16:332:541 **Stochastic Signals and Systems** **Midterm Exam**

November 5, 2008

You have 90 minutes to complete the first three problems of this exam. Find the limiting state probabilities. Items with unspecified point values are worth ten points. Please read both sides of the exam carefully and ask the instructor if you have any questions.

Preliminary: *(10 points)* Put your name and your Rutgers netid on the front of each exam bluebook. Write your random but memorable personal code (from the first quiz) on the *upper left corner of the inside front cover* of your first bluebook.

1. *40 points* Ten runners compete in a race starting at time $t = 0$. The runners' finishing times $R_1, \ldots, R_{10}$ are iid exponential random variables with expected value $1/\mu = 10$ minutes.

   (a) What is the probability that the last runner will finish in less than 20 minutes?

   (b) What is the PDF of X_1, the finishing time of the winning runner?

   (c) Find the PDF $f_Y(y)$ of $Y = R_1 + \cdots + R_{10}$.

   (d) Let $X_1, \ldots, X_{10}$ denote the runners' interarrival times at the finish line. Find the joint PDF $f_{X_1,\ldots,X_{10}}(x_1, \ldots, x_{10})$.

2. *40 points* The Gaussian random vector $\mathbf{X} = \begin{bmatrix} X_1 & X_2 \end{bmatrix}'$ has expected value $E[\mathbf{X}] = \mathbf{0}$ and covariance matrix

$$\mathbf{C}_X = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

   (a) Find the PDF of $W = X_1 + 2X_2$.

   (b) Let $V = 2X_1$. Find the conditional PDF $f_{V|W}(v|w)$.

   (c) Find the PDF $f_{\mathbf{Y}}(\mathbf{y})$ of $\mathbf{Y} = \mathbf{A}\mathbf{X}$ where $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

   (d) Does there exist a stationary Gaussian process $X(t)$ and time instances t_1 and t_2 such that $\mathbf{X}$ is actually a pair of observations $\begin{bmatrix} X(t_1) & X(t_2) \end{bmatrix}'$ of the process $X(t)$? Explain your answer.

3. Packets arrive at a forwarding node as a Poisson process of rate 1 per millisecond (ms). The forwarder simply forwards (ie transmits) packets stored in its infinite capacity buffer. When the node is working, arriving packets are queued in the buffer and packet transmission times are independent exponential random variables with expected service time of $1/\mu = 0.5$ ms. However, the forwarding node takes a break after the completion of a packet transmission. This break has an exponential duration with expected value $1/\beta$ ms, independent of the arrival process and packet transmission times. During the break, the forwarder discards all arriving packets. Following the break, the node goes back to work by transmitting a previously buffered packet.

   (a) Let M_1 denote the number of arriving packets during a one second interval of time. Find $E[M_1]$ and the PMF $P_{M_1}(m) = P[M_1 = m]$.

   (b) Sketch a continuous time Markov chain for this system. Hint: the forwarder may be *W*orking or on *B*reak when there are there are n buffered packets. For what values of β is the Markov chain irreducible?

   (c) Find the limiting state probabilities when the chain is irreducible.

   (d) After the system has been running for a long time, what is the probability $P[D]$ that an arriving packet is discarded?