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Rate of Convergence for Minimum Power Assignment Algorithms in Cellular Radio Systems

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Abstract

In wireless communication systems, mobile users adapt to a time varying radio channel by regulating transmitter powers. This power control is intended to provide each user an acceptable connection, as measured by a carrier to interference ratio (CIR), by eliminating unnecessary interference. It is important that a power control algorithm can converge quickly to a fixed point at which either all users have acceptable connections or an infeasibility can be detected. In this work, we show that an iterative power control and base station assignment algorithm based on CIR measurements converges to a unique fixed point at a geometric rate. This conclusion is shown to hold even if some or all of the users are subject to maximum power constraints. The rate of convergence is evaluated by simulation of a one dimensional CDMA system.

1 Introduction

In wireless communication systems, mobile users can adapt to a time varying radio channel by regulating transmitter power. This power control is intended to provide each user an acceptable connection, as measured by a carrier to interference ratio (CIR), by eliminating unnecessary interference. Although this model has been used in the context of TDMA and FDMA systems, it is particularly appropriate for the uplink of a CDMA system in which unsynchronized wideband CDMA signals of other users can be modeled as interfering noise signals.

Convergence properties of unconstrained fixed base station assignment power control algorithms have been discussed in [1–4]. When the system can support all users at a common target CIR, an iterative power control algorithm was shown to converge synchronously [4] as well as asynchronously [5] to a fixed point. Joint power control and base station assignment is considered in [6, 7] in the absence of transmitted power constraints. Under fixed base station assignment, [8, 9] examine the convergence problem when there are maximum power constraints. The convergence of integrated power control and base station assignment in the presence of maximum power constraints is addressed in [10]. A framework for uplink power control yielding sufficient conditions for convergence is given in [11].

This work is an effort to understand the *rate* of convergence for cellular radio systems employing iterative power control methods that may be subject to maximum power constraints. We show that a general form of iterative power control that includes the algorithms proposed in [8, 10] converges to a unique fixed point at a geometric rate. This conclusion is shown to hold when the assignment of users to base stations is fixed as well as when the users choose a base station in order to minimize transmitted power. The rate of convergence, which will depend on the number of mobile terminals, the link gains, and the transmitter power limits (if any), is then examined by simulation of a one dimensional CDMA system.

2 Interference Constraints

We assume N users and M base stations and a common radio channel. Let p_i denote the transmitted power of user i so that $\boldsymbol{p} = [p_1, ..., p_N]^\top$ denotes the power vector of the system. The corresponding received signal power of user i at base k is $h_{ki}p_i$ where h_{ki} denotes the link gain of user i to base k. The interference seen by user i at base

k is $\sum_{j \neq i} h_{kj} p_j + \eta_k$ where η_k describes the receiver noise at base k. The requirement that user i achieve a CIR of at least γ_i at base station k can be expressed as

$$\frac{h_{ki} p_i}{\sum_{j \neq i} h_{kj} p_j + \eta_k} \geq \gamma_i \tag{1}$$

When user i is assigned to base k, we can write the CIR constraint for user i as

$$p_i \geq \boldsymbol{H}_i^{(k)} \boldsymbol{p} + \delta_i^{(k)} \tag{2}$$

where $\delta_i^{(k)} = \gamma_i \eta_k / h_{ki}$ and $\boldsymbol{H}_i^{(k)}$ is a normalized N dimensional row vector with jth element

$$H_{ij}^{(k)} = \begin{cases} \dfrac{\gamma_i h_{kj}}{h_{ki}} & j \neq i \\ 0 & j = i \end{cases} \tag{3}$$

The power control algorithm we consider in this work permits the possibility that user i may choose a base station in order to minimize its transmitted power. Following [6], we will assume that each user i iteratively selects a base station k from a set K_i in order to minimize its transmitted power. The iterative algorithm can be expressed as

$$\boldsymbol{p}(n+1) = \boldsymbol{T}(\boldsymbol{p}(n)) \tag{4}$$

where for user i,

$$p_i(n+1) = T_i(\boldsymbol{p}(n)) = \min_{k \in K_i} \boldsymbol{H}_i^{(k)} \boldsymbol{p}(n) + \delta_i^{(k)} \tag{5}$$

In addition, a maximum power constraint q_i may be imposed on user i. In this case, the iteration for user i becomes

$$p_i(n+1) = \min \left\{ \min_{k \in K_i} \boldsymbol{H}_i^{(k)} \boldsymbol{p}(n) + \delta_i^{(k)}, q_i \right\} \tag{6}$$

We number the actual base stations by $k = 1, 2, \ldots, M$. In order to express the iteration (6) in the form of Equation (5), we will say that a user transmitting at maximum power is assigned to a *virtual* base station numbered $k = 0$. For this virtual base station, we define

$$\boldsymbol{H}_i^{(0)} = [0, \cdots, 0] \qquad \delta_i^{(0)} = q_i \tag{7}$$

When user i is assigned to the virtual base station 0, the CIR constraint (2) is simply $p_i \geq q_i$. In terms of the power control algorithm, once a user is transmitting at maximum power, the interference that user creates is the same no matter what the base station assignment. Of course, such a user is actually assigned to the base station where its CIR is maximized even though the resulting CIR will be below the desired target.

By including the virtual base station 0 in K_i, the constrained iteration of Equation (6) can be written in the form of Equation (5). When user i is permanently assigned to base a_i but subject to a maximum power constraint, $K_i = \{0, a_i\}$ and the iteration (5) is the fixed base station assignment constrained power control algorithm proposed in [8,9]. When $K_i = \{1, \ldots, M\}$, Equation (5) is the minimum power assignment (MPA) algorithm analyzed in [6,7]. Lastly, when $K_i = \{0, \ldots, M\}$, we have the constrained MPA algorithm considered in [10,11]. When K_i has more than one element, user i attempts to minimize its transmitter power over the set of possible base station assignments and we call the power control iteration (5) a general minimum power assignment algorithm. We see that the virtual base station approach provides a common way to consider systems with or without maximum power constraints. Furthermore, when K_i includes the virtual base station 0 for only some users i, we can consider systems where only those users are subject to maximum power limits.

A base station assignment is a vector $[a_1, \ldots, a_N]$ that specifies an assigned base $a_i \in K_i$ for each mobile i. We can number the base station assignments as $l = 1, 2, \ldots$ and define $a_i(l)$ such that under assignment l, user i is assigned to base $a_i(l)$. We can associate with each assignment l, a gain matrix $\boldsymbol{G}^{(l)}$, the normalized interference matrix under assignment l, as having i, jth element

$$G_{ij}^{(l)} = H_{ij}^{(a_i(l))} \tag{8}$$

Similarly, the normalized noise vector $\boldsymbol{\sigma}^{(l)}$ is defined to have ith component $\sigma_i^{(l)} = \delta_i^{(a_i(l))}$.

At step n of the iteration (5), each user i chooses a base station $k_i \in K_i$. There is an assignment l_n such that $a_i(l_n) = k_i$ for all i and we say that assignment l_n was used at step n. When assignment l is used, the power control iteration function $\boldsymbol{T}(\boldsymbol{p})$ becomes

$$\boldsymbol{T}^{(l)}(\boldsymbol{p}) = \boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l)} \tag{9}$$

The convergence of the power control iteration using $\boldsymbol{T}(\boldsymbol{p})$ or $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ will depend on the properties of the non-negative matrices $\boldsymbol{G}^{(l)}$. These properties, as well as some basic types of convergence, will be described in the next section.

3 Preliminaries

Before proceeding, it is worthwhile to define carefully several common forms of convergence for a synchronous iterative algorithm of the form $\boldsymbol{p}(n+1) = \boldsymbol{T}(\boldsymbol{p}(n))$. In this work, we will use the weighted maximum norm which for a positive vector $\boldsymbol{w}$ is defined as

$$\|\boldsymbol{x}\|_\infty^w = \max_i \left|\frac{x_i}{w_i}\right| \tag{10}$$

We state the following definitions for an arbitrary norm $\|\cdot\|$.

Contraction Mapping $\boldsymbol{T}(\boldsymbol{p}) : \mathcal{R}^N \to \mathcal{R}^N$ is a contraction mapping of rate α if there exists a non-negative constant $\alpha < 1$ such that for all $\boldsymbol{p}, \boldsymbol{p}'$,

$$\|\boldsymbol{T}(\boldsymbol{p}) - \boldsymbol{T}(\boldsymbol{p}')\| \leq \alpha \|\boldsymbol{p} - \boldsymbol{p}'\| \tag{11}$$

Pseudo-contraction Mapping $\boldsymbol{T}(\boldsymbol{p}) : \mathcal{R}^N \to \mathcal{R}^N$ is a pseudo-contraction of rate α if there exists a fixed point $\bar{\boldsymbol{p}}$ and a non-negative constant $\alpha < 1$ such that for all $\boldsymbol{p}$,

$$\|\boldsymbol{T}(\boldsymbol{p}) - \bar{\boldsymbol{p}}\| \leq \alpha \|\boldsymbol{p} - \bar{\boldsymbol{p}}\| \tag{12}$$

Geometric Convergence A sequence $\{\boldsymbol{p}(n)\}$ is said to converge geometrically at rate α to $\bar{\boldsymbol{p}}$ if there exist non-negative constants A and $\alpha < 1$ such that

$$\|\boldsymbol{p}(n) - \bar{\boldsymbol{p}}\| \leq A\alpha^n$$

It is easy to show that contraction mapping $\boldsymbol{T}(\boldsymbol{p})$ is also a pseudo-contraction mapping. The reader should keep in mind that a mapping $\boldsymbol{T}(\cdot)$ that is a contraction or pseudo-contraction with respect to one norm may not be with respect to another norm. However, if $\boldsymbol{T}(\boldsymbol{p})$ is either a contraction or pseudo-contraction with respect to any norm, then the sequence $\boldsymbol{p}(n)$ generated by the iteration $\boldsymbol{p}(n+1) = \boldsymbol{T}(\boldsymbol{p}(n))$ will converge to a fixed point $\bar{\boldsymbol{p}}$ at a geometric rate such that

$$\|\boldsymbol{p}(n) - \bar{\boldsymbol{p}}\| \leq \alpha^n \|\boldsymbol{p}(0) - \bar{\boldsymbol{p}}\| \tag{13}$$

This work relies on properties of non-negative matrices that derive from the Perron-Frobenius theorem. Typically, the Perron–Frobenius theorem [12, 13] is given for a non-negative matrix $\boldsymbol{A}$ that is irreducible in that for every pair i, j, there exists m such that the i, jth element of $\boldsymbol{A}^m$ is strictly positive. When the assignment l includes a mobile i assigned to the virtual base station 0, the ith row of $\boldsymbol{G}^{(l)}$ will be all zero and thus $\boldsymbol{G}^{(l)}$ is not irreducible. Consequently, we will use a "weak analogue" to the Perron-Frobenius Theorem given in [12] and restated here.[1]

Theorem 1 *If $\boldsymbol{A}$ is a square non-negative matrix, there exists an eigenvalue ρ such that*

(a) ρ is real and non-negative;

(b) with ρ can be associated non-negative left and right eigenvectors;

(c) $\rho \geq |\lambda|$ for any eigenvalue λ of $\boldsymbol{A}$;

Thus, for a non-negative matrix $\boldsymbol{A}$, the Perron-Frobenius eigenvalue is equal to the spectral radius $\rho(\boldsymbol{A})$. We will also need the following claims.

Theorem 2 *[12, Thm. 2.1] For an irreducible non-negative matrix $\boldsymbol{A}$, $\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{c}$ has a non-negative solution $\boldsymbol{x}$ for any non-negative, nonzero $\boldsymbol{c}$ iff $\rho(\boldsymbol{A}) < 1$.*

Theorem 3 *[14, Cor. 6.2 of Sec. 2.6] For a non-negative square matrix $\boldsymbol{A}$, there exists some $\boldsymbol{w} > 0$ such that $\|\boldsymbol{A}\|_\infty^w < 1$ if and only if $\rho(\boldsymbol{A}) < 1$.*

4 Convergence Results

The convergence of the iteration defined by $\boldsymbol{T}(\boldsymbol{p})$ in Equation (5) is addressed in [11]. In particular, [11] defines $\boldsymbol{I}(\boldsymbol{p})$ to be a *standard interference function* if it has the following properties:

- *Positivity* $\boldsymbol{I}(\boldsymbol{p}) > 0$

[1] See Problem 1.12 in [12].

- *Monotonicity* If $\boldsymbol{p} \geq \boldsymbol{p}'$, then $\boldsymbol{I}(\boldsymbol{p}) \geq \boldsymbol{I}(\boldsymbol{p}')$.
- *Scalability* For all $\beta > 1$, $\beta \boldsymbol{I}(\boldsymbol{p}) > \boldsymbol{I}(\beta \boldsymbol{p})$.

It is easy to show that $\boldsymbol{T}(\boldsymbol{p})$ and $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ are standard. For any standard interference function $\boldsymbol{I}(\boldsymbol{p})$, the following conclusions from [11] can be verified.

Lemma 1 *Starting with the all zero vector $\boldsymbol{z}(0)$, the sequence $\boldsymbol{z}(n) = \boldsymbol{I}(\boldsymbol{z}(n-1))$ is monotone increasing.*

Lemma 2 *If $\boldsymbol{u}(0) \geq \boldsymbol{I}(\boldsymbol{u}(0))$, then the sequence $\boldsymbol{u}(n) = \boldsymbol{I}(\boldsymbol{u}(n-1))$ is monotone decreasing.*

Theorem 4 *If there exists $\boldsymbol{u} \geq \boldsymbol{I}(\boldsymbol{u})$, then for any initial power vector $\boldsymbol{p}(0)$, the sequence $\boldsymbol{p}(n) = \boldsymbol{I}(\boldsymbol{p}(n-1))$ converges to a unique fixed point $\bar{\boldsymbol{p}}$ such that $\bar{\boldsymbol{p}} \leq \boldsymbol{u}$ for any $\boldsymbol{u} \geq \boldsymbol{T}(\boldsymbol{u})$.*

We call a power vector $\boldsymbol{u} \geq \boldsymbol{T}(\boldsymbol{u})$ feasible since in the absence of maximum power constraints each user has acceptable CIR under power vector $\boldsymbol{u}$. The same conclusion does not hold when $\boldsymbol{T}(\boldsymbol{p})$ uses virtual base station assignments to implement a maximum power constraint $\boldsymbol{q}$. For example, $\boldsymbol{q} \geq \boldsymbol{T}(\boldsymbol{q})$ holds, but users may or may not have acceptable CIR under power vector $\boldsymbol{q}$. However, by Theorem 4, $\boldsymbol{q} \geq \boldsymbol{T}(\boldsymbol{q})$ implies that the constrained iteration (4) always converges to the unique fixed point $\bar{\boldsymbol{p}}$. When $\bar{p}_i = q_i$, the fixed point is infeasible in that user i will be transmitting at maximum power with an unacceptable CIR. Although the convergence of the iteration (5) is well understood, little has been proven about the rate of convergence, the subject of this paper.

5 Rate of Convergence

In this section, we examine the rate of convergence of the general minimum power assignment algorithm (5). We will see that the existence of a fixed point will imply convergence at a geometric rate to that fixed point. Verification of this claim will require several preliminary lemmas that are proven in Appendix A.

We discuss first the rate of convergence for a fixed assignment l in the absence of maximum power constraints. The results we derive for this case will aid our study of the general minimum power assignment iteration. We will use $\lambda^{(l)} = \rho(\boldsymbol{G}^{(l)})$ to denote the Perron-Frobenius eigenvalue of $\boldsymbol{G}^{(l)}$. From Theorem 3, for an l such that $\rho(\boldsymbol{G}^{(l)}) < 1$, we will use $\boldsymbol{w}^{(l)}$ to denote a weighting vector satisfying $\left\|\boldsymbol{G}^{(l)}\right\|_{\infty}^{\boldsymbol{w}^{(l)}} < 1$. In addition, we will use $\|\boldsymbol{p}\|^{(l)}$ as a shorthand for the weighted maximum norm

$$\|\boldsymbol{p}\|^{(l)} = \|\boldsymbol{p}\|_{\infty}^{\boldsymbol{w}^{(l)}} \tag{14}$$

Throughout this work, we will denote by $\boldsymbol{z}$ the all zero vector, by $\boldsymbol{p}$ an arbitrary power vector, and by $\boldsymbol{u}$ a power vector satisfying $\boldsymbol{u} \geq \boldsymbol{T}(\boldsymbol{p})$. We will use $\boldsymbol{z}(n)$, $\boldsymbol{p}(n)$ and $\boldsymbol{u}(n)$ to denote the result of n iterations of $\boldsymbol{T}(\boldsymbol{p})$ starting $\boldsymbol{z}$, $\boldsymbol{p}$, and $\boldsymbol{u}$ respectively. Similarly, we will use $\boldsymbol{z}^{(l)}(n)$, $\boldsymbol{p}^{(l)}(n)$ and $\boldsymbol{u}^{(l)}(n)$ to denote the corresponding quantities resulting from n iterations of the fixed assignment iteration function $\boldsymbol{T}^{(l)}(\boldsymbol{p})$. Furthermore, when the system includes a maximum power limit $\boldsymbol{q}$, $\boldsymbol{q}(n)$ and $\boldsymbol{q}^{(l)}(n)$ will denote the result of n iterations from $\boldsymbol{q}$.

5.1 Fixed Base Station Assignment

Under fixed assignment, $K_i = \{a_i(l)\}$. Each user i has an assigned base $a_i(l)$ and the power control iteration is

$$\boldsymbol{p}^{(l)}(n+1) = \boldsymbol{T}^{(l)}(\boldsymbol{p}^{(l)}(n)) = \boldsymbol{G}^{(l)}\boldsymbol{p}^{(l)}(n) + \boldsymbol{\sigma}^{(l)} \tag{15}$$

The assignment l specifies a cone of feasible power vectors

$$P^{(l)} = \left\{\boldsymbol{p} \geq 0 | \boldsymbol{p} \geq \boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l)}\right\} \tag{16}$$

The region $P^{(l)}$ is a cone in the sense that if $\boldsymbol{p} \in P^{(l)}$, then $\beta\boldsymbol{p} \in P^{(l)}$ for all $\beta \geq 1$. When $P^{(l)}$ is non-empty, Theorem 4 implies the sequence $\boldsymbol{p}^{(l)}(n)$ converges to the fixed point power vector

$$\bar{\boldsymbol{p}}^{(l)} = \boldsymbol{G}^{(l)}\bar{\boldsymbol{p}}^{(l)} + \boldsymbol{\sigma}^{(l)} \tag{17}$$

The fixed point $\bar{\boldsymbol{p}}^{(l)}$ is the vertex of the cone $P^{(l)}$ since $\bar{\boldsymbol{p}}^{(l)} \leq \boldsymbol{u}^{(l)}$ for all $\boldsymbol{u}^{(l)} \in P^{(l)}$. In Appendix A, we prove the following result as a consequence of Theorems 1 and 2.

Lemma 3 *$P^{(l)}$ is empty iff $\lambda^{(l)} \geq 1$*

If under assignment l, $a_i(l) > 0$ for all i so that each user i is assigned to a real base station, then $\boldsymbol{G}^{(l)}$ is irreducible and Lemma 3 is a trivial consequence of Theorem 2. However, Lemma 3 holds even if $\boldsymbol{G}^{(l)}$ is not irreducible. In particular, Lemma 3 holds for link gain matrices $\boldsymbol{G}^{(l)}$ in which users may be assigned to virtual base stations. Under power vector $\boldsymbol{p} \in P^{(l)}$, each user i satisfying $a_i(l) > 0$ is meeting or exceeding its target CIR γ_i. If $a_i(l) = 0$, user i is transmits with constant power q_i and may or may not be meeting its target CIR.

For the fixed assignment unconstrained iteration, the following claim has been recognized by Mitra [5].

Lemma 4 *When $\lambda^{(l)} < 1$, the fixed assignment mapping $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ is a contraction mapping of rate $\left\|\boldsymbol{G}^{(l)}\right\|^{(l)}$ with respect to the maximum norm weighted by $\boldsymbol{w}^{(l)}$.*

By Lemma 3, we see that the fixed assignment power control algorithm is a contraction mapping whenever the cone $P^{(l)}$ is non-empty. That is, the fixed assignment power control is a contraction mapping iff the assignment is feasible.

5.2 General Minimum Power Assignment

For the general minimum power assignment algorithm, possibly with maximum power constraints, defined by the iteration (5), our objective will be to show that convergence to a fixed point occurs at a geometric rate. The difficulties in proving geometric convergence in this case are twofold. First, for many iterations, an infeasible base station assignment may be used even if the system eventually converges to a feasible base station assignment. Second, because iterations performed using feasible assignments are contractions with respect to a norm that depends on the assignment, a sequence of iterations using a sequence of assignments may not be a contraction with respect to any norm. Thus, Lemma 4 has no immediate generalization.

To address the rate of convergence, the algorithm must converge. Thus, we assume the existence of a fixed point $\bar{\boldsymbol{p}} = \boldsymbol{T}(\bar{\boldsymbol{p}})$. By Theorem 4, the existence of a feasible point $\bar{\boldsymbol{p}}$ implies fixed point is unique. Associated with the fixed point $\bar{\boldsymbol{p}}$ is a corresponding assignment $\bar{l}$ such that

$$\bar{\boldsymbol{p}} = \boldsymbol{G}^{(\bar{l})}\bar{\boldsymbol{p}} + \boldsymbol{\sigma}^{(\bar{l})} \tag{18}$$

Note that Lemma 3 implies that $\lambda^{(\bar{l})} = \rho(\boldsymbol{G}^{(\bar{l})}) < 1$. Hence, by Theorem 3,

$$\left\|\boldsymbol{G}^{(\bar{l})}\right\|^{(\bar{l})} < 1 \tag{19}$$

We make the assumption that the fixed point assignment $\bar{l}$ is unique. This assumption, although reasonable in practice, can be violated. For example, when two base stations share the same antenna and thus the same uplink gains from every mobile, a mobile can be assigned to either base station. The assumption is also violated when at the fixed point $\bar{\boldsymbol{p}}$, user i has power $\bar{p}_i = q_i$ at a real base station $a_i \neq 0$. In this case, user i could also be assigned to the virtual base station 0.

Our approach will be to show geometric convergence to the fixed point $\bar{\boldsymbol{p}}$ from zero power $\boldsymbol{z}$ and from a feasible power vector $\boldsymbol{u}$. This will imply geometric convergence from any power vector $\boldsymbol{p}$. The uniqueness assumption is needed to show that starting from zero power, any assignment other than the fixed point assignment will be used only a finite number of times. If there were two assignments corresponding to the fixed point, then each could be used infinitely often. We start with the following lemma that says a finite number of iterations with an assignment $l \neq \bar{l}$ will yield a power vector that is greater than $\bar{\boldsymbol{p}}$ in at least one component.

Lemma 5 *For each $l \neq \bar{l}$, there exists $M_l < \infty$ such that for some i, $p_i^{(l)}(M_l) > \bar{p}_i$.*

Let $m_l(n)$ denote the number of times assignment l is used in first n iterations starting from $\boldsymbol{z}$. We recall that $\boldsymbol{z}(n)$ denotes the power vector obtained by n iterations of $\boldsymbol{T}(\boldsymbol{p})$ starting from $\boldsymbol{z}$, the all zero vector. Also starting from $\boldsymbol{z}$, $\boldsymbol{z}^{(l)}(m_l(n))$ denotes the power vector obtained by $m_l(n)$ iterations under the fixed assignment iteration function $\boldsymbol{T}^{(l)}(\boldsymbol{p})$. In generating the sequence $\boldsymbol{z}(0), \ldots, \boldsymbol{z}(n)$, the iteration function $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ was used $m_l(n)$ times. Since $\boldsymbol{z}(n)$ is a monotone increasing sequence, the next lemma says that omitting those iterations that used assignments other than l should result in a reduced power vector.

Lemma 6 $\boldsymbol{z}(n) \geq \boldsymbol{z}^{(l)}(m_l(n))$

We will now use the result of Lemma 6 to show that the number of times an infeasible assignment l can be used is upper bounded by M_l from Lemma 5.

Lemma 7 *If $l \neq \bar{l}$, then $\lim_{n\to\infty} m_l(n) < M_l$.*

We now verify geometric convergence from zero power $\boldsymbol{z}$ and from a feasible power vector $\boldsymbol{u}$.

Lemma 8 *The sequence $\boldsymbol{z}(n)$ converges geometrically to the fixed point $\bar{\boldsymbol{p}}$.*

Proof: By Lemma 1 and Theorem 4, $\boldsymbol{z}(n)$ is monotone increasing sequence that converges to $\bar{\boldsymbol{p}}$. Thus, $\boldsymbol{z}(n) \leq \boldsymbol{z}(n+1) \leq \bar{\boldsymbol{p}}$. Since the fixed point $\bar{\boldsymbol{p}}$ is unique, if

$\boldsymbol{z}(n) \neq \bar{\boldsymbol{p}}$, then $\boldsymbol{z}(n) \neq \boldsymbol{z}(n+1)$. Thus, there exists $\alpha_n \in [0,1)$ such that

$$\|\boldsymbol{z}(n) - \bar{\boldsymbol{p}}\|^{(\bar{l})} \leq \alpha_n \|\boldsymbol{z}(n+1) - \bar{\boldsymbol{p}}\|^{(\bar{l})} \tag{20}$$

Let $l(n)$ denote the assignment used at iteration n. We define

$$I = \left\{n | l(n) \neq \bar{l}\right\} \qquad \bar{I} = \left\{n | l(n) = \bar{l}\right\} \tag{21}$$

Since an assignment $l \neq \bar{l}$ is never used more than M_l times, the cardinality of I satisfies $|I| \leq \sum_{l \neq \bar{l}} M_l < \infty$. Since $\alpha_n < 1$ for all n and since I is a finite set, $\max_{n \in I} \alpha_n < 1$. If at step n, the fixed point assignment $\bar{l}$ is used, Lemma 4 implies that $\alpha_n \leq \left\|\boldsymbol{G}^{(\bar{l})}\right\|^{(\bar{l})} < 1$. Thus, for all n,

$$\alpha_n \leq \max\left\{\max_{n \in I} \alpha_n, \left\|\boldsymbol{G}^{(\bar{l})}\right\|^{(\bar{l})}\right\} = \alpha < 1$$

Hence, from Equation (20), $\|\boldsymbol{z}(n) - \bar{\boldsymbol{p}}\|^{(\bar{l})} \leq \alpha^n \|\bar{\boldsymbol{p}}\|^{(\bar{l})}$ and the sequence $\boldsymbol{z}(n)$ converges geometrically to $\bar{\boldsymbol{p}}$. $\square$

Lemma 9 *Starting from $\boldsymbol{u} \geq \boldsymbol{T}(\boldsymbol{u})$, the sequence $\boldsymbol{u}(n)$ converges to the fixed point $\bar{\boldsymbol{p}}$ at a geometric rate.*

Proof: Let l_n denote the assignment used at step n. Since the base assignment is chosen to minimize the transmitted power, if assignment l_n is used at stage n, then for all assignments l at stage n,

$$\boldsymbol{T}^{(l_n)}(\boldsymbol{u}(n)) = \boldsymbol{G}^{(l_n)}\boldsymbol{u}(n) + \boldsymbol{\sigma}^{(l_n)} \leq \boldsymbol{T}^{(l)}(\boldsymbol{u}(n)) = \boldsymbol{G}^{(l)}\boldsymbol{u}(n) + \boldsymbol{\sigma}^{(l)} \tag{22}$$

By Lemma 2 and Theorem 4, $\boldsymbol{u}(n)$ is monotone decreasing sequence that converges to $\bar{\boldsymbol{p}}$. This implies $\boldsymbol{T}(\boldsymbol{u}(n)) \geq \bar{\boldsymbol{p}}$. Since (22) holds for $l = \bar{l}$, we can use Equation (18) to write

$$\begin{aligned}
\|\boldsymbol{T}(\boldsymbol{u}(n)) - \bar{\boldsymbol{p}}\|^{(\bar{l})} &= \left\|\boldsymbol{G}^{(l_n)}\boldsymbol{u}(n) + \boldsymbol{\sigma}^{(l_n)} - (\boldsymbol{G}^{(\bar{l})}\bar{\boldsymbol{p}} + \boldsymbol{\sigma}^{(\bar{l})})\right\|^{(\bar{l})} && (23)\\
&\leq \left\|\boldsymbol{G}^{(\bar{l})}(\boldsymbol{u}(n) - \bar{\boldsymbol{p}})\right\|^{(\bar{l})} && (24)\\
&\leq \left\|\boldsymbol{G}^{(\bar{l})}\right\|^{(\bar{l})} \|\boldsymbol{u}(n) - \bar{\boldsymbol{p}}\|^{(\bar{l})} && (25)
\end{aligned}$$

Since $\left\|\boldsymbol{G}^{(\bar{l})}\right\|^{(\bar{l})} < 1$, the sequence $\boldsymbol{p}(n)$ converges geometrically to the fixed point $\bar{\boldsymbol{p}}$.

□

Theorem 5 *Starting from any initial power vector* $\boldsymbol{p}$*, the sequence* $\boldsymbol{p}(n)$ *converges geometrically to the fixed point* $\bar{\boldsymbol{p}}$*.*

Proof: Given an initial power vector $\boldsymbol{p} = \boldsymbol{p}(0)$, we can find $\beta > 1$ such that $\boldsymbol{u} = \beta\bar{\boldsymbol{p}} \geq \boldsymbol{p}$. Scalability of $\boldsymbol{T}(\boldsymbol{p})$ implies $\boldsymbol{u} \geq \boldsymbol{T}(\boldsymbol{u})$. Monotonicity of $\boldsymbol{T}(\boldsymbol{p})$ implies

$$\boldsymbol{z}(n) \leq \boldsymbol{p}(n) \leq \boldsymbol{u}(n) \tag{26}$$

Since $\boldsymbol{z}(n) \leq \bar{\boldsymbol{p}} \leq \boldsymbol{u}(n)$, we must have $|p_i(n) - \bar{p}_i| \leq |u_i(n) - z_i(n)|$. This implies

$$\begin{aligned} \|\boldsymbol{p}(n) - \bar{\boldsymbol{p}}\|^{(\bar{l})} &\leq \|\boldsymbol{u}(n) - \boldsymbol{z}(n)\|^{(\bar{l})} && (27) \\ &= \|\boldsymbol{u}(n) - \bar{\boldsymbol{p}} - (\boldsymbol{z}(n) - \bar{\boldsymbol{p}})\|^{(\bar{l})} && (28) \\ &\leq \|\boldsymbol{u}(n) - \bar{\boldsymbol{p}}\|^{(\bar{l})} + \|\boldsymbol{z}(n) - \bar{\boldsymbol{p}}\|^{(\bar{l})} && (29) \end{aligned}$$

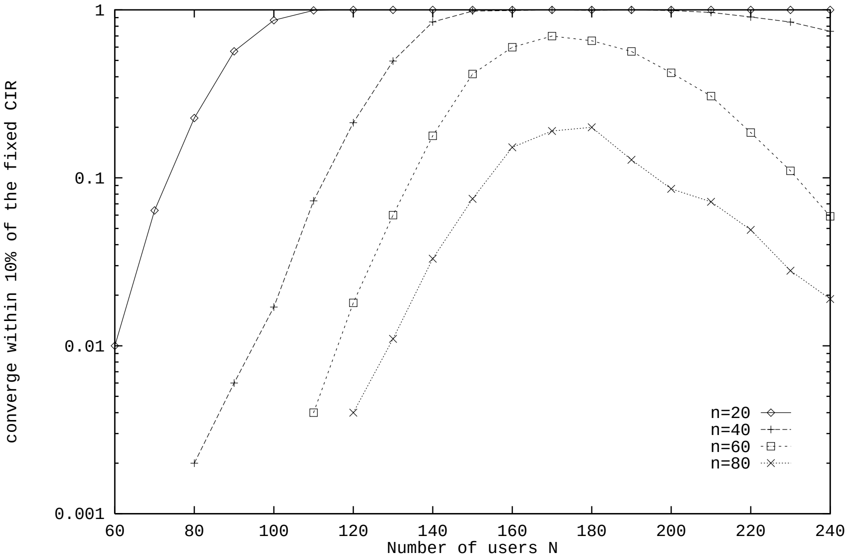
Since $\boldsymbol{z}(n)$ and $\boldsymbol{u}(n)$ converge geometrically to $\bar{\boldsymbol{p}}$, there exists $A > 0$ and $\alpha \in [0, 1)$ such that $\|\boldsymbol{u}(n) - \bar{\boldsymbol{p}}\|^{(l)} \leq (A/2)\alpha^n$ and $\|\boldsymbol{z}(n) - \bar{\boldsymbol{p}}\| \leq (A/2)\alpha^n$. Thus, $\|\boldsymbol{p}(n) - \bar{\boldsymbol{p}}\|^{(\bar{l})} \leq A\alpha^n$. □

6 Simulation Results

Although we have verified that the general minimum power assignment algorithm converges geometrically, the rate constant α depends on the number of users in the system and the uplink gains h_{ki}. In this section we compare by simulation the rate of convergence of the minimum power assignment (MPA) scheme and a simple fixed base station assignment scheme. For the simulations, we implement a maximum power limit $\boldsymbol{q}$ and the resulting constrained MPA will be called the CMPA algorithm.

The simulation employs a CDMA system with ten base stations uniformly spaced every 2000 meters on a ring. For each base station, the uplink gain included an order $\alpha = 4$ propagation loss and a position dependent lognormal shadow fading component. That is, expressed in dB, the uplink gain h_{ki} of user i to base k at a distance d_{ki} is

$$10 \log h_{ki} = -10\alpha \log d_{ki} + S_k(d_{ki}) \tag{30}$$



As a function of the number of CMPA iterations n and the number of mobiles N, this plot records the fraction of trials that fail to meet the sufficient condition (32) to guarantee for all users a CIR within 10% (0.4 dB) of the fixed point CIR $\bar{\gamma}_i$.

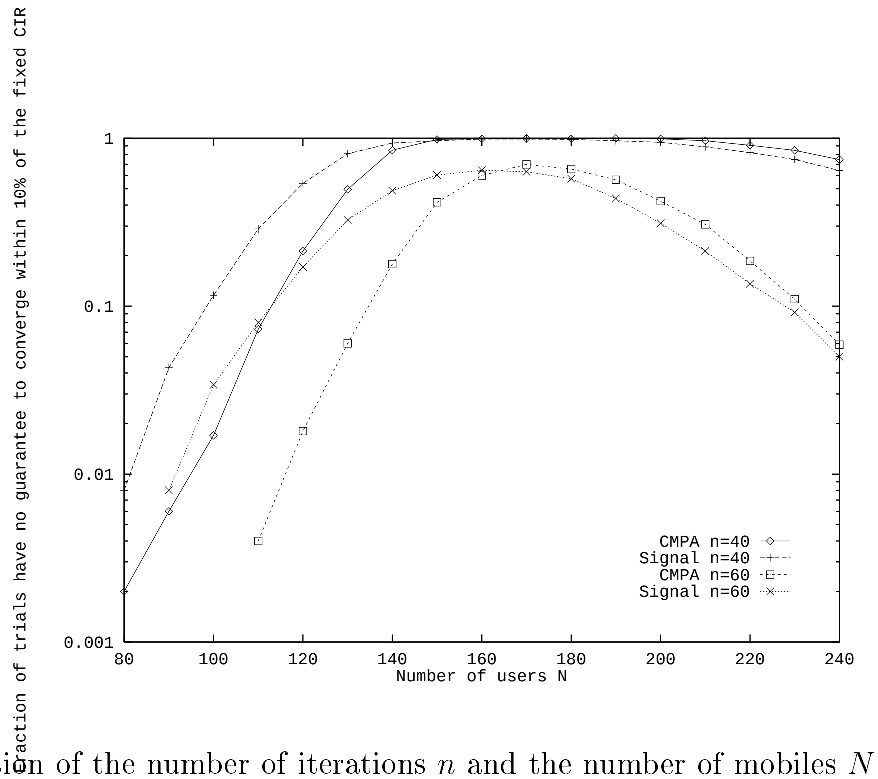
Figure 1: Convergence of the constrained MPA (CMPA) algorithm.

The shadow fading component $S_k(d_{ki})$ was taken to be a normal random variable with mean 0 and standard deviation $\eta = 8$ dB. The shadow fading processes $S_k(d)$ and $S_{k'}(d')$ for distinct base stations k and k' were assumed to be independent. The correlation over distance of the shadow fading was described by the autocorrelation function

$$\rho(d) = E[S_j(d')S_j(d'+d)] = \eta^2 e^{-|d|/d_0} \tag{31}$$

with a correlation distance of $d_0 = 45$ meters. Each mobile aimed for a CIR target of $\gamma = 1/20$ that reflected the processing gain associated with a CDMA system.

We would like power control algorithms to converge quickly since this would suggest that in the event that a set of CIR targets are feasible, the power control algorithm can track the variations in the radio propagation. Alternatively, fast convergence to the fixed point will permit fast detection of an infeasible situation. We would like to express the closeness of a power vector to the fixed point power vector in terms of the closeness of the resulting CIRs to the CIRs at the fixed point. Under power vector $\boldsymbol{p}$, let the CIR of user i at the base station where its CIR is maximized be denoted by $\Gamma_i(\boldsymbol{p})$. For the CMPA algorithm, we can use iterations starting from the zero vector



As a function of the number of iterations n and the number of mobiles N, this plot records the fraction of trials that fail to meet the sufficient condition (32) to guarantee for all users a CIR within 10% (0.4 dB) of the fixed point CIR $\bar{\gamma}_i$.

Figure 2: Convergence of the constrained MPA (CMPA) algorithm and the signal based assignment.

$\boldsymbol{z}$ and the maximum power vector $\boldsymbol{q}$ to derive bounds on $\Gamma_i(\boldsymbol{p})$.

Lemma 10 *If*

$$\boldsymbol{q}(n) \leq (1+\epsilon)\boldsymbol{z}(n) \tag{32}$$

then the CIR of each user i after n iterations satisfies

$$\frac{1-\epsilon}{1+\epsilon}\bar{\gamma}_i \leq \Gamma_i(\boldsymbol{p}(n)) \leq \frac{1+\epsilon}{1-\epsilon}\bar{\gamma}_i \tag{33}$$

Thus Lemma 10 provides a sufficient condition that guarantees that the CIR of each user i is within $10\log_{10}[(1+\epsilon)/(1-\epsilon)]$ dB of the fixed point CIR $\bar{\gamma}_i$. In particular, for $\epsilon = 0.05$, (32) implies that each user i is within 0.4 dB of the fixed point SIR.

To provide some indication of the rate of convergence, we repeatedly placed N stationary mobiles randomly on the ring and and ran the CMPA algorithm both from zero power $\boldsymbol{z}$ and maximum power $\boldsymbol{q}$. For each random placement of the mobile users, we recorded as a function of the number of synchronous CMPA iterations n whether the sufficient condition (32) for $\epsilon = 0.05$ to guarantee the mobiles were within 0.4

dB of the fixed point CIR. In Figure 1, we considered $n = 20, 40, 60, 80$ trials and a number of mobiles N that ranged from 60 to 240. We see that for a small number of users, $N = 100$ or less, 40 iterations is almost always sufficient to guarantee that the users are within 0.4 dB. As the number of users increases, the required number of MPA iterations also increases.

Note that with a CIR target of $\gamma = 1/20$, a single base station in isolation would be able to support 20 mobiles. Hence, in the simulated system of ten base stations with a CIR target of $\gamma = 1/20$, it is impossible to carry more than 200 calls, even in the absence of maximum power constraints. Thus, as the system was overloaded with more than 200 users, we observed that the required number of CMPA iterations to ensure that the users will be within 0.4 dB of the fixed point CIR began to decrease. This occurred because every possible assignment l was infeasible, and as a result, starting from zero initial power, the powers rapidly increased, and thus converged quickly, to the fixed point $\bar{\boldsymbol{p}}$.

With the same convergence criterion, Figure 2 compared the convergence of the CMPA algorithm in which the base station is iteratively chosen to minimize the transmitted power to a constrained iteration using a fixed base station assignment l that maximizes each user's uplink gain. This fixed base station approach is called the "signal" based assignment scheme since it can be implemented by having mobile users estimate the power from a downlink pilot tone signal. In making this comparison, our primary goal was to see whether always trying to minimize the transmitted power over the choice of base station assignments caused the convergence to be slower.

Because the CMPA algorithm tries to minimize the transmitter powers, $\boldsymbol{T}(\boldsymbol{p}) \leq \boldsymbol{T}^{(l)}(\boldsymbol{p})$, yielding two distinct effects. First, when starting at zero power $\boldsymbol{z}$, $\boldsymbol{z}(n) \leq \boldsymbol{z}^{(l)}(n)$. In this case, CMPA converges more slowly because powers increase as slowly as possible. Second, when starting at maximum power $\boldsymbol{q}$, $\boldsymbol{q}(n) \leq \boldsymbol{q}^{(l)}(n)$. In this case, the effort to reduce powers results in faster convergence for the CMPA algorithm.

From the simulation results of Figure 2, we find that both effects can be observed. Under a low to moderate load of $N \leq 160$ users, the fixed point $\bar{\boldsymbol{p}}$ tends to be close to $\boldsymbol{z}$ and the convergence criterion is limited by the convergence of $\boldsymbol{q}(n)$. In this case, the CMPA algorithm, which converges faster than the signal based assignment scheme when starting from maximum power, tended to meet the convergence criterion more quickly. On the other hand, as the offered load increases, the fixed point $\bar{\boldsymbol{p}}$ moves away from $\boldsymbol{z}$ and closer to the maximum power vector $\boldsymbol{q}$. According to our convergence criterion, the convergence speed will more likely be limited by the convergence from zero power $\boldsymbol{z}$. Thus, when the load was high ($N \geq 180$), the CMPA algorithm converged slower than the signal based assignment algorithm because the CMPA is always trying to use minimum power.

7 Discussion

In wireless communication systems, mobile users adapt to a time varying radio channel by regulating transmitter power. This power control is intended to provide each user an acceptable connection, as measured by a carrier to interference ratio (CIR), by eliminating unnecessary interference. It is important that a power control algorithm can quickly converge to a desirable solution when a fixed radio channel permits such a solution. We have shown that if the system can support all users at the target CIR, iterative power control guarantees convergence to a fixed point at a geometric rate for both fixed base station assignment and minimum power assignment. In the presence of maximum power constraints, geometric convergence to a unique fixed point always occurs.

Simulation results were based on a CDMA system. As described in [15], power control for CDMA systems is needed to compensate for the signal fluctuation caused by propagation loss, lognormal fading and slow Rayleigh fading. Error correction coding and interleaving can be used to mitigate the signal degradation caused by fast fading. In the proposed IS-95 CDMA system standard [16], power adjustments occur every 1.25 msec (800 times per second) for closed loop power control. In this context, our simulation results suggest that iterative power control algorithms are fairly practical in term of convergence speed.

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A Additional Proofs

Proof: Lemma 3 Under assignment l, let $I = \{i | a_i(l) = 0\}$ denote the set of mobiles assigned to the virtual base station 0. We construct the matrix $\boldsymbol{G}'$ by deleting from $\boldsymbol{G}^{(l)}$ each row and column $i \in I$. First, we will show that $\boldsymbol{G}'$ and $\boldsymbol{G}^{(l)}$ have the same positive eigenvalues. Suppose $\lambda > 0$ and $\boldsymbol{x}$ are an eigenvalue and corresponding eigenvector of $\boldsymbol{G}^{(l)}$. For all $i \in I$, $\lambda x_i = [0 \cdots 0]\boldsymbol{x} = 0$. Since $\lambda > 0$, we must have $x_i = 0$ for $i \in I$. Let $\boldsymbol{x}'$ denote the vector created by deleting all components $i \in I$ of $\boldsymbol{x}$. By construction, $\lambda \boldsymbol{x}' = \boldsymbol{G}'\boldsymbol{x}'$ and so λ is an eigenvalue of $\boldsymbol{G}'$. Given an eigenvalue $\lambda' > 0$ and a corresponding eigenvector $\boldsymbol{x}'$ of $\boldsymbol{G}'$, we can construct the vector $\boldsymbol{x}$ such that $x_i = 0$ for $i \in I$ and also removing those components $i \in I$ from $\boldsymbol{x}$ yields $\boldsymbol{x}'$. In this case, $\lambda \boldsymbol{x} = \boldsymbol{G}^{(l)}\boldsymbol{x}$ and so λ is an eigenvalue of $\boldsymbol{G}^{(l)}$. Thus, $\boldsymbol{G}^{(l)}$ and $\boldsymbol{G}'$ share the same Perron-Frobenius eigenvalue $\lambda^{(l)}$. Since $\boldsymbol{G}'$ is a matrix of all nonzero link gains, it is irreducible. Theorem 2 implies that $\boldsymbol{p}' = \boldsymbol{G}'\boldsymbol{p}' + \boldsymbol{\sigma}'$ has a non-negative solution $\boldsymbol{p}'$ for all positive $\boldsymbol{\sigma}'$ iff $\lambda^{(l)} < 1$. However, by construction of $\boldsymbol{G}'$, $\boldsymbol{p}' = \boldsymbol{G}'\boldsymbol{p}' + \boldsymbol{\sigma}'$ has a non-negative solution for all positive $\boldsymbol{\sigma}'$ iff $\boldsymbol{p} = \boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma}^{(l)}$ has a non-negative solution for all positive $\boldsymbol{\sigma}^{(l)}$. □

Proof: Lemma 4 By Theorem 3, $\lambda^{(l)} < 1$ implies $\left\|\boldsymbol{G}^{(l)}\right\|^{(l)} < 1$. For any two power vectors $\boldsymbol{p}$, $\boldsymbol{p}'$, we have

$$\begin{aligned}
\left\|\boldsymbol{T}^{(l)}(\boldsymbol{p}) - \boldsymbol{T}^{(l)}(\boldsymbol{p}')\right\|^{(l)} &= \left\|\boldsymbol{G}^{(l)}\boldsymbol{p} + \boldsymbol{\sigma} - (\boldsymbol{G}^{(l)}\boldsymbol{p}' + \boldsymbol{\sigma})\right\|^{(l)} && (34)\\
&= \left\|\boldsymbol{G}^{(l)}(\boldsymbol{p} - \boldsymbol{p}')\right\|^{(l)} && (35)\\
&\leq \left\|\boldsymbol{G}^{(l)}\right\|^{(l)} \|\boldsymbol{p} - \boldsymbol{p}'\|^{(l)} && (36)
\end{aligned}$$

□

Proof: Lemma 5 Since $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ is a monotone mapping, $\boldsymbol{p}^{(l)}(n) \geq \boldsymbol{z}^{(l)}(n)$. Thus, it is sufficient to show that there exists $M < \infty$ such that $z_i^{(l)}(M) > \bar{p}_i$. Since $l \neq \bar{l}$, either $\lambda^{(l)} < 1$ and $\boldsymbol{z}^{(l)}$ converges monotonically to $\bar{\boldsymbol{p}}^{(l)}$ or $\lambda^{(l)} \geq 1$ and assignment l is infeasible. In the former case, since $\bar{\boldsymbol{p}}^{(l)} \geq \bar{\boldsymbol{p}}$ and $\bar{\boldsymbol{p}}^{(l)} \neq \bar{\boldsymbol{p}}$, there exists i such that $\epsilon = (\bar{p}_i^{(l)} - \bar{p}_i)/2 > 0$. Moreover, there exists $M < \infty$ such that $\left|\bar{p}_i^{(l)} - z_i^{(l)}(M)\right| < \epsilon$. Thus, $z_i^{(l)}(M) > \bar{p}_i$. When $\lambda^{(l)} \geq 1$, let $\hat{\boldsymbol{G}}$, $\hat{\boldsymbol{\sigma}}$, and $\hat{\lambda}$ denote $\boldsymbol{G}^{(l)}$, $\boldsymbol{\sigma}^{(l)}$, and $\lambda^{(l)}$ respectively. We can write

$$\boldsymbol{z}^{(l)}(n) = \sum_{k=0}^{n-1} \hat{\boldsymbol{G}}^k \hat{\boldsymbol{\sigma}} \tag{37}$$

Let $\boldsymbol{e}$ denote the eigenvector of $\hat{\boldsymbol{G}}$ corresponding to the largest eigenvalue $\hat{\lambda}$. For some i, $a_i(l) > 0$ since otherwise $\boldsymbol{G}^{(l)}$ is the all zero matrix and has eigenvalue $\lambda^{(l)} = 0$ which would be a contradiction. By Theorem 1, $\boldsymbol{e}$ is non-negative and nonzero and $e_i > 0$. Since $\hat{\boldsymbol{\sigma}} > 0$, we can choose $b > 0$ such that $b\boldsymbol{e} \leq \hat{\boldsymbol{\sigma}}$. Since $\hat{\boldsymbol{G}}^k$ is a non-negative matrix, we observe that

$$\hat{\boldsymbol{G}}^k \hat{\boldsymbol{\sigma}} \geq \hat{\boldsymbol{G}}^k b\boldsymbol{e} = b\hat{\lambda}^k \boldsymbol{e}$$

Thus, from (37),

$$z_i^{(l)}(n) \geq b \sum_{k=0}^{n-1} \hat{\lambda}^k e_i$$

Since $\hat{\lambda} \geq 1$, $e_i > 0$, and $b > 0$, there exists M such $z_i^{(l)}(M) > \bar{p}_i$. $\square$

Proof: Lemma 6 We prove the claim by induction. For $n = 0$, we trivially have $\boldsymbol{z} = \boldsymbol{z}(0) = \boldsymbol{z}^{(l)}(0)$. Assuming the claim holds for $n-1$ yields $\boldsymbol{z}(n-1) \geq \boldsymbol{z}^{(l)}(m_l(n-1))$. By Lemma 1, $\boldsymbol{z}(n) \geq \boldsymbol{z}(n-1)$. If at iteration n, assignment l is not used, then $m_l(n) = m_l(n-1)$ and

$$\boldsymbol{z}(n) \geq \boldsymbol{z}(n-1) \geq \boldsymbol{z}^{(l)}(m_l(n-1)) = \boldsymbol{z}^{(l)}(m_l(n))$$

If at iteration n, assignment l is used, then $m_l(n) = m_l(n-1)+1$ and the monotone property of $\boldsymbol{T}^{(l)}(\boldsymbol{p})$ implies

$$\boldsymbol{z}(n) = \boldsymbol{T}^{(l)}(\boldsymbol{z}(n-1)) \geq \boldsymbol{T}^{(l)}(\boldsymbol{z}^{(l)}(m_l(n-1)) = \boldsymbol{z}^{(l)}(m_l(n-1)+1) = \boldsymbol{z}^{(l)}(m_l(n))$$

$\square$

Proof: Lemma 7 Suppose there exists n such that $m_l(n) \geq M_l$. By Lemma 6, $\boldsymbol{z}(n) \geq \boldsymbol{z}^{(l)}(m_l(n))$. Since $\boldsymbol{z}^{(l)}(n)$ is monotone increasing in n, $\boldsymbol{z}^{(l)}(m_l(n)) \geq \boldsymbol{z}^{(l)}(M_l)$ Since $l \neq \bar{l}$, $z_i^{(l)}(M_l) > \bar{p}_i$ for some i. Thus for that i, $z_i(n) > \bar{p}_i$ which is a contradiction since $\boldsymbol{z}(n)$ is a monotonically increasing sequence that converges to $\bar{p}$. $\square$

Proof: Lemma 10 By the monotonicity property of $\boldsymbol{T}(\boldsymbol{p})$, $\boldsymbol{z}(n) \leq \bar{\boldsymbol{p}} \leq \boldsymbol{q}(n)$. Thus, $\boldsymbol{q}(n) \leq (1+\epsilon)\boldsymbol{z}(n)$ implies

$$\boldsymbol{q}(n) \leq (1+\epsilon)\bar{\boldsymbol{p}} \tag{38}$$

and that

$$(1-\epsilon)\bar{\boldsymbol{p}} \leq \frac{\bar{\boldsymbol{p}}}{1+\epsilon} \leq \frac{\boldsymbol{q}(n)}{1+\epsilon} \leq \boldsymbol{z}(n) \tag{39}$$

Combining (38) and (39) yields

$$(1-\epsilon)\bar{\boldsymbol{p}} \leq \boldsymbol{z}(n) \leq \boldsymbol{p}(n) \leq \boldsymbol{q}(n) \leq (1+\epsilon)\bar{\boldsymbol{p}} \tag{40}$$

From Equations (1), (3) and (8), the CIR of user i under power vector $\boldsymbol{p}$ and assignment l can be written as

$$\Gamma_i^{(l)}(\boldsymbol{p}) = \frac{p_i}{\boldsymbol{G}_i^{(l)}\boldsymbol{p} + \sigma_i^{(l)}} \tag{41}$$

where $\boldsymbol{G}_i^{(l)}$ denote the ith row of $\boldsymbol{G}^{(l)}$. Let l_n denote the base station assignment after n iterations and let $\bar{l}$ denote the assignment corresponding to the fixed point $\bar{\boldsymbol{p}}$. Thus, $\Gamma_i^{(l_n)}(\boldsymbol{p}(n)) = \Gamma_i(\boldsymbol{p}(n))$ and $\Gamma_i^{(\bar{l})}(\bar{\boldsymbol{p}}) = \Gamma_i(\bar{\boldsymbol{p}})$. Since at every iteration, the MPA algorithm maximizes the CIR over all base station assignments l, we observe that $\Gamma_i^{(l_n)}(\boldsymbol{p}(n)) \geq \Gamma_i^{(l)}(\boldsymbol{p}(n))$ for all l. In this case, (40) and (41) imply that

$$\begin{aligned}
\Gamma_i^{(l_n)}(\boldsymbol{p}(n)) &\leq \frac{(1+\epsilon)\bar{p}_i}{\boldsymbol{G}_i^{(l_n)}(1-\epsilon)\bar{\boldsymbol{p}} + (1-\epsilon)\sigma_i^{(l_n)}} && (42)\\
&= \frac{1+\epsilon}{1-\epsilon}\Gamma_i^{(l_n)}(\bar{\boldsymbol{p}}) && (43)\\
&\leq \frac{1+\epsilon}{1-\epsilon}\Gamma_i^{(\bar{l})}(\bar{\boldsymbol{p}}) && (44)\\
&= \frac{1+\epsilon}{1-\epsilon}\bar{\gamma}_i && (45)
\end{aligned}$$

Next, we observe that

$$\Gamma_i^{(l_n)}(\boldsymbol{p}(n)) \geq \Gamma_i^{(\bar{l})}(\boldsymbol{p}(n)) \geq \frac{\bar{p}_i(1-\epsilon)}{\boldsymbol{G}_i^{(\bar{l})}\bar{\boldsymbol{p}}(1+\epsilon) + \sigma_i^{(\bar{l})}(1+\epsilon)} = \frac{1-\epsilon}{1+\epsilon}\bar{\gamma}_i \tag{46}$$

to complete the proof. □

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