Service Outage Based Power and Rate Allocation

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***Abstract*— This paper combines the concepts of ergodic capacity and capacity versus outage for fading channels, and explores variable rate transmissions under a service outage constraint in a block flat fading channel model. A service outage occurs when the transmission rate is below a given basic rate. We solve the problem of maximizing the expected rate subject to the average power constraint and the service outage probability constraint. When the problem is feasible, the optimum power policy is shown to be a combination of water filling and channel inversion allocation, where the outage occurs at a set of channel states below a certain threshold. The service outage approach resolves the conflicting objectives of high average rate and low outage probability.**

***Index Terms*—Adaptive transmission, block-fading channel, ergodic capacity, outage capacity, service outage**

I. Introduction

Wireless communication channels vary with time due to mobility of users and changes in the environment. For a time varying channel, dynamic allocation of resources such as power, rate, and bandwidth can yield improved performance over fixed allocation strategies. Indeed, adaptive techniques are employed in EDGE [1], GPRS [2], and HDR [3], and are proposed as standards for next generation cellular systems. The system performance criterion is usually application specific; therefore, different classes of applications will benefit from specific adaptive transmission schemes. In order to differentiate real-time service from non real-time service, three capacity measures have been defined in the literature: ergodic capacity [4], delay limited capacity [5], and capacity versus outage probability [9–11]. A comprehensive survey of these concepts can be found in [6].

The ergodic capacity [4] was developed for non real-time data services. It determines the maximum achievable rate averaged over all fading states. The corresponding optimum power allocation is the well known water filling allocation [7, 8]. The ergodic capacity may not be relevant for real-time applications in a slow fading environment, where substantial delay can occur when averaging over all fading states. Delay limited capacity [5] and the capacity versus outage probability [9–11] were developed for constant-rate real-time applications. The delay limited capacity specifies the highest achievable rate with a decoding delay independent of fading correlation structures [5]. The outage capacity in the capacity versus outage probability problem determines the ϵ-achievable rate [12] of the M-block fading channel. The corresponding optimum power allocation was derived in [10] for M parallel flat fading blocks (frequency diversity or space diversity), and in [11] for M consecutive flat fading blocks (time diversity). The zero-outage capacity in [10, 11] is also referred to as the delay-limited capacity.

Though the outage capacity studies the capacity for constant-rate real-time applications, the constant-rate assumption may not be essential for many real-time applications. For applications with simultaneous voice and data transmissions, for example, as soon as a basic rate r_o for the voice service has been guaranteed, any excess rate can be used to transmit data in a best effort fashion. For some video or audio applications, the source rate can be adapted according to the fading channel conditions to provide multiple quality of service levels. Typically these applications require a nonzero basic service rate r_o to achieve a minimum acceptable service quality. Motivated by these variable-rate real-time applications, we study variable rate transmission schemes subject to a basic rate requirement in a slow fading environment. By allowing variable rate transmissions, the variation of the fading channel can be exploited to achieve an average rate higher than the outage capacity. By imposing a basic rate requirement, the system can be guaranteed to operate properly.

Since infinite average power is needed to achieve any nonzero rate at all times in a Rayleigh fading channel, we impose a probabilistic basic service rate requirement, that we call a service outage constraint. The service is said to be in an outage when the information rate is smaller than the basic service rate r_o. Service quality is acceptable as long as the probability of the service outage is less than ϵ, a parameter indicating the outage tolerance of the application. Unlike the information outage in the capacity versus outage probability problem [10, 11], the bits transmitted during the service outage may still be valuable in that they will be transmitted reliably and will contribute to the average rate.

For variable-rate systems, the expected rate determines how much rate can be transmitted on the average and is a meaningful measure of system performance. Therefore, in this paper, the allocation problem is to find the power and rate allocation that maximize the expected rate subject to the service outage constraint and the average power constraint. Under the assumptions of a block flat fading AWGN channel model and perfect channel state information at the transmitter, we verify that the outage should occur at bad channel states below a certain threshold. The resulting optimum power allocation is shown to be a combination of channel inversion and water filling when the allocation problem is feasible. The service outage approach strikes good balance between the average rate and the outage probability. This approach has been generalized to the case of code words spanning multiple blocks in [13]. Although a continuous fading distribution is assumed in this paper, the results can be extended into the case of discrete fading distributions by employing a probabilistic power allocation, as examplified by the policies in [10].

Although our problem has been motivated by real time applications, it also characterizes coverage versus capacity tradeoffs. In particular, mobility in cellular systems results in channel variations due to changes in distance attenuation. An important objective of a cellular system is to provide a basic service rate over as much of the service area as possible. In this case, the service outage constraint characterizes the spatial coverage requirement of the system. The objective is then to maximize the average rate over all geographic locations subject to meeting the service outage constraint.

The remainder of this paper is organized as follows. In Section II, the system model and the optimization problem are presented. In Section III, the optimum allocation policy is derived. In Section IV, a supporting theorem for the optimum allocation policy is proved. Further discussion of the optimum solution is presented in Section V.

II. The Allocation Problem

In this work, we employ the block flat fading Gaussian channel (BF-AWGN) model [9]. In the BF-AWGN channel, a block of N symbols experiences the same channel state, which is constant over the whole block, but may vary from block to block. Within each block we have the time-invariant Gaussian channel $y = \sqrt{h}x + n$. Here x is the channel input, y is the channel output, n is white Gaussian noise with variance σ^2, and h is the channel state. Let $f(h)$ denote the probability density function of the channel state h and $F(h)$ denote the corresponding cumulative distribution function (CDF). Here, we consider the case where $F(h)$ is a continuous function and the power allocation is a deterministic function of channel state.

We make the following assumptions:

- The channel state information is known perfectly at both transmitter and receiver.
- One codeword spans one fading block and the block size N is large .
- The fading process is ergodic over the time scale of the application.

As pointed out in [10], it makes sense to study the BF-AWGN channel as $N \to \infty$, since for typical practical systems N is fairly large and outage is the dominant error event when using an actual code. Let $p(h)$ denote the transmission power at channel state h. Then the maximum achievable rate at each block is the capacity of Gaussian channel with received power $hp(h)$, and is denoted as $R[hp(h)]$, where

$$R[P] = \frac{1}{2}\log_2\left(1+\frac{P}{\sigma^2}\right) . \tag{1}$$

Therefore, the resource allocation problem requires finding only the optimum power allocation $p^*(h)$. Under the assumption of the ergodicity of fading process, the time average rate of the system can be characterized by the expected rate. Therefore, given the average power $P_{\rm av}$, the basic service rate $r_{\rm o}$, and the allowable service outage probability ϵ, we wish to maximize the expected code rate, as follows:

$$R^* = \max_{p(h)} E_h\left\{R[hp(h)]\right\} \tag{2}$$

$$\text{subject to:} \quad E_h\left\{p(h)\right\} \le P_{\rm av} \tag{2a}$$

$$p(h) \ge 0 \tag{2b}$$

$$\Pr\{R[hp(h)] < r_{\rm o}\} \le \epsilon . \tag{2c}$$

In the absence of the service outage constraint (2c), R^* would be the ergodic capacity for the fading channel, and the well known water filling allocation [7, 8] would be the corresponding optimum power assignment.

III. Optimum Power and Rate Allocation

In this section, we derive an optimum power allocation $p^*(h)$ for problem (2). The difficulty in deriving $p^*(h)$ is primarily due to the probabilistic nature of the constraint (2c). Here, we show how an optimum power allocation can be derived based on a problem analogous to (2) with a deterministic constraint on the assigned rate. Given a basic service rate $r_{\rm o}$ and a power policy $p(h)$, the *service set* is defined as $\mathcal{H}_{\rm s}(p(h)) = \{h|R[hp(h)] \ge r_{\rm o}\}$, and the *outage set* is $\mathcal{H}_{\rm o}(p(h)) = \{h|R[hp(h)] < r_{\rm o}\}$.

Our approach will be to show that there is an optimal solution to problem (2) with a particular form of a service set. Prior to showing this, we solve the following subproblem in which it is required that the service set contains an arbitrary set $\mathcal{H}_a$.

$$R^*(\mathcal{H}_a) = \max_{p(h)} E_h\left\{R[hp(h)]\right\} \tag{3}$$

$$\text{subject to:} \quad E_h\left\{p(h)\right\} \le P_{\rm av} \tag{3a}$$

$$p(h) \ge 0 \tag{3b}$$

$$R[hp(h)] \ge r_{\rm o} \qquad h \in \mathcal{H}_a . \tag{3c}$$

Let $p^*(h, \mathcal{H}_a)$ denote an optimum solution to problem (3). Therefore, $p^*(h, \mathcal{H}_a)$ achieves the highest average rate among all the schemes whose service set contains $\mathcal{H}_a$.

Problem (3) does not necessarily have a solution for a given $(P_{av}, r_{\rm o}, \mathcal{H}_a)$. Constraint (3c) implies that a feasible allocation $p(h)$ must satisfy

$$p(h) \ge \frac{\sigma^2(2^{2r_{\rm o}}-1)}{h} \qquad h \in \mathcal{H}_a . \tag{4}$$

This implies that the minimum average power needed to meet the constraint (3c) for a given $(r_{\rm o}, \mathcal{H}_a)$ is

$$P_{\rm min}(r_{\rm o}, \mathcal{H}_a) = \int_{\mathcal{H}_a} \frac{\sigma^2(2^{2r_{\rm o}}-1)}{h} f(h)\, dh . \tag{5}$$

Consequently, problem (3) has a solution only if $P_{\rm av} \ge P_{\rm min}(r_{\rm o}, \mathcal{H}_a)$. When $P_{\rm av} = P_{\rm min}(r_{\rm o}, \mathcal{H}_a)$ the corresponding power allocation is the on-off channel inversion policy

$$p^*(h, \mathcal{H}_a) = \begin{cases} \dfrac{\sigma^2(2^{2r_{\rm o}}-1)}{h} & h \in \mathcal{H}_a , \\ 0 & \text{otherwise} . \end{cases} \tag{6}$$

When $P_{\rm av} > P_{\rm min}(r_{\rm o}, \mathcal{H}_a)$ the corresponding power allocation is given by the following theorem. We use the notation $(x)^+ = \max(x, 0)$.

Theorem 1 *When $P_{\rm av} > P_{\rm min}(r_o, \mathcal{H}_a)$ the optimum solution for problem (3) is:*

$$p^*(h, \mathcal{H}_a) = \begin{cases} \dfrac{\sigma^2(2^{2r_o}-1)}{h} & h \in \mathcal{H}_a \cap \left\{h \le h_0 2^{2r_o}\right\} , \\ \sigma^2\left(\dfrac{1}{h_0} - \dfrac{1}{h}\right)^+ & \text{otherwise} . \end{cases} \tag{7}$$

where h_0 is the solution of $E_h\left\{p^*(h,\mathcal{H}_a)\right\} = P_{\rm av}$.

Theorem 1 follows from standard variational arguments; the proof appears in the Appendix. Note that when $P_{\rm av} = P_{\min}(r_{\rm o},\mathcal{H}_a)$ the resulting power allocation (6) can be viewed as a limiting case of expression (7) as $h_0 \to \infty$. The power allocation $p^*(h,\mathcal{H}_a)$ is a combination of channel inversion and water filling allocations. To obtain a high average rate, we would like to allocate power in the form of the water filling allocation, while to meet the service constraint (3c), we must allocate power no less than the channel inversion allocation within the set $\mathcal{H}_a$. The solution $p^*(h,\mathcal{H}_a)$ balances these two factors.

To characterize the solution to the optimization problem (2), we define h_ϵ as the solution to $F(h_\epsilon) = \epsilon$. The threshold h_ϵ partitions the channels into a set $\mathcal{H}_\epsilon = \{h \geq h_\epsilon\}$ of good channels and the complementary set $\overline{\mathcal{H}}_\epsilon = \{h < h_\epsilon\}$ of bad channels. In the following, we show that the solution of (3) with $\mathcal{H}_a = \mathcal{H}_\epsilon$, specifically $p^*(h) = p^*(h,\mathcal{H}_\epsilon)$, is an optimum solution of problem (2). In order to prove this, we define the partial ordering $\prec$ and show a number of preliminary results.

Definition 1 $\mathcal{H}_1 \prec \mathcal{H}_2$ if $h_1 < h_2$ for all $h_1 \in \mathcal{H}_1$ and $h_2 \in \mathcal{H}_2$.

Theorem 2 *Problem (2) has an optimal solution $p^*(h)$ with the outage set $\mathcal{H}_{\rm o}(p^*(h))$ and the service set $\mathcal{H}_{\rm s}(p^*(h))$ satisfying $\mathcal{H}_{\rm o}(p^*(h)) \prec \mathcal{H}_{\rm s}(p^*(h))$.*

Theorem 2 shows that there exists an optimum power allocation such that the outage occurs when the channel state is worse than a particular threshold. Proof of Theorem 2 involves a somewhat complicated two-step construction and is deferred to Section IV.

Using Theorem 2 and the fact that $\Pr\{\mathcal{H}_{\rm s}(p^*(h))\} \geq 1-\epsilon$ by constraint (2c), it is easy to show the following corollary.

Corollary 1 *Problem (2) has an optimum solution $p^*(h)$ such that $\mathcal{H}_\epsilon \subseteq \mathcal{H}_{\rm s}(p^*(h))$.*

Now we can prove $p^*(h) = p^*(h,\mathcal{H}_\epsilon)$ by showing that $R^* = R^*(\mathcal{H}_\epsilon)$. With $\mathcal{H}_a = \mathcal{H}_\epsilon$ in the outage constraint (3c), the service set of $p^*(h,\mathcal{H}_\epsilon)$ must contain $\mathcal{H}_\epsilon$. Thus $p^*(h,\mathcal{H}_\epsilon)$ satisfies the outage constraint (2c) and is a feasible power allocation scheme for problem (2), implying $R^*(\mathcal{H}_\epsilon) \leq R^*$. On the other hand, Corollary 1 implies that problem (2) has an optimal solution $p^*(h)$ achieving an average rate of R^* that satisfies constraint (3c) with $\mathcal{H}_a = \mathcal{H}_\epsilon$. Thus, $p^*(h)$ is a feasible power allocation scheme for problem (3) and $R^* \leq R^*(\mathcal{H}_\epsilon)$. Consequently, $R^* = R^*(\mathcal{H}_\epsilon)$. In conclusion, an optimum solution is $p^*(h) = p^*(h,\mathcal{H}_\epsilon)$ and the following conclusions apply to problem (2).

- Problem (2) is feasible if only if $(P_{\rm av}, r_{\rm o}, \epsilon)$ satisfies

$$P_{\rm av} \geq P_{\min}(r_{\rm o},\mathcal{H}_\epsilon) = \int_{h_\epsilon}^{\infty} \frac{\sigma^2(2^{2r_{\rm o}}-1)}{h} f(h)\,dh\,, \quad (8)$$

- When $P_{\rm av} = P_{\min}(r_{\rm o},\mathcal{H}_\epsilon)$ we have

$$p^*(h) = \begin{cases} \dfrac{\sigma^2(2^{2r_{\rm o}}-1)}{h} & h \geq h_\epsilon\,, \\ 0 & h < h_\epsilon\,. \end{cases} \quad (9)$$

- When $P_{\rm av} > P_{\min}(r_{\rm o},\mathcal{H}_\epsilon)$, we can apply Theorem 1 with $\mathcal{H}_a = \mathcal{H}_\epsilon$ yielding an optimum solution to problem (2) of the form

$$p^*(h) = \begin{cases} \dfrac{\sigma^2(2^{2r_{\rm o}}-1)}{h} & h_\epsilon \leq h \leq \min\{h_\epsilon, h_0^* 2^{2r_{\rm o}}\}\,, \\ \sigma^2\left(\dfrac{1}{h_0^*} - \dfrac{1}{h}\right)^+ & \text{otherwise}\,, \end{cases} \quad (10)$$

where h_0^* is the solution of $E_h\left\{p^*(h)\right\} = P_{\rm av}$. As $P_{\rm av}$ approaches $P_{\min}(r_{\rm o},\mathcal{H}_\epsilon)$, $h_0^* \to \infty$ and the power allocation (10) will reduce to the on-off channel inversion allocation (9).

IV. Optimum Service Sets

In this section, we will prove Theorem 2, which implies that we can find an optimal solution whose service set $\mathcal{H}_{\rm s}(p^*(h))$ includes the good channel states $\mathcal{H}_\epsilon$. Our approach will be to show that given an arbitrary feasible power allocation scheme $\hat{p}(h)$, we can always construct a better scheme $p''(h)$ which satisfies $\mathcal{H}_{\rm o}(p''(h)) \prec \mathcal{H}_{\rm s}(p''(h))$. This implies that there is an optimum power allocation scheme $p^*(h)$ with $\mathcal{H}_{\rm o}(p^*(h)) \prec \mathcal{H}_{\rm s}(p^*(h))$.

Let $\hat{\mathcal{H}}_{\rm s}$ denote the service set and $\hat{R}$ the average rate for the policy $\hat{p}(h)$. Feasibility of $\hat{p}(h)$ implies $E_h\left\{\hat{p}(h)\right\} \leq P_{\rm av}$ and $\Pr\{\hat{\mathcal{H}}_{\rm s}\} \geq 1-\epsilon$. We use a two-step construction. First, we construct $p'(h)$ from $\hat{p}(h)$ by setting $\mathcal{H}_a = \hat{\mathcal{H}}_{\rm s}$ in problem (3), yielding the solution

$$\begin{aligned} p'(h) &= p^*(h,\hat{\mathcal{H}}_{\rm s}) \\ &= \begin{cases} \dfrac{\sigma^2(2^{2r_{\rm o}}-1)}{h} & h \in \hat{\mathcal{H}}_{\rm s} \cap \left\{h < h_0' 2^{2r_{\rm o}}\right\}\,, \\ \sigma^2\left(\dfrac{1}{h_0'} - \dfrac{1}{h}\right)^+ & \text{otherwise}\,. \end{cases} \end{aligned} \quad (11)$$

where h_0' is the solution of $E_h\{p^*(h,\hat{\mathcal{H}}_{\rm s})\} = P_{\rm av}$. Here in the case of $P_{\rm av} = P_{\min}(r_{\rm o},\hat{\mathcal{H}}_{\rm s})$, $p'(h)$ can be expressed by (11) as $h_0' \to \infty$. Clearly, $p'(h)$ is feasible and achieves a higher average rate than $\hat{p}(h)$. Second, we construct $p''(h)$ by decomposing $p'(h)$ into a water filling component and a residual power component. Given h_0', we define the following functions over the whole channel state space:

$$p'_{\rm wf}(h) = \sigma^2\left(\frac{1}{h_0'} - \frac{1}{h}\right)^+ \qquad 0 \leq h \leq \infty\,, \quad (12)$$

$$p'_{\rm res}(h) = \left(\frac{\sigma^2(2^{2r_{\rm o}}-1)}{h} - p'_{\rm wf}(h)\right)^+ \qquad 0 \leq h \leq \infty\,. \quad (13)$$

The function $p'_{\rm wf}(h)$ is a water filling allocation over the whole channel space. The function $p'_{\rm res}(h)$ is the nonnegative difference of channel inversion and water filling allocations.

From (11), we observe that water filling alone meets the service condition $R[hp'(h)] \geq r_{\rm o}$ over the set of channel states

$$\mathcal{H}'_{\rm wf} = \left\{h | h \geq h_0' 2^{2r_{\rm o}}\right\}\,. \quad (14)$$

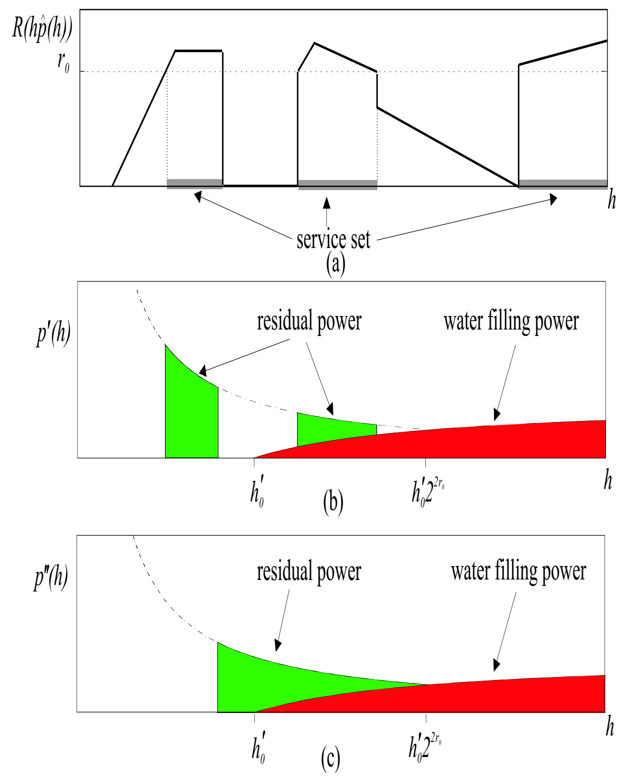

Fig. 1. (a) Rate allocation $R[h\hat{p}(h)]$ for policy $\hat{p}(h)$, (b) the improved policy $p'(h)$ given by (16) with water filling $p'_{\rm wf}(h)$ and residual power $1(h \in \mathcal{H}'_{\rm inv})p'_{\rm res}(h)$, (c) the new policy $p''(h)$ given by (17) with water filling $p'_{\rm wf}(h)$ and residual power $1(h \in \mathcal{H}''_{\rm inv})p'_{\rm res}(h)$.

In particular, $p'_{\rm res}(h) = 0$ for $h \in \mathcal{H}'_{\rm wf}$ while residual power $p'_{\rm res}(h) > 0$ is needed to meet the service condition over the channel inversion set

$$\mathcal{H}'_{\rm inv} = \hat{\mathcal{H}}_{\rm s} \backslash \mathcal{H}'_{\rm wf}\,. \tag{15}$$

Thus, in terms of the indicator function $1(x)$ such that $1(x) = 1$ when x is true, and 0 otherwise, $p'(h)$ can be rewritten in the form

$$p'(h) = p'_{\rm wf}(h) + 1(h \in \mathcal{H}'_{\rm inv})p'_{\rm res}(h)\,. \tag{16}$$

Here, we call $1(h \in \mathcal{H}'_{\rm inv})p'_{\rm res}(h)$ the residual power allocation for $p'(h)$. As shown in Figure 1, $p'(h)$ can be viewed as a two-layer allocation: the first layer is the water filling allocation over the whole channel space and the second layer is the residual power allocation over $\mathcal{H}'_{\rm inv}$.

Based on $p'(h)$, we construct $p''(h)$ by preserving the first layer water filling allocation and redistributing the residual power. Intuitively, the best allocation scheme for the residual power is to allocate it to the good channel states. Since $p'_{\rm res}(h)$ is strictly positive within $0 \le h < h'_0 2^{2r_{\rm o}}$, we will allocate the residual power over the interval $[h'_{\rm b}, h'_0 2^{2r_{\rm o}}]$ where $h'_{\rm b}$ is chosen to consume the total residual power. As shown in Figure 1, we have

$$p''(h) = p'_{\rm wf}(h) + 1(h \in \mathcal{H}''_{\rm inv})p'_{\rm res}(h)\,. \tag{17}$$

where

$$\mathcal{H}''_{\rm inv} = \left\{h'_{\rm b} \le h < h'_0 2^{2r_{\rm o}}\right\}\,, \tag{18}$$

and $h'_{\rm b}$ is the solution to

$$\int_{\mathcal{H}''_{\rm inv}} p'_{\rm res}(h) f(h)\, dh = \int_{\mathcal{H}'_{\rm inv}} p'_{\rm res}(h) f(h)\, dh\,. \tag{19}$$

Note that (16), (17), and (19) imply that $p''(h)$ has the same total power as $p'(h)$.

Let R' and R'' denote the average rates for $p'(h)$ and $p''(h)$, respectively. The following lemma gives us the properties of $p''(h)$.

Lemma 1 *The power scheme $p''(h)$ has the following properties:*

(a) $E_h\{p''(h)\} = E_h\{p'(h)\} = P_{\rm av}$
(b) $\mathcal{H}_{\rm o}(p''(h)) \prec \mathcal{H}_{\rm s}(p''(h))$
(c) $R'' \ge R'$
(d) $Pr\{\mathcal{H}_{\rm s}(p''(h))\} \ge Pr\{\mathcal{H}_{\rm s}(p'(h))\}$.

Proof of Lemma 1 is given in Appendix . Hence, we summarize the proof:

1) Start with arbitrary $\hat{p}(h)$ with average rate $\hat{R}$ and service set $\hat{\mathcal{H}}_{\rm s}$.
2) Set $\mathcal{H}_a = \hat{\mathcal{H}}_{\rm s}$ and solve (3) yielding $p'(h)$ with rate $R' \ge \hat{R}$ and service set $\mathcal{H}_{\rm s}(p'(h))$ containing $\hat{\mathcal{H}}_{\rm s}$.
3) Decompose $p'(h)$ into water filling $p'_{\rm wf}(h)$ and residual power components $1(h \in \mathcal{H}'_{\rm inv})p'_{\rm res}(h)$.
4) Fix the water filling component $p'_{\rm wf}(h)$ and reallocate the residual power to generate $p''(h)$. The power policy $p''(h)$ satisfies $\Pr\{\mathcal{H}_{\rm s}(p''(h))\} \ge \Pr\{\mathcal{H}_{\rm s}(p'(h))\}$ and $R'' \ge R'$. Hence, $p''(h)$ is a better power allocation scheme than $p'(h)$ for problem (2).

We can conclude that from any feasible $\hat{p}(h)$ we can obtain a better power allocation $p''(h)$ in which $\mathcal{H}_{\rm o}(p''(h)) \prec \mathcal{H}_{\rm s}(p''(h))$ holds. This implies that problem (2) has an optimum solution $p^*(h)$ satisfying $\mathcal{H}_{\rm o}(p^*(h)) \prec \mathcal{H}_{\rm s}(p^*(h))$.

V. Properties of the Optimum Policy

In Section III, we derived the optimum allocation scheme for problem (2). In this section, we will discuss this optimum solution, and show how problem (2) in this paper is related to the capacity versus outage probability problem.

The optimum power allocation scheme (10) includes a combination of channel inversion and water filling. For a given probability distribution $f(h)$, the optimum solution belongs to one of the following possible types depending on the value of $(P_{\rm av}, r_{\rm o}, \epsilon)$:[1]

I When $P_{\rm av} = P_{\rm min}(r_{\rm o}, \mathcal{H}_\epsilon)$, $p^*(h)$ includes no transmission for $h < h_\epsilon$ and channel inversion for $h \ge h_\epsilon$.

II When $P_{\rm av} > P_{\rm min}(r_{\rm o}, \mathcal{H}_\epsilon)$ but $h_\epsilon \le h_0^*$, $p^*(h)$ includes no transmission for $h < h_\epsilon$, channel inversion for $h_\epsilon \le h < h_0^* 2^{2r_{\rm o}}$, and water filling for $h \ge h_0^* 2^{2r_{\rm o}}$.

III When $P_{\rm av}$ is sufficiently high such that $h_\epsilon 2^{-2r_{\rm o}} < h_0^* < h_\epsilon$, $p^*(h)$ includes no transmission for $h < h_0^*$, water filling for $h_0^* \le h < h_\epsilon$, channel inversion for $h_\epsilon \le h < h_0^* 2^{2r_{\rm o}}$, and water filling for $h \ge h_0^* 2^{2r_{\rm o}}$.

[1]In the case of $r_{\rm o} = 0$ or $\epsilon = 1$, the solution types II and III will degenerate into solution type IV, which is the pure water filling allocation.

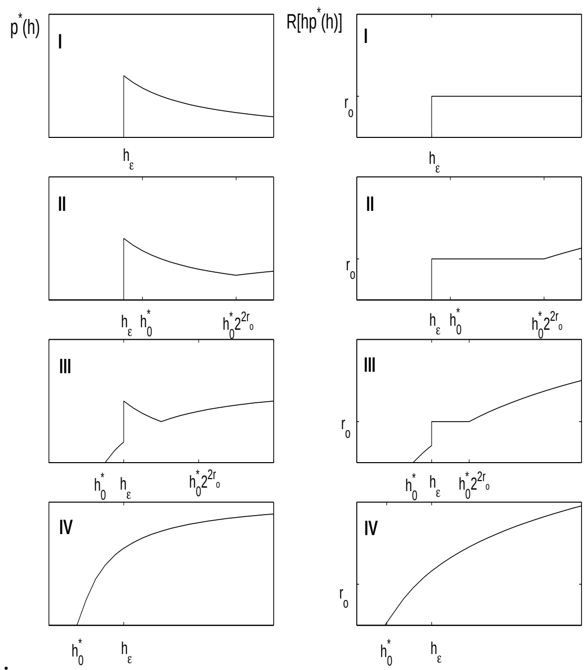

Fig. 2. For optimum solution types I-IV, power policies are given on the left and corresponding rate allocation are on the right.

IV When $P_{\rm av}$ is high enough for $h_0^* \leq h_\epsilon 2^{-2r_{\rm o}}$, $p^*(h)$ is just the water filling allocation.

These four types of power allocation schemes as well as the corresponding rate allocations are depicted in Figure 2. For solution types I, II, and III, the optimum service set is $\mathcal{H}_{\rm s}(p^*(h)) = \mathcal{H}_\epsilon$ and the resulting outage probability is ϵ, while for type IV solution $\mathcal{H}_\epsilon \subseteq \mathcal{H}_{\rm s}(p^*(h))$ and the resulting outage probability is less than ϵ. Type I solution is the on-off channel inversion allocation. In this case, we have just enough average power to satisfy the service outage constraint. When we have extra power beyond $P_{\min}(r_{\rm o}, \mathcal{H}_\epsilon)$, we can allocate the power in a more efficient way to obtain a higher average rate and, at the same time, to meet the service outage constraint. When $P_{\rm av}$ is sufficiently high for the water filling allocation to satisfy the service outage constraint, then it must also be the optimum solution for problem (2). Thus, for a given pair $(r_{\rm o}, \epsilon)$, the optimum power allocation scheme gradually changes from the on-off channel inversion allocation to the water filling allocation as $P_{\rm av}$ increases.

Now we examine the connection of the service outage problem with the outage capacity in [10] and the ergodic capacity in [4]. The outage capacity $C_\epsilon(P_{\rm av})$ in [10] specifies the maximum supportable rate for a given average power $P_{\rm av}$ with outage probability ϵ, which implies that the basic service rate in this work must satisfy $r_{\rm o} \leq C_\epsilon(P_{\rm av})$. It is easy to see that the above condition is equivalent to the feasibility condition (8), that is, $P_{\rm av} \geq P_{\min}(r_{\rm o}, \mathcal{H}_\epsilon)$. Furthermore, we can see that the resulting average rate R^* changes from $r_{\rm o}(1-\epsilon)$ to the ergodic capacity with increasing $P_{\rm av}$ for a given $(r_{\rm o}, \epsilon)$. In Fig 3, the expected rate achieved by the service outage approach is plotted against the ergodic capacity and the outage capacity in Rayleigh fading channel with normalized mean for channel gain and normalized noise variance. As we can see, for a given outage probability $\epsilon = 0.01$, the outage capacity has nearly a 5 dB loss in average power compared to the ergodic capacity for a given rate. Between the outage capacity and the ergodic capacity, a number of service outage approaches with different basic rates exist. The outage probability for different r_0 achieved by the water filling allocation is also plotted against the service outage approach with a given $\epsilon = 0.01$ in Fig 4. It can be observed that, for a range of $P_{\rm av}$, the service outage approach achieves a rate very close to the ergodic capacity, and at the same time significantly reduces the outage probability. Hence, the service outage approach strikes good balance between average rate and outage probability.

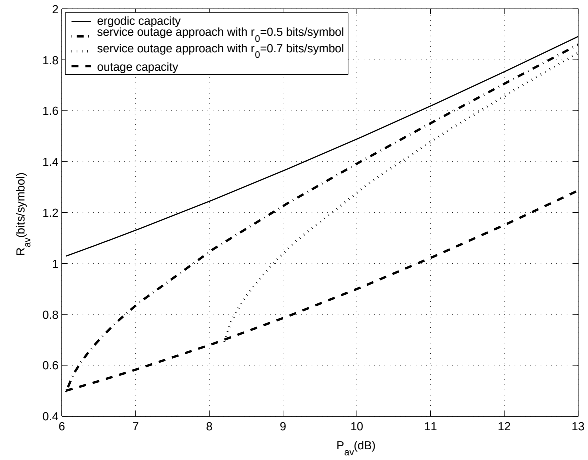

Fig. 3. Comparison of service outage approach with other capacity notions in the Rayleigh fading channel, for a fixed $\epsilon = 0.01$.

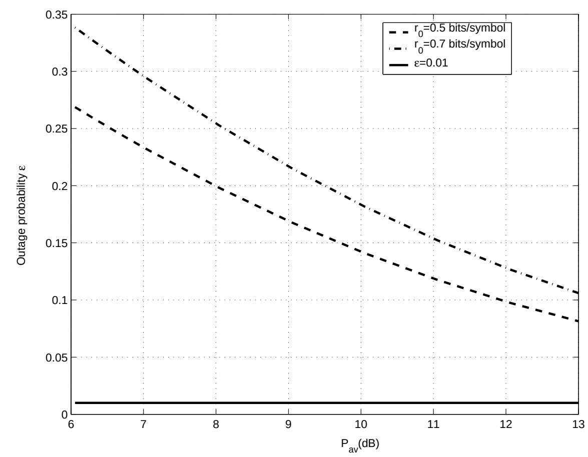

Fig. 4. Outage probability achieved by the water filling allocation for different basic rate r_0.

Appendix

Proof: Theorem 1 When $P_{\rm av} \geq P_{\min}(r_{\rm o}, \mathcal{H}_a)$, problem (3) is feasible and can be, equivalently, translated into the following problem:

$$\max_{p(h)} E_h\left\{R[hp(h)]\right\} \tag{20}$$

$$\text{subject to} \quad E_h\left\{p(h)\right\} = P_{\mathrm{av}} \tag{20a}$$

$$p(h) \geq 0 \tag{20b}$$

$$p(h) \geq \frac{\sigma^2(2^{2r_{\mathrm{o}}}-1)}{h} \qquad h \in \mathcal{H}_a\,. \tag{20b}$$

This is a standard variational optimization problem [14]. The objective function is concave on $p(h)$ and the constraints are linear functions of $p(h)$. Then $p^*(h, \mathcal{H}_a)$ is the optimum solution iff it satisfies the the Kuhn-Tucker conditions [15]. Using a Lagrange multiplier $\frac{h_0}{2log(2)\sigma^2} > 0$, we define

$$g(p(h), h, h_0) = \left[R[hp(h)] - \frac{h_0}{2\log(2)\sigma^2}p(h)\right] f(h)\,. \tag{21}$$

Let $\overline{\mathcal{H}}$ denote the set with non-boundary points as

$$\overline{\mathcal{H}} = \left\{h \in \mathcal{H}_a \middle| p^*(h, \mathcal{H}_a) > \frac{\sigma^2(2^{2r_{\mathrm{o}}}-1)}{h}\right\} \cup \left\{h \notin \mathcal{H}_a | p^*(h, \mathcal{H}_a) > 0\right\}\,. \tag{22}$$

It is easy to verify that when $h \in \overline{\mathcal{H}}$ the $p^*(h, \mathcal{H}_a)$ satisfies

$$\frac{dg(p^*(h, \mathcal{H}_a), h, h_0)}{dp^*(h, \mathcal{H}_a)} = 0, \tag{23}$$

otherwise the $p^*(h, \mathcal{H}_a)$ satisfies

$$\frac{dg(p^*(h, \mathcal{H}_a), h, h_0)}{dp^*(h, \mathcal{H}_a)} \leq 0. \tag{24}$$

Therefore, the $p^*(h, \mathcal{H}_a)$ is the optimum solution. □

Proof: Lemma 1 Power schemes $p'(h)$ and $p''(h)$ differ in the allocation of the residual power. In order to show that $p''(h)$ allocates the residual power in a better way than $p'(h)$, we define the following power efficiency function for $p'_{\mathrm{res}}(h)$ over its strictly positive space.

Definition 2 The power efficiency function $\eta(h)$ for $p'_{\mathrm{res}}(h)$ is

$$\eta(h) = \frac{r_{\mathrm{o}} - R[hp'_{\mathrm{wf}}(h)]}{p'_{\mathrm{res}}(h)} \qquad 0 \leq h < h'_0 2^{2r_{\mathrm{o}}}\,. \tag{25}$$

The power efficiency function indicates the rate increment corresponding to a unit power assigned from $p'_{\mathrm{res}}(h)$. We have the following property for $\eta(h)$.

Proposition 1 *The power efficiency function $\eta(h)$ is a strictly increasing function of h over the interval $0 \leq h < h'_0 2^{2r_{\mathrm{o}}}$.*

Proof: Proposition 1 We consider the cases $h \leq h'_0$ and $h \geq h'_0$ separately. For $h \leq h'_0$, we have $p'_{\mathrm{wf}}(h) = 0$ and

$$\eta(h) = \frac{hr_{\mathrm{o}}}{\sigma^2(2^{2r_{\mathrm{o}}}-1)}, \tag{26}$$

which is an increasing function of h.

For $h \geq h'_0$, (12), (13), and (25) imply

$$\eta(h) = \frac{r_{\mathrm{o}} - (1/2)\log_2(h/h'_0)}{\sigma^2\left(\frac{2^{2r_{\mathrm{o}}}}{h} - \frac{1}{h'_0}\right)}\,. \tag{27}$$

We define $u(h) = r_{\mathrm{o}} - 1/2\log_2(h/h'_0)$, so that $\eta(h) = \hat{\eta}(u(h))$ where

$$\hat{\eta}(u) = \frac{h'_0}{\sigma^2}\frac{u}{2^{2u}-1} \tag{28}$$

It is straightforward to verify that $\hat{\eta}(u)$ is a strictly decreasing function of u for $u \geq 0$. Since $u(h)$ is a strictly decreasing function of h and $u(h) \geq 0$ when $h \leq h'_0 2^{2r_{\mathrm{o}}}$, it follows that $\eta(h)$ is an increasing function of h for $h'_0 \leq h \leq h'_0 2^{2r_{\mathrm{o}}}$. □

We also employ the following proposition for the proof of Lemma 1.

Proposition 2 *For disjoint sets Ψ' and Ψ'', let $f(x)$ be an arbitrary function such that $f(x'') \geq f(x')$ for all $x'' \in \Psi''$ and $x' \in \Psi'$. For any nonnegative function $g(x)$ satisfying $\int_{\Psi''} g(x)dx = \int_{\Psi'} g(x)dx$, we have $\int_{\Psi''} f(x)g(x)dx \geq \int_{\Psi'} f(x)g(x)dx$.*

With these preliminaries, we now verify the claims of Lemma 1.

(a) Equations (16), (17), and (19) imply $E_h\left\{p''(h)\right\} = E_h\left\{p'(h)\right\} = P_{\mathrm{av}}$.

(b) From equations (17) and (18), the service and outage sets of $p''(h)$ are $\mathcal{H}_{\mathrm{s}}(p''(h)) = \{h | h \geq h'_{\mathrm{b}}\}$ and $\mathcal{H}_{\mathrm{o}}(p''(h)) = \{h | h < h'_{\mathrm{b}}\}$ respectively. Therefore, $\mathcal{H}_{\mathrm{o}}(p''(h)) \prec \mathcal{H}_{\mathrm{s}}(p''(h))$.

(c) Let $\Psi = \mathcal{H}'_{\mathrm{inv}} \cap \mathcal{H}''_{\mathrm{inv}}$ so that $\Psi' = \mathcal{H}'_{\mathrm{inv}} \backslash \Psi$ and $\Psi'' = \mathcal{H}''_{\mathrm{inv}} \backslash \Psi$ are two disjoint sets and nonempty when $p''(h) \neq p'(h)$. The average rate of $p'(h)$ can be expressed as

$$R' = \int_0^\infty R[hp'_{\mathrm{wf}}(h)]f(h)\,dh + \int_{\mathcal{H}'_{\mathrm{inv}}} (r_{\mathrm{o}} - R[hp'_{\mathrm{wf}}(h)])f(h)\,dh\,. \tag{29}$$

The rate contribution of the water filling component is

$$R_{\mathrm{wf}} = \int_0^\infty R[hp'_{\mathrm{wf}}(h)]f(h)\,dh\,. \tag{30}$$

Since $\mathcal{H}'_{\mathrm{inv}} = \Psi \cup \Psi'$, Definition 2 for the efficiency function $\eta(h)$ allows us to write

$$R' = R_{\mathrm{wf}} + \int_{\mathcal{H}'_{\mathrm{inv}}} \eta(h)p'_{\mathrm{res}}(h)f(h)\,dh \tag{31}$$

$$= R_{\mathrm{wf}} + \int_{\Psi} \eta(h)p'_{\mathrm{res}}(h)f(h)\,dh + \int_{\Psi'} \eta(h)p'_{\mathrm{res}}(h)f(h)\,dh\,. \tag{32}$$

Similarly, $\mathcal{H}''_{\mathrm{inv}} = \Psi \cup \Psi''$, so the average rate for $p''(h)$ can be expressed as

$$R'' = R_{\mathrm{wf}} + \int_{\Psi} \eta(h)p'_{\mathrm{res}}(h)f(h)\,dh + \int_{\Psi''} \eta(h)p'_{\mathrm{res}}(h)f(h)\,dh\,. \tag{33}$$

Thus,

$$R'' - R' = \int_{\Psi''} \eta(h) p'_{\rm res}(h) f(h)\, dh - \int_{\Psi'} \eta(h) p'_{\rm res}(h) f(h)\, dh\,. \quad (34)$$

Note that the construction of $\mathcal{H}''_{\rm inv}$ implies $\Psi' \prec \Psi''$. That is, $h'' \geq h'$ for any $h'' \in \Psi''$ and $h' \in \Psi'$. By Proposition 1, $\eta(h)$ is a strictly increasing function of h for $0 \leq h < h'_0 2^{2r_o}$. Thus, $\eta(h'') \geq \eta(h')$. Furthermore, equation (19) implies

$$\int_{\Psi''} p'_{\rm res}(h) f(h)\, dh = \int_{\Psi'} p'_{\rm res}(h) f(h)\, dh\,. \quad (35)$$

Therefore, the conditions of Proposition 2 are satisfied and we have $R'' \geq R'$.

(d) From equations (11), (15), (17) and (18), the service sets $\mathcal{H}_{\rm s}(p'(h))$ and $\mathcal{H}_{\rm s}(p''(h))$ are disjoint unions given by

$$\mathcal{H}_{\rm s}(p'(h)) = \mathcal{H}'_{\rm wf} \cup \mathcal{H}'_{\rm inv} = \mathcal{H}'_{\rm wf} \cup \Psi \cup \Psi'\,, \quad (36)$$
$$\mathcal{H}_{\rm s}(p''(h)) = \mathcal{H}'_{\rm wf} \cup \mathcal{H}''_{\rm inv} = \mathcal{H}'_{\rm wf} \cup \Psi \cup \Psi''\,. \quad (37)$$

This implies

$$\Pr\{\mathcal{H}_{\rm s}(p''(h))\} - \Pr\{\mathcal{H}_{\rm s}(p'(h))\} \quad (38)$$
$$= \Pr\{\Psi''\} - \Pr\{\Psi'\} \quad (39)$$
$$= \int_{\Psi''} \frac{1}{p'_{\rm res}(h)} p'_{\rm res}(h) f(h)\, dh - \int_{\Psi'} \frac{1}{p'_{\rm res}(h)} p'_{\rm res}(h) f(h)\, dh\,. \quad (40)$$

From equations (12) and (13), we observe that $1/p'_{\rm res}(h)$ is a increasing function of h. Since $\Psi' \prec \Psi''$, Proposition 2 implies $\Pr\{\mathcal{H}_{\rm s}(p''(h))\} \geq \Pr\{\mathcal{H}_{\rm s}(p'(h))\}$.

□

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LIST OF FIGURES

1 (a) Rate allocation $R[h\hat{p}(h)]$ for policy $\hat{p}(h)$, (b) the improved policy $p'(h)$ given by (16) with water filling $p'_{\rm wf}(h)$ and residual power $1(h \in \mathcal{H}'_{\rm inv}) p'_{\rm res}(h)$, (c) the new policy $p''(h)$ given by (17) with water filling $p'_{\rm wf}(h)$ and residual power $1(h \in \mathcal{H}''_{\rm inv}) p'_{\rm res}(h)$. 4

2 For optimum solution types I-IV, power policies are given on the left and corresponding rate allocation are on the right. 5

3 Comparison of service outage approach with other capacity notions in the Rayleigh fading channel, for a fixed $\epsilon = 0.01$. 5

4 Outage probability achieved by the water filling allocation for different basic rate r_0. 5

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