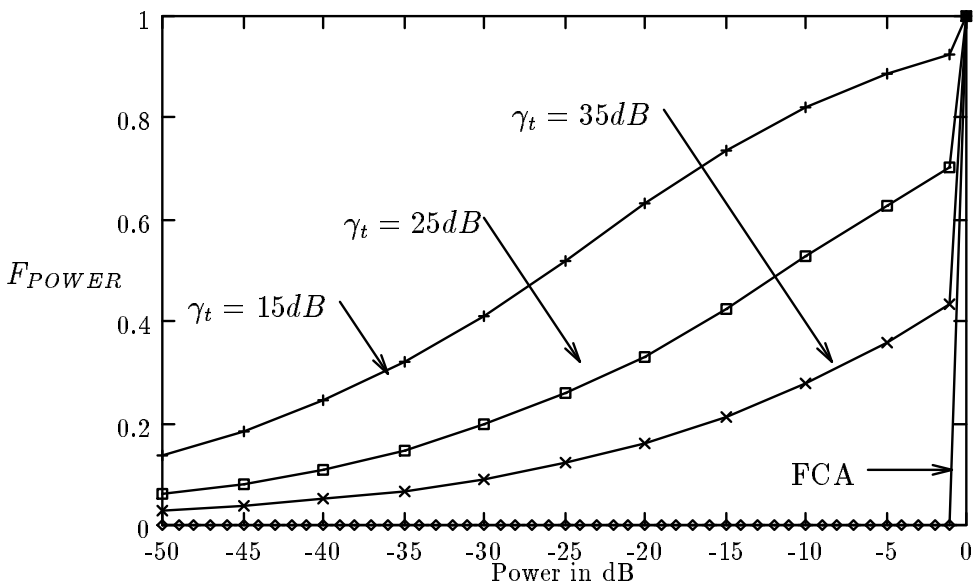

Figure 12: F_{POWER} for FCA and DCPC with γ_t =15,20,25,30,35,40 dB.

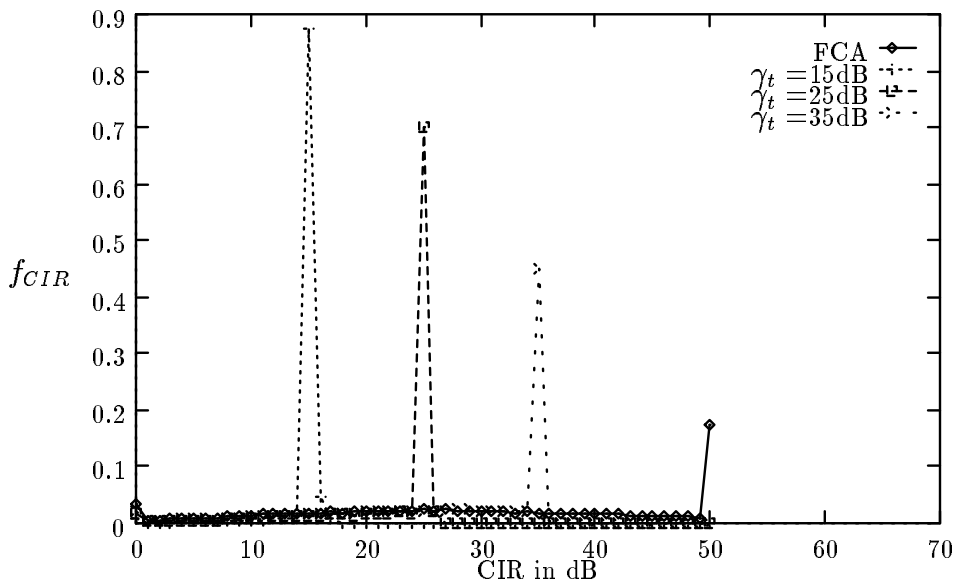

Figure 10: f_{CIR} for FCA and DCPC with γ_t =15,20,25,30,35,40 dB.

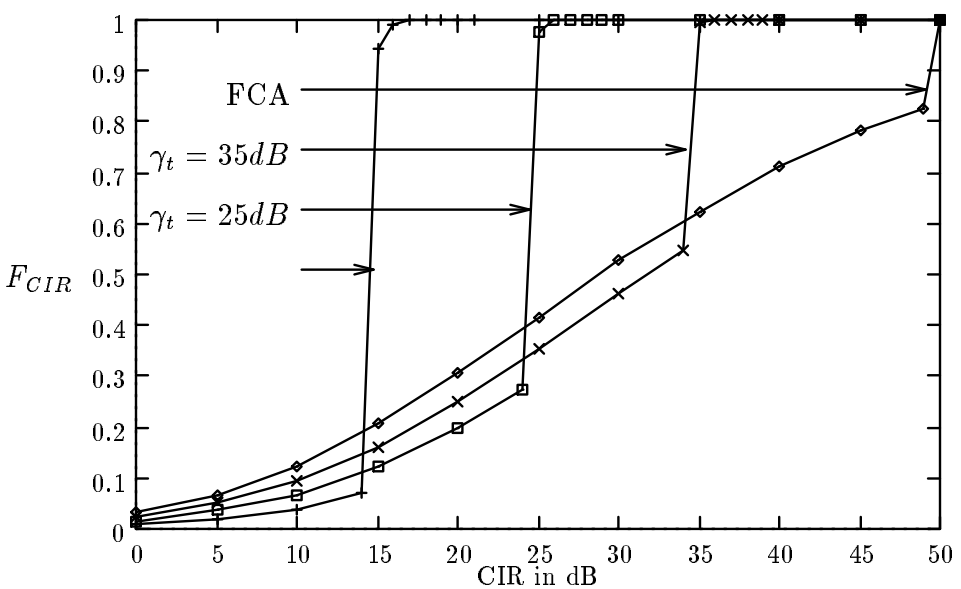

Figure 11: F_{CIR} for FCA and DCPC with γ_t =15,20,25,30,35,40 dB.

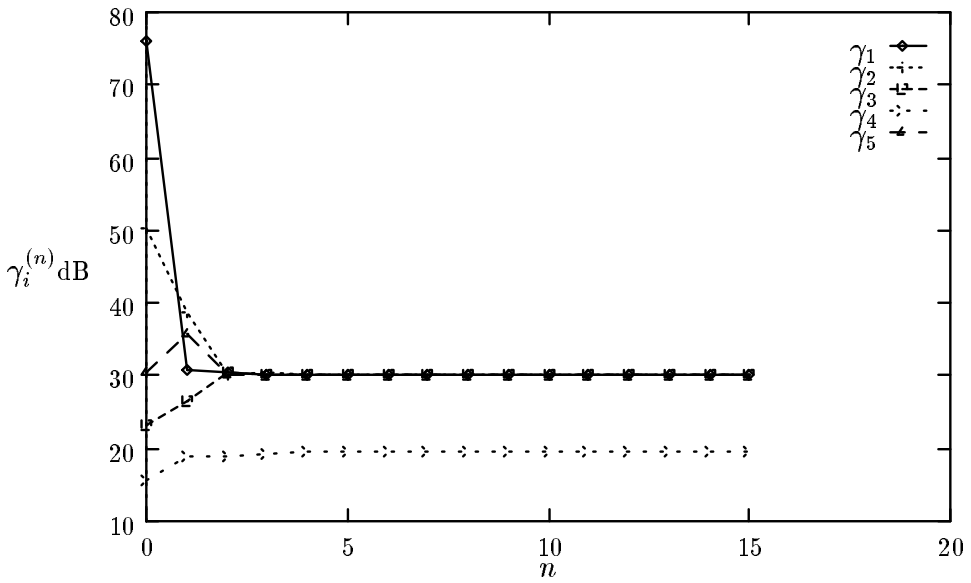

Figure 8: Convergence of CIR's in synchronous DCPC.

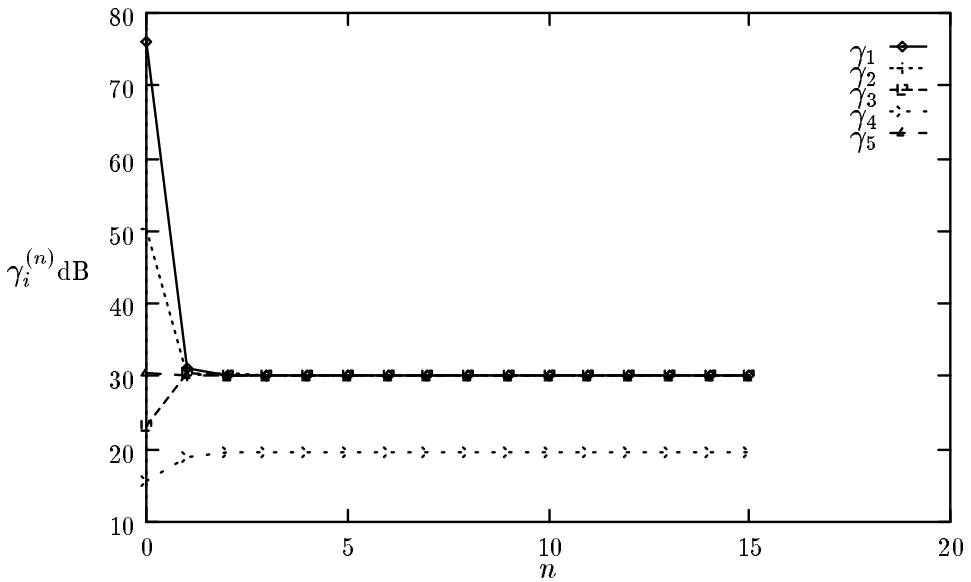

Figure 9: Convergence of CIR's in asynchronous DCPC.

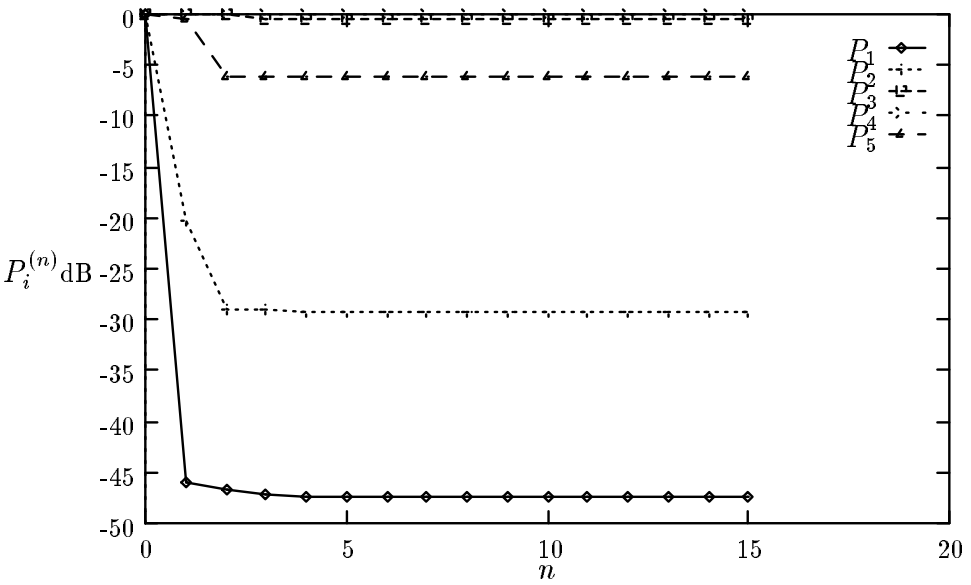

Figure 6: Convergence of Powers in synchronous DCPC.

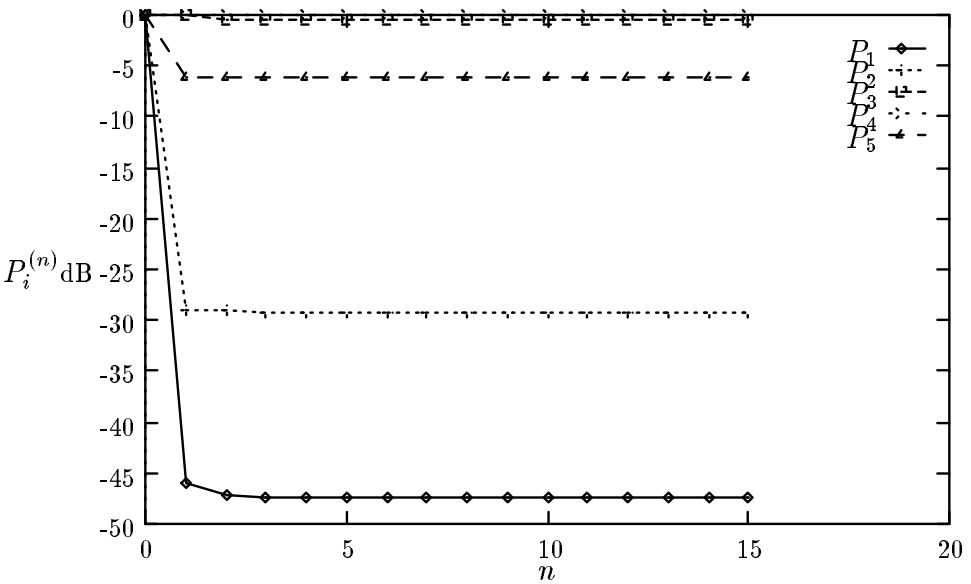

Figure 7: Convergence of Powers in asynchronous DCPC.

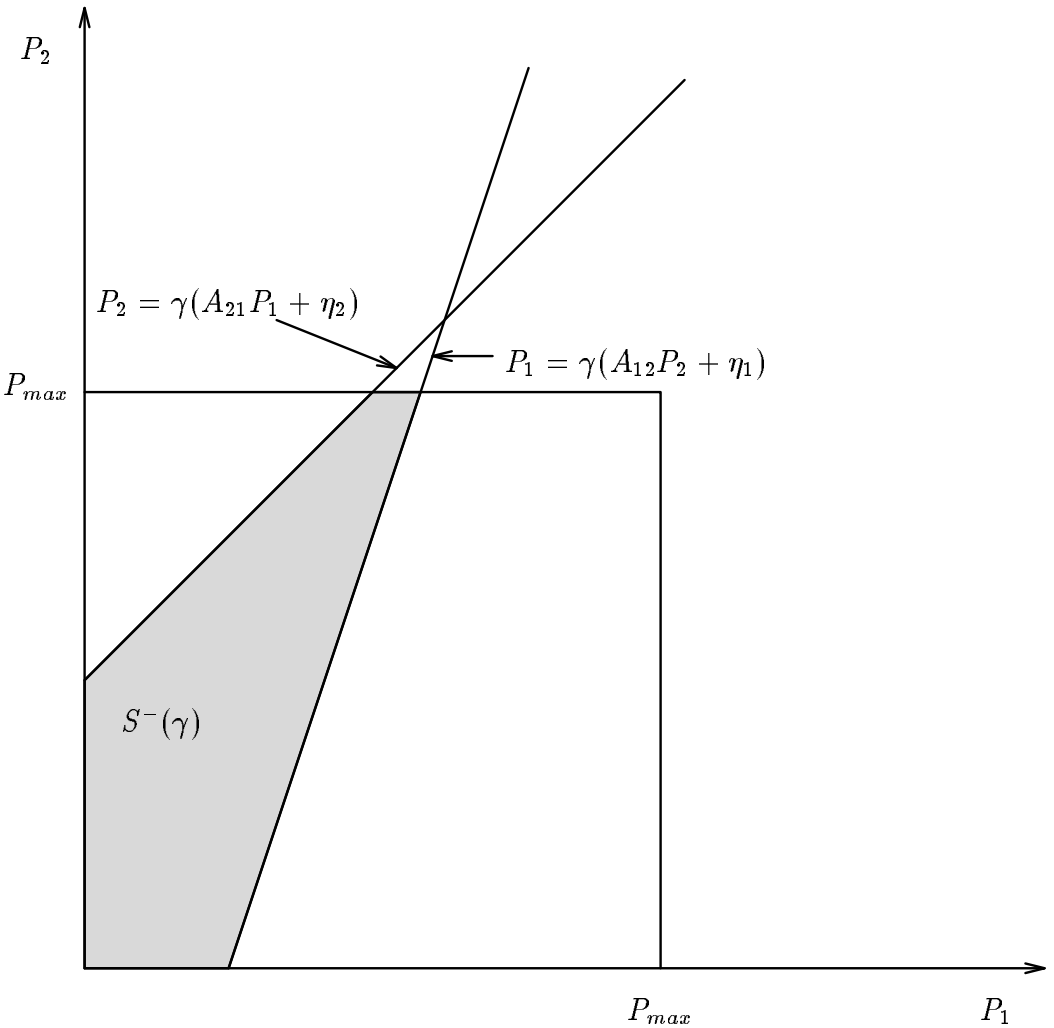

Figure 5: The set $S^-(\gamma)$ for $M = 2$.

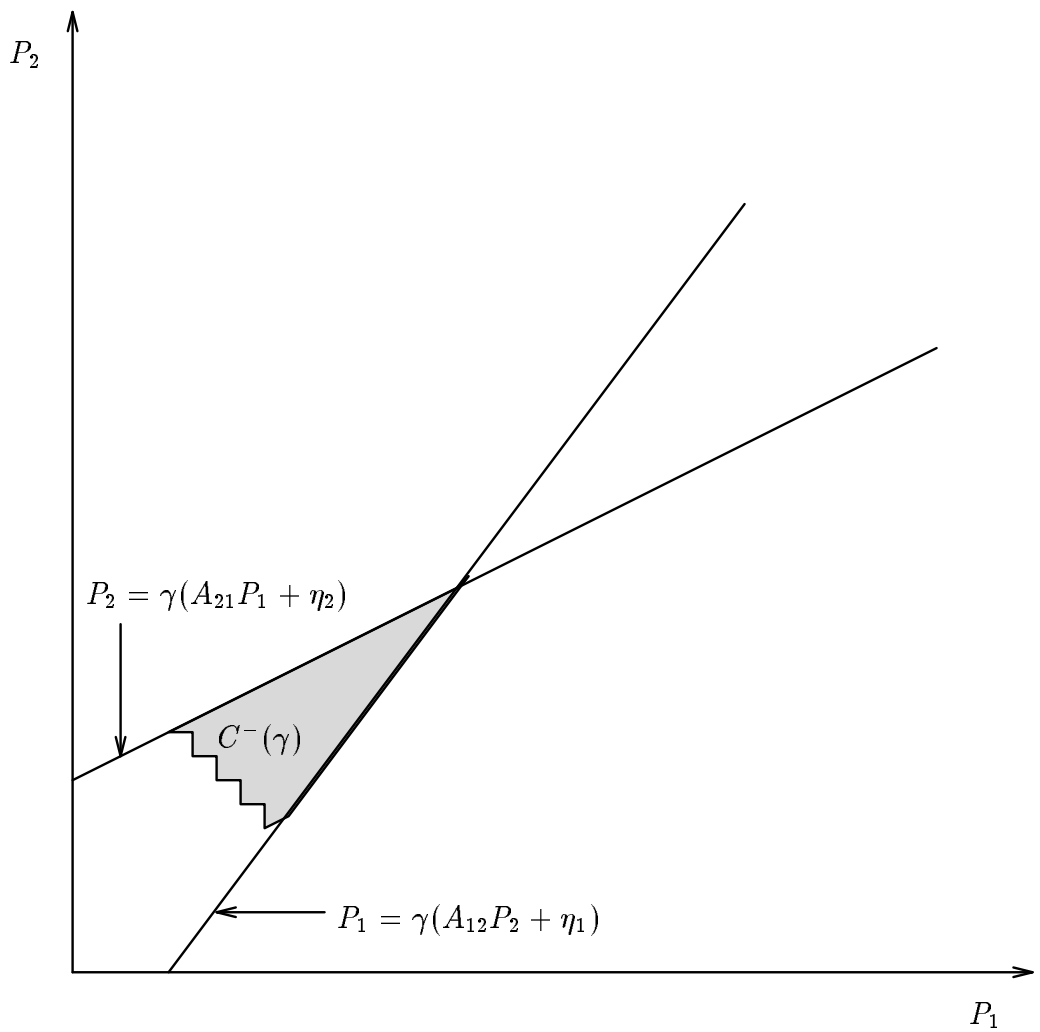

Figure 4: The polyhedral "cone" $C^-(\gamma)$ for $M = 2$.

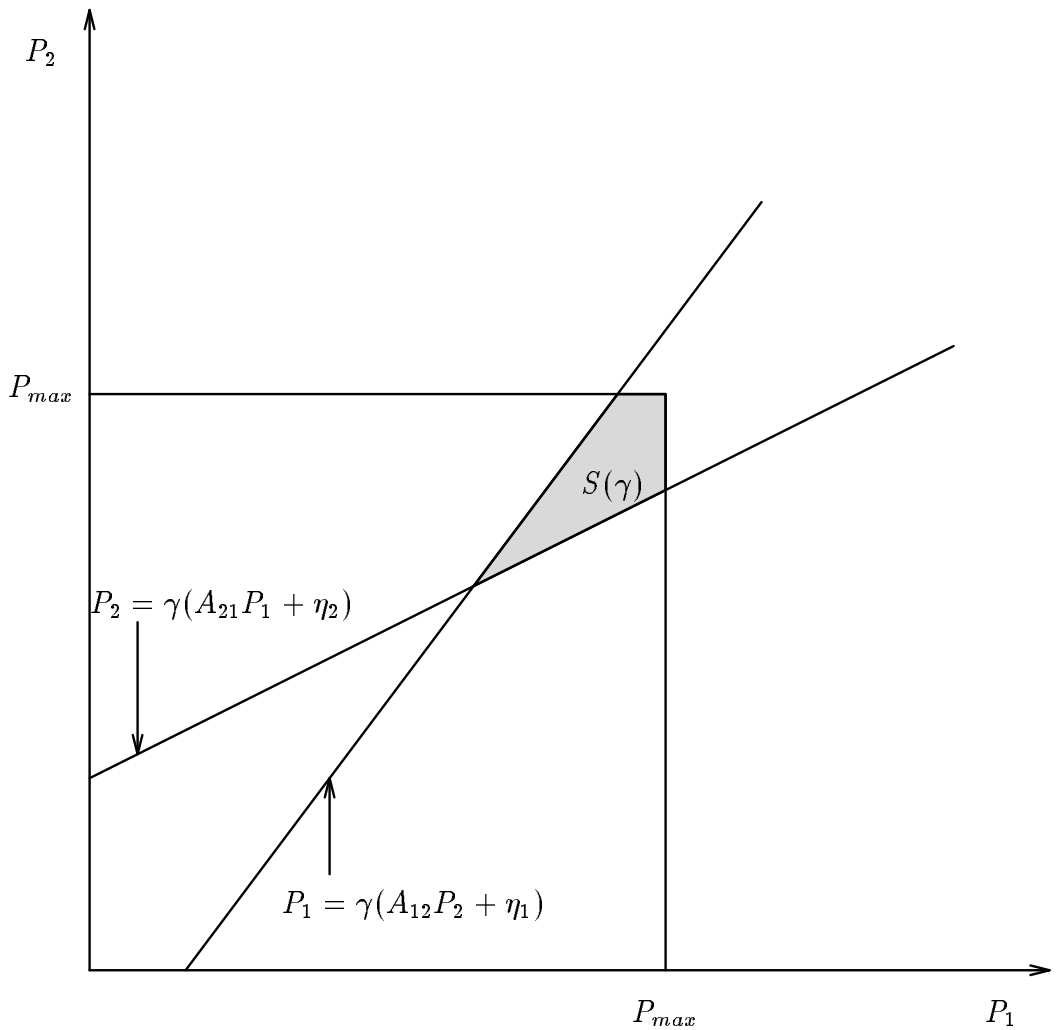

Figure 3: The set $S(\gamma)$ for $M = 2$.

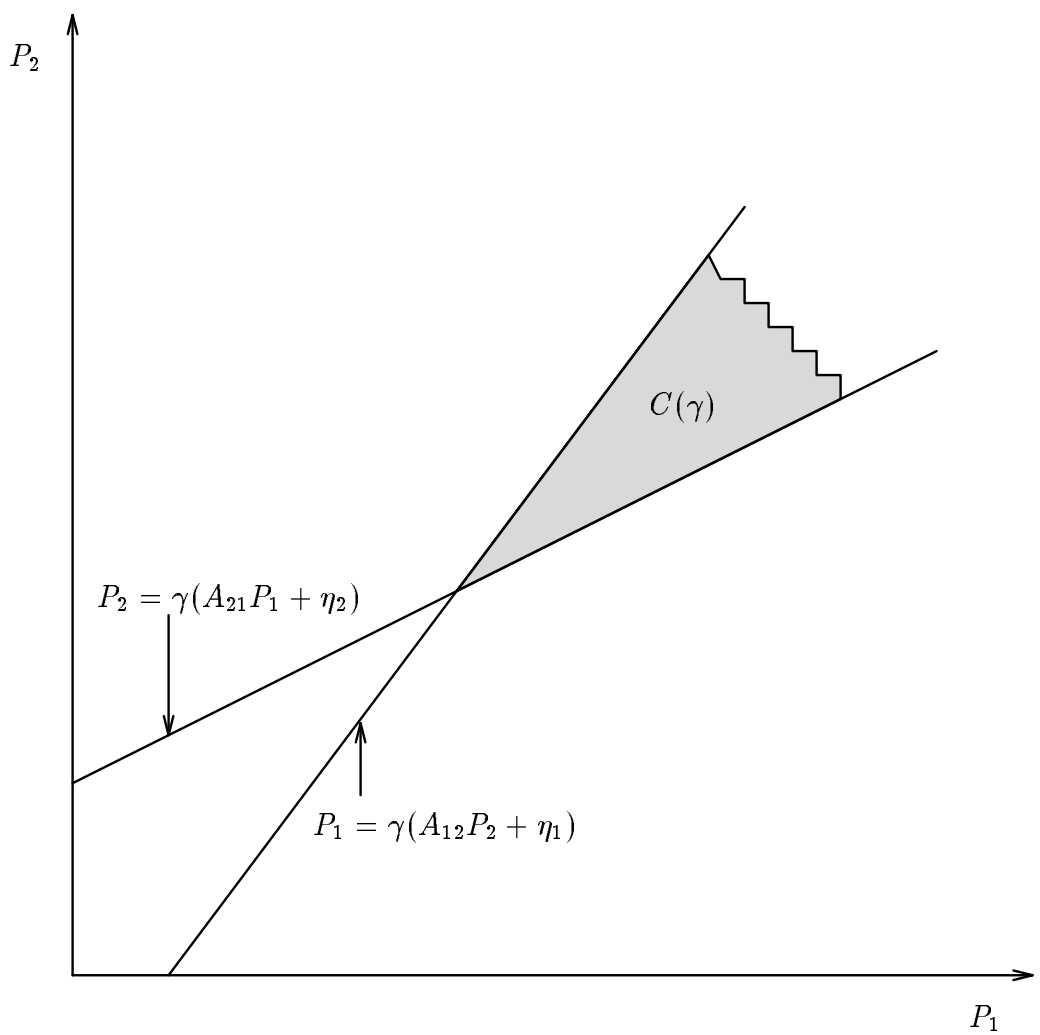

Figure 2: The polyhedral "cone" $C(\gamma)$ for $M = 2$.

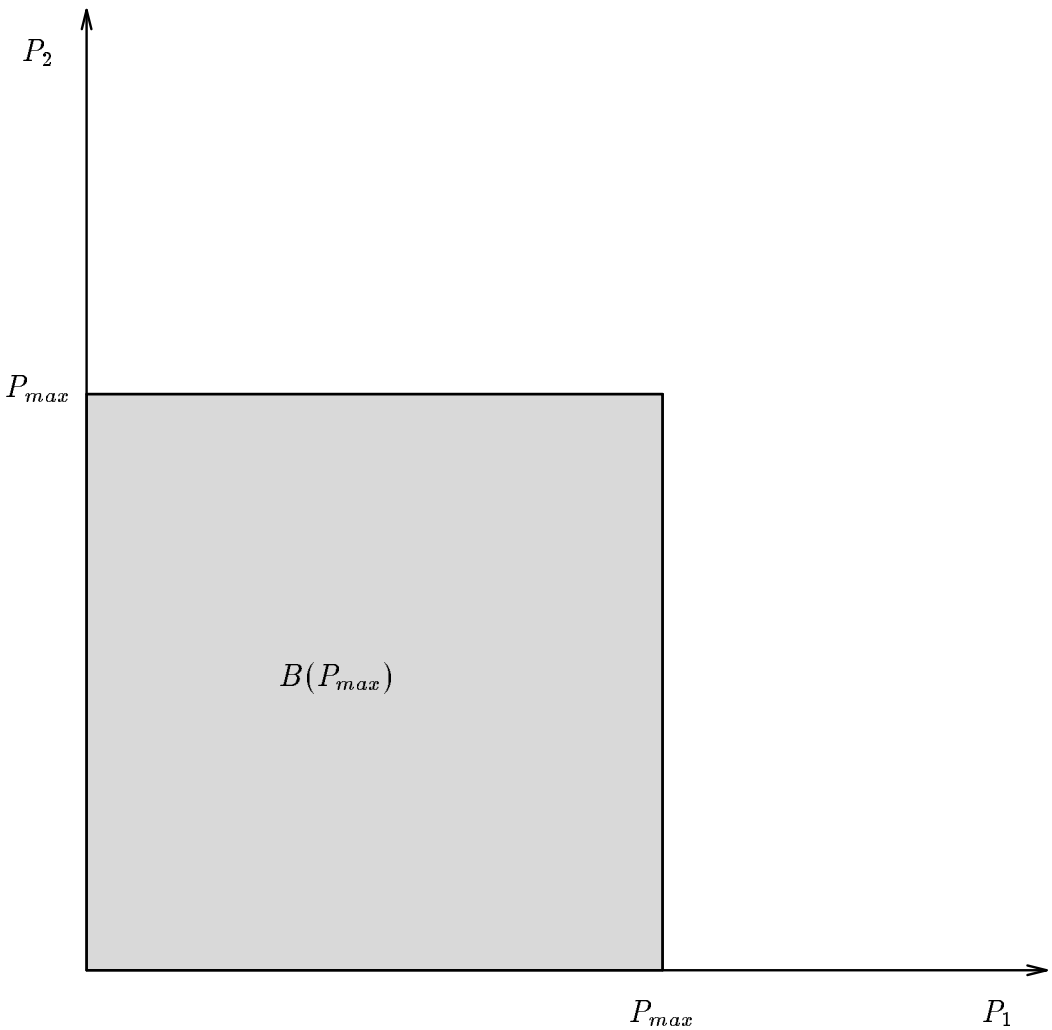

Figure 1: The "hypercube" $B(P_{max})$ for $M = 2$.

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mitter powers. The constraints do not seriously limit the performance of the DCPC scheme under moderate load conditions(γ_t close to γ^*). Under high load conditions($\gamma_t \gg \gamma^*$) the performance of the DCPC scheme will be close to (but not worse than) fixed power schemes.

A topic for further research would be call admission control in the power control environment[22]. A mobile that requires a transmitter power greater than P_{max}, for a call setup with a CIR of γ_t is a good candidate for rejection. Another topic of interest is handoff in the presence of power control. A mobile is ready for handoff when its transmitter power is at P_{max}. Signal strength measurements can be used to determine whether the handoff is to be intercell or intracell.

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the end of the five iterations. Any CIR's lower than 0 dB are counted in the bin corresponding to 0 dB. The CIR's above 50 dB are accounted for in the bin corresponding to 50 dB. This is repeated for 100 snapshots. We denote the fraction of terminals that have CIR equal to the abscissa as f_{CIR}. The fraction of terminals that have CIR (Power) less than or equal to the abscissa is denoted by F_{CIR}(F_{POWER} respectively). f_{CIR}, F_{CIR}, and F_{POWER} are plotted in Fig. 10,11, and 12 respectively. The plots include FCA(with all terminals transmitting 1W, which is the maximum power) and DCPC with γ_t =15,20,25,30,35,40 dB. From Fig. 10 we see that the CIR distribution of FCA is rather evenly spread. For DCPC as γ_t is increased fewer terminals are able to attain γ_t. From Fig. 11 we see that F_{CIR} corresponding to γ_t is always lower for DCPC than for FCA. This means that the fraction of terminals that have CIR lower than or equal to γ_t is smaller for DCPC than for FCA. In Fig. 12 we observe that increasing γ_t results in increased power levels. Finally from Fig. 10,11, and 12 we see that as γ_t increases DCPC tends to FCA. Setting $\gamma_t \gg \gamma^*$ will result in $\boldsymbol{P}^{(*)} = \boldsymbol{P}_{max}$ for DCPC which is equivalent to operating as in FCA.

7 Discussion

The optimum power vector that maximizes the minimum CIR in the system under a constraint on the maximum power level is derived. A distributed constrained power control scheme with favorable convergence properties and close to optimum performance is presented. Simulation results show that the DCPC scheme supports more number of users ,with CIR equal to or above the target CIR, than when operating with maxmimum trans-

distributed with log-variance of 10dB. The maximum transmitter power was set to 1W and the receiver noise power was assumed to be 10^{-15}W. A full load situation is assumed in all cases,i.e. all channels at each base station are in use. We consider only snapshots of the cellular system, where a snapshot is essentially a set of terminals frozen in their positions. The terminal positions are uniformly distributed in each cell. The DCPC scheme is applied to each snapshot and the powers and CIR's are recorded.

We first illustrate the convergence of the DCPC scheme. A snapshot consisting of only 10 base stations in a circle and 2 channels is generated. One terminal position was generated in each cell to create a full load situation. The powers and CIRS's of 5 terminals in one of the channels is recorded in every iteration of the DCPC scheme. Both synchronous and asynchronous versions of the DCPC are carried out with γ_t =30 dB and all starting powers at 1W. For the case studied all the terminals except terminal 4, are able to attain γ_t. In the asynchronous DCPC the powers and CIR's of all terminals are recorded just before terminal 1 makes it power update. Fig. 6 and Fig. 7 show the evolution of powers in the synchronous and asynchronous DCPC schemes respectively. The evolution of CIR's is shown in Fig. 8 and Fig. 9 for synchronous and asynchronous DCPC respectively. It was observed in various simulations besides this case that the DCPC scheme more or less converges after 5 iterations. It was also noted that the asynchronous updates converge faster.

We next illustrate the behaviour of the DCPC scheme when γ_t is varied. Here the 20 cells in a circle with 10 channels is considered. In each snapshot 5 iterations of the DCPC scheme is carried out. The Powers and CIR's, rounded to the nearest integer in dB, are recorded at

Lemma 6 that $\boldsymbol{\mathcal{T}}\ (\boldsymbol{P}) \in X(k+1), \forall \boldsymbol{P} \in X(k)$ and k. From Proposition 3

$$\lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{\,k}(0) = \lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{\,k}(\boldsymbol{P}_{max}) = \boldsymbol{P}^{(*)}.$$

It therefore follows that any sequence $\{y^k\}$ such that $y^k \in X(k), \forall k$ converges to the fixed point $\boldsymbol{P}^{(*)}$. Thus the Synchronous Convergence Condition of Theorem 1(*Asynchronous Convergence Theorem*) is satisfied. From the definition of $X(k)$ we see that the Box Condition of Theorem 1 is also satisfied. Since the set $\boldsymbol{\mathcal{B}}\ (P_{max})$ is the set $X(0)$ we have using Theorem 1 the TADCPC converges to the fixed point $\boldsymbol{P}^{(*)}$ for any $\boldsymbol{P}^{(0)} \in \boldsymbol{\mathcal{B}}\ (P_{max})$.□

The above convergence result for the TADCPC scheme renders the DCPC scheme robust for practical implementation. In the next section we will see a simulation example of the convergence the asynchronous version of the DCPC scheme.

6 Numerical Results

In this section the DCPC scheme is illustrated with examples of simulations. First we present a simulation example of the DCPC scheme with synchronous and asynchronous updates. We then show the behaviour of the DCPC scheme as γ_t is varied.

A one-dimensional cellular system with 20 cells in a circle is considered. The cell size is 2Km with the base station at the center. Fixed channel assignment is assumed with channels reused in every other cell. Each base station has 5 channels allocated to it. The link gains are given by $G_{ij} = d_{ij}^{-4} S$, where S is the slow fading factor that is log-normal

Furthermore, if $\{y^k\}$ is a sequence such that $y^k \in X(k)$ for every k, then every limit point of $\{y^k\}$ is a fixed point of f.

(b) (*Box Condition*) For every k, there exist sets $X_i(k) \subset X_i$ such that

$$X(K) = X_1(k) \times X_2(k) \times \cdots \times X_n(k). \ \Box$$

Theorem 1:[21] (*Asynchronous Convergence Theorem*) If the Synchronous Convergence and Box Conditions of Assumption 1 hold, and the initial solution estimate

$$x(0) = (x_1(0), \ldots, x_n(0))$$

belongs to the set $X(0)$, then every limit point of $\{x(t)\}$ is a fixed point of f. $\Box$

Proposition4: Starting with any $\boldsymbol{P}^{(0)} \in \boldsymbol{\mathcal{B}}\ (P_{max})$ the TADCPC scheme converges to the fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$.

Proof: We use the notation $\boldsymbol{\mathcal{T}}^{\,k}(.)$ to denote the composition of $\boldsymbol{\mathcal{T}}$ with itself k times, and $\boldsymbol{\mathcal{T}}^{\,0}(.)$ to denote the identity mapping. Let us define the sets

$$X(k) = \{\boldsymbol{P} : \boldsymbol{\mathcal{T}}^{\,k}(0) \leq \boldsymbol{P} \leq \boldsymbol{\mathcal{T}}^{\,k}(\boldsymbol{P}_{max})\}.$$

From Lemma 6 it follows that $\{\boldsymbol{\mathcal{T}}^{\,k}(0)\}$ and $\{\boldsymbol{\mathcal{T}}^{\,k}(\boldsymbol{P}_{max})\}$ are nondecreasing and nonincreasing sequences respectively. Hence we have $X(k+1) \subset X(k), \forall k$. It also follows from

powers of other terminals . Thus it is assumed that

$$P_i(t+1) = \boldsymbol{\mathcal{T}}_i(P_1(\tau_1^i(t)), \ldots, P_M(\tau_M^i(t))), \ \forall t \in T^i, \tag{29}$$

where $\tau_j^i(t)$ are times that satisfy $0 \leq \tau_j^i(t) \leq t, \forall t \in T$. When $t \notin T^i$, P_i is not affected,i.e, $P_i(t+1) = P_i(t), \forall t \notin T^i$.

We assume that the sets T^i are infinite, and if $\{t_k\}$ is a sequence of elements of T^i that tends to infinity, then $\lim_{k\to\infty} \tau_j^i(t_k) = \infty$ for every j. This assumption guarantees that each power is updated infinitely often and the old information is eventually discarded from the system. This means that given any time t_1, there exists a time $t_2 > t_1$ such that $\tau_j^i(t) > t_1$, $\forall i, j$, and $t \geq t_2$. However note that the amounts $t - \tau_j^i(t)$ by which the powers used in the iterations are outdated can become unbounded as t increases.

We use the *Asynchronous Convergence Theorem* from [21] to show that the TADCPC scheme converges.

Assumption 1:[21] There is a sequence of nonempty sets $\{X(k)\}$ with

$$\cdots \subset X(k+1) \subset X(k) \subset \cdots X \tag{30}$$

satisfying the following conditions:

(a) (*Synchronous Convergence Condition*) We have

$$f(x) \in X(k+1), \forall k \text{and} x \in X(k). \tag{31}$$

is lower than γ_t. In the DCPC scheme each terminal tries to attain γ_t. If $\gamma_t \leq \gamma'$ than all terminals attain γ_t. However if $\gamma_t > \gamma'$ then not all terminals can attain γ_t. In this case the DCPC scheme tries to support as many terminals as possible with CIR equal to γ_t and hence is a more practically useful than the CCPC scheme.

5 Totally Asynchronous Distributed Power Control

In this section we propose a totally asynchronous distributed constrained power control(TADCPC) scheme based on the totally asynchronous algorithmic model in [21]. It incorporates the same asynchrony as in [18]. Here the terminals are allowed to have different rate of power updates. Let $P_i(t)$ the transmitter power of the i^{th} terminal at time t and $P(t)$ the transmitter power vector at time t. It is now useful to consider the mapping $\boldsymbol{\mathcal{T}}$ in its component form as

$$P_i = \min\{P_{max}, \gamma_t \sum_{i=1}^{M} A_{ij} P_j + \gamma_t \eta_i\}, \ i = 1, \ldots, M,$$

or

$$P_i = \boldsymbol{\mathcal{T}}_i(P_1, \ldots, P_M) \ i = 1, \ldots, M. \tag{28}$$

We assume that there is a set of times $T = \{0, 1, 2, \ldots\}$ at which one or more of the powers are updated. The elements of T are to be viewed as the indices of the sequence of physical times at which updates are made. Let T^i be the set of times at which P_i is updated. We assume that the terminal may not have access to the most recent value of the transmitter

For any $\boldsymbol{P}^{(0)}$, such that $0 \leq \boldsymbol{P}^{(0)} \leq \boldsymbol{P}_{max}$ we have from Lemma 6

$$\lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{\,k}(0) \leq \lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{\,k}(\boldsymbol{P}^{(0)}) \leq \lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{\,k}(\boldsymbol{P}_{max}),$$

or

$$\boldsymbol{P}^{(*)} \leq \lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{\,k}(\boldsymbol{P}^{(0)}) \leq \boldsymbol{P}^{(*)},$$

or

$$\lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{\,k}(\boldsymbol{P}^{(0)}) = \boldsymbol{P}^{(*)}.\Box$$

The only requirement on γ_t in Proposition 3 and its proof is $\gamma_t \geq 0$. So Proposition 3 guarantees convergence of the DCPC scheme for any $\gamma_t \geq 0$. Note that setting $\gamma_t \geq \frac{P_{max}}{\min_i\{\eta_i\}}$ will result in $\boldsymbol{P}^{(*)} = \boldsymbol{P}_{max}$, which is equivalent to operating with constant transmitter powers. So the DCPC scheme becomes equivalent to operating at constant transmitter power P_{max} for sufficiently large γ_t. However for typical values γ_t the DCPC scheme can be expected to support more terminals with CIR no less than γ_t, than when operating with constant transmitter power P_{max}. This is illustrated with simulations in Section 6.

In Section 3 we showed that the CCPC scheme provided a maxmin CIR solution and all the terminals had CIR equal to $\gamma\prime$ the maxmin CIR. If γ_t, the target CIR required for the system, is less than or equal to $\gamma\prime$ then all terminals have acceptable transmission quality. However if $\gamma_t > \gamma\prime$, all terminals suffer unacceptable transmission quality since their CIR

For the above to be true it is necessary that

$$\boldsymbol{U}^{(l)} \leq \gamma_t \boldsymbol{A}^{\ (l)} \boldsymbol{U}^{(l)} \ . \tag{27}$$

The Inequality (27) cannot have nonnegative solutions for $\boldsymbol{U}^{(l)}$ if $\gamma_t < \gamma^*(\boldsymbol{A}^{\ (l)})$. ([20],[8],[10]). This is a contradiction, so $\boldsymbol{U} = 0$ and $\boldsymbol{P}^{(*)}{}_{sup} = \boldsymbol{P}^{(*)}{}_{inf}$. Hence the fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$ is unique. From Lemma 8 it follows that the fixed point $\boldsymbol{P}^{(*)}$ is the supremum of the set $\boldsymbol{\mathcal{S}}^{\ -}(\gamma_t)$, i.e.,

$$P_i^{(*)} = \sup\{P_i : \boldsymbol{P} \in \boldsymbol{\mathcal{S}}^{\ -}(\gamma_t)\},\ i = 1, \ldots, M. \square$$

Proposition 3: Starting with any nonnegative power vector $\boldsymbol{P}^{(0)}$ the DCPC scheme converges to the fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$.

Proof : Let us denote $\boldsymbol{P}^{(k)}$ by $\boldsymbol{\mathcal{T}}^{\ k}(\boldsymbol{P}^{(0)})$. From Lemma 8 and 10 we have

$$\lim_{k \to \infty} \boldsymbol{\mathcal{T}}^{\ k}(0) = \boldsymbol{P}^{(*)}.$$

From Lemma 9 and 10 we have

$$\lim_{k \to \infty} \boldsymbol{\mathcal{T}}^{\ k}(\boldsymbol{P}_{max}) = \boldsymbol{P}^{(*)}.$$

Lemma 6) that $\boldsymbol{P}^{(*)}{}_{inf}$ and $\boldsymbol{P}^{(*)}{}_{sup}$ are the infimum and supremum respectively of all possible fixed points of mapping $\boldsymbol{\mathcal{T}}$. Now let us assume that $\boldsymbol{P}^{(*)}{}_{sup} = \boldsymbol{P}^{(*)}{}_{inf} + \boldsymbol{U}$, where $\boldsymbol{U} \geq 0$. Since $\boldsymbol{P}^{(*)}{}_{inf}$ is a fixed point we have

$$\boldsymbol{P}^{(*)}{}_{inf} = \min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P}^{(*)}{}_{inf} + \boldsymbol{\eta}\ )\}\ .$$

Let l denote the number of components of $\boldsymbol{P}^{(*)}{}_{inf}$ which are not affected by the constraint on the maximum power. Each of these l components satisfies

$$[\boldsymbol{P}^{(*)}{}_{inf}]_i = [\gamma_t(\boldsymbol{A}\ \boldsymbol{P}^{(*)}{}_{inf} + \boldsymbol{\eta}\ )]_i\ . \tag{26}$$

Let $\boldsymbol{A}^{\ (l)}$ denote the $l \times l$ matrix formed by the deleting the i^{th} row and column of $\boldsymbol{A}$ when the i^{th} component of $\boldsymbol{P}^{(*)}{}_{inf}$ does not satisfy the above. Note that when the i^{th} component of $\boldsymbol{P}^{(*)}{}_{inf}$ does not satisfy Equation (26) the i^{th} component of $\boldsymbol{U}$ is zero and let the l dimensional vector formed by deleting these components be denoted by $\boldsymbol{U}^{(l)}$. It follows that $\gamma_t < \gamma^*(\boldsymbol{A}^{\ (l)})$([12],[19]), where $\gamma^*(\boldsymbol{A}^{\ (l)}) = \frac{1}{\lambda^*(\boldsymbol{A}^{\ (l)})}$ and $\lambda^*(\boldsymbol{A}^{\ (l)})$ is the maximum eigenvalue of $\boldsymbol{A}^{\ (l)}$. Since $\boldsymbol{P}^{(*)}{}_{sup}$ is a fixed point we have

$$\boldsymbol{P}^{(*)}{}_{sup} = \min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P}^{(*)}{}_{sup} + \boldsymbol{\eta}\ )\}\ ,$$

or

$$\boldsymbol{P}^{(*)}{}_{inf} + \boldsymbol{U} = \min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P}^{(*)}{}_{inf} + \boldsymbol{\eta}\ ) + \gamma_t \boldsymbol{A}\ \boldsymbol{U}\}\ .$$

or equivalently when $\boldsymbol{P}^{(k-1)} = \boldsymbol{P}^{(*)}$, where $\boldsymbol{P}^{(*)}$ is given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$. □

Lemma 10: The mapping $\boldsymbol{\mathcal{T}}$ has a unique fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$. The fixed point $\boldsymbol{P}^{(*)}$ has the property that

$$P_i^{(*)} = \sup\{P_i : \boldsymbol{P} \in \boldsymbol{\mathcal{S}}^{-}(\gamma_t)\},\ i = 1, \ldots, M.$$

Proof : Let us denote $\boldsymbol{P}^{(k)}$ by $\boldsymbol{\mathcal{T}}^{k}(\boldsymbol{P}^{(0)})$. Using Lemma 8 we can write

$$\lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{k}(0) = \boldsymbol{P}^{(*)}{}_{inf}\ ,$$

and from Lemma 9 we can write

$$\lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{k}(\boldsymbol{P}_{max}) = \boldsymbol{P}^{(*)}{}_{sup}\ ,$$

where $\boldsymbol{P}^{(*)}{}_{inf}$ and $\boldsymbol{P}^{(*)}{}_{sup}$ satisfy $\boldsymbol{P} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P})$. Since $0 \leq \boldsymbol{P}_{max}$ we have using Lemma 6

$$\lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{k}(0) \leq \lim_{k\to\infty} \boldsymbol{\mathcal{T}}^{k}(\boldsymbol{P}_{max}),$$

or

$$\boldsymbol{P}^{(*)}{}_{inf} \leq \boldsymbol{P}^{(*)}{}_{sup}\ .$$

Note that any fixed point $\boldsymbol{P}^{(*)}$ satisfies $0 \leq \boldsymbol{P}^{(*)} \leq \boldsymbol{P}_{max}$. It therefore follows (using

by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$. We also have $\boldsymbol{P}^{(0)} \leq \boldsymbol{P}^{(*)}$. □

Lemma 9: For $\boldsymbol{P}^{(0)} = \boldsymbol{P}_{max}$, the DCPC scheme converges to a fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$.

Proof : The range of the mapping $\boldsymbol{\mathcal{T}}\ (\boldsymbol{P})$ is always bounded by $\boldsymbol{\mathcal{B}}\ (P_{max})$. So we have

$$\boldsymbol{P}_{max} \geq \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}_{max}),$$

or

$$\boldsymbol{P}^{(0)} \geq \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(0)}),$$

or

$$\boldsymbol{P}^{(0)} \geq \boldsymbol{P}^{(1)}.$$

Applying Lemma 6 to the above equation $k-1$ times, for the k^{th} iteration of the DCPC we will have

$$\boldsymbol{P}^{(k-1)} \geq \boldsymbol{P}^{(k)}.$$

So the sequence of power vectors is nonincreasing and bounded below by the power vector with all components equal to zero. The sequence will stop decreasing at some k only when

$$\boldsymbol{P}^{(k-1)} = \boldsymbol{P}^{(k)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(k-1)}),$$

or

$$\boldsymbol{P} \leq \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}).\Box$$

Lemma 8: For all $\boldsymbol{P}^{(0)} \in \boldsymbol{\mathcal{S}}^{-}(\gamma_t)$, the DCPC scheme converges to a fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$. Also $\boldsymbol{P}^{(0)} \leq \boldsymbol{P}^{(*)}$.

Proof : For any $\boldsymbol{P}^{(0)} \in \boldsymbol{\mathcal{S}}^{-}(\gamma_t)$, from Lemma 7 we have

$$\boldsymbol{P}^{(0)} \leq \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(0)}),$$

or

$$\boldsymbol{P}^{(0)} \leq \boldsymbol{P}^{(1)}.$$

Applying Lemma 6 to the above inequality succesively, for any iteration k of the DCPC we will have

$$\boldsymbol{P}^{(k-1)} \leq \boldsymbol{P}^{(k)} \qquad \text{and} \qquad \boldsymbol{P}^{(k)} \leq \boldsymbol{P}^{(k+1)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(k)}).$$

In other words in each iteration the power vector cannot decrease in any of its components and will always lie in the set $\boldsymbol{\mathcal{S}}^{-}(\gamma_t)$. Since the sequence of power vectors is nondecreasing and bounded above by $\boldsymbol{P}_{max}$ it will converge to a power vector $\boldsymbol{P}^{(*)}$ in the set $\boldsymbol{\mathcal{S}}^{-}(\gamma_t)$, given

Proof : Let $\boldsymbol{P}_1 \leq \boldsymbol{P}_2$ then since $\boldsymbol{A}$ is a nonnegative matrix we can write

$$\boldsymbol{A}\ \boldsymbol{P}_1 \leq \boldsymbol{A}\ \boldsymbol{P}_2,$$

or

$$\boldsymbol{A}\ \boldsymbol{P}_1 + \boldsymbol{\eta}\ \leq \boldsymbol{A}\ \boldsymbol{P}_2 + \boldsymbol{\eta}\ ,$$

or

$$\min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P}_1 + \boldsymbol{\eta}\ )\} \leq \min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P}_2 + \boldsymbol{\eta}\ )\},$$

or

$$\boldsymbol{\mathcal{T}}\ (\boldsymbol{P}_1) \leq \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}_2).\square$$

Lemma 7: The power vectors in the set $\boldsymbol{\mathcal{S}}^{-}(\gamma_t)$ satisfy $\boldsymbol{P} \leq \boldsymbol{\mathcal{T}}\ (\boldsymbol{P})$.

Proof : If $\boldsymbol{P} \in \boldsymbol{\mathcal{S}}^{-}(\gamma_t)$ then $\boldsymbol{P}$ satisfies

$$\boldsymbol{P} \leq \boldsymbol{P}_{max} \qquad \text{and} \qquad \boldsymbol{P} \leq \gamma_t(\boldsymbol{A}\ \boldsymbol{P} + \boldsymbol{\eta}\ ),$$

or

$$\boldsymbol{P} \leq \min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P} + \boldsymbol{\eta}\ )\},$$

$\boldsymbol{\mathcal{B}}$ (P_{max}). Starting with $\boldsymbol{P}^{(0)} = 0$ the DCPC scheme would have powers increasing in each iteration since each terminal is adjusting its power to achieve γ_t. until the maxmimum power constraint comes into effect (for more detail see Lemma 8).

If $\boldsymbol{\mathcal{S}}$ $(\gamma_t) \neq \Phi$ then $\boldsymbol{P}_a(\gamma_t) \in \boldsymbol{\mathcal{B}}$ (P_{max}). Starting with $\boldsymbol{P}^{(0)} = 0$ the DCPC scheme would converge to $\boldsymbol{P}_a(\gamma_t)$ without the power constraint coming into effect. □

4.2 Convergence of the DCPC scheme when $\gamma_t \geq \gamma^*$

For $\gamma_t \geq \gamma^*$, $\boldsymbol{\mathcal{T}}$ is no longer a contraction mapping. However we will show in this section that the DCPC scheme still converges to the fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}$ $(\boldsymbol{P}^{(*)})$. Before we can do that we will need a few lemmas that follow.

We define the set of power vectors $\boldsymbol{\mathcal{S}}^{-}(\gamma)$ as

$$\boldsymbol{\mathcal{S}}^{-}(\gamma) \underline{\underline{\triangle}} \{\boldsymbol{P} : \boldsymbol{\mathcal{C}}^{-}(\gamma) \cap \boldsymbol{\mathcal{B}}\ (P_{max})\}\ . \tag{25}$$

The set $\boldsymbol{\mathcal{S}}^{-}(\gamma)$ is $\boldsymbol{\mathcal{C}}^{-}(\gamma)$ truncated by $\boldsymbol{\mathcal{B}}$ (P_{max}) (see Fig. 5). $\boldsymbol{\mathcal{B}}$ (P_{max}) truncates all the negative powers in $\boldsymbol{\mathcal{C}}^{-}(\gamma)$. If $\gamma \geq \gamma^*$, a part of the cone $\boldsymbol{\mathcal{C}}^{-}(\gamma)$ which includes the apex will also be truncated by $\boldsymbol{\mathcal{B}}$ (P_{max}).

Lemma 6: If $\boldsymbol{P}_1 \leq \boldsymbol{P}_2$ then $\boldsymbol{\mathcal{T}}$ $(\boldsymbol{P}_1) \leq \boldsymbol{\mathcal{T}}$ $(\boldsymbol{P}_2)$.

where the last inequality follows from the fact that $\boldsymbol{A}$ is nonnegative [21]. If $\gamma_t \lambda^* < 1$ or equivalently $\gamma_t < \gamma^*$ the result follows. $\Box$.

From the above property $\boldsymbol{\mathcal{T}}$ is a contraction mapping [21], and it follows that the algorithm $\boldsymbol{P}^{(n)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(n-1)})$ converges to the fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$ for any nonnegative starting vector $\boldsymbol{P}^{(0)}$ [21]. The following proposition proves that the DCPC scheme converges for $\gamma_t < \gamma^*$.

Proposition 2: If $\gamma_t < \gamma^*$, starting with any nonnegative power vector $\boldsymbol{P}^{(0)}$ the DCPC scheme described above converges to the fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$.

Proof : From Lemma 5, if $\gamma_t < \gamma^*$ then $\boldsymbol{P} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P})$ is a contraction mapping[21]. So the DCPC scheme

$$\boldsymbol{P}^{(n)} = \min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P}^{(n-1)} + \boldsymbol{\eta}\ )\}\ ,\ n \geq 1\ ,$$

converges [21] to the power vector solution given by $\boldsymbol{P} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P})$ for any nonnegative power vector $\boldsymbol{P}^{(0)}$ and $\gamma < \gamma^*$.$\Box$

Corollary: If $\boldsymbol{\mathcal{S}}\ (\gamma_t) = \Phi$ then the the fixed point $\boldsymbol{P}^{(*)}$ is the power vector in the set $\boldsymbol{\mathcal{C}}^{-}(\gamma_t) \cap \boldsymbol{\mathcal{B}}\ (P_{max})$ with maximum power in each component. If $\boldsymbol{\mathcal{S}}\ (\gamma_t) \neq \Phi$ then the fixed point $\boldsymbol{P}^{(*)}$ is simply $\boldsymbol{P}_a(\gamma_t)$.

Proof: The proof rests on the fact that no matter what $\boldsymbol{P}^{(0)}$ the DCPC scheme converges to the fixed point $\boldsymbol{P}^{(*)}$ given by $\boldsymbol{P}^{(*)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(*)})$.

If $\boldsymbol{\mathcal{S}}\ (\gamma_t) = \Phi$ then we have $\boldsymbol{P}_a(\gamma_t) \notin \boldsymbol{\mathcal{B}}\ (P_{max})$. In other words γ_t is not achievable in

where $\min\{.,.\}$ is component wise. We can now describe the DCPC scheme by

$$\boldsymbol{P}^{(n)} = \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}^{(n-1)}),\ n \geq 1. \tag{18}$$

4.1 Convergence of the DCPC scheme when $\gamma_t < \gamma^*$

We use contraction mapping arguments to show that the DCPC scheme converges for $\gamma_t < \gamma^*$. Consider the norm $\|\cdot\|$ defined by $\|\boldsymbol{P}\| = \max_i |P_i/P_i^*|$, where $\boldsymbol{P}^*$ is the eigenvector of $\boldsymbol{A}$ corresponding to λ^*.

Lemma 5: For $\gamma_t < \gamma^*$ the mapping $\boldsymbol{\mathcal{T}}$ has the property

$$\|\boldsymbol{\mathcal{T}}\ (\boldsymbol{P}_2) - \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}_1)\| \ \leq\ \beta\|\boldsymbol{P}_2 - \boldsymbol{P}_1\| \tag{19}$$

where β is a constant belonging to $[0, 1)$.

Proof : Consider

$$\|\boldsymbol{\mathcal{T}}\ (\boldsymbol{P}_2) - \boldsymbol{\mathcal{T}}\ (\boldsymbol{P}_1)\| \tag{20}$$

$$= \|\min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P}_2 + \boldsymbol{\eta}\ )\} - \min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P}_1 + \boldsymbol{\eta}\ )\}\| \tag{21}$$

$$\leq \|\gamma_t(\boldsymbol{A}\ \boldsymbol{P}_2 + \boldsymbol{\eta}\ ) - \gamma_t(\boldsymbol{A}\ \boldsymbol{P}_1 + \boldsymbol{\eta}\ )\| \tag{22}$$

$$= \gamma_t\|\boldsymbol{A}\ (\boldsymbol{P}_2 - \boldsymbol{P}_1)\| \tag{23}$$

$$< \gamma_t\lambda^*\|\boldsymbol{P}_2 - \boldsymbol{P}_1\| \tag{24}$$

the n^{th} time instant is given by

$$P_i^{(n)} = \min\{P_{max} \ , \ \gamma_t \ \frac{P_i^{(n-1)}}{\gamma_i^{(n-1)}}\}, \ 1 \leq i \leq M \ , \ n \geq 1 \ , \tag{15}$$

where P_{max} is the maximum transmitter power for the system. We shall refer to the above power control scheme as the distributed constrained power control (DCPC) scheme. We shall now proceed to analyse the convergence properties of the DCPC scheme.

We first define a set of powers $\boldsymbol{\mathcal{C}}^{-}(\gamma)$. The inequalities $\gamma_i \leq \gamma, \ i = 1, \ldots, M$ can be rewritten in matrix form using Equation (1) as $\boldsymbol{P} \geq \gamma(\boldsymbol{A}\ \boldsymbol{P} + \boldsymbol{\eta}\ )$. The set of all power vectors that satisfy $\gamma_i \leq \gamma, \ i = 1, \ldots, M$ is therefore defined by

$$\boldsymbol{\mathcal{C}}^{-}(\gamma) \underline{\underline{\triangle}} \{\boldsymbol{P} : (\frac{1}{\gamma}\boldsymbol{I} \ - \boldsymbol{A}\ )\boldsymbol{P} \leq \boldsymbol{\eta}\ \} \ , \tag{16}$$

The set $\boldsymbol{\mathcal{C}}^{-}(\gamma)$ is described geometrically by a polyhedral "cone" in Euclidean space(see Fig. 4) with the apex vector $\boldsymbol{P}_a(\gamma)$ given by $\boldsymbol{P}_a(\gamma) = (\frac{1}{\gamma}\boldsymbol{I} \ - \boldsymbol{A}\ )^{-1}\boldsymbol{\eta}$. Note that $\boldsymbol{\mathcal{C}}^{-}(\gamma)$ and $\boldsymbol{\mathcal{C}}\ (\gamma)$ have the same apex vector.

We define the mapping $\boldsymbol{\mathcal{T}}$ from the set of all nonnegative power vectors into itself by

$$\boldsymbol{\mathcal{T}}\ (\boldsymbol{P}) \underline{\underline{\triangle}} \ \min\{\boldsymbol{P}_{max}, \gamma_t(\boldsymbol{A}\ \boldsymbol{P} + \boldsymbol{\eta}\ )\}, \tag{17}$$

The vector $\boldsymbol{P}_a(\gamma\prime)$ achieves a balance in CIR's such that $\gamma_i = \gamma\prime,\ i = 1, \ldots, M$ in the system. Note also that of all the power vectors that can achieve $\gamma\prime$ for the system $\boldsymbol{P}_a(\gamma\prime)$ has the smallest power in each component.

A centralized constrained power control (CCPC) scheme to compute $\gamma\prime$ for the system follows naturally from Proposition 1. We start with a small value of γ (close to 0)and compute $\max_i\{P_i\}$ as

$$\max_i\{P_i\} = \max_{1\leq i\leq M}[(\frac{1}{\gamma}\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{\eta}]_i \tag{14}$$

If $\max_i\{P_i\} < P_{max}$ then we increase γ by a small factor. The above process is repeated until $|\max_i\{P_i\} - P_{max}|$ is sufficiently small. The computation of $\max_i\{P_i\}$ in the CCPC scheme involves solving for $\boldsymbol{P}$ in the system of linear equations $\boldsymbol{P} = \gamma_t(\boldsymbol{A}\,\boldsymbol{P} + \boldsymbol{\eta})$. This can be done iteratively using Equation (5).

Note that the above analysis for finding $\gamma\prime$ is for non zero receiver noise power i.e. $\boldsymbol{\eta} \neq 0$. For $\boldsymbol{\eta} = 0$ it is easy to see that all the power vectors that are scaled versions of $\boldsymbol{P}^*$ and lie within $\boldsymbol{\mathcal{B}}(P_{max})$ achieve γ^*, i.e. γ^* is always achievable in $\boldsymbol{\mathcal{B}}(P_{max})$ and $\gamma\prime = \gamma^*$.

4 Distributed Constrained Power Control

A distributed power control scheme is derived by making a simple modification to the power control of [12]. Each of the M terminals on the channel adjust their transmitter powers synchronously at discrete time instants. The power adjustment made by the i^{th} terminal at

and

$$\boldsymbol{\mathcal{S}}\ (\gamma\prime) = \{\boldsymbol{P}_a(\gamma\prime)\}\ . \tag{13}$$

Proof: Note that $\boldsymbol{\mathcal{S}}\ (0) = \boldsymbol{\mathcal{B}}\ (P_{max})$, i.e $\gamma = 0$ is achievable everywhere in $\boldsymbol{\mathcal{B}}\ (P_{max})$. The powers required to achieve γ^* tend to infinity since γ^* is a simple root of the determinant of $(\frac{1}{\gamma}\boldsymbol{I}\ - \boldsymbol{A}\ )$. So $\boldsymbol{\mathcal{S}}\ (\gamma^*) = \Phi$, i.e $\gamma = \gamma^*$ is not achievable in $\boldsymbol{\mathcal{B}}\ (P_{max})$. Note that for

$$0 < \gamma < \gamma^*$$

we have from Lemma 1

$$\boldsymbol{\mathcal{C}}\ (0) \supset \boldsymbol{\mathcal{C}}\ (\gamma) \supset \boldsymbol{\mathcal{C}}\ (\gamma^*)$$

or

$$\boldsymbol{\mathcal{S}}\ (0) \supset \boldsymbol{\mathcal{S}}\ (\gamma) \supseteq \boldsymbol{\mathcal{S}}\ (\gamma^*)$$

or

$$\boldsymbol{\mathcal{B}}\ (P_{max}) \supset \boldsymbol{\mathcal{S}}\ (\gamma) \supseteq \Phi\ .$$

So as γ increases from 0 from γ^* the feasible set $\boldsymbol{\mathcal{S}}\ (\gamma)$ decreases from $\boldsymbol{\mathcal{B}}\ (P_{max})$ to Φ. Hence there exists a $\gamma\prime$, $0 < \gamma\prime < \gamma^*$, such that $\forall\epsilon > 0$, $\boldsymbol{\mathcal{S}}\ (\gamma\prime + \epsilon) = \Phi$ and $\boldsymbol{\mathcal{S}}\ (\gamma\prime) \neq \Phi$. From Lemmas 3 and 4 we have $\boldsymbol{\mathcal{S}}\ (\gamma\prime) = \{\boldsymbol{P}_a(\gamma\prime)\}$ i.e. the apex of the cone is on the boundary of the hypercube $\boldsymbol{\mathcal{B}}\ (P_{max})$. Clearly $\max_i\{[\boldsymbol{P}_a(\gamma\prime)]_i\}_{i=1}^M = P_{max}$ respectively. So we can write

$$\max_{1\leq i\leq M}[\boldsymbol{P}_a(\gamma\prime)]_i = \max_{1\leq i\leq M}[(\frac{1}{\gamma\prime}\boldsymbol{I}\ - \boldsymbol{A}\ )^{-1}\boldsymbol{\eta}\ ]_i = P_{max}\ . \ \square$$

their components greater than P_{max}. This implies that all the vectors in the cone lie outside the hypercube $\boldsymbol{\mathcal{B}}\ (P_{max})$, or equivalently $\boldsymbol{\mathcal{S}}\ (\gamma) = \Phi$.□

Lemma 4: If $\boldsymbol{\mathcal{S}}\ (\gamma) \neq \Phi$ then there exists at least one vector $\boldsymbol{P} \in \boldsymbol{\mathcal{S}}\ (\gamma)$ such that $\max_i\{[\boldsymbol{P}]_i\}_{i=1}^{M} = P_{max}$.

Proof: By definition $\boldsymbol{\mathcal{S}}\ (\gamma) = \boldsymbol{\mathcal{C}}\ (\gamma) \cap \boldsymbol{\mathcal{B}}\ (P_{max})$. Note that all vectors $\boldsymbol{P}$ for which $\max_i\{[\boldsymbol{P}]_i\}_{i=1}^{M} = P_{max}$ lie on the upper boundary of the hypercube $\boldsymbol{\mathcal{B}}\ (P_{max})$. We need to show that if $\boldsymbol{\mathcal{S}}\ (\gamma) \neq \Phi$ then the cone $\boldsymbol{\mathcal{C}}\ (\gamma)$ has an intersection with the upper boundary of the hypercube $\boldsymbol{\mathcal{B}}\ (P_{max})$. From Lemma 2 all the vectors $\boldsymbol{P}$ in $\boldsymbol{\mathcal{C}}\ (\gamma)$ satisfy $\boldsymbol{P} \geq \boldsymbol{P}_a(\gamma)$. So $\boldsymbol{\mathcal{C}}\ (\gamma)$ is an unbounded polyhedral cone with power vectors greater than or equal to its apex $\boldsymbol{P}_a(\gamma)$. If $\boldsymbol{\mathcal{S}}\ (\gamma) \neq \Phi$ then $\boldsymbol{P}_a(\gamma) \in \boldsymbol{\mathcal{S}}\ (\gamma)$(from Lemma 3) i.e. $\boldsymbol{P}_a(\gamma)$ lies inside or on the boundary of $\boldsymbol{\mathcal{B}}\ (P_{max})$. Since $\boldsymbol{\mathcal{C}}\ (\gamma)$ is an unbounded polyhedral cone with vectors $\boldsymbol{P}$ satisfying $\boldsymbol{P} \geq \boldsymbol{P}_a(\gamma)$, it intersects the upper boundary of the hypercube $\boldsymbol{\mathcal{B}}\ (P_{max})$.□

We now use the above four lemmas in finding $\gamma\prime$. In the following proposition we prove that $\gamma\prime$ is the maximum balance (i.e. all terminals with same CIR) CIR. In [5] a similar result was stated without proof for a more general case of unequal maximum power levels.

Proposition 1: The maximum achievable CIR within $\boldsymbol{\mathcal{B}}\ (P_{max})$, $\gamma\prime$, is given by

$$\gamma\prime = \gamma : \max_{1 \leq i \leq M} [(\frac{1}{\gamma}\boldsymbol{I}\ - \boldsymbol{A}\ )^{-1}\boldsymbol{\eta}\ ]_i = P_{max} \tag{12}$$

and since $\gamma_1 < \gamma_2$ we also have

$$\boldsymbol{P'} > \gamma_1(\boldsymbol{A}\ \boldsymbol{P'} + \boldsymbol{\eta}\ )$$

which implies $\boldsymbol{P'} \in \boldsymbol{\mathcal{C}}\ (\gamma_1)$. It therefore follows that $\boldsymbol{\mathcal{C}}\ (\gamma_2) \subset \boldsymbol{\mathcal{C}}\ (\gamma_1)$.□

Lemma 2: If $\boldsymbol{P} \in \boldsymbol{\mathcal{C}}\ (\gamma)$ then $\boldsymbol{P} \geq \boldsymbol{P}_a(\gamma)$ for $\gamma < \gamma^*$.

Proof: The apex power vector $\boldsymbol{P}_a(\gamma)$ is the solution to the matrix equation $(\frac{1}{\gamma}\boldsymbol{I}\ - \boldsymbol{A}\ )\boldsymbol{P} = \boldsymbol{\eta}$. It is guaranteed to have all its elements nonnegative if $\gamma < \gamma^*$([12],[19],[14]). Let $\gamma < \gamma^*$. If $\boldsymbol{P} \in \boldsymbol{\mathcal{C}}\ (\gamma)$ then

$$(\frac{1}{\gamma}\boldsymbol{I}\ - \boldsymbol{A}\ )\boldsymbol{P} \geq \boldsymbol{\eta}$$

or

$$(\frac{1}{\gamma}\boldsymbol{I}\ - \boldsymbol{A}\ )\boldsymbol{P} \geq (\frac{1}{\gamma}\boldsymbol{I}\ - \boldsymbol{A}\ )\boldsymbol{P}_a(\gamma)$$

or

$$(\frac{1}{\gamma}\boldsymbol{I}\ - \boldsymbol{A}\ )(\boldsymbol{P} - \boldsymbol{P}_a(\gamma)) \geq 0\ .$$

From the above inequality we get $(\boldsymbol{P} - \boldsymbol{P}_a(\gamma)) \geq 0$ when $\gamma < \gamma^*$[20]. So we have $\boldsymbol{P} \geq \boldsymbol{P}_a(\gamma)$. □

Lemma 3: If $\boldsymbol{P}_a(\gamma) \notin \boldsymbol{\mathcal{B}}\ (P_{max})$ then $\boldsymbol{\mathcal{S}}\ (\gamma) = \Phi$.

Proof: From Lemma 2 the components of each vector in the cone is greater than or equal to the corresponding components in the apex vector. Let $\boldsymbol{P}_a(\gamma) \notin \boldsymbol{\mathcal{B}}\ (P_{max})$. Then at least one of its components is greater than P_{max}. So all the vectors in the cone have at least one of

The set of all power vectors that achieve γ is therefore defined by

$$\boldsymbol{\mathcal{C}}\ (\gamma) \underline{\underline{\triangle}} \{\boldsymbol{P} : (\frac{1}{\gamma}\boldsymbol{I}\ -\boldsymbol{A}\ )\boldsymbol{P} \geq \boldsymbol{\eta}\ \}\ , \tag{10}$$

The set $\boldsymbol{\mathcal{C}}\ (\gamma)$ is described geometrically by an unbounded polyhedral "cone" in Euclidean space(see Fig. 2). The apex of the "cone" $\boldsymbol{\mathcal{C}}\ (\gamma)$ is the power vector $\boldsymbol{P}_a(\gamma) = (\frac{1}{\gamma}\boldsymbol{I}\ -\boldsymbol{A}\ )^{-1}\boldsymbol{\eta}$. The set of feasible power vectors $\boldsymbol{\mathcal{S}}\ (\gamma)$ that achieve γ is thus defined by

$$\boldsymbol{\mathcal{S}}\ (\gamma) \underline{\underline{\triangle}} \{\boldsymbol{P} : \boldsymbol{\mathcal{C}}\ (\gamma) \cap \boldsymbol{\mathcal{B}}\ (P_{max})\}\ . \tag{11}$$

The set $\boldsymbol{\mathcal{S}}\ (\gamma)$ is the intersection of $\boldsymbol{\mathcal{C}}\ (\gamma)$ and $\boldsymbol{\mathcal{B}}\ (P_{max})$ (see Fig. 3). Note that γ is achievable in $\boldsymbol{\mathcal{B}}\ (P_{max})$ if $\boldsymbol{\mathcal{S}}\ (\gamma) \neq \Phi$. We can therefore write

$$\gamma\prime = \max\{\gamma : \boldsymbol{\mathcal{S}}\ (\gamma) \neq \Phi\}\ .$$

Before we can solve for $\gamma\prime$ we will need the following four lemmas.

Lemma 1: If $\gamma_2 > \gamma_1$ then $\boldsymbol{\mathcal{C}}\ (\gamma_2) \subset \boldsymbol{\mathcal{C}}\ (\gamma_1)$.

Proof: Let $\gamma_2 > \gamma_1$. If $\boldsymbol{P\prime} \in \boldsymbol{\mathcal{C}}\ (\gamma_2)$ we have

$$\boldsymbol{P\prime} \geq \gamma_2(\boldsymbol{A}\ \boldsymbol{P\prime} + \boldsymbol{\eta}\ )$$

power updates (i.e each terminal has different rate of power updates), when $\gamma_t < \gamma^*$.

3 Centralized Constrained Power control

We denote the maximum transmitter power by P_{max}. Let $\boldsymbol{P}_{max}$be an M dimensional power vector with all elements equal to P_{max}. We define the set $\boldsymbol{\mathcal{B}}\ (P_{max})$ by

$$\boldsymbol{\mathcal{B}}\ (P_{max}) \underline{\underline{\triangle}} \{\boldsymbol{P} : 0 \leq \boldsymbol{P} \leq \boldsymbol{P}_{max}\}\ . \tag{7}$$

The set $\boldsymbol{\mathcal{B}}\ (P_{max})$ is described geometrically by a "hypercube" whose side is of length P_{max} in Euclidean space(see Fig. 1).

We are interested in finding the maximum achievable CIR within $\boldsymbol{\mathcal{B}}\ (P_{max})$, $\gamma\prime$, given by

$$\gamma\prime = \max_{\boldsymbol{P} \in \boldsymbol{\mathcal{B}}\ (P_{max})}\ \min_{1 \leq i \leq M} \gamma_i\ , \tag{8}$$

An upperbound on $\gamma\prime$ is clearly γ^*[14]. $\gamma\prime$ is therefore confined to the range $[0, \gamma^*)$. Note that γ is achievable if there exist power vectors such that $\gamma_i \geq \gamma,\ i = 1, \ldots, M$. The inequalities $\gamma_i \geq \gamma,\ i = 1, \ldots, M$ can be rewritten in matrix form using Equation (1) as

$$\boldsymbol{P} \geq \gamma(\boldsymbol{A}\ \boldsymbol{P} + \boldsymbol{\eta}\ )\ . \tag{9}$$

of $\boldsymbol{A}$ corresponding to λ^*. In [12] a distributed synchronous algorithm that accounts for receiver noise power is presented. Here each of the M terminals on the channel adjust their transmittnchronously at discrete time instants. The power adjustment made by the i^{th} terminal at the n^{th} time instant is given by

$$P_i^{(n)} = \gamma_t \frac{P_i^{(n-1)}}{\gamma_i^{(n-1)}}, \ 1 \leq i \leq M \ , \ n \geq 1 \ , \tag{5}$$

where $P_i^{(n)}$ is the power transmitted by the i^{th} terminal at the n^{th} iteration, $\gamma_i^{(n)}$ is its resulting CIR, and γ_t is the target CIR. For $\gamma_t < \gamma^*$ the above algorithm converges to a power vector solution $\boldsymbol{P} = (\boldsymbol{I} - \boldsymbol{\gamma_t A})^{-1}\boldsymbol{\eta}$ which results in all links having a CIR equal to γ_t. In [19] it is shown that the power adjustments in the above algorithm can also be made asynchronously, where the M terminals can adjust powers in a round robin fashion. When $\gamma_t \geq \gamma^*$ we have [19]

$$\lim_{n\to\infty} \boldsymbol{P}^{(n)} = \boldsymbol{P}^* \lim_{n\to\infty} K(n), \tag{6}$$

and

$$\lim_{n\to\infty} \gamma_i^{(n)} = \gamma^*, \ 1 \leq i \leq M \ ,$$

where $K(n)$ is given by

$$K(n) = b\frac{(\gamma_t\lambda^*)^n - 1}{\gamma_t\lambda^* - 1} + a(\gamma_t\lambda^*)^n,$$

where the constants a and b are positive, and are decided by $\boldsymbol{P}^{(0)}$ and $\gamma_t\boldsymbol{\eta}$ respectively. In [16] it is shown that the scheme given by Equation (5) converges for totally asynchronous

Equation (1) for the CIR of the i^{th} terminal at its base can be rewritten as

$$\gamma_i = \frac{P_i}{\sum_{j=1}^{M} A_{ij} P_j + \eta_i} \ , \tag{2}$$

where the $M \times M$ matrix $\boldsymbol{A} = \{A_{ij}\}$ is defined by

$$A_{ij} = \begin{cases} \left(\frac{G_{ij}}{G_{ii}}\right) & \text{if } i \neq j \\ \\ 0 & \text{if } i = j \ . \end{cases} \ , \tag{3}$$

and $\eta_i = \nu_i / G_{ii}$. We let $\boldsymbol{\eta}$ be the M dimensional vector whose i^{th} element is η_i. The η_i's are greater than or equal to zero. In our analysis we assume that at least one of the η_i is not zero.

We introduce the notion that γ is achievable if there exist power vectors such that $\gamma_i \geq \gamma, \ i = 1, \ldots, M$. The maximum achievable CIR γ^* is defined as

$$\boldsymbol{\gamma}^* = \max_{\boldsymbol{P} \geq \mathbf{o}} \min_{1 \leq i \leq M} \boldsymbol{\gamma}_i \ , \tag{4}$$

where $\boldsymbol{P}$ is the M-dimensional power vector whose i^{th} element is P_i. The notation $\boldsymbol{P} \geq \mathbf{o}$ denotes that the elements of $\boldsymbol{P}$ are nonnegative. For interference limited systems(receiver noise power equal to zero) it is shown in ([8]-[11]) that an optimal power vector $\boldsymbol{P}^*$ exists that achieves γ^*. This optimal vector $\boldsymbol{P}^*$ is unique up to scaling by a constant, and results in all links having the same CIR, i.e., $\gamma_i = \gamma^* \ , \ 1 \leq i \leq M$. The maximum eigenvalue λ^* of matrix $\boldsymbol{A}$ is related to γ^* as $\gamma^* = 1/\lambda^*$[10]. The optimal power vector $\boldsymbol{P}^*$ is the eigenvector

achieving this bound is presented. Further a distributed algorithm that has close to optimum performance is proposed.

2 System Model

In this section we describe the system model and some relevant results needed for the analysis. We discuss power control for the uplink (from terminal to base) only. For the downlink (from base to terminal) all the results in this paper are valid with appropriate changes in the notation.

We consider a cellular radio system with a finite channel set of size N (where a channel could be a frequency or time slot). The number of terminals using the same channel is denoted by M. We assume that the channels are orthogonal i.e. terminals on different channels do not interfere with each other. We denote the transmitter power of the i^{th} terminal communicating with the i^{th} base station by P_i. The gain on the radio link from terminal j to base i is denoted by G_{ij}. All the G_{ij}'s are positive and can take values in the range $(0, 1]$. ν_i denotes the receiver noise at the i^{th} base. The link quality is measured by the carrier to interference ratio(CIR). The CIR of the i^{th} terminal at its base is given by

$$\gamma_i = \frac{P_i G_{ii}}{\sum_{\substack{j=1 \\ j \neq i}}^{M} P_j G_{ij} + \nu_i} \ , \ 1 \leq i \leq M \ . \tag{1}$$

1 Introduction

Cochannel interference, due to spectrum reuse, is one of the main limiting factors in achieving a high user density in wireless communication systems. Controlling the transmitter power level has been a frequently used tool to control this interference to achieve a high capacity as well as a high link quality (carrier to interference ratio, CIR). Previous results show that for a given link quality requirement capacity can be maximized by adjusting the transmitter power such that all admitted connections operate at the same CIR. This is referred to as CIR-balancing([5]-[8],[10]). Results show that substantial improvements in capacity, similar to the results achieved by dynamic channel allocation, can be achieved. Also distributed power control algorithms (PCA's), iteratively adjusting the power in the individual links based only on measurements in that link have been shown to achieve similar performance([9],[11]-[13]). In [18] an algorithm that integrates power control and base assignment is presented.

Most of the previously reported schemes, assume that the transmitter power can be continuously adjusted without limitations in dynamic range. This is of course very optimistic since the maximum output power of a handheld device is certainly limited, mainly due to battery capacity limitations. However, in [14] it was demonstrated that introducing transmitter power range constraints does not severely affect the performance of a centralized power control scheme. However, when using a distributed PCA, these constraints affect not only the CIR-performance but also the convergence properties of the PCA are affected. This is the topic of this paper. First, we derive an upper bound on the CIR-performance when each transmitter is confined to use at most a given power level. A centralized scheme for

Constrained Power Control

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Abstract

High system capacities can be achieved by controlling the transmitter power in multiuser radio systems. Power control with no constraint on the maximum power level has been studied extensively in earlier work([1]-[18]). Transmitter power is at a premium in radio systems such as cellular systems and PCS. There is a limit on the maximum transmitter power especially at the terminals (eg. mobile units and handsets) since the power comes from a battery. In this paper we study power control that maximizes the minimum carrier to interference ratio (CIR), with a constraint on the maximum power. The optimal power vector solution lies on the boundary of the constrained power vector set and achieves a balance in the CIR's. Results indicate that the constraints do not induce any stability problems. A distributed scheme with favourable convergence properties and close to optimum performance is presented. Simulation results show that the algorithm tries to maximize the number of terminals served with CIR greater than or equal to the target CIR, while conserving power.

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