16:332:541	**Stochastic Signals and Systems**	**Final Exam**

December 22, 2006

You have 180 minutes to complete this exam. Put your name and your Rutgers netid (but no part of your SSN or Rutger ID) on each exam book *(10 points)*. Please read *both* sides of the exam carefully and answer **all four questions**. Ask the instructor if you have any questions.

x	0.0	0.5	1.0	1.5	2.0	2.5	3
$Q(x) = 1 - \Phi(x)$	0.500	0.3085	0.1587	0.0668	0.02275	0.00621	$1.35 \cdot 10^{-3}$

1. *30 points* TRUE OR FALSE. All answers must be justified. Keep in mind that for an answer to be TRUE, it must be true in all possible cases.

   (a) X_1 and X_2 are jointly Gaussian random variables. For any constant y, there exists a constant a such that $P[X_1 + aX_2 \leq y] = 1/2$.

   (b) For identically distributed zero-mean random variables Y_1 and Y_2, $\text{Var}[Y_1+Y_2] \geq \text{Var}[Y_1]$.

   (c) If $X(t)$ and $Y(t)$ are independent zero-mean wide-sense stationary processes, then $W(t) = X(t) + Y(t)$ is wide-sense stationary.

2. *60 points* Squares are labelled 1 through n consecutively from left to right. A player starts by placing a token on square k where $1 < k < n$. On each turn, a six-sided die is rolled. With a roll of 1 or 2, the token moves one square to the left. With a roll of 3 or higher, the token moves one square to the right. The player wins if the token reaches square 1. The game is a loss if the token reaches square n. When the game ends, the token stays on its final square (1 or n).

   (a) Sketch a Markov chain that describes the position of the token and find the state transition matrix.

   (b) Identify any recurrent communicating classes, the set of transient states (if any).

   (c) Identify the set of all possible stationary distributions (if any exist).

   (d) *20 points* Let W_k denote the event that you win the game starting from position k. Outline a calculation procedure to find $P[W_k]$ for a given value of k. You do not need to write matlab code, however you must provide a sequence of steps that a programmer could translate into matlab.

   (e) For the special case of $n = 4$, find $P[W_2]$ and $P[W_3]$. Hint: Don't use the calculation procedure from part (b).

3. *50 points* The random sequence $X_0, X_1, X_2, \ldots$ is an iid sequence of Gaussian $(0, 1)$ random variables. $N(t)$ is a Poisson process of rate λ that is independent of the X_n. Let $\{Y(t)|t \geq 0\}$ denote a random process defined by $Y(t) = \sum_{n=0}^{N(t)} X_n$. Answers to the following questions must be justified.

   (a) Find the conditional CDF $F_{Y(t)|N(t)}(y|n) = P[Y(t) \leq y|N(t) = n]$. Express your answer in terms of the $\Phi(\cdot)$ function.

   (b) Is $Y(t)$ a Gaussian process?

   (c) Is $Y(t)$ a stationary process?

   (d) *20 points* Is the process wide-sense stationary? Find the expected value function $\mu_Y(t) = E[Y(t)]$ and the autocovariance function $C_Y(t, \tau)$.

4. *60 points* Suppose you have n suitcases. Suitcase i holds X_i dollars where $X_1, X_2, \ldots, X_n$ are iid continuous uniform $(0, m)$ random variables. (Think of a number like one **m**illion for the symbol m.) Unfortunately, you can't find out X_i until you open suitcase i.

(a) Suppose you can open all n suitcases and then choose the suitcase with the most money. Let Y denote the amount you receive. What is $E[Y]$?

(b) Suppose you must open the suitcases one-by-one, starting with suitcase n and going down to suitcase 1. After opening suitcase i, you can either accept or reject X_i dollars. If you accept suitcase i, the game ends. If you reject, then you get to choose only from the still unopened suitcases.

What should you do? Perhaps it is not so obvious? In fact, you can decide before the game on a policy, a set of rules to follow. We will specify a policy by a vector $(\tau_1, \ldots, \tau_n)$ of threshold parameters.

- After opening suitcase i, you accept the amount X_i if $X_i \geq \tau_i$.
- Otherwise, you reject suitcase i and open suitcase $i - 1$.
- If you have rejected suitcases n down through 2, then you must accept the amount X_1 in suitcase 1. Thus the threshold $\tau_1 = 0$ since you never reject the amount in the last suitcase.

i. Suppose you reject suitcases n through $i + 1$, but then you accept suitcase i. Find $E[X_i | X_i \geq \tau_i]$.

ii. Let W_k denote your reward given that there are k unopened suitcases remaining. What is $E[W_1]$?

iii. *20 points* As a function of τ_k, find a recursive relationship for $E[W_k]$ in terms of τ_k and $E[W_{k-1}]$.

iv. For $n = 4$ suitcases, find the optimal policy $(\tau_1^*, \ldots, \tau_4^*)$, that maximizes $E[W_4]$.