Decorrelating Detectors for a Dual Rate Synchronous DS/CDMA Channel

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ABSTRACT

This paper addresses the use of decorrelating detectors for a dual rate DS/CDMA system that serves both low bit rate and high bit rate users. All users have the same BPSK modulation technique and the same chip rate. The differences in bit transmission rates result in different processing gains for each class of user. We assume that in an interval of duration T_0, a low rate user transmits one bit while a high rate user transmits M bits. Applying a standard decorrelator to the interval of duration T_0 yields an M bit processing delay for high rate users and a computational complexity that grows with M. In this paper, we propose a decorrelator that generates bit decisions for each high rate user in every subinterval of duration T_0/M. To decode a low rate user, a *soft decoding* rule applies maximal ratio combining on M separate decorrelated outputs of each low rate user. The soft decoding dual rate decorrelator eliminates the bit processing delay for high rate users and also reduces the computational complexity of a standard decorrelator. This paper proves that the asymptotic efficiency of the standard decorrelator is greater than or equal to the proposed decorrelator. However, by evaluation, it is observed that when the signature sequences have good correlation properties, the proposed soft decoding decorrelator is found to perform nearly as well as the standard decorrelator while retaining the near-far resistance property.

1 Introduction

The rapid expansion of the mobile cellular market over the past few years has made apparent the inability of current wireless technology to meet future service requirements. The primary difficulty is that limited radio bandwidth restricts the capacity of the system. In order to optimize the existing frequency resources, FDMA, TDMA and CDMA multi-access schemes have been proposed. For CDMA systems, the structural properties of receivers have received significant attention. In 1984, Verdu [1] proposed the *optimum multiuser receiver* in which all users' signals are jointly decoded. However, the computational complexity of this receiver prompted the development of a number of suboptimal receivers [2, 3, 4, 5, 6, 7, 8, 9, 10]. Among such receivers, the decorrelating detector [3] is perhaps the simplest in structure and is reasonably easy to implement. The decorrelator improves the performance by eliminating the multiuser interference at the cost of increased noise variance. Furthermore, the decorrelator does not require the knowledge of the received signal strengths. In [2, 3, 4, 5, 6, 7, 8, 9, 10], the proposed multiuser detectors were designed for CDMA systems in which all users transmit at the same bit rate.

Significant efforts are being made to integrate the cellular network with fixed networks for communicating both voice and data messages [11, 12, 13]. When all users have the same modulation scheme, two access strategies have been proposed for multi-rate DS/CDMA [14]. These are

1. Fixed chip rate, variable processing gain,

2. Fixed processing gain, variable chip rate.

A comparative study of the above access strategies was performed in [15]. However, in [14], it is found that the access strategy 2 is complicated because the receiver must be synchronized to its particular code rate and the system needs additional frequency planning due to the unequal bandwidth spreading of different users.

Several studies regarding receiver designs for multi-rate CDMA systems have been performed. Multistage multiuser receivers for multi-rate CDMA communications were introduced in [16]. For minimum mean squared error (MMSE) performance criteria, [17] considered multi-user communication with multiple symbol rates. In [18], a successive interference cancelation scheme for multi-rate CDMA was studied by employing complex modulation techniques, such as M-ary PAM.

For the simple fixed chip rate access strategy 1, this paper addresses the use of decorrelating detectors for a dual rate DS/CDMA system. In this system, all users have the same BPSK modulation technique and high rate users transmit at M times the bit rate of the low rate users; see Figure 1. We show how the dual rate system is equivalent to a single rate system in which each high rate user is modeled as M independent low rate users. We specify a receiver called the *low rate decorrelator* for the dual rate CDMA system that is simply the standard decorrelator of [3] applied to the equivalent single rate system.

The complexity of the low rate decorrelator prompts consideration of a *high rate decorrelator*, in which during the bit interval of a high rate user, each low rate user is modeled as a high rate user. The comparative performance of the low rate and high rate decorrelators is studied. It is found that the high rate decorrelator achieves a sizable reduction in complexity though suffering a penalty in terms of bit error probability or asymptotic efficiency. Both low rate and high rate decorrelators preserve the standard decorrelator's near/far resistance property.

2 The Dual Rate Synchronous System

In our synchronous dual rate DS/CDMA system model, each bit results in the baseband transmission of a sequence of pulses, or chips, $p[t]$, each pulse having a duration of one chip period T_c. These pulses are sent over an additive white Gaussian noise channel in which the noise $n(t)$ has power spectral density $N_0/2$.

Each user group will be classified by its bit rate. The transmission rate of group g users of is denoted by R_g, where $g = 0, 1$ and $R_1 = MR_0$ for an integer $M > 1$. For the K_g group g users, the transmission time of a bit is $T_g = 1/R_g$ and the processing gain is $L_g = T_g/T_c$. The signature waveform of the nth group g customer is

$$S_{n,g}(t) = \sum_{m=1}^{L_g} \{a_{n,g}(m) \frac{1}{\sqrt{L_g}} p[t-(m-1)T_c]\} \qquad t \in [0, T_g] \tag{1}$$

where $a_{n,g}(m) \in \{-1, 1\}$ denotes the signature sequence of user n of group g. The energy of the pulse $p[t]$ is normalized so that

$$\int_0^{T_g} [S_{n,g}(t)]^2 dt = 1 \qquad g = 0, 1; n = 1, \dots, K_g \tag{2}$$

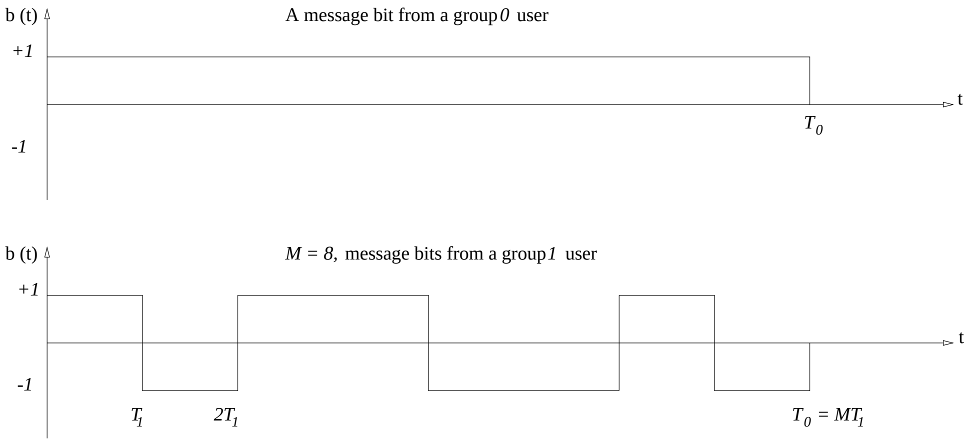

Figure 1: Messages from a user of each group in the time interval, $[0, T_0]$.

Note that a group 0 user has a signature waveform of duration T_0 while a group 1 user has a signature waveform of duration $T_1 = T_0/M$. In the interval $[0, T_0]$, each group 0 user transmits one bit while each group 1 user transmits M bits. The jth user from group 0 transmits bit $b_{j,0} \in \{-1, 1\}$ with received energy $E_{j,0}$ in the interval $[0, T_0]$. Similarly, the kth user from group 1 transmits its ith bit $b_{k,1}^{(i)} \in \{-1, 1\}$ with received $E_{k,1}^{(i)}$ in the ith subinterval $[(i-1)T_1, iT_1]$ using the signature waveform

$$S_{k,1}^{(i)}(t) = S_{k,1}(t - (i-1)T_1) \tag{3}$$

Over the interval $[0, T_0]$, the received baseband signal can be written as

$$r(t) = \sum_{j=1}^{K_0} \sqrt{E_{j,0}}\, b_{j,0} S_{j,0}(t) + \sum_{k=1}^{K_1} \left\{ \sum_{i=1}^{M} \sqrt{E_{k,1}^{(i)}}\, b_{k,1}^{(i)} S_{k,1}^{(i)}(t) \right\} + n(t) \tag{4}$$

Similar to the single rate DS/CDMA system [3], it is observed that finding the maximum likelihood receiver for the dual rate DS/CDMA system is also an NP-hard problem. Thus, we will explore the use of decorrelating receivers for the dual rate CDMA system. Equation (4) is a representation of the received signal in the dual rate system that is equivalent to a single rate system with $K = K_0 + MK_1$ group 0 users each transmitting 1 bit during $[0, T_0]$. In the next section, we use this representation to describe the low rate decorrelator.

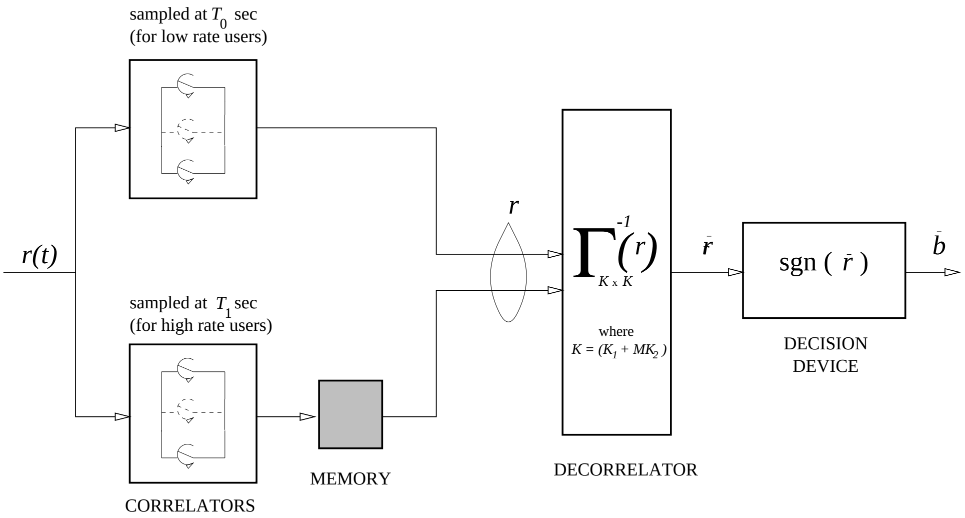

Figure 2: The low rate decorrelator, LRD, for the dual rate synchronous DS/CDMA system.

3 The Low Rate Decorrelator, LRD

The $K = K_0 + MK_1$ bits transmitted in the interval $[0, T_0]$ can be written as the K bit vector $b = [b_1, b_2, \ldots, b_K]$ where

$$b = [\overbrace{\underbrace{b_{1,0}, \ldots, b_{K_0,0}}_{\text{in } [0,T_0]}}^{\text{low rate users}} \overbrace{\underbrace{b_{1,1}^{(1)}, \ldots, b_{K_1,1}^{(1)}}_{\text{in } [0,T_1]}, \ldots \quad \ldots, \quad \underbrace{b_{1,1}^{(M)}, \ldots, b_{K_1,1}^{(M)}}_{\text{in } [(M-1)T_1, MT_1]}}^{\text{high rate users}} \;]^\top \tag{5}$$

Note that for $n \leq K_0$, the bit b_n was transmitted by user n of group 0. Furthermore, examination of the above ordering will show we can define functions $k_1(n)$ and $i_1(n)$ such that given $n > K_0$, we can identify that a group 1 user $k = k_1[n]$ transmitted bit b_n during subinterval $i = i_1[n]$. For the vector $b = [b_1, \ldots, b_K]^\top$, the bit b_n is transmitted with energy E_n using the signature waveform $s_n(t)$ where

$$s_n(t) = \begin{cases} S_{n,0}(t) & 1 \leq n \leq K_0 \\ S_{k_1[n],1}^{(i_1[n])}(t) & n > K_0 \end{cases} \tag{6}$$

$$E_n = \begin{cases} \sqrt{E_{n,0}} & 1 \leq n \leq K_0 \\ \sqrt{E_{k_1[n],1}^{(i_1[n])}} & n > K_0 \end{cases} \tag{7}$$

The received signal $r(t)$ from equation (4) can be written as

$$r(t) = \sum_{n=1}^{K} b_n \sqrt{E_n} s_n(t) + n(t) = S(t,b) + n(t) \tag{8}$$

The sampled output of a bank of correlators for the signature waveforms $s_n(t)$ is the vector $r = [r_1, r_2, \ldots, r_K]^\top$ of sufficient statistics with

$$r_n = \int_0^{T_0} r(t) s_n(t) dt \qquad n = 1, \ldots, K \tag{9}$$

As a vector, r can be written as

$$r \;=\; \Gamma \Lambda b + N \tag{10}$$

where Λ is a diagonal matrix with $\Lambda_{n,n} = \sqrt{E_n}$ and Γ is the $K \times K$ cross-correlation matrix with (j,k)th element

$$[\Gamma]_{j,k} \;=\; \int_0^{T_0} s_j(t) s_k(t) dt \tag{11}$$

In a single rate CDMA system, the bit duration of each user is the same and the correlator outputs are sampled at every bit interval. The vector of decision statistics is then obtained after decorrelating the sampled outputs with the correlation matrix of signature waveforms. Finally, all users are decoded simultaneously. We can use the low rate representation of the dual rate CDMA system to develop an equivalent decorrelator. In equation (10), each group 1 user has been replaced by M effective group 0 users with signature waveforms that are simply time shifted versions of one another. Further, each of these waveforms is non-zero in only one of the M sub-intervals corresponding to the appropriate group 1 users' bits; see Figure 3.

From equation (10), we decorrelate r by applying the transformation Γ^{-1} to yield

$$\bar{r} = \Gamma^{-1} r = \Lambda b + \bar{N} \tag{12}$$

where $\bar{N}$ is a zero mean Gaussian vector with the covariance matrix $\frac{N_0}{2}\Gamma^{-1}$. The decoding rule of the decorrelator is simply

$$\bar{b} = \operatorname{sgn}[\bar{r}] = \operatorname{sgn}\left[\Lambda b + \bar{N}\right] \tag{13}$$

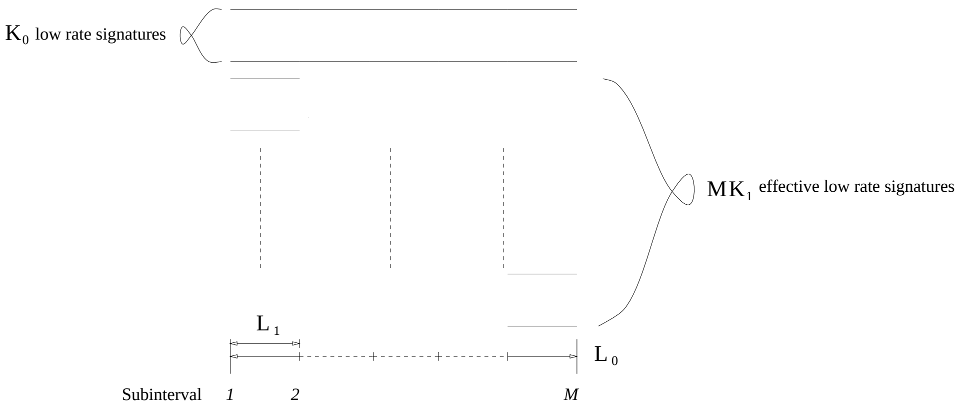

Figure 3: The figure depicts the $K_0 + MK_1$ signatures of the effective group 0 users for the interval $[0, T_0]$.

We call this receiver the *low rate decorrelator*. Since $\bar{N}_j$ has variance $\frac{N_0}{2}[\Gamma^{-1}]_{j,j}$, the probability that the jth input is decoded incorrectly can be written as

$$P_j(\sigma) = Q\left[\sqrt{\frac{E_j}{\sigma^2[\Gamma^{-1}]_{j,j}}}\right] \tag{14}$$

where $\sigma^2 = N_0/2$ and $Q[\cdot]$ denotes the standard normal complementary CDF. If $e_j(\sigma)$ denotes the energy that yields the same bit error rate $P_j(\sigma)$ of equation (14) in a single-user Gaussian channel with power spectral density $\sigma^2 = \frac{N_0}{2}$, then the *asymptotic efficiency*[3, 19, 20] of the LRD for the jth user can be written as

$$[\eta_j]^L = \lim_{\sigma \to 0} \frac{e_j(\sigma)}{E_j} = 1/\left[\Gamma^{-1}\right]_{j,j} \tag{15}$$

If in the LRD system, $[\eta_{n,0}]^L$ denotes the asymptotic efficiency for the nth group 0 user while $[\eta_{k_1[n],1}^{(i_1[n])}]^L$ is the asymptotic efficiency for the ith bit of the nth group 1 user, then equation (15) becomes

$$[\eta_n]^L = \begin{cases} [\eta_{n,0}]^L & 1 \le n \le K_0 \\ \left[\eta_{k_1[n],1}^{(i_1[n])}\right]^L & n > K_0 \end{cases} \tag{16}$$

Note that a standard decorrelator for a single rate system of $K = K_0 + MK_1$ users requires K correlators. By comparison, since the low rate decorrelator generates M effective group 0 users for each group 1 signature sequence, it requires only $K_0 + K_1$

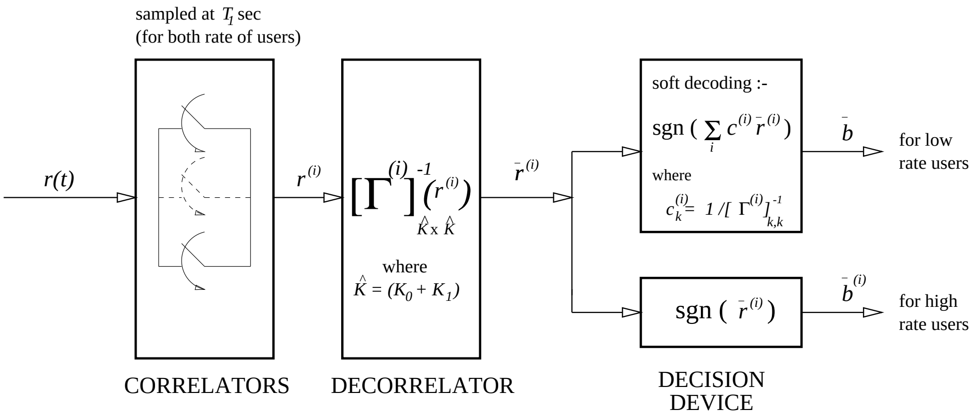

Figure 4: The high rate decorrelating detector, HRD, for the dual rate synchronous DS/CDMA system

correlators. However, the low rate decorrelator has certain disadvantages. First, the low rate decorrelator still requires inversion of the $K \times K$ matrix Γ. Although we will see that Γ has a sparse structure that makes computation of Γ^{-1} reasonably easy, the complexity of the low rate decorrelator grows with M. Second, for each high rate group 1 user, the M sampled correlator outputs in $[0, T_0]$ are stored and decoded simultaneously at time T_0 incurring an M bit processing delay; see Figure 2.

4 The High Rate Decorrelator, HRD

In this section, we describe a decorrelator in which decisions for the high rate users of group 1 are made every $T_1 = T_0/M$ units of time. In every subinterval $[(i-1)T_1, iT_1]$, there are K_1 group 1 users and K_0 group 0 users transmitting. Each high rate group 1 user k transmits one bit using the signature waveform $S_{k,1}(t)$. Each low rate group 0 user j transmits the ith segment of the signature waveform $S_{j,0}(t)$ denoted by

$$S_{j,0}^{(i)}(t) = \sum_{m=(i-1)L_1+1}^{iL_1} \left\{ a_{j,0}(m) \frac{1}{\sqrt{L_0}} p[t-(m-1)T_c] \right\} \tag{17}$$

At the receiver, the correlator outputs are sampled at the end of every subinterval of length T_1.

The high rate decorrelator, as shown in Figure 4, views the $K_0 + K_1$ transmitters as though they were $K_0 + K_1$ group 1 users each transmitting one bit in each subinterval. We enumerate these transmitting users such that the jth group 0 user is numbered j,

$1 \le j \le K_0$, while the kth high rate user is numbered $(K_0 + k)$. For subinterval i, the input vector is $\hat{b}^{(i)} = \left[\hat{b}_1, \ldots, \hat{b}_{K_0}, \hat{b}_{(K_0+1)}, \ldots, \hat{b}_{K_0+K_1}\right]^\top$. The bit $\hat{b}_n$ is transmitted using the signature waveform

$$\hat{s}_n^{(i)}(t) = \begin{cases} S_{n,0}^{(i)}(t - (i-1)T_1) & 1 \le n \le K_0 \\ S_{n-K_0,1}(t) & K_0 < n \le K_0 + K_1 \end{cases} \tag{18}$$

Note that for all i and n, $\hat{s}_n^{(i)}(t)$ is defined as a signature of duration T_1 on the interval $[0, T_1]$. In each subinterval i, each group 1 user k transmits with energy $E_{k,1}^{(i)}$; however, each group 0 user j transmits with a fraction of its total transmitted energy per bit. Since the signature waveform $S_{j,0}(t)$ is normalized to have unit energy over the interval $[0, T_0]$, the fraction of the bit energy that group 0 user j transmits in subinterval i is $\int_{(i-1)T_1}^{iT_1} [S_{j,0}(t)]^2 dt = 1/M$.

The vector of correlator output samples at time iT_1 is

$$r^{(i)} \;=\; \Gamma^{(i)}\Lambda^{(i)}\hat{b}^{(i)} + N^{(i)} \tag{19}$$

where $\Gamma^{(i)}$ is the $(K_0 + K_1) \times (K_0 + K_1)$ cross-correlation matrix with j, kth element

$$\left[\Gamma^{(i)}\right]_{j,k} = \int_0^{T_1} \hat{s}_j^{(i)}(t)\hat{s}_k^{(i)}(t)\, dt \tag{20}$$

Furthermore, $\Lambda^{(i)}$ is a diagonal matrix with

$$\Lambda_{n,n}^{(i)} = \begin{cases} \sqrt{E_{n,0}} & 1 \le n \le K_0 \\ \sqrt{E_{n-K_0,1}^{(i)}} & K_0 < n \le K_0 + K_1 \end{cases} \tag{21}$$

and $N^{(i)}$ is a $K_0 + K_1$ element Gaussian noise vector with covariance matrix $\Gamma^{(i)}$. In order to recover the vector $\hat{b}^{(i)}$, we decorrelate $r^{(i)}$ using the cross-correlation matrix $\Gamma^{(i)}$. Specifically,

$$\bar{r}^{(i)} = \left[\Gamma^{(i)}\right]^{-1} r^{(i)} = \Lambda^{(i)}\hat{b}^{(i)} + \bar{N}^{(i)} \tag{22}$$

where $\bar{N}^{(i)}$ is a zero mean Gaussian noise vector with covariance matrix $\frac{N_0}{2}\left[\Gamma^{(i)}\right]^{-1}$.

Using equation (22), the decoding rule in subinterval i for a group 1 user k is

$$\bar{b}_k^{(i)} = \operatorname{sgn}\left[\bar{r}_k^{(i)}\right] = \operatorname{sgn}\left[\Lambda_{k,k}^{(i)} b_k^{(i)} + \bar{N}_k^{(i)}\right] \qquad K_0 < k \leq K_0 + K_1 \tag{23}$$

The probability that a group 1 user k is decoded incorrectly in the ith subinterval equals the probability that bit $b_{k+K_0}^{(i)}$ is received in error which equals

$$\hat{P}_{k,1}^{(i)} = Q\left[\sqrt{\frac{E_{k,1}^{(i)}}{(N_0/2)[\Gamma^{(i)}]^{-1}_{k+K_0,k+K_0}}}\right] \tag{24}$$

and the asymptotic efficiency is

$$\left[\eta_{k,1}^{(i)}\right]^H = 1/\left[\Gamma^{(i)}\right]^{-1}_{k+K_0,k+K_0} \tag{25}$$

For a group 0 user j, we obtain the decorrelated outputs $\bar{r}_j^{(i)}$ for each subinterval i. Each sample $\bar{r}_j^{(i)}$ consists of the bit b_j weighted by the term $E_{j,0}$ in addition to an independent Gaussian noise $\bar{N}_j^{(i)}$. To decode a group 0 user j, we will use *maximal ratio combining* [21, 22] to implement the following *soft decoding* rule:

$$\bar{b}_j = \operatorname{sgn}\left[\sum_{i=1}^{M}\left(\frac{E_{j,0}^{(i)}}{[\Gamma^{(i)}]^{-1}_{j,j}}\right)\bar{r}_j^{(i)}\right] \qquad 1 \leq j \leq K_0 \tag{26}$$

Note that we assume the energy $E_{j,0}^{(i)}$ is a constant $E_{j,0}$ for all subintervals i so that under the soft decoding rule, the probability that the jth group 0 user is decoded incorrectly is

$$\hat{P}_{j,0} = Q\left[\sqrt{\frac{E_{j,0}}{(N_0/2)}\sum_{i=1}^{M}\left(\frac{1}{[\Gamma^{(i)}]^{-1}_{j,j}}\right)}\right] \qquad 1 \leq j \leq K_0 \tag{27}$$

The corresponding asymptotic efficiency of the jth low rate user under HRD is

$$[\eta_{j,0}]^H = \sum_{i=1}^{M}\left(\frac{1}{[\Gamma^{(i)}]^{-1}_{j,j}}\right) \tag{28}$$

5 A Comparison of LRD and HRD

The performance of the low rate and high rate decorrelators is not the same simply because cross-correlation matrices are different in structure. Characterizing the performance differences between the low rate and high rate decorrelators requires study of the structure of the respective inverse correlation matrices.

5.1 The Low Rate Decorrelator

First, we identify the sparse structure of the low rate correlation matrix Γ. From equation (5), we can write the vector b of transmitted bits as

$$b^\top = \left[[\tilde{b}^{(0)}]^\top, [\tilde{b}^{(1)}]^\top, \ldots, [\tilde{b}^{(M)}]^\top\right] \tag{29}$$

where $\tilde{b}^{(0)}$ is the vector of bits transmitted by group 0 users over $[0, T_0]$ and $\tilde{b}^{(i)}$ is the vector of bits transmitted by group 1 users in subinterval i. Similarly, let $\tilde{s}^{(i)}(t)$ denote the vector of signature waveforms used to transmit the bit vector $\tilde{b}^{(i)}$. Note that each component of $\tilde{s}^{(0)}(t)$, is a signature of duration T_0. From equation (1), we can write

$$\tilde{s}^{(0)}(t) = [S_{1,0}(t), \cdots, S_{K_0,0}(t)] \tag{30}$$

For $i > 0$, each component of $\tilde{s}^{(i)}(t)$ is a signature that is nonzero only on the ith subinterval. In particular,

$$\tilde{s}^{(i)}(t) = \left[S_{1,1}^{(i)}(t), \cdots, S_{K_1,1}^{(i)}(t)\right] \qquad (i > 0) \tag{31}$$

Since each group 1 user applies the same signature in each subinterval,

$$\tilde{s}^{(i+1)}(t) = \tilde{s}^{(i)}(t - T_1) \qquad i > 1 \tag{32}$$

The LRD cross-correlation matrix Γ will be denoted by $\Gamma = \Gamma_M$ where for $k = 0, \ldots, M$, Γ_k is the block matrix

$$\Gamma_k = \begin{pmatrix} \Gamma_{00} & \Gamma_{01} & \Gamma_{02} & \cdots & \Gamma_{0k} \\ \Gamma_{10} & \Gamma_{11} & 0 & \cdots & 0 \\ \Gamma_{20} & 0 & \Gamma_{22} & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ \Gamma_{k0} & 0 & \cdots & 0 & \Gamma_{kk} \end{pmatrix} \tag{33}$$

Note that Γ_{ij} is the cross-correlation matrix generated from the signature vectors $\tilde{s}^{(i)}(t)$ and $\tilde{s}^{(j)}(t)$. That is, element m, n of Γ_{ij} is

$$[\Gamma_{ij}]_{m,n} = \int_0^{T_0} \tilde{s}_m^{(i)}(t) \tilde{s}_n^{(j)}(t)\, dt \tag{34}$$

As expected, $\Gamma_{ij} = \Gamma_{ji}^\top$. In addition, since the high rate users have the same signature in each subinterval, $\Gamma_{ii} = \Gamma_{kk}$ for all $i, k > 1$. The sparse structure of Γ arises because for $i, j > 0$ and $i \neq j$, the signature vector $\tilde{s}^{(i)}(t)$ is nonzero only in subinterval i while $\tilde{s}^{(j)}(t)$ is nonzero over the subinterval j. The inverse of Γ_k can be written in a corresponding block form:

$$\Gamma_k^{-1} = \begin{pmatrix} B_{00}^{(k)} & \cdots & B_{0M}^{(k)} \\ \vdots & \ddots & \vdots \\ B_{M0}^{(k)} & \cdots & B_{MM}^{(k)} \end{pmatrix} \tag{35}$$

The matrix elements of Γ_k^{-1} satisfy $B_{ij}^{(k)} = \left[B_{ji}^{(k)}\right]^\top$ and also the matrix equations

$$B_{00}^{(k)} = \left(\Gamma_{00} - \sum_{i=1}^{k} \Gamma_{0i}\Gamma_{ii}^{-1}\Gamma_{i0}\right)^{-1} \tag{36}$$

$$B_{0i}^{(k)} = -B_{00}^{(k)}\Gamma_{0i}\Gamma_{ii}^{-1} \qquad i > 0 \tag{37}$$

$$B_{ij}^{(k)} = \Gamma_{ii}^{-1}\Gamma_{i0}B_{00}^{(k)}\Gamma_{0j}\Gamma_{jj}^{-1} \qquad i, j > 0, i \neq j \tag{38}$$

$$B_{ii}^{(k)} = \Gamma_{ii}^{-1} + \Gamma_{ii}^{-1}\Gamma_{i0}B_{00}^{(k)}\Gamma_{0i}\Gamma_{ii}^{-1} \qquad i > 0 \tag{39}$$

5.2 The High Rate Decorrelator

For the high rate decorrelator, we can view each group 0 user j as transmitting the same bit b_j in each subinterval i using the signature $\hat{s}_j^{(i)}(t)$ which we recall is the segment of its actual signature $S_{j,0}(t)$ that occurs in subinterval i. Each group 1 user k transmits bit $b_{k,1}^{(i)}$ in subinterval i using the signature $S_{k,1}(t) = \hat{s}_{K_0+k}^{(i)}(t)$. Alternatively, we can view the group 0 users as transmitting a repetition of the vector $\tilde{b}^{(0)}$ while the group 1 users are transmitting $\tilde{b}^{i}$. The correlation matrix $\Gamma^{(i)}$ for the ith subinterval reflects the interaction of group 0 and group 1 users and can be written as

$$\Gamma^{(i)} = \begin{pmatrix} \Gamma_{00}^{(i)} & \Gamma_{0i} \\ \Gamma_{i0} & \Gamma_{ii} \end{pmatrix} \tag{40}$$

Note that Γ_{0i}, Γ_{i0} and Γ_{ii} are the same as in equations (33) and (34) for the low rate cross correlation matrix Γ. However, $\Gamma_{00}^{(i)} \neq \Gamma_{00}$ since $\Gamma_{00}^{(i)}$ represents the cross correlations of the group 0 users just in just the ith subinterval while Γ_{00} describes the cross correlations of the group 0 users over $[0, T_0]$. For $\Gamma_{00}^{(i)}$, element m, n is

$$\left[\Gamma_{00}^{(i)}\right]_{m,n} = \int_0^{T_1} S_{m,0}^{(i)}(t-(i-1)T_1) S_{n,0}^{(i)}(t-(i-1)T_1)\,dt \tag{41}$$

We will repeatedly use the property that an invertible 2×2 block matrix has an inverse of the form

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} (A - BD^{-1}C)^{-1} & -(A - BD^{-1}C)^{-1}BD^{-1} \\ -(D - CA^{-1}B)^{-1}CA^{-1} & (D - CA^{-1}B)^{-1} \end{pmatrix} \tag{42}$$

Thus, by defining $L^{(i)}$ and $H^{(i)}$ as

$$L^{(i)} = \left(\Gamma_{00}^{(i)} - \Gamma_{0i}\Gamma_{ii}^{-1}\Gamma_{i0}\right)^{-1} \qquad H^{(i)} = \left(\Gamma_{ii} - \Gamma_{i0}\left[\Gamma_{00}^{(i)}\right]^{-1}\Gamma_{0i}\right)^{-1} \tag{43}$$

the inverse of the correlation matrix Γ_i can be written as

$$\left[\Gamma^{(i)}\right]^{-1} = \begin{pmatrix} L^{(i)} & -L^{(i)}\Gamma_{0i}\Gamma_{ii}^{-1} \\ -H^{(i)}\Gamma_{i0}\left[\Gamma_{00}^{(i)}\right]^{-1} & H^{(i)} \end{pmatrix} \tag{44}$$

Using equation (39) and (43), we will establish the following results:

Theorem 1 *For the group 0 user, the asymptotic efficiency of the LRD is greater than or equal to that of the HRD. Equality holds when the cross-correlation matrix of the HRD is the same in every subinterval i.*

Proof: For the LRD, the asymptotic efficiency of the jth group 0 user is

$$[\eta_{j,0}]^L = \frac{1}{[\Gamma^{-1}]_{j,j}} = \frac{1}{[B_{00}]_{j,j}} \tag{45}$$

In [3], it has been shown that $\frac{1}{[B_{00}]_{j,j}}$ can be expressed as a constrained minimization of the form

$$[\eta_{j,0}]^L = \min_{\substack{\vartheta \in R^{K_0} \\ \vartheta_j = 1}} \vartheta^\top [B_{00}]^{-1} \vartheta \tag{46}$$

Since $\Gamma_{00} = \sum_{i=1}^M \Gamma_{00}^{(i)}$, equation (36) permits us to write

$$B_{00}^{-1} = \sum_{i=1}^{M} \left(\Gamma_{00}^{(i)} - \Gamma_{0i} \Gamma_{ii}^{-1} \Gamma_{i0} \right) = \sum_{i=1}^{M} \left[L^{(i)} \right]^{-1} \tag{47}$$

Suppose ϑ_o minimizes the objective function of equation (46), then equation (46) can be written as

$$\begin{aligned} [\eta_{j,0}]^L &= \sum_{i=1}^{M} \left(\vartheta_o^\top \left[L^{(i)} \right]^{-1} \vartheta_o \right) & (48) \\ &\geq \sum_{i=1}^{M} \min_{\substack{\nu^{(i)} \in R^{K_0} \\ \nu_j^{(i)} = 1}} \left(\left[\nu^{(i)} \right]^\top \left[L^{(i)} \right]^{-1} \nu^{(i)} \right) & (49) \\ &= \sum_{i=1}^{M} \frac{1}{[L^{(i)}]_{j,j}} & (50) \end{aligned}$$

From equation (44), we see that for $1 \leq j \leq K_0$, $\left[L^{(i)} \right]_{jj} = \left[\Gamma^{(i)} \right]_{jj}^{-1}$. Thus, by equation (28),

$$[\eta_{j,0}]^L \geq \sum_{i=1}^{M} \frac{1}{[\Gamma^{(i)}]_{j,j}^{-1}} = [\eta_{j,0}]^H \tag{51}$$

When the cross-correlation matrix of the HRD is the same for all i, then $L^{(i)} = L$, for all i, and the inequality (49) becomes an equality and $[\eta_{j,0}]^L = [\eta_{j,0}]^H$. □

Theorem 2 *For the group 1 user, the asymptotic efficiency of the LRD is strictly greater than that of the HRD.*

Proof: Without loss of generality, we can consider just the Mth subinterval. Similar to equation (45), the asymptotic efficiency of the LRD for decoding the Mth transmitted bit of the nth group 1 customer is

$$\left[\eta_{k_1[n],1}^{(M_1[n])}\right]^L = \frac{1}{\left[B_{MM}^{(M)}\right]_{n,n}} = \min_{\substack{\vartheta \in R^{K_1} \\ \vartheta_n = 1}} \vartheta^\top \left[B_{MM}^{(M)}\right]^{-1} \vartheta \tag{52}$$

From equation (33), we can write the LRD cross correlation matrix $\Gamma = \Gamma_M$ in the 2×2 block form

$$\Gamma_M = \begin{pmatrix} \Gamma_{M-1} & \bar{\Gamma} \\ \bar{\Gamma}^\top & \Gamma_{MM} \end{pmatrix} \tag{53}$$

where $\bar{\Gamma}^\top = [\Gamma_{M0}, 0, \ldots, 0]$. By equations (35) and (42),

$$B_{MM}^{(M)} = \left(\Gamma_{MM} - \bar{\Gamma}^\top \Gamma_{M-1}^{-1} \bar{\Gamma}\right)^{-1} \tag{54}$$

Equations (36) and (35) imply

$$\bar{\Gamma}^\top \Gamma_{M-1}^{-1} \bar{\Gamma} = \Gamma_{M0} B_{00}^{(M-1)} \Gamma_{0M} = \Gamma_{M0} \left(\Gamma_{00} - \sum_{j=1}^{M-1} \Gamma_{0j} \Gamma_{jj}^{-1} \Gamma_{j0}\right)^{-1} \Gamma_{0M} \tag{55}$$

Since $\Gamma_{00} = \sum_{j=1}^{M} \Gamma_{00}^{(j)}$, combining equations (54) and (55) yields

$$\left[B_{MM}^{(M)}\right]^{-1} = \Gamma_{MM} - \Gamma_{M0} \left(\Gamma_{00}^{(M)} + \sum_{j=1}^{M-1} \left(\Gamma_{00}^{(j)} - \Gamma_{0j} \Gamma_{jj}^{-1} \Gamma_{j0}\right)\right)^{-1} \Gamma_{0M} \tag{56}$$

By defining

$$\Upsilon = \sum_{j=1}^{M-1} \left(\Gamma_{00}^{(j)} - \Gamma_{0j} \Gamma_{jj}^{-1} \Gamma_{j0}\right) \tag{57}$$

$$A = \left(\Gamma_{00}^{(M)} \Upsilon^{-1} \Gamma_{00}^{(M)} + \Gamma_{00}^{(M)}\right)^{-1} \tag{58}$$

some algebra will verify that

$$\left[B_{MM}^{(M)}\right]^{-1} = \Gamma_{MM} - \Gamma_{M0}\left(\left[\Gamma_{00}^{(M)}\right]^{-1} - A\right)\Gamma_{0M} \tag{59}$$

Using ϑ_o to denote the vector which minimizes the objective function of equation (52), equation (59) implies

$$\left[\eta_{k_1[n],1}^{(M_1[n])}\right]^L = \vartheta_o^\top\left(\Gamma_{MM} - \Gamma_{M0}\left[\Gamma_{00}^{(M)}\right]^{-1}\Gamma_{0M}\right)\vartheta_o + \vartheta_o^\top\Gamma_{M0}A\Gamma_{0M}\vartheta_o \tag{60}$$

Since A is a positive definite matrix, $\vartheta_o^\top\Gamma_{M0}A\Gamma_{0M}\vartheta_o > 0$, implying

$$\left[\eta_{k_1[n],1}^{(M_1[n])}\right]^L > \vartheta_o^\top\left(\Gamma_{MM} - \Gamma_{M0}\left[\Gamma_{00}^{(M)}\right]^{-1}\Gamma_{0M}\right)\vartheta_o = \vartheta_o^\top\left[H^{(M)}\right]^{-1}\vartheta_o \tag{61}$$

Thus,

$$\left[\eta_{k_1[n],1}^{(M_1[n])}\right]^L > \min_{\substack{\nu\in R^{K_1}\\ \nu_n=1}} \nu^\top\left[H^{(M)}\right]^{-1}\nu = \left[\eta_{n,1}^{(M)}\right]^H \tag{62}$$

□

6 Numerical Analysis

In sections (3) and (4), it has been observed that in comparison to the HRD, the LRD has higher computational complexity which is a increasing function of M. On the other hand, without quantifying the difference in asymptotic efficiencies of the dual rate decorrelators, section (5) established that the LRD outperforms the HRD.

In this section, the performance of the HRD will be evaluated numerically and compared to that of the LRD. In the first part of this section, we shall see that for a small CDMA system where the signature sequences have good correlation properties, then the HRD will perform as well as the LRD. In the second part, a general study is performed on the performance of both decorrelators as a function of the number of users.

Let us consider a CDMA system which supports five users where $K_0 = 3$, and $K_1 = 2$. A group 1 user transmits $M = 8$ bits while a group 0 user transmits one bit. The processing gain for the group 1 user is $L_1 = 31$, while the processing gain for the group 0 user is $L_0 = 8 \times 31 = 248$. Users 1, 2 and 3 are group 0 users, while users 4

Five Maximum-length Shift Register Sequences of Length 31	
s_1	+++++−−−++−+++−+−+−−−−+−−+−++−−
s_2	+++++−+−−−+−−+−+−++−−−−+++−−++−
s_3	+++++−++−−+++−−−−++−+−+−−+−−−+−
s_4	+++++−−++−+−−+−−−−+−+−+++−++−−−
s_5	+++++−−+−−++−−−−+−++−+−+−−−+++−

Signature Sequences of the users for the interval $[0, T_0]$		
User Number	Group	Cyclic Signature Assignment
1	0	$[s_1, s_2, s_3, s_1, s_2, s_3, s_1, s_2]$
2	0	$[s_2, s_3, s_1, s_2, s_3, s_1, s_2, s_3]$
3	0	$[s_3, s_1, s_2, s_3, s_1, s_2, s_3, s_1]$
4	1	$[s_4, s_4, s_4, s_4, s_4, s_4, s_4, s_4]$
5	1	$[s_5, s_5, s_5, s_5, s_5, s_5, s_5, s_5]$

Figure 5: Signature sequences for the synchronous DS/CDMA system with five users. Note that + and − denote +1 and −1 respectively. The signature waveform of the low rate is normalized for the interval $[0, T_0]$ while the signature waveform of the high rate is normalized for the interval $[(i-1)T_1, iT_1]$.

and 5 are group 1. The SNR of the kth user of the system will be $10\log(\frac{E_k}{N_0/2})$.

We use $\log_2(L_1+1)$ or 5 shift registers to generate the signature sequences for both group of users. In each subinterval i, the signature sequences of the group 1 users are the maximum-length shift register sequences [23, 24] of length 31. It is assumed that each of the two group 1 users will have fixed signature sequences over the time interval, $[0, T_0]$. We generate the group 0 signature sequences of length 248 by using using a maximum-length shift register sequences of length 31 in each subinterval i; see Figure 5. Under the LRD as well as HRD system, the BER of a user is not only a function of it's own received energy but also the cross-correlation of the signature sequences; see equations (14), (24) and (27). Therefore, in order to provide balanced BER's among all the group 0 users, we assign three signature sequences cyclically to the group 0 users over the time interval $[0, T_0]$. This cyclic assignment technique provides equal BERs to the group 1 users at every subinterval. In addition, the above signature sequence assignment strategies guarantee the linear independence of the transmitted signature waveforms at each subinterval and as a result, the HRD as well as the LRD will exist.

In Figure 6, the BER of group 0 user 1 is plotted as a function of his SNR. In this

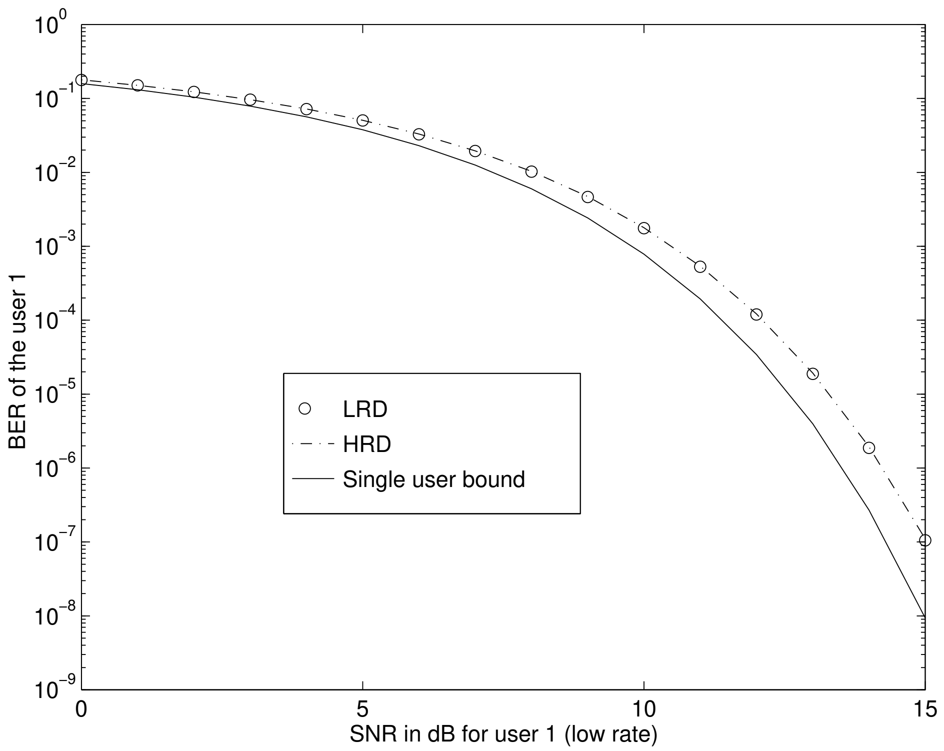

Figure 6: Five user system: BER of the 1st group 0 user when $K_0 = 3$, $K_1 = 2$, $L_0 = 248$ and $L_1 = 31$.

figure, it is observed that the HRD performs as well as LRD. In Figure 7, the low rate and high rate decorrelators are compared to a conventional detector. The BER of the group 0 user 1 is plotted as a function of the increasing SNR of the other users. Under the conventional receiver, the BER of the desired user drastically deteriorates as the interfering users increase their transmitted power, while the BER of the desired user is fixed for the decorrelators. As expected, both low rate and high rate decorrelators eliminated the near/far effect of the conventional detector. Figure 8 shows that for the group 1 users, the low rate decorrelator outperforms the high rate decorrelator although the improvement is insignificant. The same result is also evident in Figure 9 where the performance of the decorrelators is compared to that of the conventional receiver. Since equations (14), (24) and (27) verify that the BER of each user is independent of the interfering users' energies, the energies of the interfering users are not mentioned in Figures 6 and 8.

To build the low rate decorrelator, it is necessary that $K_0 + MK_1 \leq L_0$, since otherwise it will not be possible to find $K_0 + MK_1$ independent code words for the users of the equivalent single rate system. Thus for the low rate decorrelator, removing

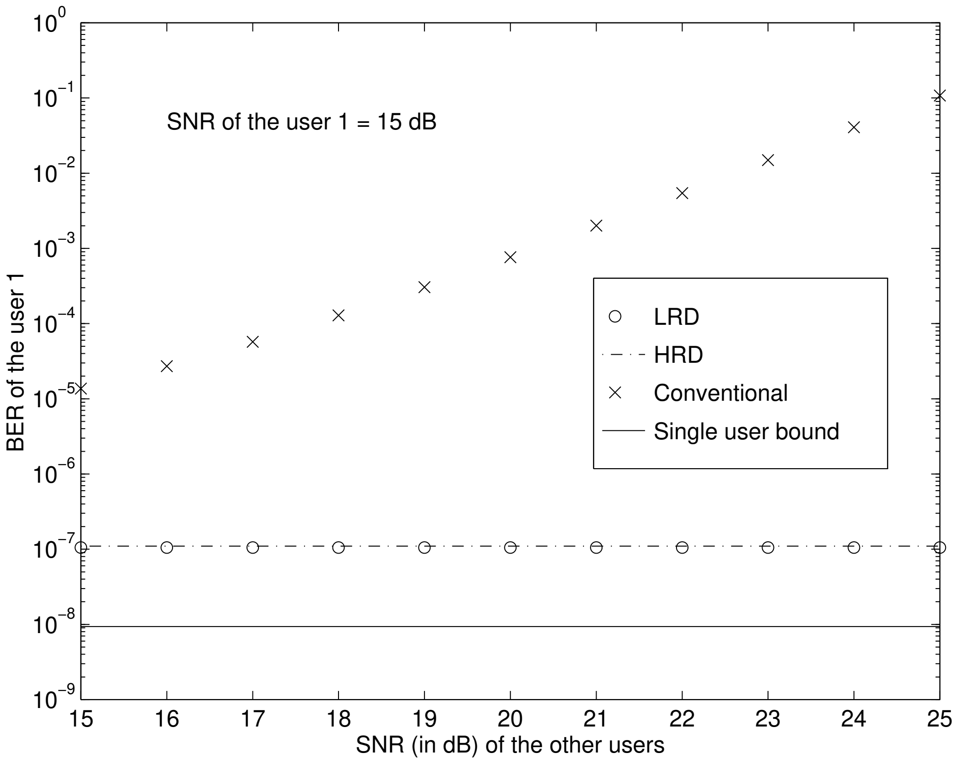

Figure 7: Five user system: BER of the 1st group 0 user when $K_0 = 3$, $K_1 = 2$, $L_0 = 248$ and $L_1 = 31$.

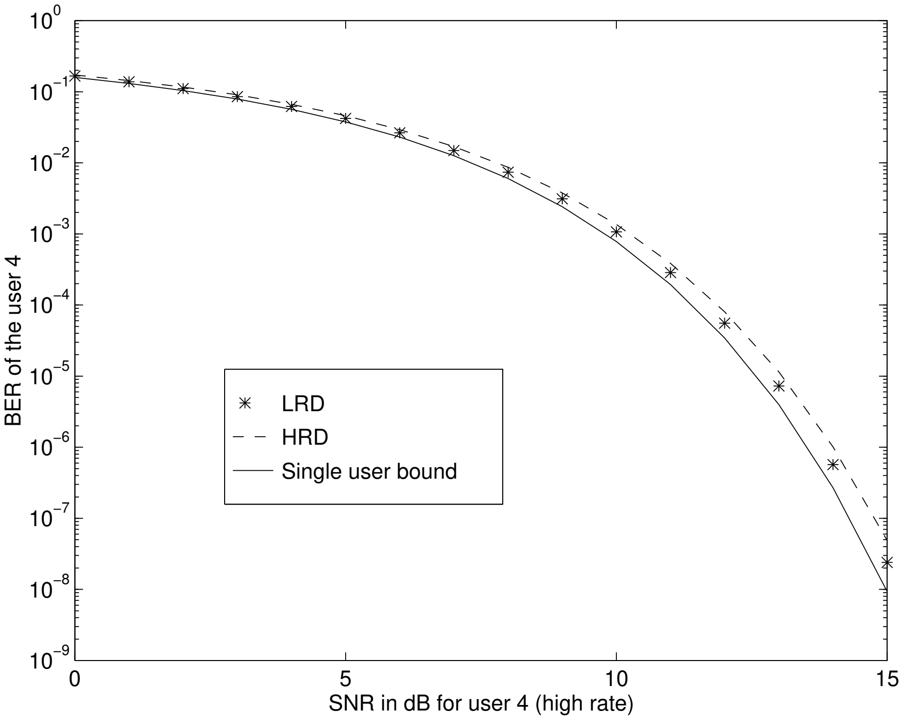

Figure 8: Five user system: BER of the 1st group 1 user when $K_0 = 3$, $K_1 = 2$, $L_0 = 248$ and $L_1 = 31$.

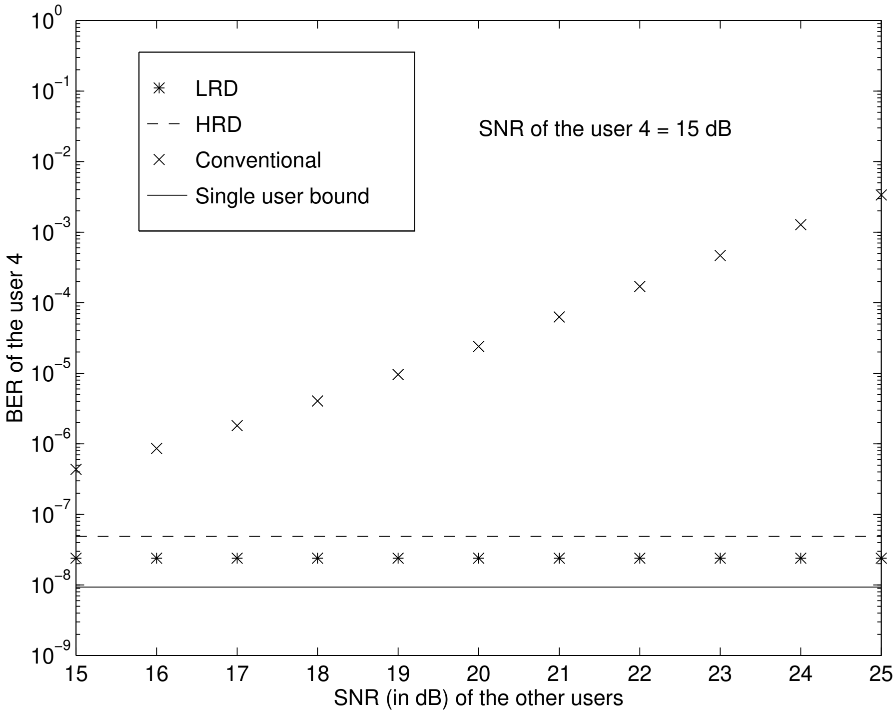

Figure 9: Five user system: BER of the 1st group 1 user when $K_0 = 3$, $K_1 = 2$, $L_0 = 248$ and $L_1 = 31$.

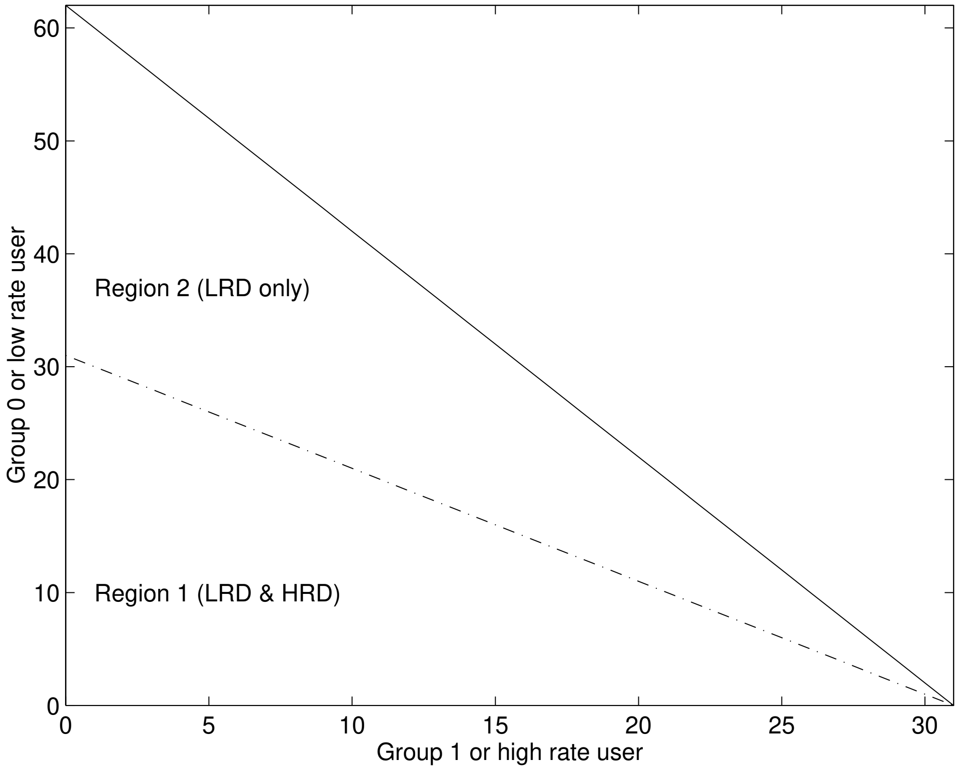

Figure 10: The figure describes the feasible region of the dual rate decorrelators for $L_1 = 31$ and $L_0 = 62$.

a group 1 user will permit us to add M group 0 users. On the other hand, for the high rate decorrelator, we must have $K_0 + K_1 \leq L_1$. In this case, removing one group 1 user permits us to carry only one additional group 0 user. Hence, the low rate decorrelator is more bandwidth efficient for the group 0 users. Alternatively, the high rate decorrelator wastes bandwidth to carry low rate users.

The bandwidth efficiency of the dual rate decorrelators also influences their performance. Figure 10 sketches the feasible regions of both dual rate decorrelators for $L_1 = 31$ and $L_0 = 62$. In this figure, it is observed that the LRD can exist in both regions 1 and 2. while the HRD exists in region 1 only. THIS IS NOT ENOUGH TO MAKE THE NEXT STATEMENT This also implies that the LRD always exists if the HRD exists, however, the converse is not true. Moreover, increasing the number of users will cause both dual rate decorelators to approach infeasibility. However, the HRD becomes infeasibile sooner. Therefore, for the fixed K_0 and K_1, we would expect that the performance of the HRD will not be as good as that of the LRD. However, the performance of the LRD will be more penalized with the increase of K_1 than the increase of K_0, where as, the performance of the HRD will be equally sensitive to the increase of K_0 and K_1.

7 Conclusion

We have found that a dual rate system with K_0 low rate users and K_1 high rate users can be modeled as an equivalent system of $K_0 + MK_1$ low rate users. However, the corresponding low rate decorrelator for this system introduces an M bit processing delay for the high rate users and has a complexity that grows with M. The high rate decorrelator eliminates the M bit processing delay of the high rate users and is significantly less complex than the low rate decorrelator. On the other hand, the low rate decorrelator is more bandwidth efficient for the low rate user which results in lower bit error rate or higher asymptotic efficiency. Hence, it will be desirable to identify multi-rate detectors that are bandwidth efficient as the low rate decorrelator while maintaining complexity comparable to the high rate decorrelator.

This work has used the restrictive model of an AWGN synchronous channel. It would be desirable to extend this work to consider both asynchronous systems as well as frequency selective Rayleigh fading multi-path channels. For the single rate system with multi-path channels, two receiver structures have been developed in [25].

These approaches can also be used for the both dual rate decorrelators. In the first approach, a bank of correlators matched to delayed versions of the users' signature waveforms which is followed by a linear transformation followed by optimal combiner for each user. The linear transformation will decorrelate the multi-paths of each user. In the second approach, a bank of RAKE receivers is followed by a linear transformation. To implement dual rate decorrelators for the asynchronous channel, the interfering users in the system can be modeled as follows. The LRD system will generate $2(K_0 - 1) + (M + 1)K_1$ interfering users for a low rate user while a high rate user will observe $2K_0 + (M + 1)(K_1 - 1)$ interfering users. Whereas, in the HRD system, each effective high rate user will experience interference from $2(K_0 + K_1 - 1)$ effective high rate users. All these extensions are areas for future investigation.

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