Therefore, we can now compute the probability distribution of r_1 and ϕ_1 as follows:

$$p(r_1) = \int r_1 p(r_1, \phi_1) d\phi_1 = \frac{2\pi^n}{V\Gamma(n-1)} r_1 (n - r_1)^{n-1}$$

$$= \frac{2}{n^{n-1}} r_1 (n - r_1^2)^{n-1}$$

$$p(\phi_1) = \int r_1 p(r_1, \phi_1) dr_1 = \frac{1}{2\pi}.$$

Also, since $p(r_1, \phi_1) = p(r_1)p(\phi_1)$, r_1 and ϕ_1 are independent. □

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User Capacity of Asynchronous CDMA Systems With Matched Filter Receivers and Optimum Signature Sequences

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***Abstract*—For a symbol-asynchronous (but chip-synchronous) single-cell code-division multiple-access (CDMA) system, we define a system-wide quantity called the total squared asynchronous correlation (TSAC) which, for arbitrary signature sets, depends on the users' delay profile. We develop a lower bound for TSAC that is independent of the users' delays. We show that if the signature set achieves this TSAC lower bound, then the user capacity of the asynchronous CDMA system using matched filters becomes the same as that of a single-cell synchronous CDMA system; in this case, there is no loss in user capacity due to asynchronism. We present iterative signature adaptation algorithms, which, when executed sequentially by the users, appear to converge to these optimum signature sequences; however, the existence, for all user delay profiles, of signature sequences achieving this lower bound remains a significant open problem.**

***Index Terms*—Asynchronous code-division multiple access (CDMA), CDMA user capacity, interference avoidance, minimum mean-square error (MMSE) filters, optimum signature sequence sets, Welch bound equality (WBE) sequences, Welch bound.**

I. INTRODUCTION

For code-division multiple-access (CDMA) systems, there has been recent progress in understanding the influence of signature sequences on the overall system capacity [3]–[5]. In particular, for a single-cell synchronous CDMA system with equal received powers, [3] showed that one can always choose the signature sequences to be Welch bound equality (WBE) sequences [6]–[8] and that WBE sequences maximize the user capacity, i.e., the maximum number of supportable users at a common signal-to-interference ratio (SIR) target level for a fixed processing gain. A generalized version of this problem where users have arbitrary (unequal) received powers was solved in [4].

In this correspondence, we investigate the user capacity of an *asynchronous* single-cell CDMA system with matched filters, under the assumption that the users' signature sequences can be optimized. Even though the system is symbol-asynchronous, we assume that it is chip-synchronous (e.g., as in [9]) in order to make the analysis tractable. We also assume that short sequences are used; that is, the length of the signature sequences is equal to one symbol duration, and that the signature sequences are repeated at every symbol interval. We define a quantity called the total squared asynchronous correlation (TSAC) of a signature sequence set. For arbitrary signature sequences, the TSAC depends on the users' delay profile. We identify a lower bound on the TSAC that is independent of the users' delay profile. For those delay profiles for which there exist signature sets that achieve the TSAC lower bound, we show that an asynchronous system in which each user employs a matched filter receiver over a single-symbol interval has the same user

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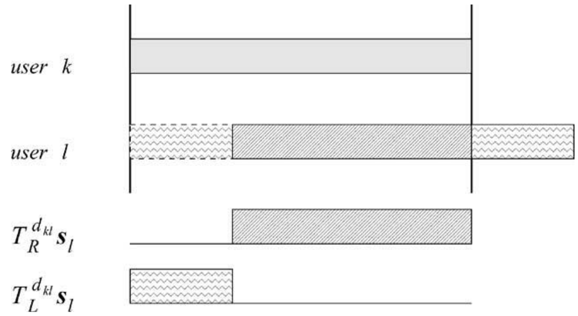

Fig. 1. Asynchronous interference calculation.

capacity as a symbol synchronous system; in this case, there is no loss in user capacity due to asynchronism. We emphasize, however, that the existence, for all user delay profiles, of signature sets achieving this lower bound on the TSAC is a significant open problem.

By extending techniques for iterative signature optimization for synchronous CDMA systems, [10]–[14], we present signature adaptation algorithms where, at each iteration, only one user updates its signature sequence to decrease the TSAC of the entire set. Experimentally, we observe that these distributed adaptation algorithms converge to signature sets that achieve the lower bound on the TSAC.

II. User Capacity for the Asynchronous System

We consider a single-cell symbol-asynchronous (but chip-synchronous) CDMA system with K users and processing gain N. The received signal in the nth symbol interval of user k is given as (see Fig. 1)

$$\boldsymbol{r}_k(n)=\sqrt{p_k}b_k(n)\boldsymbol{s}_k+\sum_{l\neq k}\sqrt{p_l}\Big(b_l(n)T_L^{d_{kl}}\boldsymbol{s}_l+b_l(n+1)T_R^{d_{kl}}\boldsymbol{s}_l\Big)+\boldsymbol{n}_k \tag{1}$$

where p_k, $b_k(n)$, and $\boldsymbol{s}_k$ are the received power, the nth transmitted symbol, and the signature sequence of user k, respectively, and $\boldsymbol{n}_k$ is a zero-mean Gaussian random vector with $E[\boldsymbol{n}_k\boldsymbol{n}_k^\top]=\sigma^2\boldsymbol{I}_N$, where $\boldsymbol{I}_N$ denotes the N-dimensional identity matrix. The signature sequences of all users are of unit energy, i.e., $\boldsymbol{s}_k^\top\boldsymbol{s}_k=1$, for all k. For users k and l, d_{kl} represents the relative time delay of user l with respect to the time delay of user k, that is, $d_{kl}=d_l-d_k$, where d_k and d_l are the time delays of users k and l, respectively. Symbols T_R^d and T_L^d denote the operations of shifting, to right and left, respectively, of a vector by d and $N-d$ chips (components). For both operators, the vacated positions in the vector are filled with zeros. That is, for a vector $\boldsymbol{x}=[x_1,\ldots,x_N]^\top$ and integer $d\geq 0$, we define

$$T_L^d\boldsymbol{x}=[x_{N-d+1},\ldots,x_N,0^{N-d}]^\top \text{ and } T_R^d\boldsymbol{x}=[0^d,x_1,\ldots,x_{N-d}]^\top \tag{2}$$

where 0^d denotes d consecutive zeros.

We will use one-shot matched filters as the receivers. The decision statistics for the kth user in the nth symbol interval is $y_k(n)=\boldsymbol{s}_k^\top\boldsymbol{r}_k(n)$, where we do assume that the matched filter receiver of each user is perfectly aligned with the symbol interval of the user. Since $\boldsymbol{s}_k^\top\boldsymbol{s}_k=1$, the SIR of the kth user is then given by

$$\mathrm{SIR}_k=\frac{p_k}{\sum_{l\neq k}A_{kl}p_l+\sigma^2} \tag{3}$$

where we define the $K\times K$ matrix $\boldsymbol{A}$ with the following entries:

$$A_{kl}=\begin{cases}(\boldsymbol{s}_k^\top T_L^{d_{kl}}\boldsymbol{s}_l)^2+(\boldsymbol{s}_k^\top T_R^{d_{kl}}\boldsymbol{s}_l)^2, & k\neq l\\ 0, & k=l.\end{cases} \tag{4}$$

The common SIR target β is said to be feasible iff one can find nonnegative powers $\{p_k\}_{k=1}^K$ such that $\mathrm{SIR}_k\geq\beta$ for all k, which can be written in an equivalent matrix form as

$$\boldsymbol{p}\geq\beta\left(\boldsymbol{A}\boldsymbol{p}+\sigma^2\boldsymbol{1}\right) \tag{5}$$

where $\boldsymbol{1}$ is the vector of all ones. It is well known that if the common SIR target β is feasible, then the optimum power vector, i.e., the componentwise smallest feasible power vector, is found by solving (5) with equality [15]. Furthermore, the power control problem is feasible iff [16]

$$\beta<\frac{1}{\rho_A} \tag{6}$$

where ρ_A is the largest (also called the Perron–Frobenius) eigenvalue of the symmetric nonnegative matrix $\boldsymbol{A}$. We define the matrix $\boldsymbol{R}=\boldsymbol{A}+\boldsymbol{I}$ so that $R_{kk}=(\boldsymbol{s}_k^\top\boldsymbol{s}_k)^2=1$ and $\boldsymbol{R}$ represents the squared asynchronous cross correlations of the signature sequences. The Perron–Frobenius eigenvalue of $\boldsymbol{R}$ satisfies $\rho_R=\rho_A+1$, and the feasibility condition in (6) can also be expressed as

$$\beta<\frac{1}{\rho_R-1}. \tag{7}$$

That is, for a single-cell CDMA system, the range of common achievable SIR values is determined only by the Perron–Frobenius eigenvalue of the squared asynchronous cross-correlation matrix $\boldsymbol{R}$ which depends only on the signature sequences of the users and their relative time delays. For a given signature sequence set $\{\boldsymbol{s}_k\}_{k=1}^K$ and a set of time delays $\{d_k\}_{k=1}^K$, the supremum of common achievable SIR targets equals $1/(\rho_R-1)$. Our aim is to choose the signature sequences of the users, for any given set of time delays, such that the common achievable SIR is maximized. Therefore, we seek the signature sequence set that maximizes $1/(\rho_R-1)$, or, equivalently, minimizes ρ_R.

We note that it is hard to characterize the dependence of ρ_R on individual signature sequences. If this were not the case, one could devise an algorithm to update the signature sequences of the users in the direction that decreases ρ_R. Instead, our approach is to tie the Perron–Frobenius eigenvalue of $\boldsymbol{R}$, ρ_R, to another parameter of $\boldsymbol{R}$ which can be related to the signature sequences in a more direct way. By this approach, we will be able to characterize the optimum signature sequences in a closed-form expression in addition to being able to devise an iterative and distributed signature sequence update algorithm that will construct progressively better signature sequence sets.

To this end, we start our derivation with the following bounds on the Perron–Frobenius eigenvalue of $\boldsymbol{R}$ in terms of its row-sums [16]

$$\min_k\sum_{l=1}^K R_{kl}\leq\rho_R\leq\max_k\sum_{l=1}^K R_{kl}. \tag{8}$$

Similar bounds that can be obtained using column-sums of $\boldsymbol{R}$ are identical to (8) since $\boldsymbol{R}$ is symmetric. We also have the following bound from a simple application of the Rayleigh quotient [17]:

$$\frac{1}{K}\sum_{k=1}^K\sum_{l=1}^K R_{kl}\leq\rho_R \tag{9}$$

which is equivalent to $(\boldsymbol{1}^\top\boldsymbol{R}\boldsymbol{1})/(\boldsymbol{1}^\top\boldsymbol{1})\leq\rho_R$. Combining (8) and (9) and the fact that the minimum row-sum lower-bounds the average of the row-sums yields

$$\min_k\sum_{l=1}^K R_{kl}\leq\frac{1}{K}\sum_{k=1}^K\sum_{l=1}^K R_{kl}\leq\rho_R\leq\max_k\sum_{l=1}^K R_{kl}. \tag{10}$$

We define the TSAC as

$$\text{TSAC} = \sum_{k=1}^{K}\sum_{l=1}^{K} R_{kl}. \tag{11}$$

Note that the TSAC is equal to the sum of the entries of the asynchronous squared correlation matrix $\boldsymbol{R}$. Since we want to minimize ρ_R, and since ρ_R is lower-bounded by TSAC$/K$, it is reasonable to try to minimize the TSAC over the space of all possible signature sequences. Although it is not clear that ρ_R decreases as TSAC decreases, we will show that the signature sequence sets that achieve a particular lower bound on TSAC are precisely those that minimize ρ_R.

III. The Synchronous Problem Revisited

In order to motivate the solution of the asynchronous problem, we will first revisit the synchronous problem solved in [3]. In the synchronous case, $R_{kl} = (\boldsymbol{s}_k^\top \boldsymbol{s}_l)^2$. The following two theorems guarantee that the signature sequences that minimize the TSC (equivalent of the TSAC in the synchronous case) are those that minimize ρ_R.

Theorem 1 (Welch [6], Massey [7], Massey–Mittelholzer [8]): For any given set of unit energy sequences $\{\boldsymbol{s}_k\}_{k=1}^K$

$$\text{TSC} = \sum_{k=1}^{K}\sum_{l=1}^{K} (\boldsymbol{s}_k^\top \boldsymbol{s}_l)^2 \geq \frac{K^2}{N}. \tag{12}$$

Theorem 2 (Massey–Mittelholzer [8]): (The Uniformly Good Property) If the sequences $\{\boldsymbol{s}_k\}_{k=1}^K$ are such that the equality holds in (12) then

$$\sum_{l=1}^{K} (\boldsymbol{s}_k^\top \boldsymbol{s}_l)^2 = \frac{K}{N}, \qquad k = 1, \ldots, K. \tag{13}$$

For a synchronous system, $R_{kl} = (\boldsymbol{s}_k^\top \boldsymbol{s}_l)^2$ and Theorem 1 combined with (10) says that

$$\rho_R \geq \frac{K}{N}. \tag{14}$$

Since our aim is to minimize ρ_R, the best we can do is to choose the signature sequences so as to achieve (14) with equality. Theorem 2 says that when the signature sequences are chosen such that the TSC is minimized, i.e., the bound on the TSC is achieved with equality, then all of the row-sums equal K/N. Since from (10) the row-sums sandwich ρ_R, (14) is satisfied with equality, and the lowest possible ρ_R is obtained: $\rho_R = K/N$. Therefore, using (7), in the synchronous case, the bound on the common achievable SIR target is $\beta < 1/(K/N - 1)$, which is equivalent to the user capacity expression, derived in [3]

$$\frac{K}{N} < 1 + \frac{1}{\beta}. \tag{15}$$

Theorems 1 and 2 apply to the $K > N$ case. When $K < N$, the bound in Theorem 1 is loose; the K^2/N bound cannot be achieved, and Theorem 2 loses its applicability. When $K \leq N$, the equivalent of Theorem 1 is TSC $\geq K$. In this case, the equivalent of Theorem 2 is the following: if the sequences $\{\boldsymbol{s}_k\}_{k=1}^K$ are such that TSC $= K$, then $\sum_l (\boldsymbol{s}_k^\top \boldsymbol{s}_l)^2 = 1$ for $k = 1, \ldots, K$. That is, all of the row-sums of $\boldsymbol{R}$ are equal to 1, and therefore, $\rho_R = 1$. The implication of this result, from (7), is that any (arbitrarily large) common SIR target β is feasible with sufficiently large transmit powers. Note that TSC $= K$ is achieved with K orthonormal signature sequences, in which case $R_{kl} = 0$ for $k \neq l$.

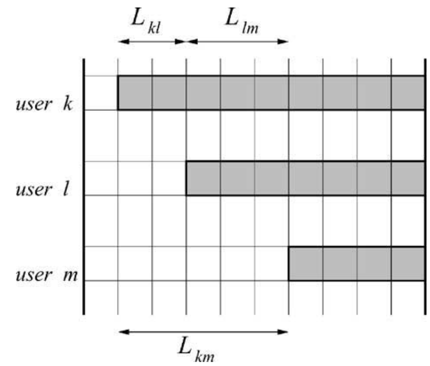

Fig. 2. Asynchronous system representation.

IV. The Asynchronous Problem

In this section, we derive asynchronous versions of Theorems 1 and 2. In order to make the derivations more tractable, we will use an alternate representation for the users' signature sequences. In Section II, the actual signature sequences of the users $\{\boldsymbol{s}_k\}_{k=1}^K$ are used as main variables, and the asynchronous cross correlations of the users are represented by using the right and left shift operations T_R and T_L; see for example, (4). Furthermore, each user aligns its receiver to its own symbol interval.

In the sequel, we will concentrate on a time duration which is equal to one symbol interval, i.e., N chip intervals. However, this symbol interval is not assumed to be time-aligned to any particular user's symbol period. This symbol interval is depicted in Fig. 2, where each box represents a chip interval. For each user, the white and gray chips in Fig. 2 correspond to symbols with time stamps n and $n+1$ of that user. In particular, for user k, the white chips on the left represent the last d_k chips in the signature $\boldsymbol{s}_k$ used to transmit symbol n, and the gray chips on the right are the first $N - d_k$ chips of $\boldsymbol{s}_k$ used to transmit symbol $n+1$.

In this fixed one-symbol-duration interval, we represent the sequence of N chips (both white and gray) of user k by the vector $\boldsymbol{x}_k$, even though the first d_k elements of $\boldsymbol{x}_k$ contribute to the transmission of symbol n while the remaining $N - d_k$ elements were used to transmit symbol $n+1$. The actual signature sequence of user k will then be equal to $\boldsymbol{s}_k = C^{d_k}\boldsymbol{x}_k$, where

$$C^d\boldsymbol{x} = [x_{d+1}, \ldots, x_N, x_1, \ldots, x_d]^\top$$

represents the cyclic shift of sequence $\boldsymbol{x}$ to the left by d positions. In Fig. 2, for example, user k is represented by the vector

$$\boldsymbol{x}_k = [x_{k1}, x_{k2}, \ldots, x_{kN}]^\top$$

whereas its actual signature sequence

$$\boldsymbol{s}_k = [x_{k2}, \ldots, x_{kN}, x_{k1}]^\top.$$

With this new representation, we can obtain a more concise expression for the squared asynchronous cross-correlation terms A_{kl} in (4). As in (4), the interference a pair of users create to each other has two components: these two components can be expressed as correlations of two vectors restricted to two sets of chip indexes. For users k and l, let L_{kl} denote the set of chip indexes for which users k and l transmit symbols with different time stamps. In terms of Fig. 2, L_{kl} is the set of columns (chip indexes) for which rows (users) k and l have different colors (e.g., white and gray). With this new representation, A_{kl} given in (4) can be expressed as

$$A_{kl} = \begin{cases} \left(\sum_{i\in L_{kl}} x_{ki}x_{li}\right)^2 + \left(\sum_{i\notin L_{kl}} x_{ki}x_{li}\right)^2, & k \neq l \\ 0, & k = l. \end{cases} \quad (16)$$

By definition of L_{kl}, we observe that $L_{kk} = \emptyset$. Thus,

$$\sum_{i\notin L_{kk}} x_{ki}x_{kl} = \boldsymbol{x}_k^\top \boldsymbol{x}_k = \boldsymbol{s}_k^\top \boldsymbol{s}_k = 1$$

and we observe that $\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{I}$ has (k,l)th element

$$R_{kl} = \left(\sum_{i\in L_{kl}} x_{ki}x_{li}\right)^2 + \left(\sum_{i\notin L_{kl}} x_{ki}x_{li}\right)^2. \quad (17)$$

By thinking of the users' signatures in terms of the rows of Fig. 2, computing the TSAC using (11) with R_{kl} from (17) is equivalent to summing the squared partial correlations over these rows. For our purposes, we develop an alternate way of calculating the TSAC that can be viewed as summing the squared partial correlations over the columns of Fig. 2. In particular, we use M_{ij} to denote the set of user indexes for which the chip positions i and j lie in the same symbol interval. That is, given any two chip indexes i and j, M_{ij} is the set of users for whom chips i and j are either both "gray" or both "white" in Fig. 2. For instance, for the three users in Fig. 2, $M_{12} = \{l, m\}$, $M_{19} = \emptyset$, and $M_{47} = \{k, l\}$. In the Appendix , we prove the following claim.

Lemma 1: TSAC, as given by (11) with R_{kl} from (17) can equivalently be expressed as

$$\text{TSAC} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left\{\left(\sum_{k\in M_{ij}} x_{ki}x_{kj}\right)^2 + \left(\sum_{k\notin M_{ij}} x_{ki}x_{kj}\right)^2\right\}. \quad (18)$$

As an aside, for a synchronous system, $L_{kl} = \emptyset$ for all k and l, and $M_{ij} = \{1, \ldots, K\}$ for all i and j, and equating (11) and (18) for the TSAC yields

$$\text{TSAC}_{\text{synch}} = \sum_{k=1}^{K}\sum_{l=1}^{K}\left(\sum_{i=1}^{N} x_{ki}x_{li}\right)^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(\sum_{k=1}^{K} x_{ki}x_{kj}\right)^2. \quad (19)$$

When the users are synchronous, $x_{ki} = s_{ki}$ for all k and i, implying $\text{TSAC}_{\text{synch}} = \text{TSC}$; in this case, the identity (19) was used in [7], [8] to reprove Welch's bound which was originally proved in [6].

The following two theorems, proven in the Appendix, guarantee that the signature sequences achieving the TSAC lower bound are those that minimize ρ_R.

Theorem 3: For any delay profile $\{d_k\}_{k=1}^K$ and any given set of unit energy sequences $\{\boldsymbol{s}_k\}_{k=1}^K$

$$\text{TSAC} \geq \frac{K^2}{N} \quad (20)$$

with equality iff the sum $\sum_{k=1}^K x_{ki}^2$ is the same for all i, and for all $i \neq j$

$$\sum_{k\in M_{ij}} x_{ki}x_{kj} = 0 \quad \text{and} \quad \sum_{k\notin M_{ij}} x_{ki}x_{kj} = 0. \quad (21)$$

Theorem 4: (The Asynchronous Uniformly Good Property) If the sequences $\{\boldsymbol{s}_k\}_{k=1}^K$ are such that they achieve the TSAC lower bound of Theorem 3 with equality then

$$\sum_{l=1}^{K} R_{kl} = \frac{K}{N}, \qquad k = 1, \ldots, K. \quad (22)$$

We will continue our derivation similar to the synchronous case. For this asynchronous CDMA system, Theorem 3 combined with (10) says that

$$\rho_R \geq \frac{K}{N}. \quad (23)$$

Similar to the synchronous case, our aim is to minimize ρ_R, and we cannot do better than to choose signature sequences that achieve (23) with equality. Theorem 4 says that when the signature sequences can be chosen such that the TSAC lower bound is achieved with equality, then all of the row-sums equal K/N. By (10), the row-sums sandwich ρ_R, and so (23) is satisfied with equality, yielding the lowest possible ρ_R: $\rho_R = K/N$. Therefore, using (7), the bound on the common achievable SIR target in this asynchronous case is

$$\beta < \frac{1}{K/N - 1} \quad (24)$$

which is the same as the bound found in the synchronous case. In this case, the user capacity expression (15) is valid for the asynchronous case as well.

Similar to the synchronous case, Theorems 3 and 4 apply to the $K > N$ case. When $K < N$, the bound in Theorem 3 is loose; the K^2/N bound cannot be achieved, and Theorem 4 loses its applicability. When $K \leq N$, the equivalent of (20) in Theorem 3 is

$$\text{TSAC} \geq K. \quad (25)$$

The bound is achieved with equality when R_{kl} satisfies $R_{kl} = 0$ for all $k \neq l$. In this case, the equivalent of Theorem 4 is the following: if the sequences $\{\boldsymbol{s}_k\}_{k=1}^K$ are such that the equality holds in (25) then $\sum_l R_{kl} = 1$ for $k = 1, \ldots, K$. That is, all of the row-sums of $\boldsymbol{R}$ are equal to 1, and, therefore, $\rho_R = 1$. The implication of this result, from (7), as in the synchronous case, is that any (arbitrarily large) common SIR target β is feasible. In the asynchronous case, for R_{kl} to be equal to zero for $k \neq l$, from (4), the two partial correlations between the signature sequences of users k and l should be equal to zero, which is clearly a stronger condition than the orthogonality condition in the synchronous case.

Next we note that the optimum received powers of the users are equal. We state this as a corollary to Theorem 3, and give a proof in the Appendix.

Corollary 1: If the sequences $\{\boldsymbol{s}_k\}_{k=1}^K$ are such that they achieve the TSAC lower bound of Theorem 3 with equality, then all users have SIRs equal to β, by using the received powers

$$p_k = p = \frac{\sigma^2}{1 + 1/\beta - K/N}, \qquad k = 1, \ldots, K. \quad (26)$$

Note that $p > 0$ as long as K, N, and β satisfy the user capacity inequality (24).

V. TSAC Reduction: Iterative Algorithms

Following the closed-form expressions for the signature sequence sets maximizing the information theoretic sum capacity [4], [5] and user capacity [3], [10]–[12] introduced the iterative adaptation of signature sequences for synchronous CDMA systems. Since the optimum signature sequences minimize the TSC in the synchronous case, the algorithms in [10]–[14], [18] were designed to decrease (more precisely, not to increase) the TSC at each iterative step. Here, we will design algorithms which decrease the TSAC at each iteration. To this end, we first separate the terms that depend on the signature sequence of the

kth user in the TSAC. From the TSAC definition (11), the definition of $\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{I}$, and the fact that $\boldsymbol{A}$ in (4) is symmetric, we can write

$$\text{TSAC} = (\boldsymbol{s}_k^\top \boldsymbol{s}_k)^2 + 2\boldsymbol{s}_k^\top \boldsymbol{B}_k \boldsymbol{s}_k + \gamma_k \tag{27}$$

where $\gamma_k = \sum_{i \neq k} \sum_{j \neq k} R_{ij}$ denotes the squared asynchronous correlation terms that do not depend on $\boldsymbol{s}_k$ and where in terms of $\bar{\boldsymbol{s}}_{kl} = T_L^{d_{kl}} \boldsymbol{s}_l$ and $\tilde{\boldsymbol{s}}_{kl} = T_R^{d_{kl}} \boldsymbol{s}_l$, the left and right signatures of the lth asynchronous user with respect to the kth user

$$\boldsymbol{B}_k = \sum_{l \neq k} \left(\bar{\boldsymbol{s}}_{kl} \bar{\boldsymbol{s}}_{kl}^\top + \tilde{\boldsymbol{s}}_{kl} \tilde{\boldsymbol{s}}_{kl}^\top \right). \tag{28}$$

In order to minimize the TSAC, we are looking for updates of the signature sequence of the kth user from $\boldsymbol{s}_k$ to some $\boldsymbol{c}_k$ that is guaranteed to decrease (not to increase) the TSAC. Let us denote the TSAC after the $\boldsymbol{s}_k \to \boldsymbol{c}_k$ update as $\overline{\text{TSAC}}$. Then

$$\overline{\text{TSAC}} = (\boldsymbol{c}_k^\top \boldsymbol{c}_k)^2 + 2\boldsymbol{c}_k^\top \boldsymbol{B}_k \boldsymbol{c}_k + \gamma_k. \tag{29}$$

Restricting the new (updated) signature sequence of the kth user to be of unit energy as well, i.e., $\boldsymbol{c}_k^\top \boldsymbol{c}_k = 1$, we note that $\overline{\text{TSAC}} \leq \text{TSAC}$ iff

$$\boldsymbol{c}_k^\top \boldsymbol{B}_k \boldsymbol{c}_k \leq \boldsymbol{s}_k^\top \boldsymbol{B}_k \boldsymbol{s}_k. \tag{30}$$

Although there are many possible $\boldsymbol{s}_k \to \boldsymbol{c}_k$ updates that would guarantee that (30) holds, we will propose two of them here. The two similar updates used in the synchronous CDMA context were given in [10]–[12] and in [13], [14]. We call the first update the asynchronous minimum mean-square error (MMSE) update which we define as

$$\boldsymbol{c}_k = \frac{\left(\boldsymbol{B}_k + a^2 \boldsymbol{I}_N\right)^{-1} \boldsymbol{s}_k}{\left[\boldsymbol{s}_k^\top \left(\boldsymbol{B}_k + a^2 \boldsymbol{I}_N\right)^{-2} \boldsymbol{s}_k\right]^{1/2}} \tag{31}$$

and we call the second update the asynchronous eigen update which we define as the normalized eigenvector of $\boldsymbol{B}_k$ corresponding to its smallest eigenvalue. Note that, in the asynchronous MMSE update, the new signature sequence of user k, $\boldsymbol{c}_k$, is the normalized one-shot MMSE receiver filter for that user when the signature sequences of all other users are fixed. Similar to the synchronous MMSE update [10]–[12], the new signature sequence can be obtained using an adaptive [19]–[22] or a blind [23] implementation of the one-shot MMSE filter. The proof that the asynchronous eigen update decreases the TSAC follows from the Rayleigh quotient applied to the matrix $\boldsymbol{B}_k$ [17]. The proof that the asynchronous MMSE update decreases the TSAC can be carried out in a very similar fashion to the proof that the MMSE update decreases the TSC [10]–[12].

VI. Further Remarks

In a synchronous CDMA system with $K > N$, the optimum signature sequences are those that minimize the TSC (i.e., WBE sequences), and they are completely characterized by the matrix equation $\boldsymbol{S}\boldsymbol{S}^\top = (K/N)\boldsymbol{I}_N$, where $\boldsymbol{S}$ is a matrix containing the signature sequences of the users in its columns. That is, for a set of signature sequences to be optimum they need to satisfy three sets of conditions: 1) each column of $\boldsymbol{S}$ must have unit length, 2) rows of $\boldsymbol{S}$ must be orthogonal to each other, and 3) each row of $\boldsymbol{S}$ must have length $\sqrt{K/N}$. These conditions were identified in [7], [8] in rederiving the Welch's bound, but the existence of sequences satisfying these conditions for arbitrary N and K with $K > N$ was first addressed in [3].

For asynchronous systems, we have found that if a set of unit energy signature sequences achieves the TSAC lower bound of Theorem 3 with equality, then the user capacity is the same as in the synchronous CDMA system. However, we have not addressed the issue of the existence of such signature sequence sets for arbitrary K, N, and user delay profile. As expected, optimality conditions on the signature sequences in the asynchronous case are stricter compared to the synchronous case. It is worth noting that the optimum signature sequences for an arbitrary user delay profile constitute a WBE set when each signature sequence in the set is shifted appropriately. This can be deduced from the equality conditions of Theorem 3 which imply that

$$\sum_{k=1}^{K} x_{ki} x_{kj} = \sum_{k \in M_{ij}} x_{ki} x_{kj} + \sum_{k \notin M_{ij}} x_{ki} x_{kj} = (K/N)\delta_{ij} \tag{32}$$

where $\delta_{ij} = 1$ if $i = j$, and 0 if $i \neq j$. This is equivalent to $\boldsymbol{X}\boldsymbol{X}^\top = (K/N)\boldsymbol{I}_N$ where $\boldsymbol{X}$ is an $N \times K$ matrix containing $\boldsymbol{x}_k$'s as its columns. Therefore, $\{\boldsymbol{x}_k\}_{k=1}^K$ is a WBE set. One must be careful, however, since the $\boldsymbol{x}_k$ are cyclicly shifted versions of the actual signature sequences $\boldsymbol{s}_k$. Therefore, a signature sequence set that is optimum for a certain user delay profile can be cyclicly shifted to obtain a WBE set which is optimum for a synchronous system. However, the converse is not true; one cannot cyclicly shift an arbitrary WBE set to obtain an optimum signature sequence set for an arbitrary asynchronous system.

It is not obvious that the sets of signature sequences satisfying the equality conditions of Theorem 3 should exist for all K, N, and the user delay profile. When $K \leq N$, a trivial set of optimal sequences can be constructed. In particular, in an observation interval where the users are ordered according to their delays in an ascending order (as in the example of Fig. 2 where the delays of the users are one, three, and six chips), and assuming that the delays of the users are distinct, let $\boldsymbol{x}_k = \boldsymbol{e}_k$ where $\boldsymbol{e}_k$ is the unit vector with a single 1 in position k and zeros elsewhere. It is easy to verify that such an assignment guarantees that the two partial correlations between any two users are zero, and, therefore, $A_{kl} = 0$ for all k and l, and we have $\text{TSAC} = K$. The actual signature sequence of user k will be $\boldsymbol{s}_k = C^{d_k} \boldsymbol{e}_k$. In fact, depending on the delay profile, some users may have the same signature; however, different delays result in the desired property that $A_{kl} = 0$ among the signatures. Consider a simple example where $K = N$, and the delay of user k is $d_k = k - 1$ for $k = 1, \ldots, K$. In this case, an optimum signature assignment is $\boldsymbol{x}_k = \boldsymbol{e}_k$, for $k = 1, \ldots, K$, which implies that the actual signature sequences of the users are $\boldsymbol{s}_k = \boldsymbol{e}_1$, for $k = 1, \ldots, K$, i.e., the signature sequences of all users are the same. Although these sequences are not typical CDMA signatures, they prove the existence of minimum-TSAC sequences for an asynchronous system with an arbitrary user delay profile when $K \leq N$. For $K > N$, the issue of existence is an important open problem which has not been addressed in this correspondence.

In particular, although the algorithms of Section V monotonically decrease the TSAC, we have not proved that these algorithms will converge to a set that achieves $\text{TSAC} = K^2/N$. We can report, however, that through a large number of numerical experiments with randomly generated initial signature sequence sets, we have observed that the TSAC reduction algorithms we have proposed here have always converged to signature sequences meeting the TSAC lower bound. That is, we have not only observed the existence of such sequence sets, but also the convergence of our proposed algorithms to these sets through many numerical experiments.

Finally, we provide a simple experimental result here. In this experiment, we implement both the asynchronous MMSE update and the asynchronous eigen update algorithms and plot the TSAC and the supremum of the common achievable SIR target $1/(\rho_R - 1)$ as a function of the iteration index in Fig. 3. In this example system, $K = 20$, $N = 10$, and the initial signature sequence set and the delays of the users are generated randomly. In Fig. 3, one iteration is equivalent to K intermediate iterations where at each intermediate iteration only one user updates its signature sequence. As we see from Fig. 3, the TSAC decreases to the bound $K^2/N = 40$ and ρ_R decreases to

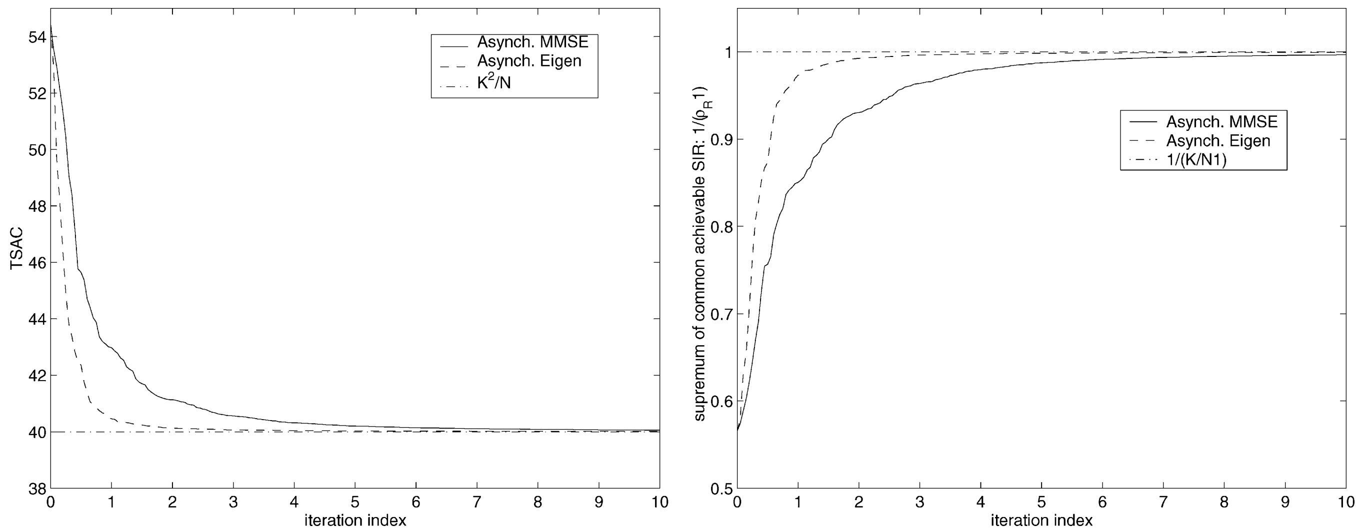

Fig. 3. TSAC and $1/(\rho_R - 1)$ versus iteration index.

$K/N = 2$, and the supremum of the common achievable SIRs increases to $1/(K/N-1) = 1$. Fig. 3 verifies, for this instance, that the signature sequences achieving the lower bound on the TSAC exist, and the two algorithms we proposed here converge to those signature sequences.

Appendix
Proofs

Proof of Lemma 1

Let us define the indicator variables

$$Y_i^{kl} = \begin{cases} 1, & i \in L_{kl} \\ 0, & i \notin L_{kl} \end{cases} \tag{33}$$

and

$$Z_{ij}^k = \begin{cases} 1, & k \in M_{ij} \\ 0, & k \notin M_{ij}. \end{cases} \tag{34}$$

With the help of this definition for Y_i^{kl}, it follows from (17) that

$$R_{kl} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_{ki}x_{li}x_{kj}x_{lj}\left[Y_i^{kl}Y_j^{kl} + \left(1 - Y_i^{kl}\right)\left(1 - Y_j^{kl}\right)\right]. \tag{35}$$

By defining the function f of two binary variables as

$$f(x,y) = xy + (1-x)(1-y) = 1 - (x-y)^2 \tag{36}$$

we can write

$$\text{TSAC} = \sum_{k=1}^{K}\sum_{l=1}^{K} R_{kl} = \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{ki}x_{li}x_{kj}x_{lj} f(Y_i^{kl}, Y_j^{kl}). \tag{37}$$

We now will show that

$$f(Y_i^{kl}, Y_j^{kl}) = f(Z_{ij}^k, Z_{ij}^l). \tag{38}$$

To prove (38), we define the indicator

$$X_i^k = \begin{cases} 1, & d_k < i \\ 0, & \text{otherwise.} \end{cases} \tag{39}$$

That is, X_i^k is 1 iff the $(n+1)$st symbol of user k has started on or before chip i; or equivalently, X_i^k is 1 iff the ith chip position of user k is "gray" in Fig. 2. Then Y_i^{kl} and Z_{ij}^k can be written in terms of X_i^k, X_i^l, X_j^k, and X_j^l as

$$Y_i^{kl} = (X_i^k - X_i^l)^2 \tag{40}$$
$$Z_{ij}^k = 1 - (X_i^k - X_j^k)^2. \tag{41}$$

To show that (38) holds, (40) and (41) imply that it is sufficient to show that

$$\left((X_i^k - X_i^l)^2 - (X_j^k - X_j^l)^2\right)^2 = \left((X_i^k - X_j^k)^2 - (X_i^l - X_j^l)^2\right)^2 \tag{42}$$

whose correctness can be verified using straightforward manipulations, and the observation that $b^2 = b$ for any binary variable b. Thus, (36)–(38) imply

$$\text{TSAC} = \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{ki}x_{li}x_{kj}x_{lj}\left[Z_{ij}^k Z_{ij}^l + \left(1 - Z_{ij}^k\right)\left(1 - Z_{ij}^l\right)\right] \tag{43}$$

which is the statement of Lemma 1 expressed in terms of the Z_{ij}^k indicator variables. □

Proof of Theorem 3

From Lemma 1, we can write the TSAC using (18) as

$$\text{TSAC} = \sum_{i=1}^{N}\left(\sum_{k=1}^{K} x_{ki}^2\right)^2 + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\left\{\left(\sum_{k\in M_{ij}} x_{ki}x_{kj}\right)^2 + \left(\sum_{k\notin M_{ij}} x_{ki}x_{kj}\right)^2\right\} \tag{44}$$

$$\geq \sum_{i=1}^{N}\left(\sum_{k=1}^{K} x_{ki}^2\right)^2 \tag{45}$$

$$\geq \frac{1}{N}\left(\sum_{i=1}^{N}\sum_{k=1}^{K} x_{ki}^2\right)^2 \tag{46}$$

$$= \frac{1}{N}\left(\sum_{k=1}^{K}\sum_{i=1}^{N} x_{ki}^2\right)^2 \tag{47}$$

$$= \frac{K^2}{N} \tag{48}$$

$$\sum_{l=1}^{K} R_{kl} = \sum_{l=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{ki}x_{li}x_{kj}x_{lj}\left[Z_{ij}^{k}Z_{ij}^{l} + \left(1 - Z_{ij}^{k}\right)\left(1 - Z_{ij}^{l}\right)\right] \tag{54}$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} x_{ki}x_{kj}\left\{Z_{ij}^{k}\left(\sum_{l=1}^{K} x_{li}x_{lj}Z_{ij}^{l}\right) + (1 - Z_{ij}^{k})\left(\sum_{l=1}^{K} x_{li}x_{lj}(1 - Z_{ij}^{l})\right)\right\}. \tag{55}$$

where, in going from (45) to (46), we used the inequality

$$\sum_{i=1}^{N} a_i^2 \geq \frac{1}{N}\left(\sum_{i=1}^{N} a_i\right)^2 \tag{49}$$

which is satisfied with equality iff all $a_i = \sum_{k=1}^{K} x_{ki}^2$ are equal. Thus, the overall inequality (48) is satisfied with equality iff the inequalities (45) and (46) are both satisfied with equality. These are precisely the equality conditions given in Theorem 3. □

Proof of Theorem 4

By the hypothesis of the theorem, the signature sequences achieve the lower bound on the TSAC with equality. Thus, by Theorem 3, for all $i \neq j$

$$\sum_{k=1}^{K} x_{ki}x_{kj}Z_{ij}^{k} = \sum_{k\in M_{ij}} x_{ki}x_{kj} = 0 \tag{50}$$

and

$$\sum_{k=1}^{K} x_{ki}x_{kj}(1 - Z_{ij}^{k}) = \sum_{k\notin M_{ij}} x_{ki}x_{kj} = 0 \tag{51}$$

and $\sum_{k=1}^{K} x_{ki}^2$ is the same for all i. Note that

$$\sum_{i=1}^{N}\left(\sum_{k=1}^{K} x_{ki}^2\right) = \sum_{k=1}^{K}\left(\sum_{i=1}^{N} x_{ki}^2\right) = K \tag{52}$$

where we used $\boldsymbol{x}_k^\top \boldsymbol{x}_k = 1$ for all k. Thus, when all $\sum_{k=1}^{K} x_{ki}^2$ are all the same, we have

$$\sum_{k=1}^{K} x_{ki}^2 = K/N. \tag{53}$$

From (35), (36), and (38), we get (54) and (55) at the top of the page. Thus, (50) and (51) imply that all $i \neq j$ terms vanish, and we have only the $i = j$ terms. Noting $Z_{ii}^{l} = 1$ for all l

$$\sum_{l=1}^{K} R_{kl} = \sum_{l=1}^{K}\sum_{i=1}^{N} x_{ki}^2 x_{li}^2 = \sum_{i=1}^{N} x_{ki}^2\left(\sum_{l=1}^{K} x_{li}^2\right) = \frac{K}{N} \tag{56}$$

where we inserted (53) and noted that $\boldsymbol{x}_k^\top \boldsymbol{x}_k = 1$. □

Proof of Corollary 1

From (3), the SIR of the kth user is

$$\mathrm{SIR}_k = \frac{p}{p\left(\sum_{l\neq k} A_{kl}\right) + \sigma^2} = \frac{p}{p\left(\sum_{l} R_{kl} - 1\right) + \sigma^2}. \tag{57}$$

If the signature sequences satisfy (20) with equality in Theorem 3, then using (22) in Theorem 4, we can write (57) as

$$\mathrm{SIR}_k = \frac{p}{p\,(K/N - 1) + \sigma^2}. \tag{58}$$

It is now straightforward to show that when (26) is inserted into (58), $\mathrm{SIR}_k = \beta$ for all k. □

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