An Asynchronous Multi-rate Decorrelator

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Abstract— **This paper examines truncated window decorrelators for an asynchronous DS/CDMA system supporting users transmitting at different bit rates. We decode a user by extending the observation window over sufficient number of its bits. To characterize practical window sizes, simple upper and lower bounds for the asymptotic efficiency of both the truncated window and infinite window decorrelators are developed. Empirical results show that as the length of the observation window increases, the bounds converge rapidly to the asymptotic efficiency of the infinite window decorrelator. The complexity of the receiver depends strongly on the ratio of the maximum to minimum bit rates.**

I. Introduction

Future CDMA systems must be designed for integrated voice and data access, implying that the network must support different traffic rates. In a DS/CDMA system in which all users have the same modulation scheme, methods for supporting multi-rate services include assigning multiple codes per user, varying the chip rate with a fixed processing gain, and varying the processing gain with a fixed chip rate. For a synchronous dual rate DS/CDMA system with a fixed chip rate and variable processing gain, decorrelators [1,2] and decision feedback detectors [3] have been studied. For multi-rate DS/CDMA systems, jointly optimal detection was analyzed in [4].

In [5], the asynchronous decorrelator for a single rate DS/CDMA system was proposed. Although this decorrelator can achieve the optimum near-far resistance [5] by extending the observation window to infinity, only the *truncated* or *finite window decorrelator* [5,6], can be implemented. Generalizing the synchronous dual-rate CDMA system model of [1,2], this paper considers an asynchronous multi-rate CDMA system and develops a decentralized, truncated window decorrelator called the *asynchronous multi-rate decorrelator* (AMD). The AMD will decode the bits of a user by employing a set of modified correlators whose signals are simply time shifted versions of one another. Due to the decentralization of our decorrelator, each user will be able to suppress the multiuser interference as long as that user has non-zero asymptotic efficiency [5] over its observation window. For a given observation window, we will derive upper and lower bounds on the performance of the infinite window decorrelator. These bounds, which follow naturally from the implementation of the AMD, are functions of the observation window length and will permit performance comparisons of the truncated window and infinite window decorrelators.

We note that for synchronous multi-rate systems, references [1,2] construct decorrelating receivers that exploit the properties of the synchronous system. One of the observations of this work is that there are no such similar receiver structures for an asynchronous multi-rate system. In particular, to decode a desired bit within the observation window of an asynchronous multi-rate system, other bits transmitted in that window must be viewed as interferers in an equivalent single rate system. Consequently, the reader of this paper will see that Theorems 2 and 3 in this work apply equally well to decorrelators for single rate asynchronous systems. Only in the empirical performance results of Section IV will the effect of multi-rate transmission become evident.

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II. Asynchronous multi-rate system

In our asynchronous multi-rate DS/CDMA system model, each bit results in the baseband transmission of a sequence of pulses, or chips, $p[t]$, each pulse having a duration of one chip period T_c. These pulses are sent over an additive white Gaussian noise channel in which the noise $N(t)$ has power spectral density σ^2.

For a system with K users, the transmission rate of user j is denoted by $R_j = M_j R$ for an integer $M_j \geq 1$, where R denotes the system base rate. The bit transmission time of user j is $T_j = 1/R_j$ and the processing gain is $L_j = T_j/T_c$. The jth user has signature waveform

$$S_j(t) = \frac{1}{\sqrt{L_j}} \sum_{m=1}^{L_j} a_j(m) p[t-(m-1)T_c] \tag{1}$$

where $a_j(m) \in \{-1, 1\}$ denotes the signature sequence of user j. The energy of the pulse $p[t]$ is normalized so that for all j, $\int_0^{T_j} S_j^2(t)\,dt = 1$. Let us denote by Δ_j the delay of the jth user. In the asynchronous channel, the received signal due to the jth user is given by

$$r_j(t) = \sum_{i=-\infty}^{+\infty} b_j^{(i)} \sqrt{E_j^{(i)}} S_j(t - iT_j - \Delta_j) \qquad 1 \leq j \leq K \tag{2}$$

where $b_j^{(i)} \in \{-1, +1\}$ is the ith bit and $E_j^{(i)}$ is the received energy of the jth user. We assume that the receiver knows the time delay Δ_j for each user j and the received signal is

$$r(t) = \sum_{k=1}^{K} r_k(t) + N(t) \tag{3}$$

III. Asynchronous multi-rate decorrelator

Linear multiuser receivers, such as the decorrelator or MMSE detector, can be implemented in a decentralized fashion by modifying the correlator signal. It is well known that the modified correlators of a decorrelator can be found by applying Gram-Schmidt orthogonalization on the interfering users' signature sequences [7].

Without loss of generality, the rest of the paper will consider user j of equation (2) as the user of interest. Among the M_j bits of user j, now we will concentrate on the bit $b_j^{(0)}$ that was

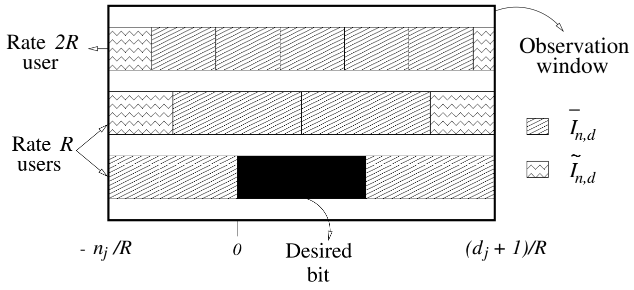

Fig. 1. Blocks denote whole and partial signatures transmitted in the truncated window.

transmitted using the signature sequence $S_j(t)$ of equation (1). For the sake of simplicity, the superscript which denotes the bit number will be ignored. To decode b_j, the observation window extends n_j bits to the past and d_j bits to the future. As depicted in Figure 1, let I_{n_j,d_j} denote the set of signature waveforms used to transmit the interfering users' bits in the observation window $[-n_jT_j,(d_j+1)T_j]$. We note that I_{n_j,d_j} includes both *direct interferers* that use signatures with nonzero cross-correlation with $S_j(t)$ and *indirect interferers* that have zero cross-correlation with $S_j(t)$.

Let L_{n_j,d_j} denote the size of the set I_{n_j,d_j}. We assume the Gram-Schmidt procedure on the set I_{n_j,d_j} yields a set of $l_{n_j,d_j} \leq L_{n_j,d_j}$ orthonormal basis functions $\Psi_{n_j,d_j} = \{\Psi_m(t) : m = 1,\ldots,l_{n_j,d_j}\}$. Now we append $S_j(t)$ to the set I_{n_j,d_j} such that $S_j(t)$ is signal $L_{n_j,d_j}+1$ in the set $Q_j = I_{n_j,d_j} \cup \{S_j(t)\}$. If the projection of $S_j(t)$ onto the subspace orthogonal to the interference I_{n_j,d_j} is non-zero, then the Gram-Schmidt procedure on Q_j will yield a new non-zero normalized basis function $\Phi_0(t) = \Psi_{l_{n_j,d_j}+1}(t)$ in addition to the set Ψ_{n_j,d_j}.

Before proceeding, for the correlation of $v(t)$ and $w(t)$ over $[-n_jT_j,(d_j+1)T_j]$, we adopt the notation

$$\langle v(t), w(t)\rangle_{n_j,d_j} = \int_{-n_jT_j}^{(d_j+1)T_j} v(t)w(t)\,dt \tag{4}$$

With this notation, we state the following significant property of $\Phi_0(t)$.

Theorem 1: The Gram-Schmidt orthonormal basis function $\Phi_0(t)$ is the solution to the following optimization problem:

$$\begin{aligned} \sqrt{\zeta_{n_j,d_j}} = \max_{\Phi(t)} \quad & \langle \Phi(t), S_j(t)\rangle_{n_j,d_j} \\ \text{subject to} \quad & \langle \Phi(t), I_m(t)\rangle_{n_j,d_j} = 0 \quad I_m(t) \in I_{n_j,d_j} \\ & \langle \Phi(t), \Phi(t)\rangle_{n_j,d_j} = 1 \end{aligned} \tag{5}$$

Proof of Theorem 1 can be found in [8]. With respect to the interference I_{n_j,d_j}, $\Phi_0(t)$ is simply the decorrelator for bit b_j expressed in the form of a modified correlator. Theorem 1 simply says that among all unit energy signals orthogonal to the interference I_{n_j,d_j}, the decorrelator maximally captures the energy of the desired bit.

Since $\Phi_0(t)$ is orthogonal to all interfering signatures $I_m(t) \in I_{n_j,d_j}$, the output of the desired modified correlator is

$$\rho_j = \langle r_j(t), \Phi_0(t)\rangle_{n_j,d_j} = \sqrt{\zeta_{n_j,d_j}E_j}\,b_j + N_j \tag{6}$$

where N_j is a Gaussian random variable with mean zero and variance σ^2. The term $\zeta_{n_j,d_j} = [\langle \Phi_0(t), S_j(t)\rangle_{n_j,d_j}]^2$ is the asymptotic efficiency [5] of the AMD for decoding the bit b_j in the window $[-n_jT_j,(d_j+1)T_j]$. For the decoding rule $\bar{b}_j = \mathrm{sgn}(\rho_j)$, the probability of bit error is $q_{n_j,d_j} = Q(\sqrt{\zeta_{n_j,d_j}E_j}/\sigma)$, where $Q(\cdot)$ is the standard normal complementary CDF. Thus for a given bit energy, the AMD performance is characterized by the asymptotic efficiency. We now establish that it is always possible to improve the asymptotic efficiency of the AMD by expanding the observation window.

Theorem 2: The asymptotic efficiency, ζ_{n_j,d_j} is an increasing function of both n_j and d_j.

Proof: Suppose $n'_j \geq n_j$ and $d'_j \geq d_j$. Since $\Phi_0(t)$ is zero outside the window $[-n_jT_j,(d_j+1)T_j]$, it is straightforward to verify for the expanded window $[n'_jT_j,(d'_j+1)T_j]$ that $\Phi_0(t)$ is feasible with respect to the constraints of the optimization problem (5) and still provides asymptotic efficiency ζ_{n_j,d_j}. For the expanded window, however, $\Phi_0(t)$ may be or may not be the optimal solution. Hence $\zeta_{n_j,d_j} \leq \zeta_{n'_j,d'_j}$. □

Theorem 2 encourages the receiver designer to extend the observation window for improved receiver performance. However, further extension of the observation window may provide a negligible performance gain compared to the increase in receiver complexity. Therefore, based on the construction of the truncated window decorrelator, we will develop a simple upper bound on the performance of the infinite window decorrelator. This upper bound will indicate whether the observation window should be extended.

We decompose I_{n_j,d_j} into the disjoint union $I_{n_j,d_j} = \bar{I}_{n_j,d_j} \cup \tilde{I}_{n_j,d_j}$ where $\bar{I}_{n_j,d_j}$ is the set of normalized interfering signatures corresponding to complete bit transmissions within the observation window and $\tilde{I}_{n_j,d_j}$ is the set of interfering partial signature waveforms. Each signature in $\tilde{I}_{n_j,d_j}$ is truncated either at the left boundary or at the right boundary of the observation window and, hence, is not normalized. We will say that the observation window *covers* the interferers transmitting bits using the complete signatures from the set $\bar{I}_{n_j,d_j}$. We note that the other bits of user j transmitted within the window $[-n_jT_j,(d_j+1)T_j]$ represent indirect interferers that are covered by the observation window.

We assume the elements of the set I_{n_j,d_j} are ordered such that the signatures in the subset $\bar{I}_{n_j,d_j}$ come first. Using $\psi_m = \langle \Psi_m(t), S_j(t)\rangle_{n_j,d_j}$, we define $\bar{\zeta}_{n_j,d_j} = 1 - \sum_{m=1}^{\bar{l}_{n_j,d_j}} \psi_m^2$. Recall that the first $\bar{l}_{n_j,d_j}$ basis functions of the set Ψ_{n_j,d_j} are generated by the subset $\bar{I}_{n_j,d_j}$. The term $1-\bar{\zeta}_{n_j,d_j} = \sum_{m=1}^{\bar{l}_{n_j,d_j}} \psi_m^2$ is the energy of the projection of the desired signature waveform onto the subspace spanned by $\bar{I}_{n_j,d_j}$. Thus $\bar{\zeta}_{n_j,d_j}$ would be the asymptotic efficiency if it were only necessary to decorrelate against the complete signatures in the observation window $[-n_jT_j,(d_j+1)T_j]$. In Theorem 3, we show that $\bar{\zeta}_{n_j,d_j}$ is an upper bound to the asymptotic efficiency of the infinite window decorrelator. In the proof of Theorem 3, we make repeated use of the following well known property of the Gram-Schmidt procedure.

Lemma 1: Let $S_1 \subset S_2$ be two sets of functions defined over $[-\bar{T},\hat{T}]$ such that the Gram-Schmidt procedure on S_i generates

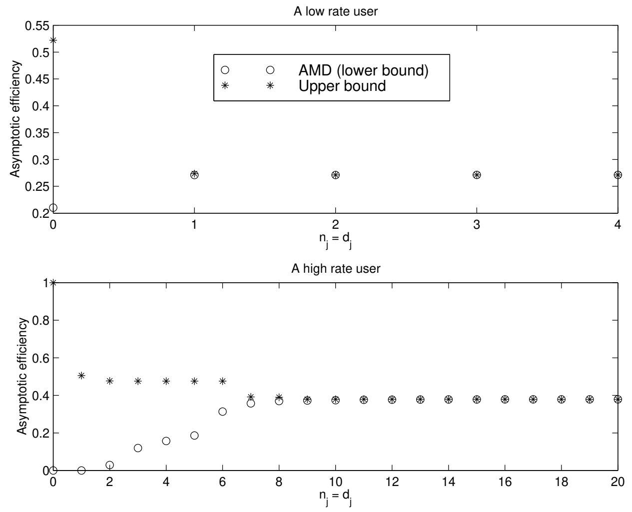

Fig. 2. For a dual rate system supporting low rate R users and high rate $8R$ users, the asymptotic efficiencies of a low rate bit and a high rate bit are plotted as a function of $n_j = d_j$. Note that each bit is decoded using a window of $W = n_j + d_j + 1$ bits. When $n_j = d_j = 0$, we have a one-shot decorrelator.

the orthonormal basis V_i with l_i basis functions. If a function $v(t)$ with unit energy on $[-\bar{T}, \hat{T}]$ has a projection $v_{i,m}$ on the mth basis function of V_i, then $1 - \sum_{m=1}^{l_1} v_{1,m}^2 \geq 1 - \sum_{m=1}^{l_2} v_{2,m}^2$. Lemma 1 simply says that if $S_1 \subset S_2$, then the projection of a signal onto the subspace orthogonal to S_1 has more energy than the projection onto the subspace orthogonal to S_2.

Theorem 3: For any observation window $[-n'_j T_j, (d'_j + 1)T_j]$, with $n'_j \geq n_j$ and $d'_j \geq d_j$, the asymptotic efficiency $\zeta_{n'_j,d'_j}$ will be bounded by

$$\zeta_{n_j,d_j} \leq \zeta_{n'_j,d'_j} \leq \bar{\zeta}_{n_j,d_j} \tag{7}$$

and $\bar{\zeta}_{n_j,d_j}$ is a decreasing function of both n_j and d_j.

Proof: Theorem 2 implies that $\zeta_{n_j,d_j} \leq \zeta_{n'_j,d'_j}$. The observation window $[-n'_j T_j, (d'_j + 1)T_j]$ covers the complete signature waveforms of the observation window $[-n_j T_j, (d_j + 1)T_j]$ or $\bar{I}_{n_j,d_j} \subset I_{n'_j,d'_j}$. Therefore, Lemma 1 implies $\zeta_{n'_j,d'_j} \leq \bar{\zeta}_{n_j,d_j}$. Similarly, since $\bar{I}_{n_j,d_j} \subset \bar{I}_{n'_j,d'_j}$, Lemma 1 also implies $\bar{\zeta}_{n'_j,d'_j} \leq \bar{\zeta}_{n_j,d_j}$. □

Theorem 3 states that in an asynchronous channel, the infinite window decorrelator performs better than any truncated window decorrelator; however, if it were not necessary to decorrelate the partial interferers, then any truncated window decorrelator would outperform the infinite window decorrelator. In [5, equation 4.23], Lupas and Verdú derive the asymptotic efficiency of the infinite window decorrelator but evaluation of the resulting expression is quite challenging. Thus, for a user's given finite window decorrelator, it would be difficult to know if the asymptotic efficiency is close to that of the infinite window decorrelator. From the construction of the finite window decorrelator, the upper bound of Theorem 3 provides a tight upper bound on the infinite window asymptotic efficiency essentially for free. We will see in the following section that this upper bound converges quickly to the infinite window asymptotic efficiency.

Users		Trial 1			Trial 2		
L	H	*LB*	*UB*	*W*	*LB*	*UB*	*W*
1	1	1.0000	1.0000	1	0.9090	1.0000	3
2	2	0.9065	0.9682	5	0.9065	0.9682	3
3	3	0.6767	0.6804	15	0.5656	0.6089	3
4	4	0.4839	0.5011	15	0.3706	0.3757	13
5	5	0.3573	0.3917	15	0.2212	0.2315	13
6	6	0.0802	0.0899	21	0.1962	0.2090	19

Fig. 3. In these experiments, we determined the minimum window size W that guaranteed a high rate bit with at least 90% of the maximum asymptotic efficiency. In the Users column, L and H denote the number of low rate and high rate users respectively. For each trial, $LB = \zeta_{n_j,d_j}$ and $UB = \bar{\zeta}_{n_j,d_j}$ and window size, $W = n_j + d_j + 1$.

IV. Empirical results

In our experiments, we modeled a dual-rate asynchronous DS/CDMA system. The processing gain of the low rate users was 64 and the processing gain of the high rate users was 8. The signatures and time offsets of different users were generated randomly and independently. To simplify computations, we assumed that the time offsets were multiples of the chip duration. The desired bit was decoded by employing a symmetric observation window with $n_j = d_j$.

First, we considered an asynchronous dual rate system with five low rate and five high rate users. For a low rate user, Figure 2 shows upper and lower bounds of Theorem 3 as a function of n_j. Note that for each low rate user j, $M_j = 1$. Thus in each sliding window of fixed n_j and d_j, the low rate user saw the same interfering signatures. This implies that for fixed n_j and d_j, each bit of the low rate user experienced the same asymptotic efficiency. When $n_j = d_j = 0$, the difference between the upper bound and lower bound was significant. However, when $n_j = d_j = 1$, the lower and upper bounds became very close. In this case, Theorem 3 indicated that the asymptotic efficiency was very close to that of the infinite window decorrelator and that a further increase of the observation window would not yield a significant performance gain. Note that at $n_j = d_j = 1$, the observation window covered all direct interferers. From Figure 2, we can also draw similar conclusions for a high rate user in the same experiment. Since the bit duration of a low rate user was 8 times that of a high rate user, a high rate user j needs a window that extends $d_j = 8$ bits into the future and $n_j = 8$ bits into the past to include completely all direct interferers of the desired high rate bit. In Figure 2, we see that when $n_j = d_j = 8$, the closeness of the lower and upper bounds demonstrate that the asymptotic efficiency for a high rate user was close to that of the infinite window decorrelator.

In the next experiment, we characterize practical window sizes as a function of the number of users in the system. For two different sets of signatures and time offsets, we found the minimum length of the observation window which guaranteed a high rate user with at least 90% of the maximum asymptotic efficiency, $\zeta_{\infty,\infty}$. This quality of service (QoS) objective was met by satisfying $\zeta_{n_j,d_j} \geq 0.9\bar{\zeta}_{n_j,d_j}$; see Theorem 3. Our experimental results are shown in Figure 3. In each experimental trial, we started with one low rate and one high rate user and then sequentially added a pair of users, one low rate and one high rate.

The signatures and time offsets were randomly generated. Here the experimental results showed that increasing the number of users resulted in smaller values of $\zeta_{\infty,\infty}$ which required longer observation windows. As expected, when the number of users in each group is fixed, the necessary window sizes depended on the users' cross-correlations.

In the above experiments, we observe that when the infinite window decorrelator did not suffer from poor performance, we obtained an asymptotic efficiency close to that of the infinite window decorrelator by expanding the observation window to cover the direct interferers. For a user with bit rate M_jR, this required $n_j = d_j = M_j$ and a corresponding window of size of $2M_j + 1$ bits. In the context of a multirate decorrelator supporting a minimum rate R and a maximum rate MR, we can draw the following conclusions.

- For a low rate user with rate R, a window size of three bits is likely to be sufficient. In such a window, an asynchronous high rate user of rate MR contributes $3M + 1$ bits and for purposes of decorrelation, appears as $3M + 1$ interferers.
- For a high rate user, a window size of $2M + 1$ bits will cover the low rate direct interferers. In such a window, every other high rate user transmits $2M + 2$ bits.

When $M = 1$, we have a single rate system and a window of 3 bits is likely to be sufficient to decode a particular bit. In that three bit window, each user contributes interference with two whole bits and 2 partial bits. Effectively, in the multi-rate system, the number of interferers that must be decorrelated is increased by a factor that grows linearly with M, the ratio of the highest rate to the lowest rate supported service. One interpretation of this observation is that supporting multiple rates can dramatically increase the decorrelator complexity.

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