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A Framework for Uplink Power Control in Cellular Radio Systems*

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Abstract

In cellular wireless communication systems, transmitted power is regulated to provide each user an acceptable connection by limiting the interference caused by other users. Several models have been considered including: (1) fixed base station assignment where the assignment of users to base stations is fixed, (2) minimum power assignment where a user is iteratively assigned to the base station at which its signal to interference ratio is highest, and (3) diversity reception, where a user's signal is combined from several or perhaps all base stations.

For the above models, the uplink power control problem can be reduced to finding a vector $\boldsymbol{p}$ of users' transmitter powers satisfying $\boldsymbol{p} \geq \boldsymbol{I}(\boldsymbol{p})$ where the jth constraint $p_j \geq I_j(\boldsymbol{p})$ describes the interference that user j must overcome to achieve an acceptable connection. This work unifies results found for these systems by identifying common properties of the interference constraints. It is also shown that systems in which transmitter powers are subject to maximum power limitations share these common properties. These properties permit a general proof of the synchronous and totally asynchronous convergence of the iteration $\boldsymbol{p}(t+1) = \boldsymbol{I}(\boldsymbol{p}(t))$ to a unique fixed point at which total transmitted power is minimized.

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1 Introduction

In wireless communication systems, mobile users adapt to a time varying radio channel by regulating transmitter powers. This power control is intended to provide each user an acceptable connection by eliminating unnecessary interference. This work intends to unify and extend convergence results for cellular radio systems employing iterative power control methods. For a variety of systems, we show that interference constraints derived from the users' signal to interference ratio (SIR) requirements share certain simple properties. These properties imply that an iterative power control algorithm converges not only synchronously but also totally asynchronously [1] when users perform power adjustments with outdated or incorrect interference measurements.

This emphasis on meeting SIR constraints would appear to be particularly appropriate for the uplink of a CDMA system in which unsynchronized signals of other users can be modeled as interfering noise signals. Previous analyses of power control algorithms have assumed users' locations and radio channel characteristics are fixed. However, proposed iterative algorithms have been designed for distributed implementation in dynamic systems with time varying radio channels.

Power control has been shown to increase the call carrying capacity of cellular systems for channelized systems [2–4] as well as single channel CDMA systems [5–7]. In [4, 5, 8–12], analytical approaches to attaining a common signal to interference ratio (SIR) or maximizing the minimum SIR are considered. In these works, the assignment of users to base stations is fixed or specified by outside means. In [13–16], an integrated approach to power control and base station assignment is analyzed. Power control under the assumption that all users are received by all base stations is studied in [17].

For the most part, analytical methods have derived convergence results for iterative power control algorithms that meet an SIR requirement for each user. In this work, we will see that for a broad class of power controlled systems, the users' SIR requirements can be described by a vector inequality of interference constraints of the form

$$\boldsymbol{p} \geq \boldsymbol{I}(\boldsymbol{p}) \tag{1}$$

In this case, $\boldsymbol{p} = (p_1, \ldots, p_N)$ where p_j denotes the transmitter power of user j and $\boldsymbol{I}(\boldsymbol{p}) = (I_1(\boldsymbol{p}), \cdots, I_N(\boldsymbol{p}))$ where $I_j(\boldsymbol{p})$ denotes the effective interference of other users that user j must overcome. We will say that a power vector $\boldsymbol{p} \geq 0$ is a feasible solution if $\boldsymbol{p}$ satisfies the constraints (1) and that an interference function $\boldsymbol{I}(\boldsymbol{p})$ is feasible if (1) has a feasible solution. In addition, if under power vector $\boldsymbol{p}$, $p_j \geq I_j(\boldsymbol{p})$, then we say user j has an acceptable connection.

For a system with interference constraints (1), we will examine the iterative power control algorithm

$$\boldsymbol{p}(t+1) = \boldsymbol{I}(\boldsymbol{p}(t)) \tag{2}$$

We will speak interchangeably of a system with interference constraints (1) or power control algorithm (2). We will show that synchronous and totally asynchronous convergence of the iteration (2) can be proven when $\boldsymbol{I}(\boldsymbol{p})$ satisfies three simple properties.

In section 2, we express the interference constraints of five systems in the form of equation (1) and we identify the three properties common to these systems. In sections 3 and 4, we derive the synchronous and asynchronous convergence of the iteration (2). Section 5 shows how this framework permits a number of extensions. In particular, we will show how to incorporate maximum and minimum power constraints, hybrid interference functions, and a general form of the active link protection in [18].

2 General Interference Constraints

We assume N users, M base stations and a common radio channel. The transmitted power of user j is p_j. Let h_{kj} denote the gain of user j to base k. At base k, the received signal power of user j is $h_{kj}p_j$ while the interference seen by user j at base k is $\sum_{i \neq j} h_{ki}p_i + \sigma_k$ where σ_k denotes the receiver noise power at base station k. Hence, under power vector $\boldsymbol{p}$, the SIR of user j at base station k is $p_j \mu_{kj}(\boldsymbol{p})$ where

$$\mu_{kj}(\boldsymbol{p}) = \frac{h_{kj}}{\sum_{i \neq j} h_{ki}p_i + \sigma_k} \tag{3}$$

We now express the interference constraints of a number of systems in the form of (1).

- *Fixed Assignment* We will denote by a_j the assigned base of user j, which we assume to be fixed or specified by outside means such as the received signal strength of base station pilot tone signals. The SIR requirement of user j at its assigned base a_j can be written $p_j \mu_{a_j j}(\boldsymbol{p}) \geq \gamma_j$. That is, we can write

$$p_j \geq I_j^{\mathrm{FA}}(\boldsymbol{p}) = \frac{\gamma_j}{\mu_{a_j j}(\boldsymbol{p})} \tag{4}$$

Under fixed assignment, [5, 8, 11] have considered the maxmin SIR problem in which $\gamma_j = \gamma$ for all j and the objective is to maximize γ subject to $\boldsymbol{p} \geq \boldsymbol{I}^{\mathrm{FA}}(\boldsymbol{p})$. In this work, the desired common SIR γ is embedded in the interference function $\boldsymbol{I}^{\mathrm{FA}}(\boldsymbol{p})$. For a fixed SIR target and fixed base station assignment, Grandhi et al. [19], Zander [20], and Foschini and Miljanic [12] use $\boldsymbol{p}(t+1) = \boldsymbol{I}^{\mathrm{FA}}(\boldsymbol{p}(t))$ to solve the subproblem of finding a feasible power vector $\boldsymbol{p}$. In [21], Mitra proves geometric convergence for an asynchronous implementation of Foschini's algorithm. These methods find the unique power vector $\boldsymbol{p} = \boldsymbol{I}^{\mathrm{FA}}(\boldsymbol{p})$.

- *Minimum Power Assignment (MPA)* At each step of this iterative procedure, user j is assigned to the base station at which its SIR is maximized. The convergence of the MPA iteration has been analyzed by Yates and Huang [14,15] and Hanly [13] for continuous power adjustments, and by Stolyar [16] for discrete power adjustments. MPA can also be considered a generalization of soft handoff; see [22]. The SIR constraint of user j is $\max_k p_j \mu_{kj}(\boldsymbol{p}) \geq \gamma_j$, which can be written

  $$p_j \geq I_j^{\mathrm{MPA}}(\boldsymbol{p}) = \min_k \frac{\gamma_j}{\mu_{kj}(\boldsymbol{p})} \tag{5}$$

  An example of the MPA interference constraints are depicted in Figure 1. In the MPA iteration $\boldsymbol{p}(t+1) = \boldsymbol{I}^{\mathrm{MPA}}(\boldsymbol{p}(t))$, user j is assigned to the base station k where minimum power is needed to attain the target SIR γ_j, under the assumption that the other users hold their powers fixed.

- *Macro Diversity* In [17], Hanly considers the combining of the received signals of user j at all base stations k. Under the assumption that the interfering signals at base stations k and k' appear to user j as independent noises, maximal ratio combining of the received signals for user j at all base stations yields an SIR constraint for user j of the form

  $$p_j \sum_k \mu_{kj}(\boldsymbol{p}) \geq \gamma_j \tag{6}$$

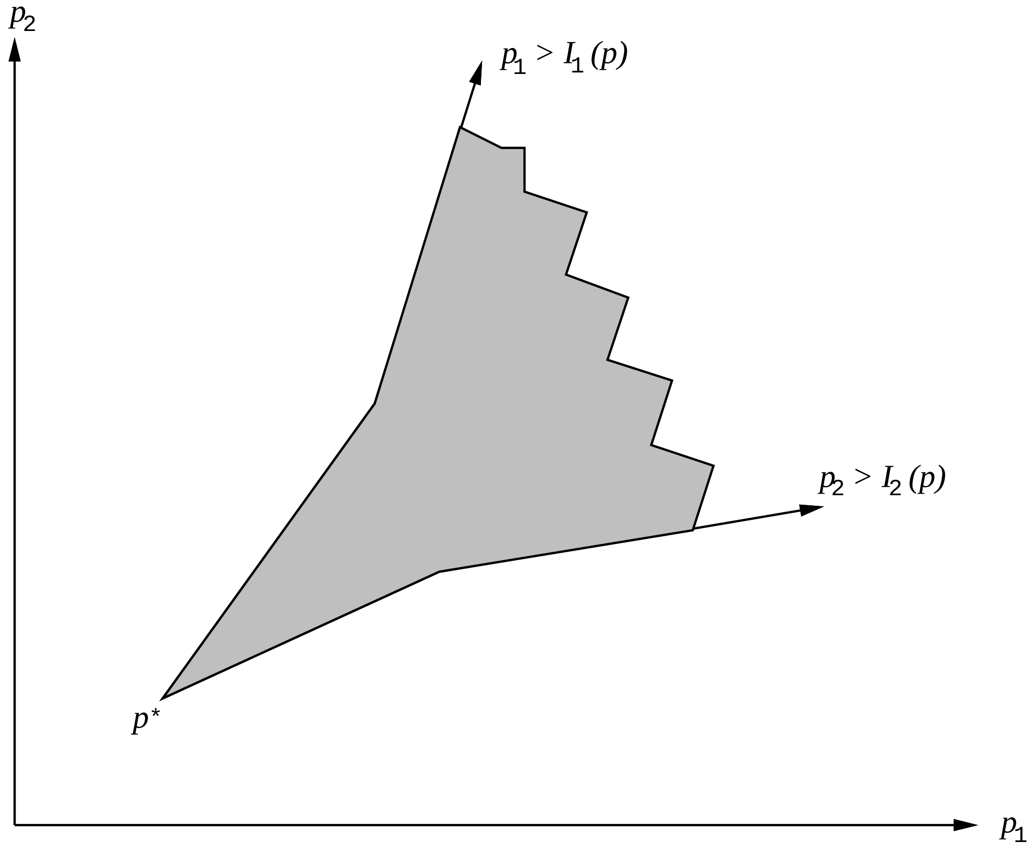

Figure 1: The interference constraints under MPA. The shaded region denotes the set of feasible power vectors under MPA for a system of two users and two base stations. The breakpoints of the constraints occur when the base station assignment changes. We observe that the set of feasible power vectors is nonconvex.

In this case we have

$$p_j \geq I_j^{\mathrm{MD}}(\boldsymbol{p}) = \frac{\gamma_j}{\sum_k \mu_{kj}(\boldsymbol{p})} \tag{7}$$

- *Limited Diversity* We can also consider a strategy in which the received signal of user j is combined from d_j base stations. We define $K_j(\boldsymbol{p})$ to be the d_j element set with the property that for all $k \in K_j(\boldsymbol{p}), k' \notin K_j(\boldsymbol{p})$, $\mu_{kj}(\boldsymbol{p}) \geq \mu_{k'j}(\boldsymbol{p})$. That is, $K_j(\boldsymbol{p})$ consists the d_j base stations at which user j has highest SIR. When $d_j = 1$ for all j, we have the ordinary MPA. When $d_j = M$ for all j, we have the macro diversity model. By using base stations $k \in K_j(\boldsymbol{p})$ to receive the signal of user j, we can write the SIR constraint of user j as

$$p_j \geq I_j^{\mathrm{LD}}(\boldsymbol{p}) = \frac{\gamma_j}{\sum_{k \in K_j(\boldsymbol{p})} \mu_{kj}(\boldsymbol{p})} \tag{8}$$

- *Multiple Connection Reception* In this approach, user j is required to maintain an acceptable SIR γ_j at d_j distinct base stations. To describe this method, we adopt the notation that $\langle n \rangle \max_k a_k$ and $\langle n \rangle \min_k a_k$ equal the nth largest and nth smallest elements of the set $\{a_k\}$. Using this notation, the SIR requirement of user j can be written $\langle d_j \rangle \max_k p_j \mu_{kj} \geq \gamma_j$. We can also express this constraint as

$$p_j \geq I_j^{\mathrm{MC}}(\boldsymbol{p}) = \langle d_j \rangle \min_k \frac{\gamma_j}{\mu_{kj}(\boldsymbol{p})} \tag{9}$$

For an arbitrary interference function $\boldsymbol{I}(\boldsymbol{p}) = (I_1(\boldsymbol{p}), \cdots, I_N(\boldsymbol{p}))$, we make the following definition.

> **Definition:** Interference function $\boldsymbol{I}(\boldsymbol{p})$ is *standard* if for all $\boldsymbol{p} \geq 0$ the following properties are satisfied.
>
> - *Positivity* $\boldsymbol{I}(\boldsymbol{p}) > 0$
> - *Monotonicity* If $\boldsymbol{p} \geq \boldsymbol{p}'$, then $\boldsymbol{I}(\boldsymbol{p}) \geq \boldsymbol{I}(\boldsymbol{p}')$.
> - *Scalability* For all $\alpha > 1$, $\alpha \boldsymbol{I}(\boldsymbol{p}) > \boldsymbol{I}(\alpha \boldsymbol{p})$.

We adopt the convention that the vector inequality $\boldsymbol{p} > \boldsymbol{p}'$ is a strict inequality in all components. The positivity property is implied by a nonzero background receiver noise. The scalability property implies that if $p_j \geq I_j(\boldsymbol{p})$ then $\alpha p_j \geq \alpha I_j(\boldsymbol{p}) > I_j(\alpha \boldsymbol{p})$ for $\alpha > 1$. That is, if user j has an acceptable connection under power vector $\boldsymbol{p}$, then user j will have a more than acceptable connection when all powers are scaled up uniformly. Note that positivity and convexity of $I_j(\boldsymbol{p})$ for all j implies scalability; however, the converse does not hold.

We note that $\mu_{kj}(\boldsymbol{p})$ satisfies

$$\mu_{kj}(\boldsymbol{p}) \leq \mu_{kj}(\boldsymbol{p}') \qquad (\boldsymbol{p} \geq \boldsymbol{p}') \tag{10}$$

$$\mu_{kj}(\alpha \boldsymbol{p}) > \frac{\mu_{kj}(\boldsymbol{p})}{\alpha} \qquad (\alpha > 1) \tag{11}$$

From Equations (10) and (11), it is easily verified that the interference functions $\boldsymbol{I}^{\mathrm{FA}}$, $\boldsymbol{I}^{\mathrm{MPA}}$, $\boldsymbol{I}^{\mathrm{MD}}$, $\boldsymbol{I}^{\mathrm{LD}}$ and $\boldsymbol{I}^{\mathrm{MC}}$ are standard.

3 Synchronous Iterative Power Control

When $\boldsymbol{I}(\boldsymbol{p})$ is a standard interference function, the iteration (2) will be called the *standard power control algorithm.* In this section, we examine the convergence properties of standard power control under the assumption that $\boldsymbol{I}(\boldsymbol{p})$ is feasible. When we consider maximum power constraints in section 5, we shall see that that feasibility of $\boldsymbol{I}(\boldsymbol{p})$ is not a significant restriction. Moreover, when $\boldsymbol{I}(\boldsymbol{p})$ is infeasible, we have a call admission problem [18, 23, 24] in finding a subset of users that can obtain acceptable connections. In addition, the feasibility of $\boldsymbol{I}(\boldsymbol{p})$ is highly dependent on the underlying wireless system implementation while this work emphasizes the common properties of interference based systems.

Starting from an initial power vector $\boldsymbol{p}$, n iterations of the standard power control algorithm produces the power vector $\boldsymbol{I}^n(\boldsymbol{p})$. We now present convergence results for the sequence $\boldsymbol{I}^n(\boldsymbol{p})$.

Theorem 1 *If the standard power control algorithm has a fixed point, then that fixed point is unique.*

Proof: Theorem 1 Suppose $\boldsymbol{p}$ and $\boldsymbol{p}'$ are distinct fixed points. Since $\boldsymbol{I}(\boldsymbol{p}) > 0$ for all $\boldsymbol{p}$, we must have $p_j > 0$ and $p'_j > 0$ for all j. Without loss of generality, we can assume there exists j such that $p_j < p'_j$. Hence, there exists $\alpha > 1$ such that $\alpha\boldsymbol{p} \geq \boldsymbol{p}'$ and that for some j, $\alpha p_j = p'_j$. The monotonicity and scalability properties imply

$$p'_j = I_j(\boldsymbol{p}') \leq I_j(\alpha\boldsymbol{p}) < \alpha I_j(\boldsymbol{p}) = \alpha p_j \tag{12}$$

Since $p'_j = \alpha p_j$, we have found a contradiction, implying the fixed point must be unique. $\square$

Lemma 1 *If $\boldsymbol{p}$ is a feasible power vector, then $\boldsymbol{I}^n(\boldsymbol{p})$ is a monotone decreasing sequence of feasible power vectors that converges to a unique fixed point $\boldsymbol{p}^*$.*

Proof: Let $\boldsymbol{p}(0) = \boldsymbol{p}$ and $\boldsymbol{p}(n) = \boldsymbol{I}^n(\boldsymbol{p})$. Feasibility of $\boldsymbol{p}$ implies that $\boldsymbol{p}(0) \geq \boldsymbol{p}(1)$. Suppose $\boldsymbol{p}(n-1) \geq \boldsymbol{p}(n)$. Monotonicity implies $\boldsymbol{I}(\boldsymbol{p}(n-1)) \geq \boldsymbol{I}(\boldsymbol{p}(n))$. That is, $\boldsymbol{p}(n) \geq \boldsymbol{I}(\boldsymbol{p}(n)) = \boldsymbol{p}(n+1)$. Hence $\boldsymbol{p}(n)$ is a decreasing sequence of feasible power vectors. Since the sequence $\boldsymbol{p}(n)$ is bounded below by zero, Theorem 1 implies the sequence must converge to a unique fixed point $\boldsymbol{p}^*$. $\square$

Lemma 1 implies $\boldsymbol{p} \geq \boldsymbol{p}^*$ for any feasible vector $\boldsymbol{p}$. That is, the fixed point $\boldsymbol{p}^*$ is the solution of $\boldsymbol{p} \geq \boldsymbol{I}(\boldsymbol{p})$ corresponding to minimum total transmitted power. For the uplink in cellular radio systems, this is particularly desirable in that users may have limited battery power.

Lemma 2 *If $\boldsymbol{I}(\boldsymbol{p})$ is feasible, then starting from $\boldsymbol{z}$, the all zero vector, the standard power control algorithm produces a monotone increasing sequence of power vectors $\boldsymbol{I}^n(\boldsymbol{z})$ that converges to the fixed point $\boldsymbol{p}^*$.*

Proof: Lemma 2 Let $\boldsymbol{z}(n) = \boldsymbol{I}^n(\boldsymbol{z})$. We observe that $\boldsymbol{z}(0) < \boldsymbol{p}^*$ and that $\boldsymbol{z}(1) = \boldsymbol{I}(\boldsymbol{z}) \geq \boldsymbol{z}$. Suppose $\boldsymbol{z} \leq \boldsymbol{z}(1) \leq ... \leq \boldsymbol{z}(n) \leq \boldsymbol{p}^*$, monotonicity implies

$$\boldsymbol{p}^* = \boldsymbol{I}(\boldsymbol{p}^*) \geq \boldsymbol{I}(\boldsymbol{z}(n)) \geq \boldsymbol{I}(\boldsymbol{z}(n-1)) = \boldsymbol{z}(n) \tag{13}$$

That is, $\boldsymbol{p}^* \geq \boldsymbol{z}(n+1) \geq \boldsymbol{z}(n)$. Hence the sequence of $\boldsymbol{z}(n)$ is nondecreasing and bounded above by $\boldsymbol{p}^*$. Theorem 1 implies $\boldsymbol{z}(n)$ must converge to $\boldsymbol{p}^*$. $\square$

Theorem 2 *If $\boldsymbol{I}(\boldsymbol{p})$ is feasible, then for any initial power vector $\boldsymbol{p}$, the standard power control algorithm converges to a unique fixed point $\boldsymbol{p}^*$.*

Proof: Feasibility of $\boldsymbol{I}(\boldsymbol{p})$ implies the existence of the unique fixed point $\boldsymbol{p}^*$. Since $p_j^* > 0$ for all j, for any initial $\boldsymbol{p}$, we can find $\alpha \geq 1$ such that $\alpha\boldsymbol{p}^* \geq \boldsymbol{p}$. By the scalability property, $\alpha\boldsymbol{p}^*$ must be feasible. Since $\boldsymbol{z} \leq \boldsymbol{p} \leq \alpha\boldsymbol{p}^*$, the monotonicity property implies

$$\boldsymbol{I}^n(\boldsymbol{z}) \leq \boldsymbol{I}^n(\boldsymbol{p}) \leq \boldsymbol{I}^n(\alpha\boldsymbol{p}^*) \tag{14}$$

Lemmas 1 and 2 imply $\lim_{n\to\infty} \boldsymbol{I}^n(\alpha\boldsymbol{p}^*) = \lim_{n\to\infty} \boldsymbol{I}^n(\boldsymbol{z}) = \boldsymbol{p}^*$ and the claim follows. $\square$

We have shown that for any initial power vector $\boldsymbol{p}$, the standard power control algorithm converges to a unique fixed point whenever a feasible solution exists.

4 Asynchronous Power Control

In this section, we examine an asynchronous version of the standard power control algorithm using the totally asynchronous algorithm model of Bertsekas and Tsitsiklis [1]. The asynchronous iteration allows some users to perform power adjustments faster and execute more iterations than others. In addition, the asynchronous iteration allows users to perform these updates using outdated information on the interference caused by other users.

Let $p_j(t)$ denote the transmitted power of user j at time t so that the power vector at time t is $\boldsymbol{p}(t) = (p_1(t), \dots, p_N(t))$. We assume that user j may not have access to the most recent values of the components of $\boldsymbol{p}(t)$. This occurs when user j has outdated information about the received power at certain bases. At time t, let $\tau_i^j(t)$ denote the most recent time for which p_i is known to user j. Note that $0 \leq \tau_i^j(t) \leq t$. If user j adjusts its transmitter power at time t, that adjustment is performed using the power vector

$$\boldsymbol{p}(\tau^j(t)) = (p_1(\tau_1^j(t)), p_2(\tau_2^j(t)), ..., p_N(\tau_N^j(t))) \tag{15}$$

We assume a set of times $T = \{0, 1, 2, ...\}$ at which one or more components $p_j(t)$ of $\boldsymbol{p}(t)$ are updated. Let T^j be the set of times at which $p_j(t)$ is updated. At times $t \notin T^j$, $p_j(t)$ is left unchanged. Given the sets $T_1, \dots, T_N$, the totally asynchronous standard power control algorithm is defined by

$$p_j(t+1) = \begin{cases} I_j(\boldsymbol{p}(\tau^j(t))) & t \in T^j \\ p_j(t) & \text{otherwise} \end{cases} \tag{16}$$

We assume the sets T^j are infinite and given any time t_0, there exists t_1 such that $\tau_i^j(t) \geq t_0$ for all $t \geq t_1$. Convergence of the totally asynchronous standard power control algorithm will be proven by the Asynchronous Convergence Theorem from [1] as stated below in Theorem 3. We note that x and $f(x)$ in the statement of Theorem 3 represent the power vector $\boldsymbol{p}$ and iteration function $\boldsymbol{I}(\boldsymbol{p})$ in the context of this work.

Theorem 3 *(Asynchronous Convergence Theorem) If there is a sequence of nonempty sets $\{X(n)\}$ with $X(n+1) \subset X(n)$ for all n satisfying the following two conditions:*

1. *Synchronous Convergence Condition: For all n and $x \in X(n)$, $f(x) \in X(n+1)$. If $\{y^n\}$ is a sequence such that $y^n \in X(n)$ for all n, then every limit point of $\{y^n\}$ is a fixed point of f.*

2. *Box Condition: For every n, there exists sets $X_i(n) \in X_i$ such that $X(n) = X_1(n) \times X_2(n) \times \cdots \times X_N(n)$.*

and the initial solution estimate $x(0)$ belongs to the set $X(0)$, then every limit point of $\{x(t)\}$ is a fixed point of f.

Theorem 4 *If $\boldsymbol{I}(\boldsymbol{p})$ is feasible, then from any initial power vector $\boldsymbol{p}$, the asynchronous standard power control algorithm converges to $\boldsymbol{p}^*$.*

Proof: Let $\boldsymbol{z}$ denote the all zero vector. Feasibility implies the existence of the fixed point $\boldsymbol{p}^*$. Given an initial power vector $\boldsymbol{p}$, we can choose $\alpha \geq 1$ such that $\alpha\boldsymbol{p}^* \geq \boldsymbol{p}$. We define

$$\boldsymbol{X}(n) = \{\boldsymbol{p} | \boldsymbol{I}^n(\boldsymbol{z}) \leq \boldsymbol{p} \leq \boldsymbol{I}^n(\alpha\boldsymbol{p}^*)\} \tag{17}$$

For all n, the set $\boldsymbol{X}(n)$ satisfies the box condition. Lemmas 1 and 2 imply $\boldsymbol{X}(n+1) \subset \boldsymbol{X}(n)$ for all n and $\lim_{n\to\infty} \boldsymbol{I}^n(\boldsymbol{z}) = \lim_{n\to\infty} \boldsymbol{I}^n(\alpha\boldsymbol{p}^*) = \boldsymbol{p}^*$. Hence any sequence $\{\boldsymbol{p}(n)\}$ such that $\boldsymbol{p}(n) \in \boldsymbol{X}(n)$ for all n must converge to $\boldsymbol{p}^*$. Since the initial power vector $\boldsymbol{p}$ satisfies $\boldsymbol{p} \in \boldsymbol{X}(0)$, the asynchronous convergence theorem implies convergence to the fixed point $\boldsymbol{p}^*$. $\Box$

5 Extensions to the Framework

In this section, we describe a number of extensions of the basic framework. Based on standard interference functions, it is possible to generalize a number of iterative power control enhancements.

5.1 Interference Alternatives

Suppose user j is given a choice between two standard interference functions $I_j(\boldsymbol{p})$ and $I'_j(\boldsymbol{p})$. For example, $I_j(\boldsymbol{p})$ and $I'_j(\boldsymbol{p})$ may describe the powers required for user j to communicate with bases k and k' at SIR requirements γ_j and γ'_j respectively. In this case, user j can choose between alternative bases and SIR targets. We will use this notion of interference alternatives to derive some useful structural properties of standard interference functions.

Suppose each user always makes the minimum power choice between $\boldsymbol{I}(\boldsymbol{p})$ and $\boldsymbol{I}'(\boldsymbol{p})$. That is, we define $\boldsymbol{I}^{\min}(\boldsymbol{p})$ by

$$I_j^{\min}(\boldsymbol{p}) = \min\left\{I_j(\boldsymbol{p}), I'_j(\boldsymbol{p})\right\} \tag{18}$$

We can also consider the case in which user j makes the less desirable choice. We define $\boldsymbol{I}^{\max}(\boldsymbol{p})$ by

$$I_j^{\max}(\boldsymbol{p}) = \max\left\{I_j(\boldsymbol{p}), I'_j(\boldsymbol{p})\right\} \tag{19}$$

The trivial verification of the positivity, monotonicity and scalability properties of $\boldsymbol{I}^{\min}(\boldsymbol{p})$ and $\boldsymbol{I}^{\max}(\boldsymbol{p})$ yields the following claim.

Theorem 5 *If $\boldsymbol{I}(\boldsymbol{p})$ and $\boldsymbol{I}'(\boldsymbol{p})$ are standard, then $\boldsymbol{I}^{\min}(\boldsymbol{p})$ and $\boldsymbol{I}^{\max}(\boldsymbol{p})$ are standard.*

It is perhaps not so surprising that $\boldsymbol{I}^{\min}(\boldsymbol{p})$ is standard since under the iteration of $\boldsymbol{I}^{\min}$, each user always chooses the more desirable minimum power alternative. Our interest in $\boldsymbol{I}^{\max}(\boldsymbol{p})$ is that it allows users to choose the less desirable maximum power alternative among $\boldsymbol{I}(\boldsymbol{p})$ and $\boldsymbol{I}'(\boldsymbol{p})$ and permits some consideration of systems in which each user is not required to minimize transmitted power.

5.2 Maximum and Minimum Power Constraints

In real systems, transmitters may be subject to either maximum or minimum power constraints. In this section, we verify the convergence of power constrained iterations that are based on standard interference functions.

Before proceeding, we consider the trivial constant power control in which each user j maintains a fixed power level $q_j > 0$. We define $\boldsymbol{I}^{(q)}(\boldsymbol{p})$ such that for all $\boldsymbol{p} \geq 0$, $\boldsymbol{I}^{(q)}(\boldsymbol{p}) = \boldsymbol{q}$. Although the convergence of $\boldsymbol{p}(t+1) = \boldsymbol{I}^{(q)}(\boldsymbol{p}(t))$ is obvious, we will make use of the following simple observation.

Theorem 6 *$\boldsymbol{I}^{(q)}(\boldsymbol{p})$ is a standard interference function.*

Given a standard interference function $\boldsymbol{I}(\boldsymbol{p})$ and a maximum power vector $\boldsymbol{q}$, we can define the constrained interference function $\hat{\boldsymbol{I}}^{(q)}(\boldsymbol{p}) = (\hat{I}_1^{(q)}(\boldsymbol{p}), \cdots, \hat{I}_N^{(q)}(\boldsymbol{p}))$ by

$$\hat{I}_j^{(q)}(\boldsymbol{p}) = \min\{q_j, I_j(\boldsymbol{p})\} \tag{20}$$

We define the standard constrained power control iteration as

$$\boldsymbol{p}(t+1) = \hat{\boldsymbol{I}}^{(q)}(\boldsymbol{p}(t)) \tag{21}$$

Under the iteration (21), user j transmits with maximum power q_j whenever its SIR requirement calls for transmitter power exceeding q_j. The convergence of (21) has been considered in [25] under fixed base station assignment and in [15] under the minimum power assignment. We note that $\hat{\boldsymbol{I}}^{(q)}(\boldsymbol{p})$ is not truly an interference function in the sense that satisfying $\boldsymbol{p} \geq \hat{\boldsymbol{I}}^{(q)}(\boldsymbol{p})$ does not imply that each user has an acceptable connection. However, we can verify the convergence of (21) by the following result.

Theorem 7 *If $\boldsymbol{I}(\boldsymbol{p})$ is standard, then $\hat{\boldsymbol{I}}^{(q)}(\boldsymbol{p})$ is standard.*

Proof: We observe that $\hat{I}_j^{(q)}(\boldsymbol{p}) = \min\left\{I_j(\boldsymbol{p}), I_j^{(q)}(\boldsymbol{p})\right\}$ is the minimum of two standard interference functions. Hence, the claim follows from Theorem 5. □

We note that $\boldsymbol{p} \geq \hat{\boldsymbol{I}}^{(q)}(\boldsymbol{p})$ always has the trivial feasible solution $\boldsymbol{p} = \boldsymbol{q}$. Hence, Theorems 2 and 7 imply the following corollary.

Corollary 1 *From any initial power vector $\boldsymbol{p}$, the standard constrained power control iteration always converges to a unique fixed point.*

We observe that the fixed point $\boldsymbol{p}^*$ of (21) will satisfy $\boldsymbol{p}^* \geq \boldsymbol{I}(\boldsymbol{p}^*)$ iff $\boldsymbol{p} \geq \boldsymbol{I}(\boldsymbol{p})$ has a feasible solution $\boldsymbol{p}$ that is bounded above by $\boldsymbol{q}$. When this is not the case, $\boldsymbol{p}^*$ has the property that if user j is transmitting at power $p_j^* < q_j$, then user j will have its desired SIR γ_j.

Minimum power requirements can be incorporated in a similar way. Let $\boldsymbol{\epsilon}$ denote a minimum power vector such that user j must transmit with power $p_j \geq \epsilon_j$. For a standard interference function $\boldsymbol{I}(\boldsymbol{p})$, we define $\tilde{\boldsymbol{I}}^{(\epsilon)}(\boldsymbol{p})$ by

$$\tilde{I}_j^{(\epsilon)}(\boldsymbol{p}) = \max\left\{\epsilon_j, I_j(\boldsymbol{p})\right\} \tag{22}$$

In this case, the convergence of

$$\boldsymbol{p}(t+1) = \tilde{\boldsymbol{I}}^{(\epsilon)}(\boldsymbol{p}(t)) \tag{23}$$

is verified by the following theorem.

Theorem 8 *If $\boldsymbol{I}(\boldsymbol{p})$ is standard, then $\tilde{\boldsymbol{I}}^{(\epsilon)}(\boldsymbol{p})$ is standard.*

Proof: Since $\tilde{I}_j^{(\epsilon)}(\boldsymbol{p}) = \max\left\{I_j^{(\epsilon)}(\boldsymbol{p}), I_j(\boldsymbol{p})\right\}$ is the maximum of two standard interference functions, the claim follows from Theorem 5. □

5.3 Active Link Protection

In [18], Bambos et al. describe a fixed base station assignment power control algorithm called DPC-ALP (Distributed Power Control with Active Link Protection). In DPC-ALP, a user with an acceptable SIR is called *active*. In [18], it is shown that under

DPC-ALP, active users are guaranteed to remain active while each inactive user steadily raises its transmitted power in an effort to become active. In this work, we generalize DPC-ALP to standard interference functions.

We assume the SIR requirements $\gamma_1, \ldots, \gamma_N$ of the users are described by a standard interference function $\boldsymbol{I}(\boldsymbol{p})$. We express the standard ALP iteration as

$$\boldsymbol{p}(t+1) = \boldsymbol{I}^{\mathrm{ALP}}(\boldsymbol{p}(t)) \tag{24}$$

where for a constant $\delta > 1$ and a constant vector $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_N) > 0$,

$$I_j^{\mathrm{ALP}}(\boldsymbol{p}) = \min\left\{\delta p_j + (\delta - 1)\epsilon_j, \delta I_j(\boldsymbol{p} + \boldsymbol{\epsilon})\right\} \tag{25}$$

In the definition of DPC-ALP in [18], the constant vector $\boldsymbol{\epsilon}$ was taken to be zero. Here $\boldsymbol{\epsilon}$ is assumed to be a very small positive vector whose sole purpose is to prevent $\boldsymbol{p} = 0$ from being a fixed point of the ALP iteration; otherwise, $\boldsymbol{\epsilon}$ has no practical significance.

We say that user j is *active* at time t if $p_j(t) \geq I_j(\boldsymbol{p}(t) + \boldsymbol{\epsilon})$. That is, an active user j achieves its required SIR γ_j. During an ALP iteration, an inactive user j' increases its power from $p_{j'}$ to $\delta p_{j'} + (\delta - 1)\epsilon_{j'}$. At the same time, an active user j aims for an SIR target of $\delta\gamma_j$ in order to ensure that an SIR of γ_j is maintained. The synchronous and asynchronous convergence of (24) are verified by the following claim.

Theorem 9 *If $\boldsymbol{I}(\boldsymbol{p})$ is standard, then $\boldsymbol{I}^{\mathrm{ALP}}(\boldsymbol{p})$ is standard.*

Proof: Note that $\boldsymbol{I}'(\boldsymbol{p}) = \delta\boldsymbol{p} + (\delta - 1)\boldsymbol{\epsilon}$ and $\delta\boldsymbol{I}(\boldsymbol{p} + \boldsymbol{\epsilon})$ both satisfy the requirements of a standard interference function. Since $I_j^{\mathrm{ALP}}(\boldsymbol{p}) = \min\left\{I_j'(\boldsymbol{p}), \delta I_j(\boldsymbol{p} + \boldsymbol{\epsilon})\right\}$, the claim follows from Theorem 5. □

If the ALP iteration (24) converges, then it must converge to $\boldsymbol{p} = \delta\boldsymbol{I}(\boldsymbol{p} + \boldsymbol{\epsilon})$. In this case, user j will have SIR $\delta\gamma_j$, exceeding the nominal required SIR γ_j of the underlying standard interference function.

We now verify that an active link always stays active under the synchronous ALP iteration.

Theorem 10 *If $p_j(t) \geq I_j(\boldsymbol{p}(t) + \boldsymbol{\epsilon})$, then $p_j(t+1) \geq I_j(\boldsymbol{p}(t+1) + \boldsymbol{\epsilon})$.*

Proof: First, we observe that (25) implies $\boldsymbol{p}(t+1) \leq \delta\boldsymbol{p}(t) + (\delta - 1)\boldsymbol{\epsilon}$. Hence, $\boldsymbol{p}(t+1) + \boldsymbol{\epsilon} \leq \delta(\boldsymbol{p}(t) + \boldsymbol{\epsilon})$. Second, if user j is active, scalability and monotonicity of

$\boldsymbol{I}(\boldsymbol{p})$ imply

$$\begin{aligned} p_j(t+1) &= \delta I_j(\boldsymbol{p}(t)+\boldsymbol{\epsilon}) && (26)\\ &> I_j(\delta(\boldsymbol{p}(t)+\boldsymbol{\epsilon})) && (27)\\ &\geq I_j(\boldsymbol{p}(t+1)+\boldsymbol{\epsilon}) && (28)\end{aligned}$$

□

We note that these results will also hold when we place maximum power constraints on either $\boldsymbol{I}^{\mathrm{ALP}}$ or the underlying $\boldsymbol{I}(\boldsymbol{p})$. In these cases, convergence is guaranteed but the active link protection property is fictitious in that a user transmitting at maximum power trivially satisfies the requirements of the constrained interference function although that user's actual SIR requirement is not necessarily being met. Furthermore, although the ALP iteration (24) is guaranteed to converge asynchronously, the active link protection property holds only for the synchronous iteration.

5.4 Interference Averaging

To reduce fluctuations in users' transmitter powers possibly due to inaccurate power measurements, it may be desirable to average a user's current power p_j with the needed power $I_j(\boldsymbol{p})$. Given a standard $\boldsymbol{I}(\boldsymbol{p})$ and a constant $0 \leq \beta < 1$, we define the standard interference averaging power control iteration as

$$\boldsymbol{p}(t+1) = \bar{\boldsymbol{I}}(\boldsymbol{p}(t)) = \beta\boldsymbol{p}(t) + (1-\beta)\boldsymbol{I}(\boldsymbol{p}(t)) \tag{29}$$

We call this approach interference averaging because $\boldsymbol{p}(t)$ is based on previous interference measurements. Note that p_j and $I_j(\boldsymbol{p})$ may differ by several orders of magnitude. In this case, it may be more appropriate to to average $\log p_j$ and $\log I_j(\boldsymbol{p})$. Hence, we define the logarithmic interference averaging function as $\bar{\boldsymbol{I}}^{\mathrm{dB}}(\boldsymbol{p})$ where

$$\bar{I}_j^{\mathrm{dB}}(\boldsymbol{p}) = \exp\left(\beta \ln p_j + (1-\beta)\ln I_j(\boldsymbol{p})\right) \tag{30}$$

From Equations (29) and (30), the following claim is readily verified.

Theorem 11 *If $\boldsymbol{I}(\boldsymbol{p})$ is standard, then $\bar{\boldsymbol{I}}(\boldsymbol{p})$ and $\bar{\boldsymbol{I}}^{\mathrm{dB}}(\boldsymbol{p})$ are standard interference functions with a fixed point $\boldsymbol{p}^*$ satisfying $\boldsymbol{p}^* = \boldsymbol{I}(\boldsymbol{p}^*)$.*

5.5 Hybrid Interference Functions

Suppose that the interference constraints faced by user j are described by $p_j \geq I_j^{(s_j)}(\boldsymbol{p})$ where the superscript s_j indicates whether user j has a fixed assignment, or is using the minimum power assignment, or has some form of diversity reception. From the definition of standard interference functions, we make the following claim.

Theorem 12 *If* $(I_1^{(s_j)}(\boldsymbol{p}), \cdots, I_N^{(s_j)}(\boldsymbol{p}))$ *is standard for all* j*, then the hybrid interference function* $\boldsymbol{I}(\boldsymbol{p}) = (I^{(s_1)}(\boldsymbol{p}), \cdots, I^{(s_N)}(\boldsymbol{p}))$ *is standard.*

6 Discussion

When it is possible to provide each user an acceptable connection, as defined by the interference function of the system, the synchronous and asynchronous standard power control algorithms will find the minimum power solution. When $\boldsymbol{I}(\boldsymbol{p})$ is infeasible, then the constrained power control iteration of (21) is guaranteed to converge, permitting the system to detect the infeasibility.

The asynchronous convergence results give an indication of the robustness of the standard power control iteration. In addition, we observe that $\mu_{kj}(\boldsymbol{p})$ can be expressed as

$$\mu_{kj}(\boldsymbol{p}) = \frac{h_{kj}}{R_k(\boldsymbol{p}) - h_{kj}p_j} \tag{31}$$

where $R_k(\boldsymbol{p}) = \sum_j h_{kj}p_j + \sigma_k$ denotes the total received power at base k. Hence, the power controlled systems described in section 2 can be implemented by each user knowing only its own uplink gains and the total received power at each base station. It is not necessary to know all uplink gains or transmitted powers of the other mobiles. This suggests that these standard power control algorithms may be suitable for distributed asynchronous implementation in real systems in which users must perform updates with wrong or outdated interference measurements.

We believe that the properties of the standard interference function should hold for the uplink of any single channel interference based power controlled system. In addition, this framework is also valid under fixed base station assignment for the downlink power control problem. However, we must emphasize that the standard interference function approach does have certain limitations. The monotonicity property implies that whenever a user can reduce its transmitted power, all other users will benefit from that power reduction. This property does not hold for all cases of interest. For example, in a multichannel system, a power reduction associated with user j changing from channel c to c' would create greater interference for mobiles currently using channel c'. For a second example, on the downlink of a system in which one base station must be chosen to transmit to each mobile, the power reduction associated with changing the base station assignment of user j from k to k' may create greater interference for those mobiles near base k'.

We observe that this framework permits simple system comparisons to be made on the basis of interference functions. In particular, we observe that for all power vectors $\boldsymbol{p}$,

$$\boldsymbol{I}^{\mathrm{MC}}(\boldsymbol{p}) \geq \boldsymbol{I}^{\mathrm{MPA}}(\boldsymbol{p}) \geq \boldsymbol{I}^{\mathrm{LD}}(\boldsymbol{p}) \geq \boldsymbol{I}^{\mathrm{MD}}(\boldsymbol{p}) \tag{32}$$

Hence, if we denote by $\boldsymbol{\Pi} = \{\boldsymbol{p} \geq 0 | \boldsymbol{p} \geq \boldsymbol{I}(\boldsymbol{p})\}$ the set of feasible power vectors under interference function $\boldsymbol{I}(\boldsymbol{p})$, then we have

$$\boldsymbol{\Pi}^{\mathrm{MC}} \subseteq \boldsymbol{\Pi}^{\mathrm{MPA}} \subseteq \boldsymbol{\Pi}^{\mathrm{LD}} \subseteq \boldsymbol{\Pi}^{\mathrm{MD}} \tag{33}$$

As expected, increasing diversity increases the space of feasible power vectors. However, it remains unclear whether these capacity improvements are significant in actual systems in which the interactions between user mobility, channel fading and power control must be considered.

We hope this work provides a framework for understanding the convergence of common power control algorithms. As more sophisticated power control methods are developed, standard interference functions may be an aid in verifying the convergence properties of these methods.

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