16:330:541   **Stochastic Signals and Systems**   **First Exam**

October 26, 1998

You have 100 minutes to complete this exam. The point value of each part is indicated. Please read the exam carefully and ask the instructor if you have any questions.

1. A certain device contains n circuits and it works if one or more of those circuits work. Each circuit fails with probability q, independent of any other circuit. A working device, even if it contains one or more broken circuits, can be sold for \$50 but a broken device must be discarded.

    (a) *10 points* Suppose you test devices until you find a working device. What is the PMF of K, the number of devices you test?

    (b) *10 points* If each circuit costs \$5, what is the expected profit $E[R]$ per device?

    (c) *10 points* Suppose $n = 5$. What is the PMF of D, the number of working circuits in a device? Given the event W that the device works, find the conditional PMF of D given the event W.

    (d) *10 points* Ultrareliable circuits cost \$10 each but fail with probability $q/2$. Suppose $n = 2$ and you have the option of substituting zero, one or two ultrareliable circuits for the ordinary circuits. Let $E[R_i]$ denote your expected profit using i ultrareliable circuits. For what values of q is it best to use exactly 1 ultrareliable circuit?

    (e) *10 points* Let M equal the number of circuits tested in order to find enough good circuits for 100 devices. What is the PMF of M?

2. *50 points* Random variable X is Gaussian with zero mean and unit variance. Given $X = x$, Y is a Gaussian random variable with mean $10x$ and variance 1. Note that some parts of this problem are easiest using conditional expectations. Find $E[Y]$, $E[Y^2]$, $\sigma_{X,Y}$, and $f_{X,Y}(x,y)$. Do X and Y have a bivariate Gaussian PDF? Justify your answer carefully.

3. *50 points* $X_1, X_2, \ldots, X_n$ are iid random variables, each uniformly distributed over $[0,1]$.

    (a) *10 points* Find the joint PDF $f_{X_1,X_2}(x_1,x_2)$.

    (b) *10 points* Find the mean and variance of $W = X_1 + \cdots + X_n$.

    (c) *10 points* Find the probability $P[A] = P[X_1 \le X_2 \le \cdots \le X_n]$.

    (d) *20 points* Find the PDF of $Y = X_3 X_4$.