Power Control for an Asynchronous Multi-rate Decorrelator*

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ABSTRACT

The varied Quality of Service (QoS) or bit error rate (BER) requirements of multimedia traffic require the use of power control for CDMA wireless systems employing multiuser detection. Using a decorrelator in an asynchronous multi-rate DS/CDMA system, it may be necessary for different users to combat the noise enhancement and the propagation losses to varying degrees depending on individual requirements. In this context, we propose a power control algorithm for a multi-rate decorrelator that is suitable for a class of BER based link quality objectives. If the uplink channel gain of the desired user is known, then it is straightforward for each user to choose the transmitter power needed to meet its target BER objective. In practice, however, the uplink channel gain is often difficult to measure. To avoid this measurement, we employ stochastic approximation methods to develop a simple iterative power control algorithm. In this algorithm, each mobile will use the output of its own decorrelator to update its transmitted power in order to achieve its QoS objective. We will show that when a user's bits have nonzero asymptotic efficiencies, then the power control algorithm converges quickly in the mean square sense to the minimum power at which a user achieves its QoS objective.

1 Introduction

Extensive efforts are being made to integrate code division multiple access (CDMA) cellular networks with fixed networks for communicating both voice and data messages. Re-

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ceiver design for an asynchronous multi-rate CDMA system has become an important new area of investigation. For multi-rate DS/CDMA systems, several access strategies have been proposed. These include fixed chip rate with variable processing gain [1, 2], fixed processing gain with variable chip rate [1], a multi-modulation scheme [2], and multi-code methods [2, 3]. For the variable processing gain strategy, references [4, 5] addressed the use of decorrelators for a synchronous dual rate DS/CDMA system while reference [6] extended the results to a multi-rate decorrelator. For the same synchronous dual-rate system model, a decision feedback receiver was proposed in [7, 8]. The performance for jointly optimal detection for multi-rate DS/CDMA systems is found in [9]. This study was performed in the context of the multi-code scheme [2, 3] where high rate users multiplex their information bits onto multiple signature waveforms and the above variable processing gain access strategy.

For a single rate DS/CDMA system, [10] showed that the asynchronous decorrelator achieves optimum near-far resistance [10, 11] by extending the observation window to infinity. However, practical truncated window decorrelators [12, 13] may suffer from poor performance or infeasibility. The complexity of the infinite window decorrelator [10] has led researchers to investigate alternative solutions [14–17] with sub-optimum near-far resistance.

Reference [18, 19] proposed the *asynchronous multi-rate decorrelator* (AMD) for an asynchronous multi-rate CDMA system. Here the asynchronous multi-rate system was derived by generalizing the synchronous dual-rate system model of [4, 5]. The AMD is decentralized and decodes users by employing sliding finite length observation windows. In [18, 19], it was proven that the performance of any finite window decorrelator improves monotonically with the observation window length, although a longer observation window results in greater computational complexity. The AMD employs a set of modified correlators and can decode a user by suppressing the multiuser interference as long as that user has non-zero asymptotic efficiency. As with all decorrelators, the multiuser interference is suppressed at the cost of reduced received signal energy. As a consequence of supporting multiple rates in an asynchronous system, different bits of an individual user are subject to varying degrees of energy degradation and, as a consequence, may experience different probabilities of error.

Recently, several studies [20–22] have integrated power control with multiuser detectors. The motivation of these works was to achieve a performance gain over multiuser detection by controlling the interference that must be suppressed. References [20, 21] addressed power control for the linear MMSE multiuser receiver in a single-rate system. The proposed schemes are synchronous and require SIR (signal-to-interference ratio) measurements for the power updates.

In this paper, a class of QoS objectives for the AMD are proposed. The choice of a QoS objective will dictate a corresponding power control algorithm. The criterion that is used for power control is minimize the total transmitted power while at the same time guaranteeing each user's QoS requirement. In this study, we consider the following QoS objectives 1) for each bit of a user, meet a bit error rate (BER) objective, 2) using a fixed transmitted power, provide an individual user with an average BER and 3) using a fixed transmitted power, guarantee a target BER to the bit with the lowest asymptotic efficiency or worst BER.

If the uplink channel gain of the mobile is known, then it is simple to provide each user its QoS objective. In practice, however, the measurement of the channel gain is non-trivial and neglects the stochastic behavior of the channel. This practical situation leads us to use stochastic approximation methods [23] to develop an asynchronous, distributed, iterative power control algorithm for the AMD that finds the correct powers without knowledge of the uplink gain. In this stochastic power control algorithm, a mobile needs to know only the output of the AMD to update its transmitted power in order to achieve its QoS objective. The proposed power control algorithm is feasible as long as asymptotic efficiencies of the bits of the desired user are nonzero. In this case, the algorithm converges in the mean square sense to an optimal power at which the user achieves its QoS requirement.

The power control algorithm in this work is fundamentally different from power control algorithms that have been presented in the literature, for example [24–27]. These works have assumed the availability of perfect measurements of deterministic quantities such as the SIR, received power, or interference in order to prove that the transmitter powers converge deterministically to an optimum power vector. In practice, however, these quantities cannot be estimated perfectly and deterministic convergence results are invalid when these deterministic variables are replaced with their random estimates. Consequently, we adopt from [28] the term *stochastic power control* for algorithms that use actual random measurements to converge stochastically to the optimal transmitter power vector. For conventional matched filter receivers, [28] has studied stochastic power control for matched filter receivers where each user aims for a target SIR using its matched filter output. However, [28] required each user to estimate its uplink channel gain to its assigned base station. Following the approach of [28], this work also uses stochastic measurements at each user's filter output to derive stochastic mean square convergence results. Unlike [28] however, this work exploits the decorrelator's effective orthogonality of users at the receiver to develop an algorithm that does not require a user to know its uplink channel gain.

The rest of the paper is organized as follows. Section 2 will derive a continuous time model for an asynchronous multi-rate system. Section 3 will briefly describe the working principle of the AMD. In section 4, we will propose a power control algorithm that is suitable for a class of BER based QoS objectives for the AMD. Stochastic convergence of the proposed power control algorithm will be proved in section 5. Performance results will be presented in sections 6 while proofs are deferred to Appendix A.

2 Asynchronous multi-rate system

In the asynchronous multi-rate DS/CDMA system model of [18], each bit results in the baseband transmission of a sequence of pulses, or chips, $p[t]$, each pulse having a duration of one chip period T_c. These pulses are sent over an additive white Gaussian noise channel in which the noise $N(t)$ has power spectral density σ^2.

For a system with K users, the transmission rate of user j is denoted by $R_j = M_j R$ for an integer $M_j \geq 1$, where R denotes a system base rate. The bit transmission time of user j is $T_j = 1/R_j$ and the processing gain is $L_j = T_j/T_c$. The jth user has signature waveform

$$S_j(t) = \frac{1}{\sqrt{L_j}} \sum_{m=1}^{L_j} a_j(m) p[t-(m-1)T_c] \tag{1}$$

where $a_j(m) \in \{-1, 1\}$ denotes the signature sequence of user j. The energy of the pulse $p[t]$ is normalized so that each $S_j(t)$ has unit energy over $[0, T_j]$. Let Δ_j denote the delay of the jth user. In the asynchronous channel, the received signal due to the jth user is given by

$$r_j(t) = \sum_{i=-\infty}^{+\infty} b_j^{(i)} \sqrt{E_j^{(i)}} S_j(t - iT_j - \Delta_j) \tag{2}$$

where $b_j^{(i)} \in \{-1, +1\}$ is the ith bit and $E_j^{(i)}$ is the received energy of the jth user. Since the received power is proportional to the received energy per bit, we will use energy and power as synonyms. It is assumed that the receiver knows the time delay Δ_j for each user

j and the received signal is

$$r(t) = \sum_{k=1}^{K} r_k(t) + N(t) \tag{3}$$

We now describe how the ith bit $b_j^{(i)}$, $0 \leq i \leq M_j - 1$, of user j is decoded. In order to decode $b_j^{(i)}$, we assume $\Delta_j = 0$ and that the other Δ_k are computed relative to user j. It may be desirable for different bits to be decorrelated using different window sizes. In terms of the unit step function $u(t)$, the bit $b_j^{(i)}$ will be decoded by processing the windowed received signal

$$R_j^{(i)}(t) = r(t)\left[u\left(t - [i - n_j^{(i)}]T_j\right) - u\left(t - [i + 1 + d_j^{(i)}]T_j\right)\right] \tag{4}$$

With respect to bit $b_j^{(i)}$, the window of $R_j^{(i)}(t)$ extends $n_j^{(i)}$ bits into the past and $d_j^{(i)}$ bits into the future and thus covers $n_j^{(i)} + d_j^{(i)} + 1$ bits of user j.

3 Asynchronous multi-rate decorrelator, AMD

The decorrelator [10, 11] can be implemented in a decentralized fashion using modified correlator signals. In [16], it was noted that the modified correlators of a decorrelator can be generated by applying the Gram-Schmidt orthogonalization to the interfering users' signature sequences. Reference [18] focused its attention on the Gram-Schmidt procedure to develop the AMD which we now briefly describe.

Among the M_j bits of user j, we present the AMD decoding technique for bit $b_j^{(i)}$. Let $I_{n_j^{(i)},d_j^{(i)}}$ denote the set of signature waveforms that are generated by the interfering users within the observation window $\left[(i - n_j^{(i)})T_j, (i + 1 + d_j^{(i)})T_j\right]$. Note that $I_{n_j^{(i)},d_j^{(i)}}$ includes the partial signatures corresponding to bits whose transmission times extend outside the observation window. The AMD decodes the bit $b_j^{(i)}$ using a modified correlator $\Phi_0^{(i)}(t)$ such that $\Phi_0^{(i)}(t)$ is orthogonal to each interfering signal in the set $I_{n_j^{(i)},d_j^{(i)}}$. The output of the modified correlator $\Phi_0^{(i)}(t)$ is

$$r_j^{(i)} = \int_{(i-n_j^{(i)})T_j}^{(i+1+d_j^{(i)})T_j} R_j^{(i)}(t)\Phi_0^{(i)}(t)\,dt = \sqrt{\zeta_j^{(i)} E_j^{(i)}}\, b_j^{(i)} + N_j^{(i)} \tag{5}$$

where $N_j^{(i)}$ is a Gaussian random variable with mean zero and variance σ^2 and

$$\zeta_j^{(i)} = \left\{ \int_{(i-n_j^{(i)})T_j}^{(i+1+d_j^{(i)})T_j} \Phi_0^{(i)}(t) S_j(t - iT_j)\, dt \right\}^2 \tag{6}$$

is the asymptotic efficiency of the AMD for decoding the bit $b_j^{(i)}$ in the observation window $\left[(i-n_j^{(i)})T_j, (i+1+d_j^{(i)})T_j\right]$. The corresponding decoding rule is $\bar{b}_j^{(i)} = \text{sgn}(r_j^{(i)})$ and the probability that the bit $b_j^{(i)}$ is decoded incorrectly is

$$q_j^{(i)} = Q\left(\sqrt{\zeta_j^{(i)} E_j^{(i)}/\sigma^2}\right) \tag{7}$$

where $Q(\cdot)$ is the standard normal complementary CDF.

We have described the decoding technique for the ith bit, $i = 1, \ldots, M_j - 1$, of user j. After an interval of M_j bits, user j will observe the same pattern of interfering signature waveforms. Thus, the receiver dedicated to the user j needs to know the specifications of M_j modified correlators; see Figure 1. Furthermore, user j will experience M_j different asymptotic efficiencies or BERs.

4 Quality of Service for the AMD

In this section, we will propose a class of QoS objectives for the AMD. Let us denote the uplink channel gain of user j by h_j and its transmitted power in the ith slot by $P_j^{(i)}$. Then the received power in the ith slot becomes $E_j^{(i)} = h_j P_j^{(i)}$. The criterion that is used for power control is to guarantee each user's link quality objective or QoS requirement. We will consider the following link quality objectives.

- *The fixed BER (FB) objective:* Provide each bit of user j with a target probability of error q_j^*. Mathematically, we must choose a set of transmitter powers $P_j^{(i)}$, $i = i, \ldots, M_j - 1$, such that

$$Q\left(\sqrt{\zeta_j^{(i)} h_j P_j^{(i)}/\sigma^2}\right) = q_j^* \qquad i = 0, \ldots, M_j - 1 \tag{8}$$

Note that the FB objective requires that the transmitter power of user j be adjusted from bit to bit. When the target BER q_j^* is low, then small variations in $\zeta_j^{(i)}$ will significantly change the transmitted power. Thus it may not be possible for user j to adjust the transmitter power quickly enough. This situation leads us to propose the following link quality objective.

- *Average BER (AB) objective:* Using a fixed transmitted power, guarantee user j an average bit error target q_j^* over M_j transmitted bits. Here $P_j^{(i)} = P_j$, for each bit $b_j^{(i)}$ and the link quality objective of user j is

$$\frac{1}{M_j}\sum_{i=0}^{M_j-1} Q\left(\sqrt{\zeta_j^{(i)} h_j P_j/\sigma^2}\right) = q_j^* \tag{9}$$

We observe that the AB objective achieves the same average BER as the FB objective by the use of a fixed transmitter power. The AB objective yields different BERs for different bits. The bit with the lowest asymptotic efficiency will experience the highest BER. In an asynchronous multi-rate channel, several bits of user j may experience significantly lower asymptotic efficiencies than others. In this case, user j may prefer to protect those bits which are suffering from low asymptotic efficiencies. For this situation, we propose the following link quality objective.

- *Worst case BER (WB) objective:* Using a fixed transmitted power, provide target BER q_j^* to the bit with the lowest asymptotic efficiency. That is, we choose P_j such that

$$\max_i Q\left(\sqrt{\zeta_j^{(i)} h_j P_j/\sigma^2}\right) = q_j^* \tag{10}$$

Compared to the AB objective, the WB objective will demand higher transmitted power from the mobile. We will see that the same power control algorithm can be used to achieve either the AB or WB objective. For either objective, user j will have a target received power E_j^* that guarantees the corresponding link QoS. Of course, E_j^* will depend on whether user j desires the AB or WB objective. Since $Q(\cdot)$ is a monotone decreasing function, a straightforward offline calculation can determine E_j^* for either objective. If the uplink channel gain h_j is known, then the target transmitted power P_j^* is simply E_j^*/h_j.

In practice, the uplink channel gain is often difficult to measure. The base station may require training sequences from the mobile in order to measure h_j. We develop a stochastic approximation method [23] that uses the AMD outputs to iteratively converge to a target received power E_j^* without the knowledge of the uplink channel gain h_j. Let us refer this stochastic approximation method as the *target power* or *TP* algorithm. Based on the value of E_j^*, the TP algorithm will achieve the AB or WB objective. In the TP algorithm, the base station will use the modified correlator outputs to estimate the average received power and the mobile uses this estimated average received power to update its transmitted power in order to meet the link quality objective. It is easy to verify that the TP algorithm is feasible as long as the asymptotic efficiencies of user j are nonzero.

In the TP algorithm, we assume that both in the uplink and the downlink that bits are transmitted in packets. It is also assumed that the base station measures J modified correlator outputs and estimates the average received power over J bits. Let us concentrate on the J modified correlators which are used to decode bits $0,\dots,J-1$. From equation (5), the output for the ith bit of user j is

$$r_j^{(i)} = \sqrt{\zeta_j^{(i)} h_j P_j} b_j^{(i)} + N_j^{(i)} \tag{11}$$

Under transmitted power P_j, the received power of user j, averaged over the outputs of J modified correlators of user j, is

$$\overline{Y}_j(P_j) = \frac{h_j P_j}{J} \sum_{i=0}^{J-1} \zeta_j^{(i)} + \sigma^2 \tag{12}$$

In equation (12), we see that $P_j = P_j^*$ iff $\overline{Y}_j(P_j) = \overline{Y}_j(P_j^*) = Y_j^*$. Therefore, the target power P_j^* will be achieved if the average power at the output of modified correlators equals Y_j^*. In the TP algorithm, the base station will transmit an estimate of $\overline{Y}_j(P_j)$ to user j who then updates its transmitted power in order to achieve the target average power Y_j^*.

Squaring equation (11), we get

$$\left(r_j^{(i)}\right)^2 = \zeta_j^{(i)} h_j P_j + 2\sqrt{\zeta_j^{(i)} h_j P_j} b_j^{(i)} N_j^{(i)} + \left(N_j^{(i)}\right)^2 \tag{13}$$

The noise components at the J modified correlator outputs can be represented by the noise vector

$$\mathbf{N}_j = \left[N_j^{(0)}, \dots, N_j^{(J-1)}\right] \tag{14}$$

Note that the autocorrelation matrix of the noise vector $\mathbf{N}_j$ will be a function of the cross-correlations of the J modified correlators. Using m to index the power control iterations, we will need to assume that the AMD and the mobile terminal are synchronized such a way that random noise vectors $\mathbf{N}_j(m)$, $m = 1, 2, \dots$, are independent and identically distributed (iid). In section 6, we will show that a set of iid noise vectors $\mathbf{N}_j(m)$, $m = 1, 2, \ldots$, can be easily obtained by carefully choosing the J bits for power control.

Using equations (13) and (14), we denote the sample mean of the average power at the outputs of modified correlators as

$$Y_j\left[P_j, \mathbf{N}_j\right] = \frac{1}{J}\sum_{i=0}^{J-1}\left\{\zeta_j^{(i)} h_j P_j + 2\sqrt{\zeta_j^{(i)} h_j P_j} b_j^{(i)} N_j^{(i)} + \left(N_j^{(i)}\right)^2\right\} \tag{15}$$

Taking the conditional expectation on both sides of equation (15), we can write the average power at the outputs of modified correlators as

$$E\left[Y_j\left[P_j, \mathbf{N}_j\right] | P_j = \xi\right] = \frac{h_j \xi}{J}\sum_{i=0}^{J-1}\zeta_j^{(i)} + \sigma^2 = \overline{Y}_j(\xi) \tag{16}$$

When $\xi = P_j^*$, we obtain the target average power Y_j^* at the outputs of modified correlators. We have mentioned earlier that in the TP algorithm, our goal is to obtain $\overline{Y}_j(\xi) = Y_j^*$ to guarantee user j the target power P_j^*. Note that exact calculation of Y_j^* requires perfect estimation of the noise power or variance, σ^2. Using $Y_j\left[P_j, \mathbf{N}_j\right]$ and Y_j^*, the TP algorithm for user j implements the stochastic approximation algorithm

$$P_j(m+1) = P_j(m) - a_m\left(Y_j\left[P_j(m), \mathbf{N}_j(m)\right] - Y_j^*\right) \tag{17}$$

Note that a_m may be a constant or a function of the iteration index m. The properties of a_m that are necessary for the convergence of the proposed equation (17) will be discussed shortly. Note that either AB or WB objectives can be achieved by appropriately setting the target average received power Y_j^*. The mobile executes the TP algorithm without knowledge of the asymptotic efficiencies $\zeta_j^{(i)}$.

5 Convergence of the TP algorithm

We denote the *mean square error* of the transmitter power at the mth iteration by

$$\Theta_m = E\left[\left[P_j(m) - P_j^*\right]^2\right] \tag{18}$$

Our goal will be to verify that the TP algorithm converges by showing that $\lim_{m\to\infty}\Theta_m = 0$. From equation (17), the evolution of the stochastic power control algorithm is dictated by the received power error

$$X_j(m) = Y_j\left[P_j(m), \mathbf{N}_j(m)\right] - Y_j^* \tag{19}$$

We will use upper bounds on the first and second conditional moments of $X_j(m)$ given $P_j(m)$ in order to prove convergence. Using

$$\mu_k(\xi) = E\left[X_j^k(m)|P_j(m) = \xi\right] \tag{20}$$

to denote the kth conditional moment, we prove the following lemmas in Appendix A.

Lemma 1 *There exists a positive constant c_0 such that*

$$\mu_1(\xi) = E\left[X_j(m)|P_j(m) = \xi\right] = c_0\left(\xi - P_j^*\right) \tag{21}$$

Lemma 2 *There exist positive constants c_1 and c_2 such that*

$$\mu_2(\xi) = E\left[X_j^2(m)|P_j(m) = \xi\right] \le c_1\left(\xi - P_j^*\right)^2 + c_2 \tag{22}$$

To prove the convergence of Θ_m, the following intermediate result will be used.

Theorem 1 *At iteration $m+1$, the mean square error Θ_{m+1} satisfies the upper bound*

$$\Theta_{m+1} \le \left(1 - 2a_m c_0 + a_m^2 c_1\right)\Theta_m + a_m^2 c_2 \tag{23}$$

Proof of Theorem 1 can be found in Appendix A. In the following section, we use Theorem 1 to study the convergence of the TP algorithm for fixed a_m.

5.1 Convergence of the TP algorithm for a fixed a_m

To study the convergence of the TP algorithm for a fixed a_m, we need to prove the following theorem. The proof is given in Appendix A.

Theorem 2 *If $a_m = a$ and $|1-2ac_0+a^2c_1| < 1$, then*

$$\lim_{m\to\infty} \Theta_m \leq \frac{ac_2}{2c_0 - ac_1} \tag{24}$$

Since c_0 and c_1 are positive, we can choose a sufficiently small so that $ac_1 < 2c_0$ and $|1-2ac_0+a^2c_1| \leq 1$. Further, equation (24) implies that by choosing an arbitrarily small a, we can obtain $ac_2/(2c_0 - ac_1) \approx 0$. However, as a approaches zero, $|1-2ac_0+a^2c_1|$ approaches 1 and the convergence speed slows down. This problem also arises when c_0 and c_1 are very small. In this case, we need to choose a very large a to achieve $|1-2ac_0+a^2c_1| < 1$. Suitable selection of a requires the knowledge of c_0, c_1 and c_2. As c_0 and c_1 are functions of the uplink channel gain h_j, thus, the selection of a is as difficult as the measurement of h_j. This practical situation leads us to search a coefficient sequence a_m for which Θ_m converges quickly to zero.

5.2 Convergence of the TP algorithm for $a_m = f(m)$

In the following theorem, we observe that if a_m satisfies the well known Robbins-Munro conditions, then the limiting mean squared error in the transmiter power converges to zero.

Theorem 3 *If $\sum_{m=1}^{\infty} a_m = \infty$ and $\sum_{m=1}^{\infty} a_m^2 < \infty$, then $\lim_{m\to\infty}\Theta_m = 0$.*

Note that $a_m = a/m$ meets the conditions of Theorem 2. From Lemmas 1 and 2, the proof of Theorem 3 follows Sakrison's procedure [23, pages 60-61].

Although the TP algorithm will converge, a number of issues must be resolved in order to implement the algorithm. In practice, the uplink channel gain h_j and the noise variance σ^2 are very small. Therefore, the received power measurement $Y_j\left[P_j, \mathbf{N}_j(m)\right]$ and the target received power Y_j^* in equation (17) will be very small. The TP algorithm uses these small values to update the transmitted power which typically is several orders of magnitude larger. If the initial transmitted power varies significantly from P_j^*, then the convergence speed of the TP algorithm will be extremely slow. For fast convergence, we need to choose a sufficiently large. A heuristic procedure for the selection of a will be discussed in the following section.

A second practical concern is that the channel gain h_j, the time offsets of equation (2), and the asymptotic efficiencies of user j can vary with time. Therefore, to track these variations, we would initialize the TP algorithm using $a_m = a/m$. When a_m becomes small, we would use a small fixed value for a_m. A similar technique is also used in [29] to implement the blind adaptive MMSE receiver.

6 Empirical results

We modeled a circular single cell of a dual-rate rate asynchronous DS/CDMA system. The radius of the cell was chosen to be $r_0 = 1000$ meters. It was assumed that the mobiles were uniformly distributed in the cell. This assumption yielded a probability density function $f(r) = 2ru(r)/r_0^2$ for the distance r of a user from the base station. We used a path loss exponent $\alpha = 4$. The height of the base station was 30 meters so that the uplink channel gain of user j, h_j was $1/\left(r^2+30^2\right)^{\alpha/2}$. In our experiments, we first randomly generated h_j from $f(r)$ and used that value of h_j in all the experiments. It was assumed that there were five low rate and five high rate users in the system. The processing gain of the low rate users was 64 and the processing gain of the high rate users was 8. The signature waveforms were generated using random signature sequences. The time offsets of different users were randomly generated. We assumed that the time offsets were multiples of the chip duration and the Gaussian noise had variance 6×10^{-14}.

Recall that in equation (14), iid noise vectors $\mathbf{N}_j(m)$, $m = 1, 2, \ldots$, are assumed for the sake of the convergence of the TP algorithm. In our experiments, this requirement was satisfied by carefully choosing J bits for our measurements such that for $m \neq m'$, $\mathbf{N}_j(m)$ and $\mathbf{N}_j(m')$ are derived from non-overlapping time intervals. Such a choice of J bits is depicted in Figure 2. In practice, the number of outputs J will depend on the length of the uplink packet, the length of modified correlators and the quantization levels of the modified correlator outputs. It is apparent from Figure 2 that for fixed J the convergence time of the TP algorithm will vary linearly with the uplink packet length.

In the experiment, user j was a high rate customer. The observation window of the decorrelator extended n_j bits into the past and d_j bits into the future so that the window length was $n_j + d_j + 1$ bits. In our experiments, we employed $n_j = d_j$. In the first experiment, our goal was to observe the transmitter power requirement of user j as a function of the observation window length; see Figure 3 where transmitter power requirements for different observation windows are shown with respect to the transmitter power requirement at $n_j = d_j = 2$. Here we considered the AB link quality objective and $q_j^* = 10^{-i}$, for $i = 1, \ldots, 4$. In this particular example, the AMD was not able to decode each bit of the desired user with a non-zero asymptotic efficiency at $n_j = d_j = 0$ or $n_j = d_j = 1$. Thus the transmitter power requirement at $n_j = d_j = 2$ was considered the baseline. As expected, we observed that the transmitter power requirement decreased as we increased the observation window length. However, increasing the observation window beyond $n_j = d_j = 7$ provided very insignificant transmitter power reductions compared to the increase in receiver complexity. This result is consistent with the observation in [18] that when the observation window extends over all signatures with nonzero cross correlation with the signature of the desired bit, then further increases in the window size provide little improvement in the asymptotic efficiency.

The result of Figure 3 led us to fix the observation window at $n_j = d_j = 7$ for the rest of our experiments where we observed the convergence speed of the proposed TP algorithm by varying different system parameters. We noticed that at $n_j = d_j = 7$, user j experienced the following 8 different asymptotic efficiencies

$$\left\{\zeta_j^{(i)} | 0 \leq i \leq 7\right\} = \left\{0.357 \;\; 0.332 \;\; 0.339 \;\; 0.378 \;\; 0.454 \;\; 0.330 \;\; 0.332 \;\; 0.277\right\} \tag{25}$$

These $\zeta_j^{(i)}$ depended on the randomly chosen signature sequences of both high rate and low rate users. We assumed that user j applied the AB link quality objective with a target average BER $q_j^* = 10^{-2}$. The corresponding target received power E_j^* was 9.5356×10^{-13}. Note that for each of the following experiments one trial was performed.

In the experiment of Figure 4, the average BER was determined as a function of the number of power control iterations for fixed $a_m = a$ as well as $a_m = 10^{13}/m$. In each iteration, only the output of modified correlator 0 was used to update the transmitted power, or $J = 1$. The mobile's initial transmitted power was $P_j(0) = 0$. The empirical results showed that for $a_m = 10^{13}/m$, the mobile achieved its QoS objective quickly compared to fixed $a_m = a$ with $a = 0.1 \times 10^{13}$, or $a = 0.01 \times 10^{13}$. These large values of a were needed, because the errors $X_j(m)$ of equation (19) were very small and $P_j^* \gg P_j(0)$. In this experiment, we found that in the case of fixed $a_m = a$, smaller values of a suffered from slow convergence, as suggested by the discussion in section 5.1. In the following experiments, we only studied the sequence $a_m = a/m$.

In Figure 5, we plot the average BER as a function of the number of power control iterations for $J = 1, 8, 16$, with $a = 10^{13}$ and $P_j(0) = 0$. In this experiment, we observed that the convergence speed of the TP algorithm improved at J increased. For both $J = 8$ and $J = 16$, the mobile varied the transmitter power slowly to achieve q_j^*. Similar results were also observed in [28] where the stochastic power control algorithm for the matched filter receiver was studied.

Figure 6 shows the effect of a on the convergence when $J = 8$ and $P_j(0) = 0$. As expected, our empirical results showed that smaller values of a resulted in a slower convergence speed. On the other hand, larger values of a led to a wide variation in the initial transmitted power. These experimental results implied that neither a large a nor a small a was desirable for the mobile. This practical situation leads us to propose the following heuristic procedure to select the parameter a.

Let us assume that $P_j(0) > 0$. Using the target average power, Y_j^* and equation (15), an estimate of the target transmitted power, P_j^* is

$$\hat{P}_j^* = \frac{\left(Y_j^* - \sigma^2\right) P_j(0)}{Y_j\left[P_j(0), \mathbf{N}_j(0)\right] - \sigma^2} \tag{26}$$

In order to be in the vicinity of P_j^* in the very first iteration, we will choose a so that $P_j(1) = \hat{P}_j^*$. Equations (17) and (26) imply that $P_j(1) = \hat{P}_j^*$ if

$$a = \frac{P_j(0)}{Y_j\left[P_j(0), \mathbf{N}_j(0)\right] - \sigma^2} \tag{27}$$

In equation (28), since $\mathbf{N}_j(0)$ is a random vector, a is a random variable and its magnitude depends on the initial transmitted power $P_j(0)$. For large J, equation (15) implies that $a \approx J/\left(h_j \sum_{i=0}^{J-1} \zeta_j^{(i)}\right)$ which is proportional to $1/h_j$. Thus equation (27) can be used to approximate h_j.

For small $P_j(0)$ and h_j, equation (27) may yield $a < 0$. However, Theorem 3 requires the parameter a to be positive. To handle this situation, we will modify equation (27) as follows.

$$a = \frac{P_j(0)}{\left|Y_j\left[P_j(0), \mathbf{N}_j(0)\right] - \sigma^2\right|} \tag{28}$$

In Figure 7, the effect of $P_j(0)$ on the TP algorithm is shown when equation (28) was used to select a. Here $J = 8$ and $P_j(0) = \kappa P_j^*$, where κ takes on values 10^{-2}, 10^{-1}, 10, and 10^2. Larger values of $P_j(0)$ yielded better estimates of P_j^* or smaller values of $|P_j(1) - P_j^*|$. As a result, the convergence speed of the power control was improved as $P_j(0)$ increased. For all values of $P_j(0)$, we observed that the mobile varied the transmitted power slowly to achieve q_j^*.

7 Conclusions

For the varied QoS requirements of multimedia traffic in an asynchronous DS/CDMA system, this paper addresses a class of link quality objectives for a decentralized, truncated window multi-rate decorrelator. Both the average BER and worst case BER objectives can be achieved using the TP stochastic power control algorithm. The proposed algorithm is iterative, asynchronous and distributed and does not require a measurement of the uplink channel gain. It is proved that when each bit of a user has nonzero asymptotic efficiency, then the TP algorithm converges in the mean square sense to an optimal power. This theoretical result was verified by our empirical studies. In this paper, a heuristic technique that assists the proposed stochastic power control algorithm in converging quickly was also presented.

A Additional proofs

Proof: Lemma 1 Using $Y_j^* = (E_j^*/J)\sum_{i=0}^{J-1}\zeta_j^{(i)} + \sigma^2$ in equation (19), we can write

$$\mu_1(\xi) = E\left[X_j(m)|P_j(m)=\xi\right] = \left\{\frac{h_j}{J}\sum_{i=0}^{J-1}\zeta_j^{(i)}\right\}\left(\xi - P_j^*\right) = c_0\left(\xi - P_j^*\right) \tag{29}$$

where $c_0 = (h_j/J)\sum_{i=0}^{J-1}\zeta_j^{(i)} > 0$ is a constant related to the system parameters. $\square$

Proof: Lemma 2 From equation (19) and the definition (20) of $\mu_2(\xi)$, we can write

$$\mu_2(\xi) = E\left[\left[\frac{1}{J}\sum_{i=0}^{J-1}\left(\zeta_j^{(i)}h_j\left(\xi - P_j^*\right) + 2\sqrt{\zeta_j^{(i)}h_j\xi}\,b^{(i)}N_j^{(i)} + \left\{\left(N_j^{(i)}\right)^2 - \sigma^2\right\}\right)\right]^2\right] \tag{30}$$

It is straightforward to show that for a set of real numbers $\{x_i\}$, the Schwarz inequality yields

$$\left[\sum_{i=0}^{J-1}x_i\right]^2 \le J\sum_{i=0}^{J-1}x_i^2 \tag{31}$$

We will apply (31) twice to equation (30) in the following fashion.

$$\begin{aligned}
\mu_2(\xi) &\le \frac{1}{J^2}E\left[\sum_{i=0}^{J-1} J\left(\zeta_j^{(i)}h_j\left(\xi-P_j^*\right)+2\sqrt{\zeta_j^{(i)}h_j\xi}b^{(i)}N_j^{(i)}+\left\{\left(N_j^{(i)}\right)^2-\sigma^2\right\}\right)^2\right] \\
&\le \frac{3}{J}\sum_{i=0}^{J-1} E\left[\left(\zeta_j^{(i)}h_j\right)^2\left(\xi-P_j^*\right)^2+4\zeta_j^{(i)}h_j\xi\left(N_j^{(i)}\right)^2+\left\{\left(N_j^{(i)}\right)^2-\sigma^2\right\}^2\right]
\end{aligned} \tag{32}$$

Taking the expectation of equation (32) with respect to $N_j^{(i)}$, we can write

$$\mu_2(\xi) \le \frac{3}{J}\sum_{i=0}^{J-1}\left\{\left(\zeta_j^{(i)}h_j\right)^2\left(\xi-P_j^*\right)^2+4\zeta_j^{(i)}h_j\xi\sigma^2+2\sigma^4\right\} \tag{33}$$

$$= \frac{3}{J}\sum_{i=0}^{J-1}\left\{\left(\zeta_j^{(i)}h_j\left(\xi-P_j^*\right)+2\sigma^2\right)^2+4\zeta_j^{(i)}h_jP_j^*\sigma^2-2\sigma^4\right\} \tag{34}$$

Applying (31) to the first term inside the sum in (34) yields

$$\mu_2(\xi) \le \frac{3}{J}\sum_{i=0}^{J-1}\left\{2\left(\zeta_j^{(i)}h_j\right)^2\left(\xi-P_j^*\right)^2+8\sigma^4+4\zeta_j^{(i)}h_jP_j^*\sigma^2-2\sigma^4\right\} \tag{35}$$

$$= \frac{6}{J}\sum_{i=0}^{J-1}\left\{\left(\zeta_j^{(i)}h_j\right)^2\left(\xi-P_j^*\right)^2+2\zeta_j^{(i)}h_jP_j^*\sigma^2+3\sigma^4\right\} \tag{36}$$

The result of Lemma 2 follows from equation (36) by defining constants

$$c_1=\frac{6h_j^2}{J}\sum_{i=0}^{J-1}\left(\zeta_j^{(i)}\right)^2 \qquad c_2=\frac{12E_j^*\sigma^2}{J}\sum_{i=0}^{J-1}\zeta_j^{(i)}+18\sigma^4 \tag{37}$$

□

Proof: Theorem 1 Subtracting P_j^* from both sides of equation (17), squaring the result, and then using equation (19), we obtain

$$\left[P_j(m+1)-P_j^*\right]^2=\left[P_j(m)-P_j^*\right]^2-2a_mX_j(m)\left[P_j(m)-P_j^*\right]+a_m^2X_j^2(m) \tag{38}$$

We denote the conditional second moment of the transmitter power error by

$$\Theta_{m+1}(\xi)=E\left[\left[P_j(m+1)-P_j^*\right]^2|P_j(m)=\xi\right] \tag{39}$$

Taking the conditional expectation of equation (38) given $P_j(m) = \xi$ yields

$$\Theta_{m+1}(\xi) \quad = \quad \left(\xi - P_j^*\right)^2 - 2a_m\mu_1(\xi)\left(\xi - P_j^*\right) + a_m^2\mu_2(\xi) \tag{40}$$

Lemmas 1 and 2 yield an upper bound on equation (40) as follows.

$$\Theta_{m+1}(\xi) \leq \left(1 - 2a_mc_0 + a_m^2c_1\right)\left(\xi - P_j^*\right)^2 + a_m^2c_2 \tag{41}$$

Taking the expectation with respect to $P_j(m)$ of equation (41) and using equation (18), we obtain the desired result of Theorem 1. $\square$

Proof: Theorem 2 We can express Θ_m of inequality (23) in terms of Θ_0 as follows.

$$\Theta_m \leq \left(1 - 2ac_0 + a^2c_1\right)^m \Theta_0 + a^2c_2 \sum_{i=0}^{m} \left(1 - 2ac_0 + a^2c_1\right)^m \tag{42}$$

Since $|1 - 2ac_0 + a^2c_1| < 1$, as m approaches to infinity, inequality (42) yields

$$\begin{aligned} \lim_{m\to\infty} \Theta_m \quad &\leq \quad \Theta_0 \lim_{m\to\infty} \left(1 - 2ac_0 + a^2c_1\right)^m \\ &\qquad + a^2c_2 \lim_{m\to\infty} \sum_{i=0}^{m} \left(1 - 2ac_0 + a^2c_1\right)^m && (43) \\ &= \frac{ac_2}{2c_0 - ac_1} && (44) \end{aligned}$$

$\square$

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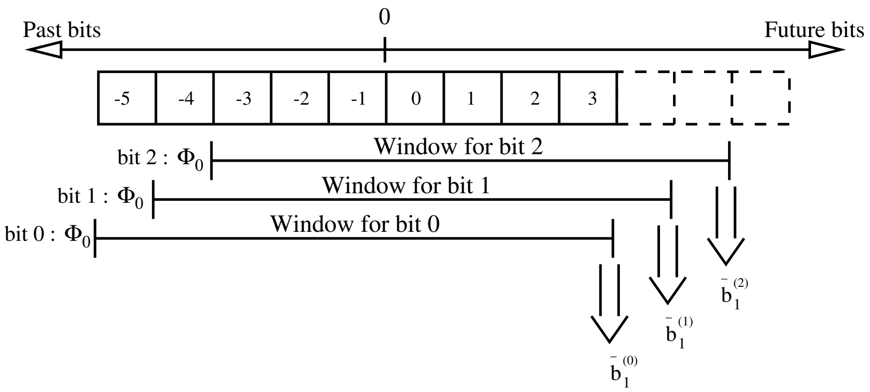

Figure 1: Here we have assumed that user j is a rate R user and $n_j^{(i)} = 5$ and $d_j^{(i)} = 3$. This picture depicts operation of the correlators which decode $b_j^{(0)}, b_j^{(1)}$ and $b_j^{(2)}$. The modified correlator signals are time shifted versions of one another.

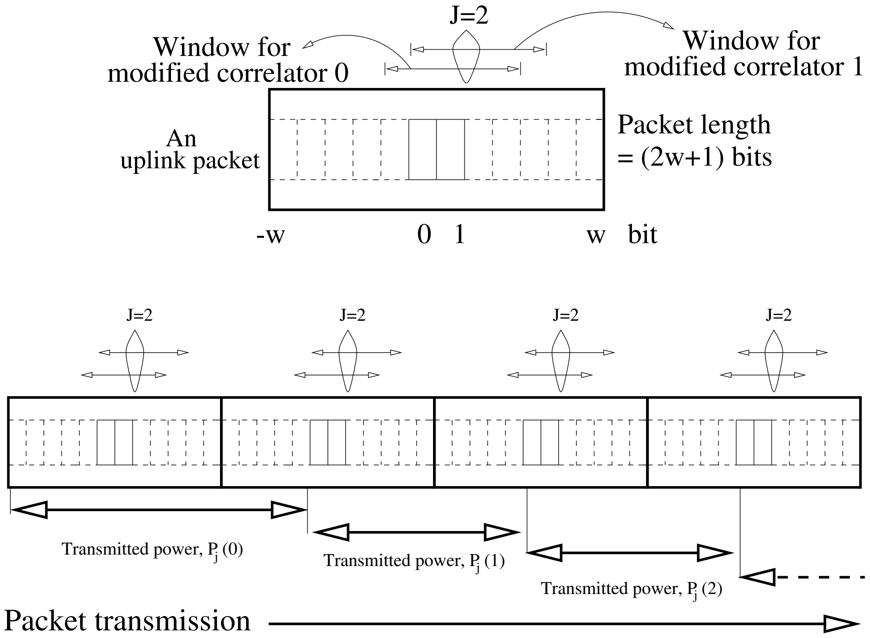

Figure 2: The modified correlators whose outputs are used in different power control iterations operate over different time intervals. Therefore, random noise vectors $\mathbf{N}_j(m)$, $m = 1, 2, \ldots$, are identically distributed and statistically independent or i.i.d. In this figure $J = 2$. However, in practice, the value of J will depend on the packet length, the length of modified correlators and the quantization levels of the correlator outputs.

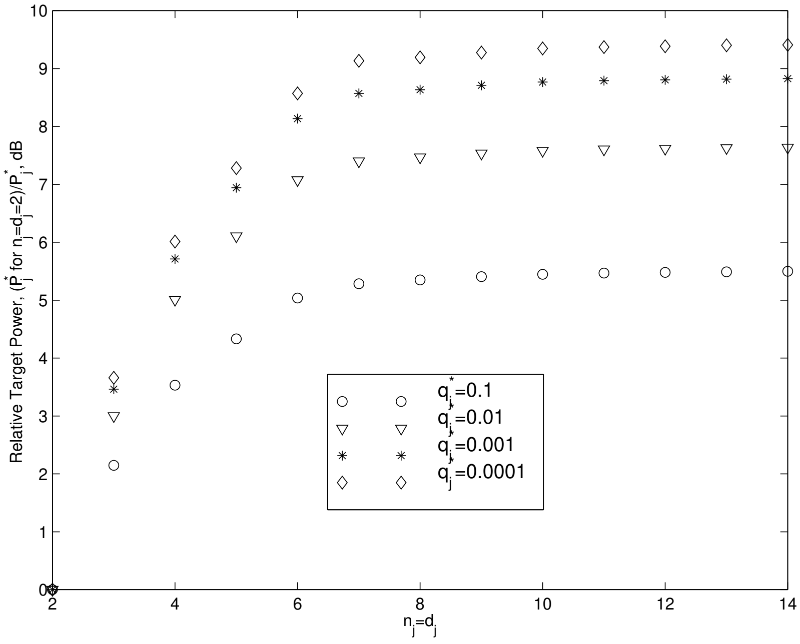

Figure 3: Relative transmitter power requirement is shown as a function of the observation window length for the AB link quality objective at different q_j^*.

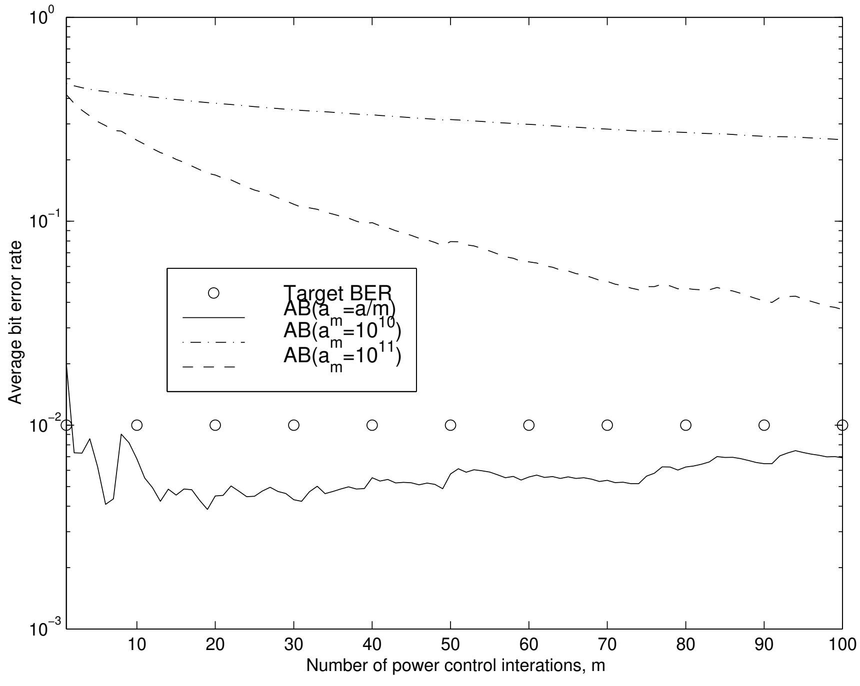

Figure 4: The average BER is plotted as a function of the number of power control iterations for fixed a_m as well as $a_m = 10^{13}/m$. Here $J = 1$ and $P_j(0) = 0$.

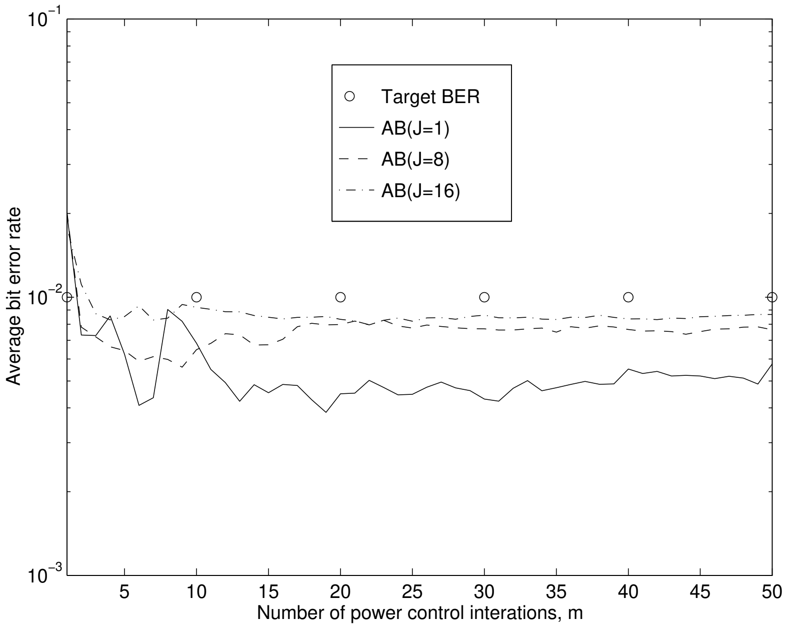

Figure 5: The average BER is shown as a function of the number of power control iterations for variable J. Here $a_m = 10^{13}/m$ and $P_j(0) = 0$.

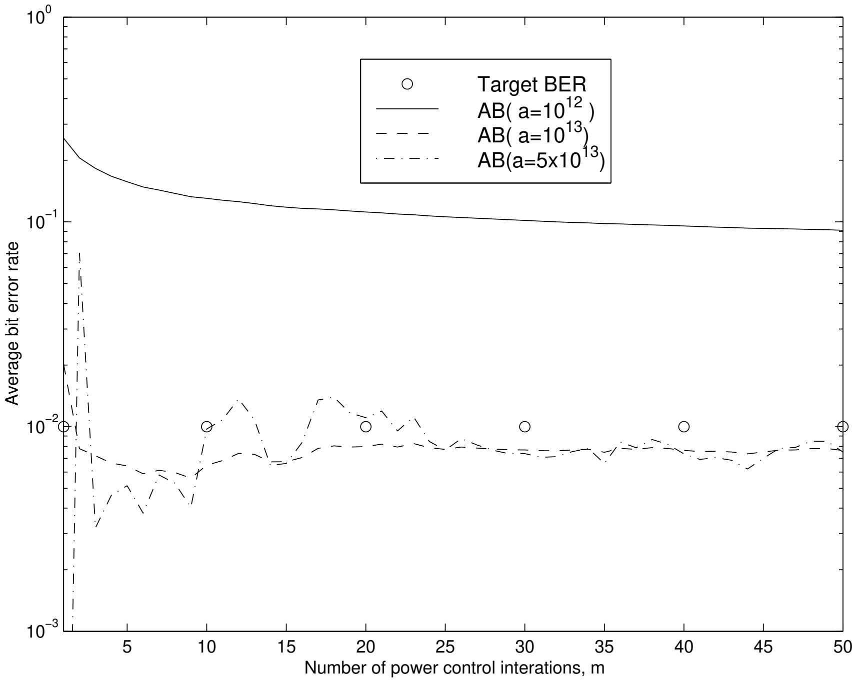

Figure 6: The average BER is plotted as a function of the number of power control iterations for variable a when $a_m = a/m$. Here $P_j(0) = 0$.

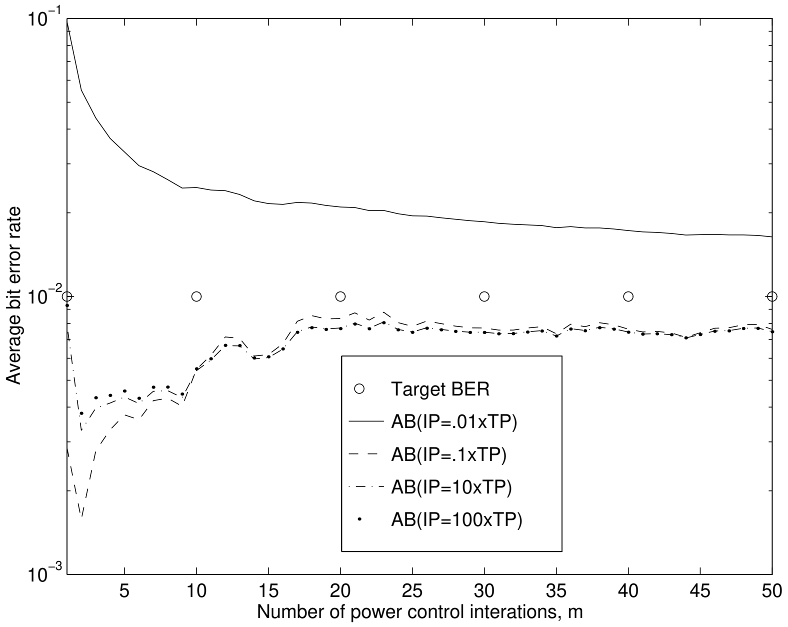

Figure 7: The average BER is shown as a function of the number of power control iterations for various $P_j(0)$, when $J = 8$ and equation (28) was used to select a. In the legend, IP and TP denote $P_j(0)$ and P_j^*, respectively.