A Decision Feedback Decorrelator for a Dual Rate Synchronous DS/CDMA System

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ABSTRACT

A dual rate synchronous DS/CDMA system provides service to low bit rate and high bit rate users. In a fixed duration interval, a low rate user transmits one bit while a high rate user transmits M bits. The differences in the bit transmission rates result in different processing gains for each class of user. In this paper, we propose a decision feedback decorrelator for the dual rate synchronous DS/CDMA system which uses modified correlators and initiates the bit decision process at the end of the high rate bit interval. Each step of the decision process is executed by utilizing the decisions of all previously decoded users. This dual rate decision feedback receiver (evaluated by simulation) is found to outperform two types of decorrelators for dual rate CDMA systems. It is also observed that as the interferers grow stronger relative to the desired user, the performance of the decision feedback receiver for decoding the desired user approaches the single user bound.

1 Introduction

Beginning with the *optimum multiuser receiver* of Verdú [1], multiuser detection for CDMA systems has received a great deal of attention in the past decade. In particular, the computational complexity of the optimum multiuser receiver prompted the development of high performance suboptimal receivers [2–10]. All of these proposed multiuser detectors were designed for CDMA systems in which all users transmit at the same bit rate. Among such receivers, the decorrelating detector [3] is perhaps the simplest in structure and is reasonably easy to implement. The decorrelator eliminates the multiuser interference at the cost of increased noise variance. Furthermore, the decorrelator does not require the knowledge of the received signal strengths. A somewhat more complex receiver is the decorrelating decision-feedback detector [2]. The decision feedback receiver achieves better performance by utilizing bit decisions of the stronger users to reduce multiuser interference for the weaker users. An adaptive fractionally spaced decision feedback equalizer for a CDMA system in an indoor wireless Rayleigh fading environment is presented in [11].

Significant efforts are being made to integrate the cellular network with fixed networks for communicating both voice and data messages [12–14]. When all users have the same modulation scheme, two access strategies have been proposed for multi-rate DS/CDMA [15]. These are

1. Fixed chip rate, variable processing gain,
2. Fixed processing gain, variable chip rate.

A comparative study of the above access strategies was performed in [16]. However, in [15], it is found that the access strategy 2 is complicated because the receiver must be synchronized to its particular code rate and the system needs additional frequency planning due to the unequal bandwidth spreading of different users.

Several studies regarding receiver designs for multi-rate CDMA systems have been performed. Multistage multiuser receivers for multi-rate CDMA communications were introduced in [17]. For minimum mean squared error (MMSE) performance criteria, [18] considered multi-user communication with multiple symbol rates. In [19], a successive interference cancelation scheme for multi-rate CDMA was studied by employing complex modulation techniques, such as M-ary PAM.

In [20, 21], it is shown that the dual rate system is equivalent to a single rate system in which each high rate user is modeled as M independent low rate users. This equivalent representation permits the specification of a receiver called the *low rate decorrelator* (*LRD*) for the dual rate CDMA system that is simply the decorrelator of [3] applied to the equivalent single rate system. The complexity of the low rate decorrelator prompts consideration of a *high rate decorrelator* (*HRD*), in which during the bit interval of a high rate user, each low rate user is modeled as a high rate user. It is found that the high rate decorrelator achieves a sizable reduction in complexity while suffering a penalty in terms of bit error probability. Both low rate and high rate decorrelators preserve the standard decorrelator's near/far resistance properties.

Motivated by the decision feedback detector of [2], this paper develops a decision feedback decorrelating detector for the dual rate synchronous CDMA system. This proposed receiver is referred to as the *high rate decision feedback decorrelator* or *HRDF*. This paper will study the performance of the HRDF and compare that with the performance of the dual rate decorrelators proposed in [20, 21].

2 The Dual Rate Synchronous System

In the dual rate CDMA system model, each bit results in the baseband transmission of a sequence of pulses, or chips, $p[t]$, each pulse having a duration of one chip period T_c. These pulses are sent over an additive white Gaussian noise channel in which the noise $n(t)$ has power spectral density $N_0/2$.

Each user group will be classified by its bit rate. The transmission rate of users of group g is denoted by R_g, where $g = 0, 1$ and $R_1 = MR_0$ for an integer $M > 1$. For the K_g group g users, the duration of a bit is $T_g = 1/R_g$ and the processing gain is $L_g = T_g/T_c$. The signature waveform of the nth group g customer is

$$S_{n,g}(t) = \sum_{m=1}^{L_g} \{a_{n,g}(m) \frac{1}{\sqrt{L_g}} p[t - (m-1)T_c]\} \qquad t \in [0, T_g] \tag{1}$$

where $a_{n,g}(m) \in \{-1, 1\}$ denote the signature sequence of user n of group g. The energy of the pulse $p[t]$ is normalized so that

$$\int_0^{T_g} [S_{n,g}(t)]^2 dt = 1 \qquad g = 0, 1; n = 1, \ldots, K_g \tag{2}$$

In the interval $[0, T_0]$, each group 0 user transmits one bit while each group 1 user

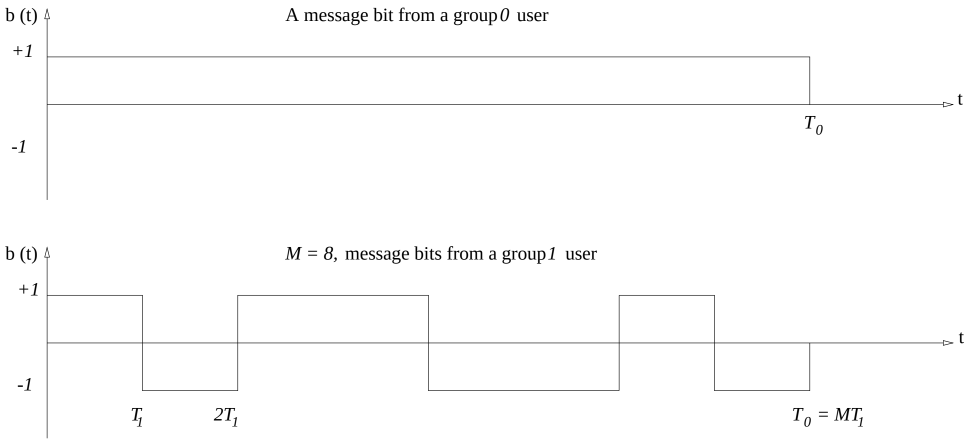

Figure 1: Messages from a user of each group in the time interval, $[0, T_0]$.

transmits M bits. The jth user from group 0 transmits bit $b_{j,0} \in \{-1, 1\}$ with received energy $E_{j,0}$ in the interval $[0, T_0]$. Similarly, the kth user from group 1 transmits its ith bit $b_{k,1}^{(i)} \in \{-1, 1\}$ with received energy $E_{k,1}^{(i)}$ in the ith subinterval $[(i-1)T_1, iT_1]$ using the signature waveform

$$S_{k,1}^{(i)}(t) = S_{k,1}[t - (i-1)T_1] \tag{3}$$

Over the interval $[0, T_0]$, the received baseband signal can be written as

$$r(t) = \sum_{j=1}^{K_0} \sqrt{E_{j,0}} b_{j,0} S_{j,0}(t) + \sum_{k=1}^{K_1} \left\{ \sum_{i=1}^{M} \sqrt{E_{k,1}^{(i)}} b_{k,1}^{(i)} S_{k,1}^{(i)}(t) \right\} + n(t) \tag{4}$$

The $K = K_0 + MK_1$ bits transmitted in the interval $[0, T_0]$ can be written as the K bit vector $b = [b_1, b_2, \ldots, b_n, \ldots, b_K]^\top$ where

$$b = [\overbrace{\underbrace{b_{1,0}, \ldots, b_{K_0,0}}_{\text{in } [0,T_0]}}^{\text{group 0 users}}, \overbrace{\underbrace{b_{1,1}^{(1)}, \ldots, b_{K_1,1}^{(1)}}_{\text{in } [0,T_1]}, \cdots \quad \ldots, \quad \underbrace{b_{1,1}^{(M)}, \ldots, b_{K_1,1}^{(M)}}_{\text{in } [(M-1)T_1, MT_1]}}^{\text{group 1 users}}]^\top \tag{5}$$

Note that for $n \leq K_0$, the bit b_n was transmitted by user n of group 0. Furthermore, examination of the above ordering will show we can define functions $k_1[n]$ and $i_1[n]$ such that given $n > K_0$, we can identify that a group 1 user $k = k_1[n]$ transmitted bit b_n during subinterval $i = i_1[n]$. For the vector $b = [b_1, b_2, \ldots, b_n, \ldots, b_K]^\top$, the bit b_n is transmitted with energy E_n using the signature waveform $s_n(t)$ where

$$s_n(t) = \begin{cases} S_{n,0}(t) & 1 \leq n \leq K_0 \\ S_{k_1[n],1}^{(i_1[n])}(t) & n > K_0 \end{cases} \tag{6}$$

$$E_n = \begin{cases} \sqrt{E_{n,0}} & 1 \leq n \leq K_0 \\ \sqrt{E_{k_1[n],1}^{(i_1[n])}} & n > K_0 \end{cases} \tag{7}$$

In terms of the vector b, the received signal $r(t)$ from equation (4) can be written as

$$r(t) = \sum_{n=1}^{K} b_n \sqrt{E_n} s_n(t) + n(t) = S(t, b) + n(t) \tag{8}$$

It has been observed [3] that finding the maximum-likelihood estimate of b from equation (8) is an NP-hard problem. Thus, the use of sub-optimum receivers for the dual rate CDMA system is explored. In section 3, we summarize the dual rate decorrelators of [20, 21]. In section 4, we develop the high rate decision feedback decorrelator (HRDF) for the dual rate synchronous DS/CDMA system.

3 Dual Rate Detectors

Equation (8) is a representation of the received signal in the dual rate system that is equivalent to a single rate system with $K = K_0 + MK_1$ group 0 users each transmitting 1 bit during $[0, T_0]$. Now we use this representation to describe the low rate decorrelator (LRD). The sampled output of a bank of correlators for the signature waveforms $s_n(t)$ is the vector $r = [r_1, r_2, \ldots, r_K]^\top$ of sufficient statistics with

$$r_n = \int_0^{T_0} r(t) s_n(t) dt \qquad n = 1, \ldots, K \tag{9}$$

As a vector, r can be written as

$$r = \Gamma \Lambda b + N \tag{10}$$

where Λ is a diagonal matrix with $\Lambda_{n,n} = \sqrt{E_n}$ and Γ is the $K \times K$ cross-correlation matrix with (j,k)th element

$$[\Gamma]_{j,k} = \int_0^{T_0} s_j(t)s_k(t)dt \tag{11}$$

The decorrelator applies the transformation Γ^{-1} to r, yielding the decision rule

$$\bar{b} = \text{sgn}[\Gamma^{-1}r] = \text{sgn}[\Lambda b + \bar{N}] \tag{12}$$

where $\bar{N}$ is a zero mean Gaussian vector with the covariance matrix $(N_0/2)\Gamma^{-1}$. Since $\bar{N}_j$ has variance $(N_0/2)[\Gamma^{-1}]_{j,j}$, the probability that the jth input is decoded incorrectly will be

$$P_j = Q\left[\sqrt{\frac{E_j}{(N_0/2)[\Gamma^{-1}]_{j,j}}}\right] \tag{13}$$

Note that Γ is a square matrix of dimension $K_0 + MK_1$. Thus, the complexity of the low rate decorrelator grows with M. Second, for each high rate group 1 user, the M sampled correlator outputs in $[0, T_0]$ are stored and decoded simultaneously at time T_0, incurring an M bit processing delay.

Now we describe the high rate decorrelator (HRD) which decodes the group 1 users of group 1 at every $T_1 = T_0/M$ units of time. During the ith subinterval, each group 0 user j transmits the ith segment of the signature waveform $S_{j,0}(t)$ denoted as

$$S_{j,0}^{(i)}(t) = \sum_{m=(i-1)L_1+1}^{iL_1} \left\{ a_{j,0}(m)\frac{1}{\sqrt{L_0}}p[t-(m-1)T_c] \right\} \tag{14}$$

On the other hand, each group 1 user k transmits one bit using the signature $S_{k,1}^{(i)}(t)$ given by equation (3). We order the users such that the jth group 0 user is numbered j while the kth group 1 user is numbered (K_0+k). For subinterval i, the input vector is $\hat{b}^{(i)} = [\hat{b}_1, \dots, \hat{b}_{K_0}, \hat{b}_{(K_0+1)}, \dots, \hat{b}_{K_0+K_1}]^\top$. The corresponding signature sequences can be denoted as

$$\hat{s}_n^{(i)}(t) = \begin{cases} S_{n,0}^{(i)}(t) & 1 \le n \le K_0 \\ S_{n-K_0,1}^{(i)}(t) & K_0 < n \le K_0 + K_1 \end{cases} \tag{15}$$

The sampled output of a bank of correlators for the signature waveforms $\hat{s}_n^{(i)}(t)$ is the vector $r^{(i)}$, where

$$r^{(i)} = \Gamma^{(i)}\Lambda^{(i)}b^{(i)} + N^{(i)} \tag{16}$$

where $\Gamma^{(i)}$ is the $(K_0 + K_1) \times (K_0 + K_1)$ cross-correlation matrix, where

$$[\Gamma^{(i)}]_{n,m} = \int_0^{T_1} \hat{s}_n^{(i)}(t)\hat{s}_m^{(i)}(t)dt \tag{17}$$

and $\Lambda^{(i)}$ is a diagonal matrix with

$$\Lambda_{n,n}^{(i)} = \begin{cases} \sqrt{E_{n,0}} & 1 \le n \le K_0 \\ \sqrt{E_{n-K_0,1}^{(i)}} & K_0 < n \le K_0 + K_1 \end{cases} \tag{18}$$

and $N^{(i)}$ is a $K_0 + K_1$ element Gaussian noise vector with covariance matrix $\Gamma^{(i)}$. In order to recover the vector $b^{(i)}$, the vector $r^{(i)}$ is decorrelated using the cross-correlation matrix $\Gamma^{(i)}$. Specifically,

$$\bar{r}^{(i)} = [\Gamma^{(i)}]^{-1}r^{(i)} = \Lambda^{(i)}b^{(i)} + \bar{N}^{(i)} \tag{19}$$

where $\bar{N}^{(i)}$ is a zero mean Gaussian noise vector with covariance matrix $(N_0/2)[\Gamma^{(i)}]^{-1}$. Using equation (19), the decoding rule in subinterval i for a group 1 user k is

$$\bar{b}_k^{(i)} = \text{sgn}[\bar{r}_k^{(i)}] = \text{sgn}[\Lambda_{k,k}^{(i)}b_k^{(i)} + \bar{N}_k^{(i)}] \qquad K_0 < k \le K_0 + K_1 \tag{20}$$

The probability that a group 1 user k is decoded incorrectly in the ith subinterval is the probability that bit $b_{k+K_0}^{(i)}$ is received in error which equals

$$\hat{P}_{k,1}^{(i)} = Q\left[\sqrt{\frac{E_{k,1}^{(i)}}{(N_0/2)[\Gamma^{(i)}]_{k+K_0,k+K_0}^{-1}}}\right] \tag{21}$$

To decode a group 0 user j, *maximal ratio combining* [22,23] is used to implement the following *soft decoding* rule:

$$\bar{b}_j = \text{sgn}\left[\sum_{i=1}^{M}\left(\frac{1}{[\Gamma^{(i)}]_{j,j}^{-1}}\right)\bar{r}_j^{(i)}\right] \qquad 1 \le j \le K_0 \tag{22}$$

Under the soft decoding rule, the probability that the jth low rate user is decoded incorrectly in the time interval $[0, T_0]$ is

$$\hat{P}_{j,0} = Q\left[\sqrt{\frac{E_{j,0}}{(N_0/2)}\sum_{i=1}^{M}\left(\frac{1}{[\Gamma^{(i)}]^{-1}_{j,j}}\right)}\right] \qquad 1 \le j \le K_0 \tag{23}$$

In comparison to LRD, HRD is less complex and eliminates the bit processing delay for the group 1 user. Both low rate and high rate decorrelators suppress the multiuser interference by enhancing the noise variance, see equations (13), (21) and (23). However, in [21], it is shown that $\eta_{LRD} \ge \eta_{HRD}$, where η_{LRD} and η_{HRD} are the *asymptotic efficiencies* [3, 24, 25] of the LRD and HRD respectively. Generalization to this result for multi-rate decorrelators is observed in [26].

4 High Rate Decision Feedback Decorrelator, HRDF

The main goal of this paper is to develop the structure of the high rate decision feedback decorrelator, or HRDF, which retains the desirable properties of the HRD. The HRDF exploits the fact that a group 0 user transmits the same bit during each subinterval i in which a group 1 user transmits one bit. The working principle of our proposed receiver is as follows:

1. At the end of each subinterval $i < M$:
   (a) Decorrelate the partial signature sequences of the group 0 users over $[0, iT_1]$.
   (b) For $i > 1$, subtract away group 1 users for the interval $[0, (i-1)T_1]$ based on bit decisions in each of the $(i-1)$ subintervals.
   (c) Using decision feedback, make bit decisions for the group 1 users in the subinterval $[(i-1)T_1, iT_1]$.
2. At the end of subinterval M:
   (a) Repeat step (1b) and use decision feedback to decode the group 0 user bits in $[0, MT_1]$ together with the group 1 user bits in $[(M-1)T_1, MT_1]$.

4.1 Structure of HRDF

Recall that during the ith subinterval, the jth group 0 user has transmitted using the signature segment $S_{j,0}^{(i)}(t)$ as given by equation (14). Furthermore, over the interval $[0, iT_1]$, the jth group 0 user has transmitted using the *partial signature*

$$\tilde{S}_{j,0}^{(i)}(t) = \sum_{m=1}^{i} S_{j,0}^{(m)}(t) \qquad j = 1, \ldots, K_0 \tag{24}$$

Using equation (24), the baseband signal which is received during the interval $[0, iT_1]$ can be written as

$$r^{(i)}(t) \quad = \quad \sum_{j=1}^{K_0} \sqrt{E_{j,0}} b_{j,0} \tilde{S}_{j,0}^{(i)}(t) + \sum_{k=1}^{K_1} \left\{ \sum_{m=1}^{i} \sqrt{E_{k,1}^{(m)}} b_{k,1}^{(m)} S_{k,1}^{(m)}(t) \right\} + n(t) \tag{25}$$

The group 0 users' partial signatures segments transmitted during the interval $[0, iT_1]$, where $i < M$, are not normalized and may not be linearly independent. Therefore, we generate the effective group 0 users from the actual group 0 users by applying the Gram-Schmidt procedure to the group 0 partial signatures of equation (24).

Let us assume that the Gram-Schmidt procedure on the partial signatures of the group 0 users in the interval $[0, iT_1]$, where $i < M$, yields $K_0^{(i)}$ basis functions, $\left\{ \Psi_{j,0}^{(i)}(t) : j = 1, \ldots, K_0^{(i)} \right\}$, where $\Psi_{j,0}^{(i)}(t)$ is normalized to have unit energy on the interval $[0, iT_1]$. Using these basis functions, we can write each low rate partial signature as a linear combination

$$\tilde{S}_{j,0}^{(i)}(t) = \sum_{j'=1}^{K_0^{(i)}} a_{jj'}^{(i)} \Psi_{j',0}^{(i)}(t)$$

Thus, the received signal during the interval $[0, iT_1]$, where $i < M$, can be written

$$r^{(i)}(t) \quad = \quad \sum_{j'=1}^{K_0^{(i)}} \sqrt{\epsilon_{j',0}^{(i)}} \beta_{j',0}^{(i)} \Psi_{j',0}^{(i)}(t) + \sum_{k=1}^{K_1} \left\{ \sum_{m=1}^{i} \sqrt{E_{k,1}^{(m)}} b_{k,1}^{(m)} S_{k,1}^{(m)}(t) \right\} + n(t) \tag{26}$$

where

$$\epsilon^{(i)}_{j',0} = \left(\sum_{j=1}^{K_0} a^{(i)}_{jj'} \sqrt{E_{j,0}} b_{j,0} \right)^2$$

$$\beta^{(i)}_{j',0} = \mathrm{sgn} \left(\sum_{j=1}^{K_0} a^{(i)}_{jj'} \sqrt{E_{j,0}} b_{j,0} \right)$$

Effectively, we have reduced K_0 group 0 users with possibly dependent partial signatures to $K_0^{(i)}$ effective group 0 users with orthogonal signatures. Each effective group 0 user transmits one bit $\beta^{(i)}_{j,0}$ with energy $\epsilon^{(i)}_{j,0}$ in the interval $[0, iT_1]$, where $i < M$. We observe that the energy $\epsilon^{(i)}_{j,0}$ of a group 0 effective user depends on the bits transmitted by the group 0 users and also on the projections of the partial signatures on the basis function $\Psi^{(i)}_{j,0}$. Thus, the energies of the group 0 effective users are not known.

Let us denote $k_1^{(i)}[n] = n - K_0^{(i)}$ if $n > K_0^{(i)}$, $i < M$ and redefine the $K^{(i)}$ effective users' signature waveforms, energies and transmitted bits of equation (26) as follows:

$$s_n^{(i)}(t) = \begin{cases} \Psi^{(i)}_{n,0}(t) & 1 \le n \le K_0^{(i)} \\ S^{(i)}_{k_1^{(i)}[n],1}(t) & K_0^{(i)} < n \le K^{(i)} \end{cases} \tag{27}$$

$$E_n^{(i)} = \begin{cases} \sqrt{\epsilon^{(i)}_{n,0}} & 1 \le n \le K_0^{(i)} \\ \sqrt{E^{(i)}_{k_1^{(i)}[n],1}} & K_0^{(i)} < n \le K^{(i)} \end{cases} \tag{28}$$

$$b_n^{(i)} = \begin{cases} \beta^{(i)}_{n,0} & 1 \le n \le K_0^{(i)} \\ b^{(i)}_{k_1^{(i)}[n],1} & K_0^{(i)} < n \le K^{(i)} \end{cases} \tag{29}$$

Using equations (27)-(29), equation (26) can be written as

$$r^{(i)}(t) = \sum_{n=1}^{K^{(i)}} \sqrt{E_n^{(i)}} b_n^{(i)} s_n^{(i)}(t) + \sum_{m=1}^{i-1} \left\{ \sum_{k=1}^{K_1} \sqrt{E^{(m)}_{k,1}} b^{(m)}_{k,1} S^{(m)}_{k,1}(t) \right\} + n(t) \qquad t \in [0, iT_1] \tag{30}$$

The proposed receiver of Figure 2 will decode each group 1 bit after the end of its transmission. We see in equation (30) that the received signal $r^{(i)}(t)$ over the interval $[0, iT_1]$ is the sum of the following three components, i) the baseband signals due to the $K^{(i)} = K_0^{(i)} + K_1$ effective users, ii) cumulative interference from group 1 bits

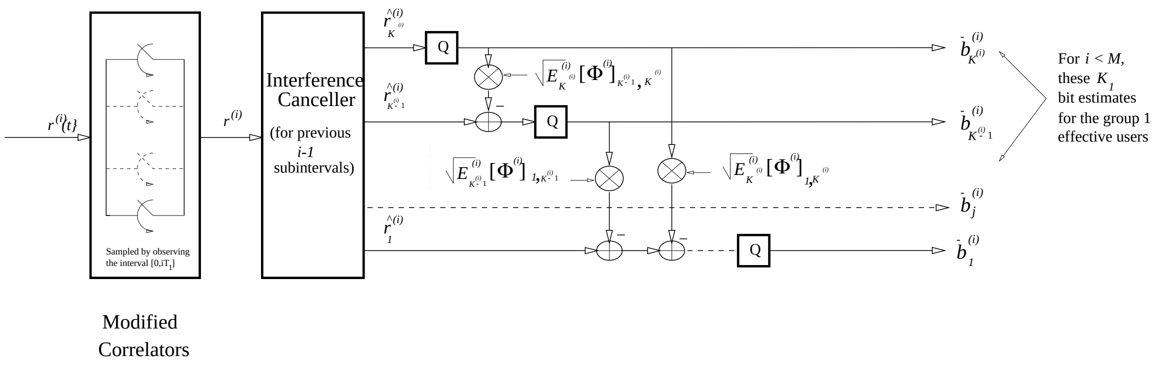

Figure 2: This diagram shows the operational principle of the high rate decision feedback decorrelating detector, HRDF. The "Interference Canceller" estimates $\Upsilon^{(i)}$ of equation (31) and subtract from the outputs of the modified correlators.

of the previous subintervals and iii) the additive Gaussian noise. Note that equation (30) is generated by dividing the effective users into groups according their processing gains. For a single rate DS/CDMA system, a group detector is presented in [27].

At the end of the subinterval M, the group 0 bits of the interval $[0, MT_1]$ and group 1 bits of the subinterval $[(M-1)T_1, MT_1]$ will be jointly decoded. At this time, this joint decoding will use the actual signature waveform, energy, transmitted bit of each group 0 user. In this case, the effective number of users $K^{(M)}$ equals the actual number of users $K_0 + K_1$. In this regard, it is assumed that during the interval $[0, MT_1]$, the $K^{(M)}$ signature waveforms of the effective users will be linearly independent.

Let us assume that the Gram-Schmidt procedure on the signature waveforms $\left\{s_n^{(i)}(t)\right\}$ given in equation (27) yields the set of basis functions $\left\{\Psi_j^{(i)}(t) : 1 \leq j \leq K^{(i)}\right\}$, where

$$\Psi_j^{(i)}(t) \quad = \quad \Psi_{j,0}^{(i)}(t) \qquad \text{for } 1 \leq j \leq K_0^{(i)}$$

i.e., for the effective group 0 users. Now let us modify the correlators so that the received signal will be correlated with $\left\{\Psi_j^{(i)}(t)\right\}$. However, in the subinterval $i < M$, as we want to decode the group 1 bits only, thus, we shall correlate the received signal with $\left\{\Psi_j^{(i)}(t); K_0^{(i)} < j \leq K^{(i)}\right\}$, whereas, for $i = M$, the group 0 users will be decoded together with the group 1 users and the complete set of basis functions $\left\{\Psi_j^{(i)}(t)\right\}$ will be used in the correlators. For $i < M$, the HRDF can be viewed as a partial decision feedback multiuser detector for the dual-rate synchronous DS/CDMA system. A partial decision feedback multiuser detector for the asynchronous single rate DS/CDMA system is proposed in [28].

We now examine the decision feedback receiver over subintervals $i < M$. The sampled output of the bank of modified correlators at subinterval $i < M$ is the vector $r^{(i)}$, where

$$r^{(i)} = \Phi^{(i)}\Lambda^{(i)}b^{(i)} + \Upsilon^{(i)} + n^{(i)} \tag{31}$$

where $\Phi^{(i)}$ is an upper triangular matrix of size $K_1 \times K_1$, the (j,n)th component of $\Phi^{(i)}$ is

$$\Phi^{(i)}_{j,n} = \begin{cases} 0 & j > n \\ \int_0^{iT_1} s^{(i)}_{j+K_0^{(i)}}(t)\Psi^{(i)}_{n+K_0^{(i)}}(t)\,dt & j \leq n \end{cases} \tag{32}$$

Note that, $\Phi^{(i)}$ is an the upper triangular matrix because of Gram-Schmidt orthogonalization on the signature waveforms $\left\{s^{(i)}_n(t)\right\}$ given in equation (27). The term $\Lambda^{(i)}$ is a $K_1 \times K_1$ diagonal matrix with $\Lambda^{(i)}_{n,n} = \sqrt{E^{(i)}_{n+K_0^{(i)}}}$ for $i < M$ and $n^{(i)}$ is a Gaussian noise vector of size K_1 with cross-correlation matrix $\frac{N_0}{2}I$, where I is an identity matrix of size $K_1 \times K_1$. The term $\Upsilon^{(i)}$, is a $K_1 \times 1$ column vector which is the residual multiuser interference resulting from the group 1 users of the previous $i-1$ subintervals. The nth component of $\Upsilon^{(i)}$ is

$$\Upsilon^{(i)}_n = \begin{cases} 0 & i = 1 \\ \sum_{m=1}^{i-1}\left[\sum_{j=1}^{K_1}\phi^{(mi)}_{j,n}\sqrt{E^{(m)}_{j,1}}\,b^{(m)}_{j,1}\right] & 1 \leq i \leq M-1 \end{cases} \tag{33}$$

where

$$\phi^{(mi)}_{j,n} = \int_0^{iT_1} S^{(m)}_{j,1}(t)\Psi^{(i)}_{n+K_0^{(i)}}(t)\,dt \tag{34}$$

In subinterval M, the complete set of basis functions $\left\{\Psi^{(M)}_j(t)\right\}$ will be used in the correlators and the dimension of the sampled output of bank of modified correlators of equation (31), $r^{(M)}$ will be $K^{(M)} \times 1$, where, the corresponding upper triangular matrix $\Phi^{(M)}$ and the diagonal matrix $\Lambda^{(M)}$ are $K^{(M)} \times K^{(M)}$. The residual multiuser interference vector $\Upsilon^{(M)}$ and the independent Gaussian noise vector $n^{(M)}$ are $K^{(M)}\times 1$.

In general, for any subinterval i $(1 \leq i \leq M)$, the kth element of the vector $r^{(i)}$ will be

$$r_k^{(i)} = [\Phi^{(i)}]_{k,k}\sqrt{E_k^{(i)}}b_k^{(i)} + \sum_{j=k+1}^{K^{(i)}} \left\{[\Phi^{(i)}]_{k,j}\sqrt{E_j^{(i)}}b_j^{(i)}\right\} + \Upsilon_k^{(i)} + n_k^{(i)} \tag{35}$$

Note that in equation (27), for $n > K_0^{(i)}$, $s_n^{(i)}(t)$ denotes a signature waveform that is nonzero only over the ith subinterval and has zero cross-correlation with $S_{k,1}^{(m)}(t)$ for $m < i$. However, as we have used modified correlator signals of duration $[0, iT_1]$, therefore, the transmitted signal of group 1 users in previous $i-1$ subintervals will have an effect on the group 1 bit decisions at the ith subinterval.

We have assumed that we know the actual bit energies $E_{j,0}$ and $E_{k,1}$ of each group 0 user j and group 1 user k. At the receiver end, the users will be ordered and decoded sequentially in a decreasing order. The bit decision process will exploit previous decisions to decode the current user. In subinterval i, given the group 1 bit decisions, $\bar{b}_{j,1}^{(m)}$ from the previous $(i-1)$ subintervals, if we have already made bit decisions $\bar{b}_{k+1}^{(i)}, \ldots, \bar{b}_{K^{(i)}}^{(i)}$ in the ith subinterval, then we can write the decision statistic for user k as

$$\bar{r}_k^{(i)} = r_k^{(i)} - \sum_{m=1}^{i-1}\left[\sum_{j=1}^{K_1}\phi_{j,k}^{(mi)}\sqrt{E_{j,1}^{(m)}}\bar{b}_{j,1}^{(m)}\right] - \sum_{j=k+1}^{K^{(i)}}[\Phi^{(i)}]_{k,j}\sqrt{E_j^{(i)}}\bar{b}_j^{(i)} \tag{36}$$

The corresponding decoding rule is

$$\bar{b}_k^{(i)} = \operatorname{sgn}[\bar{r}_k^{(i)}] \tag{37}$$

In [2,4,5], it is observed that the feedback in the receiver of the single rate CDMA system is primarily beneficial if the interfering users are stronger. Therefore, these receivers rank and decode the users according to nondecreasing strengths of received energies. However, this ranking is not necessary. In our receiver design, we rank the $K^{(i)}$ users as follows.

We define the *orthogonal energy*, or *OE*, of a user as the fraction of it's received energy associated with the subspace orthogonal to the interfering users. We write the orthogonal energy as

$$\mathrm{OE}_n^{(i)} \;=\; E_n^{(i)} / \left(\bar{\Gamma}^{(i)}\right)^{-1}_{n,n} \tag{38}$$

where $\Gamma^{(i)}$ denotes $K^{(i)} \times K^{(i)}$ cross-correlation matrix of equation (27). For the interval $[0, iT_1]$, where $i < M$, we order the K_1 group 1 users such that $\mathrm{OE}_n^{(i)}$ is increasing in n. However, for $i = M$, group 0 users will be decoded together with group 1 users. In particular, without classifying the user by it's rate, all $K^{(M)} = K_0 + K_1$ users will be ranked according to non decreasing OEs.

This heuristic ordering procedure is motivated by the known performance of the decorrelator. For the very last user, there is no feedback and a decision feedback receiver is equivalent to a decorrelator. In a bandwidth efficient CDMA system with many users, the signature waveforms may have high cross-correlations which could lead to erroneous decisions to the earlier ranked users. From equations (13), (21) and (23), it is seen that the bit error rate of the decorrelator is a decreasing function of OE. Therefore, we rank the users by their decreasing OEs rather than their energies in order to reduce the likelihood of early decision feedback errors which have an impact on the decoding of higher ordered users.

4.2 Alternative Implementations of HRDF

The implementation of the HRDF requires $K_0 + MK_1$ modified correlators and two interference cancellers. We can implement the HRDF by using $\sum_{i=1}^{M} K_0^{(i)} + K_1$ correlators matched to the effective signature waveforms rather than using $K_0 + MK_1$ modified correlators. However, this technique requires an additional linear transformation $([\Phi^{(i)}]^\top)^{-1}$ at the outputs of the correlators. Note that this alternative method is used in the decision feedback receiver of the single rate system [2]. In order to process the current bit decision, the first interference canceller cancels out the multiuser interference caused by the high rate bits of the previous subintervals and the second one subtracts the multiuser interference caused by the previously decoded users of the current subinterval. Every step of the bit decision process is executed by measuring the cross-correlation among the current modified correlator signal and the signature waveforms corresponding to the bits which are decoded previously. However, these operations of interference cancellers can be observed by adapting the following method applied in [4]. At the end of each bit decision, the corresponding baseband signal will be regenerated and subtracted from the received signal, and then, this new signal will be correlated with the correlator corresponding to the bit which will be decoded next.

4.3 Bandwidth Efficiency of HRDF

In [20,21], it has been observed that to build the low rate decorrelator, it is necessary that $K_0 + MK_1 \le L_0$, since otherwise it will not be possible to find $K_0 + MK_1$ independent code words for the users of the equivalent single rate system. Thus, for the low rate decorrelator, removing a group 1 user will permit adding M group 0 users. On the other hand, for the high rate decorrelator, we must have $K_0 + K_1 \le L_1$. In this case, removing one group 1 user permits us to carry only one additional group 0 user. Hence, the low rate decorrelator is more bandwidth efficient for the group 0 users. However, the HRDF system model allows us to support M group 0 users by using one group 1 signature sequence. In other words, using a group 1 signature sequences of length L_1, we can generate M independent group 0 signature sequences of length L_0. It is always possible to have a $M \times M$ matrix A, where $A_{j,i} \in \{1, -1\}$, with full rank M. Therefore, if the row vector $S_{k,1}^{\top}$ of size $1 \times L_1$ denotes the signature sequences of the kth group 1 user, then, there always exist M independent row vectors $\{S_{j,0}^{\top} : j = 1, \ldots, M\}$ of size $1 \times L_0$ such that

$$S_{j,0}^{\top} = \left[A_{j,1} S_{k,1}^{\top}, A_{j,2} S_{k,1}^{\top}, \ldots, A_{j,M} S_{k,1}^{\top}\right] \tag{39}$$

For example, if $M = 2$ and A has jth row $[1, -1]$, then using equation (39), we can write $S_{j,0}^{\top} = \left\{S_{k,1}^{\top}, -S_{k,1}^{\top}\right\}$. Note that the above signature sequence assignment strategy is similar to the sub-code concatenation scheme of [29] which generates additional orthogonal signature sequences for different bits of a high rate user.

5 Performance Measure

Now, the performance of the simulated HRDF system will be compared with the evaluated performance of LRD and HRD. For the purpose of comparisons, we will use a set of signature waveforms derived from Gold sequences of length seven [2,5]; see Figure 3. As the HRDF system model permit us to use group 1 signature sequences to support group 0 users, we will use the same set of signature sequences to support

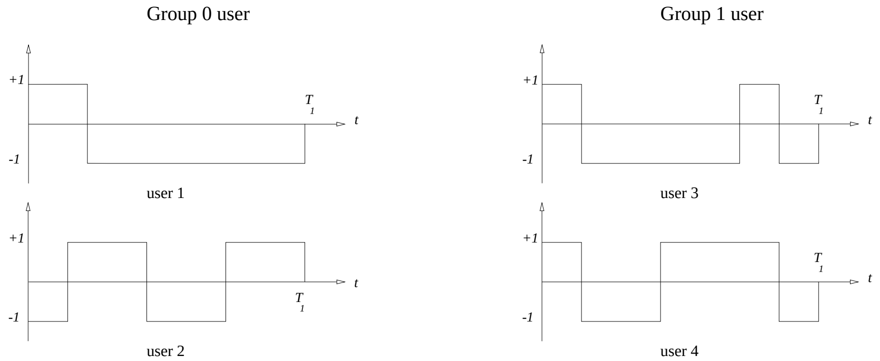

Figure 3: Gold signature sequences of length seven for the dual rate DS/CDMA system: These signature sequences are used to support the users in the simulation study.

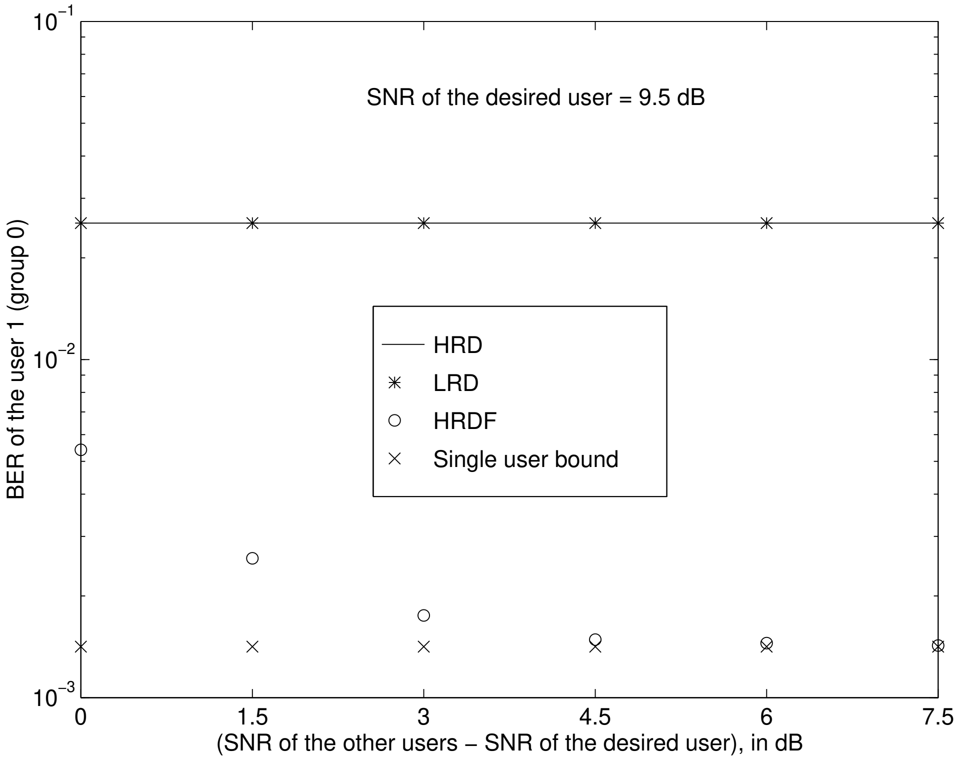

Figure 4: Four user system: BER of the user 1 (group 0) when $K_0 = 2$, $K_1 = 2$, $L_0 = 56$ and $L_1 = 7$.

the group 1 users as well group 0 users and assume four users are active. Among the four users, there are $K_0 = 2$ group 0 users, and $K_1 = 2$ group 1 users. Users 1 and 2 are group 0 users while users 3 and 4 are group 1 users. It is also assumed that the group 0 users have the same signature sequences in every subinterval of duration of T_1. A group 1 user transmits $M = 8$ bits while a group 0 user transmits one bit. The SNR of the kth user of the system will be $10\log(\frac{E_k}{N_0/2})$ and it is assumed that the received energies of group 1 users will not vary from bit to bit.

In Figure 4, the bit error rate (BER) of the group 0 user 1 is plotted as a function of the SNR of the other users, while the SNR of the desired group 0 user is fixed at 9.5 dB. Here, it is observed that the proposed receiver, HRDF significantly outperforms the LRD and HRD. As the other users become stronger relative to the desired user, the feedback data for decoding the desired user becomes more accurate. As a consequence, the BER of the user 1 approaches the single user bound; see Figure 4. Note that the high rate and low rate decorrelators yield the same bit error probability for the group 0 user. This was observed in [20, 21] to be a consequence of using the same cross-correlation matrix in every subinterval i. From Figure 4, we see that the HRDF performs better than either of the dual rate decorrelators of [20, 21] for all operating points we have considered. Until the Mth subinterval, the group 1 users are decoded by decorrelating the group 0 users which enhances the noise variance of the group 1 users. However, in this specific example (using the signature sequences shown in Figure 3), the enhancement of noise variance becomes smaller as i increases. In other words, in successive subintervals, the group 1 user experiences less multi-user interference from group 0 users and hence decreasing BER; see Figure 5 where the BER of the user 3 (a group 1 user) in different subintervals, i, is observed when the SNR of the other users is 12.5 dB and the SNR of the desired group 1 user is fixed at 9.5 dB. Here, it is also noted that the user 3 observes sharp improvement in the BER at the Mth subinterval compared to the $(M-1)$th subinterval. This result shows the gain from using decision feedback for the group 0 users over decorrelating the group 0 users at the Mth subinterval.

In order to quantify the relative performance of the HRDF with respect to the HRD and the LRD for decoding the group 1 user, let us define the *average BER* of the kth group 1 user as

$$\bar{P}_{k,1} = \frac{1}{M}\sum_{i=1}^{M} P_{k,1}^{(i)}$$

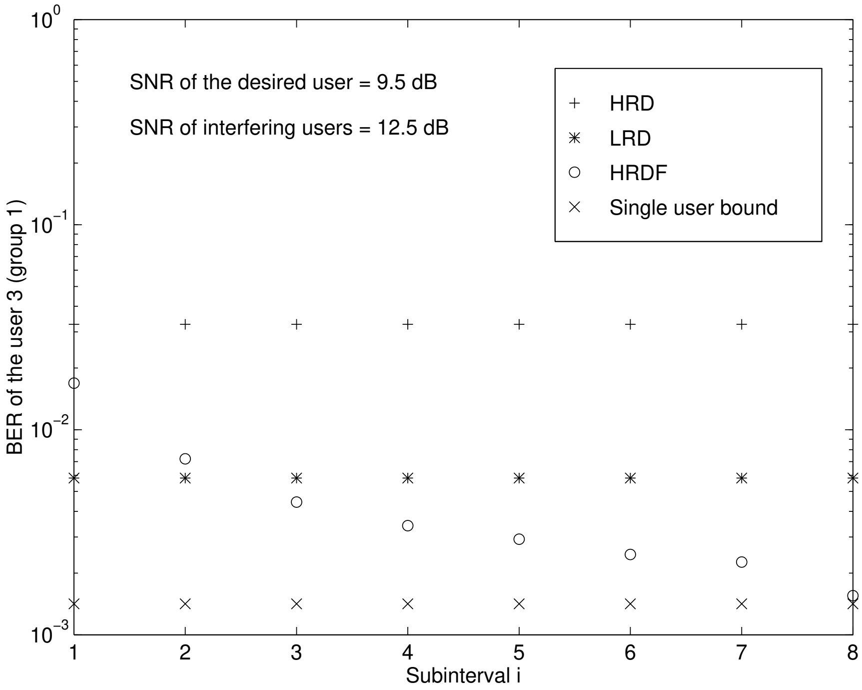

Figure 5: Four user system: BER of the user 3 (group 1) when $K_0 = 2$, $K_1 = 2$, $L_0 = 56$ and $L_1 = 7$.

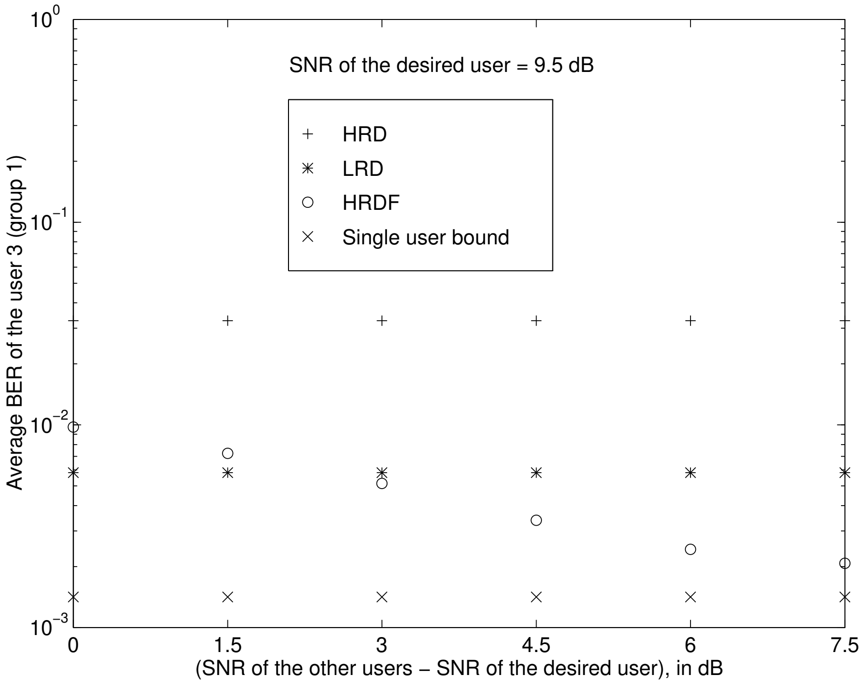

Figure 6: Four user system: Average BER of the user 3 (group 1) when $K_0 = 2$, $K_1 = 2$, $L_0 = 56$ and $L_1 = 7$.

where $P_{k,1}^{(i)}$ denotes the BER of the kth group 1 user at the ith subinterval under the HRDF system. Since, interference cancellation is done in the HRDF system by increasing the observation window from $i = 1$ to M, the above performance measure yields a composite performance metric for the entire window. In Figure 6, we have plotted the average BER of user 3 as a function of the SNR of the interfering users. Here, it is observed that as the interfering users become stronger, the performance of the HRDF approaches to the single user bound.

6 Conclusion

We have found that similar to the high rate decorrelator, HRD, our proposed high rate decision feedback decorrelating detector, HRDF, eliminates the bit processing delay of the low rate decorrelator, LRD and outperforms both the LRD and the HRD. It is observed that as the interfering users become stronger, the bit error rate of the weaker user approaches the single user bound. In addition, it is also observed that HRDF system is as bandwidth efficient as the LRD system.

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