq \boldsymbol{T}(\boldsymbol{v}^*) = \boldsymbol{v}^* \tag{43}$$

That is, the increasing sequence of power vectors $\boldsymbol{z}(n)$ is bounded above by $\boldsymbol{v}^*$ and so must converge to the unique fixed point $\boldsymbol{v}^*$. ■

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Roy D. Yates received the B.S.E. degree in 1983 from Princeton University, and the S.M. and Ph.D. degrees in 1986 and 1990 from MIT, all in Electrical Engineering. Since September 1990, he has been an Assistant Professor with the Wireless Information Networks Laboratory (WINLAB) in the department of Electrical and Computer Engineering, Rutgers University. His research interests include power control and media access protocols for wireless communications systems, high speed networks, and reversible queueing systems.

Ching-Yao Huang received the B.S. degree in Physics from the National Taiwan University in 1987 and the M.S. degree in Electrical Engineering from the New Jersey Institute of Technology in 1991. He is currently a graduate assistant for the Wireless Information Networks Laboratory (WINLAB) in the Department of Electrical and Computer Engineering, Rutgers University. His research interests include power control and radio resource management for wireless communication systems.