MINIMUM ENERGY TRANSMISSION SCHEMES FOR MOBILE AD-HOC NETWORKS

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Abstract

This thesis proposes several novel relay transmission schemes that lead to minimum energy consumption in a mobile ad-hoc network. The study introduces mobility in the form of a node that is capable of collecting data from sources randomly distributed in space, and delivering it to a fixed gateway. After defining an energy cost function based on transmission powers, it describes ways to minimize the cost for a host of network configurations and mobile trajectories. These include motion in a straight line, piece-wise linear motion, and finally, random motion in bounded and unbounded space. The proposed algorithms are supported by proofs based on the theories of analytic geometry and Markov processes.

Acknowledgments

The author would like to begin by expressing his sincere gratitude to Professor Korman, Professor Subramaniam and Professor Ahmadi for the amount of time they have shared in countless meetings and discussions and for steering the work in the right direction. Without their guidance and invaluable input, this research could not have been completed in this time-frame.

Thanks are also due to the other Professors in the Department who have played a role in kindling and keeping the interest in wireless networks alive— the name of Dr. Saul Torrico stands out in this respect— and to colleagues and friends, particularly Yang Cao, whose help and suggestions made the work much easier.

Finally, the author is deeply indebted to his parents for their inspiration and support through the years, and humbly dedicates this work to them.

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Chapter 1

Introduction

A wireless ad-hoc network is a multi-hop network in which all nodes cooperatively maintain network connectivity without a centralized infrastructure. If these nodes change their positions dynamically, it is called a mobile ad-hoc network (MANET). Thus, a MANET may be defined as a collection of mobile nodes that maintain inter-connection without the intervention of a centralized access point. Different routing protocols use different metrics to dynamically determine the optimal path between the sender and the recipient. This study considers energy consumption as the primary criterion for routing decisions.

A typical scenario may comprise a group of wireless static nodes, called *sources*, randomly distributed in a region as shown in Figure 1.1. At each of these nodes, information is generated at random times and this data needs to be delivered to a destination node, called the *gateway*. However, each source has limited battery supply for the transmission of this data and therefore, to maximize its lifetime, it would have to choose a routing scheme that would consume the least amount of energy, subject to a maximum allowed time for

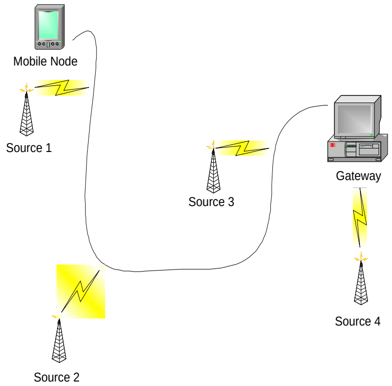


Figure 1.1: A wireless ad-hoc network where the source nodes transmit information to the gateway either through direct transmission (Source 4) or relay transmission (Sources 1, 2, 3) through the mobile node.

the transmission. This is possible by making a relay transmission through a mobile node that has the capacity to receive and transmit information from sources without a substantial increase in transmission cost.

An example of this kind of setup is a system of wireless *source* nodes distributed randomly on a battleground that transmit information to the army hub or *gateway* either directly or by relay transmission through a reconnaisance aircraft that flies overhead periodically. Apart from battlefield applications, the source may also be sensors for gathering environmental information and transmitting it to the gateway for processing.

This thesis proposes several novel routing algorithms that minimize energy consumption in a mobile ad-hoc wireless network allowing one-hop relay transmission. For simplicity of analysis, only a single mobile node is considered for transmission of information to the gateway so that it results in lower energy consumption.

The thesis report is organized as follows: After a brief introduction to ad-hoc networks in Chapter 2, the model and assumptions are described in detail in Chapter 3. Chapter 4 discusses the transmission algorithms for deterministic mobile trajectories, while Chapter 5 extends the idea to random motion. Finally, Chapter 6 concludes the study with suggestions for future areas of work.

Chapter 2

Ad Hoc Networks

2.1 Introduction

An ad-hoc network is a multi-hop wireless network where all nodes cooperatively maintain network connectivity without any centralized infrastructure. If these nodes change their positions dynamically, it is called a mobile ad-hoc network (MANET). Due to the limited transmission range of wireless nodes, as well as the rapid change in network topology, multiple network hops may be needed for one node to exchange data with another across the network. Thus, each mobile node operates not only as a host but also as a router, forwarding packets for other mobile nodes in the network that may not be within transmission range of the destination. Each node participates in an ad-hoc routing protocol that allows it to discover multi-hop paths though the network to any other node.

MANETs have the following salient characteristics [17, 2]:

- *Dynamic topologies:* Nodes are free to move arbitrarily; thus network topology— which is typically multihop— may change randomly and rapidly at unpredictable times. Adjustment of transmission and reception parameters such as power may also impact the topology.

- *Bandwidth-constrained, variable capacity links:* Wireless links typically have significantly lower capacity than their wired counterparts. One effect of this relatively low to moderate link capacity is that congestion is typically the norm rather than the exception, *i.e.*, aggregate application demand is likely to exceed network capacity frequently.
- *Power-constrained operation:* Some or all the nodes in a MANET rely on batteries for their power supply. Thus, the most important design criteria may be that of energy conservation.
- *Limited physical security:* Mobile wireless networks are generally more prone to security threats than fixed, wired networks. Existing link security techniques are often applied within wireless networks to reduce security threats.

2.2 Historical Development

Mobile wireless distributed multi-hop networking developed out of the military need for operation without preplaced infrastructure and connectivity beyond line-of-sight communications. Although packet-switching technology was first introduced by ARPANet in the 1960s, it was not until the growth of the Internet infrastructure and the microcomputer revolution that packet radio network ideas became truly applicable and feasible [22]. Mobile ad-hoc networking (also known as Mobile Packet Radio Networking) is the name given to a technology under development for the past twenty years principally by the Defence Advanced Research Projects Agency (DARPA), the United States Army and the Office of Naval Research (ONR) [17].

One of the original motivations for MANET is found in the military need for battlefield survivability [12]. Arrangements should be available for soldiers to move about freely without any of the restrictions imposed by wired communication devices. Moreover, the military cannot rely on access to a fixed, pre-placed communications infrastructure in battlefield environments. A rapidly deployable self-organizing mobile network is the primary factor that differentiates MANET design issues from those associated with commercial cellular systems.

Today, almost any commercially successful network application is a candidate for useful deployment with nodes that can form ad-hoc networks [22]. Some potential applications include:

- *Conferencing:* Mobile computer users can exchange ideas even outside the business network infrastructure.

- *Home networking:* An ad-hoc network may maintain connection between the home PC and a laptop that is moved between the home and office environments. Automation may also be introduced by forming a wireless network with other network-compatible devices such as motion detectors and security cameras [29].

- *Emergency services:* An ad-hoc network can also come in handy when the existing infrastructure is damaged or out of service due to power shortage or natural calamities.

- *Personal area networks (PAN):* The idea of a PAN is to create a very localized network populated by some network nodes that are closely associated with a single person. For instance, Bluetooth [13] is a short-range radio technology targeted at eliminating wires between devices like personal digital assistants (PDAs).

2.3 Routing

Routing protocols provide the information necessary for each node to forward packets to the next node along the most optimum route from the source to the destination. They are typically self-starting, adaptable to changing network conditions, and offer multi-hop paths across a network. While early routing protocols for ad-hoc networks were adapted from exisiting protocols for infrastructure-based networks, ad-hoc network routing protocols today comprise a complex and active field of research, and can be broadly classified into two categories.

1. **Proactive or Table-Driven Routing**

   Such a routing protocol keeps track of routes for all destinations in the network. It is based on traditional wired LAN/WAN routing in which the routing information is disseminated among all nodes in the network throughout the operating time irrespective of the need for such a route.

   - *Advantage:* Communications with arbitrary destinations experience minimum initial delay since the route can be immediately selected from the route table.
   - *Disadvantage:* Additional control traffic is needed to continually update stale route entries. Unlike the Internet, an ad-hoc network may contain mobile nodes and therefore links are continuously broken and reestablished.

Proactive or table-driven routing protocols may again be subdivided depending on the manner in which route tables are constructed, maintained and updated. The two primary classes are as follows [21]:

- *Link State:* In this protocol, each node maintains a view of the entire network topology with a cost for each link. To keep these views consistent, each node periodically broadcasts the link costs of its outgoing links to all other nodes using a protocol such as flooding. As a node receives the information, it updates its view of the network topology and applies a minimum-cost algorithm to choose its next hop for each destination.
- *Distance Vector (DV):* In DV routing, each node maintains a routing table consisting of a destination address, distance to the destination (number of hops) and the next node in the path. Each router periodically broadcasts this table information to each of its neighboring routers, and uses similar routing updates received from its neighbors to update its own table.

2. **Reactive or On-Demand Routing**

   Since a node in an ad-hoc network does not need a route to a destination until that destination is to be a recipient of packets sent by the node (either as the actual source of the packet or as an intermediate node along a path from the source to the destination), these protocols are defined to acquire information only when needed.

   - *Advantage:* Uses far less bandwidth to maintain route tables at each node.
   - *Disadvantage:* Since the route to a destination will have to be acquired before communications can begin, the latency period for most applications is likely to

increase drastically.

Thus, the problem of routing is essentially a distributed version of the minimum cost algorithm [21]. Each node in the network maintains or calculates for each destination a preferred neighbor (or next hop), the selection of which minimizes the cost function. The cost function that is used differs from protocol to protocol and is discussed in more detail in the next section.

2.4 Cost Metrics

The problem of routing in MANETs is compounded by node mobility [26], which results in two conflicting goals— frequent topology updates are required to optimize routes, yet frequent updates result in higher message overhead, bandwidth wastage and power loss. In this section the different metrics used for routing are enumerated, and their effects on node and network life briefly examined.

Different routing protocols use one or more of a small set of metrics to determine the optimum path:

- The most common cost metric used is shortest distance or fewest number of hops, as in the case of Dynamic Source Routing (DSR) [14], Destination-Sequenced Distance Vector (DSDV) [21], Temporally-Ordered Routing Algorithm (TORA) [20], Wireless Routing Protocol (WRP) [18] and the DARPA packet radio protocol [12]. These protocols can also use shortest delay as the metric, since the shortest distance leads to the shortest amount of time.

- Link quality is a metric that is used by Signal Stability based Adaptive Routing (SSA) [10] and by the DARPA protocol. Since link quality information is used to select one among several different routes, sometimes a shortest hop may not be used. In addition to link quality, SSA also uses location stability to bias selections towards routes with relatively stationary nodes (which will require fewer updates).

- The Spline Routing Algorithm (SRA) [9] attempts to minimize the message and time overhead for computing routes. In this protocol nodes are assigned to clusters (one or two hops in diameter) and clusters are joined together by a virtual backbone, so that packets destined for other clusters get routed by this backbone.

- In Associativity Based Routing (ABR) [27], each mobile node periodically transmits beacons to identify itself and constantly updates its associativity ticks in accordance with the mobile hosts sighted (*i.e.*, hearing others' beacons) in the neighborhood.

- Transmission energy is one of the more recent cost metrics for ad-hoc networks and the most popular algorithms in this field include Power-Aware Multi-Access Protocol with Signaling (PAMAS) [25], Minimum Energy Mobile Wireless Networks [24] and Routing for Maximum System Lifetime (MSL) [6, 7, 8]. While the Minimum Energy Protocol aims at designing a network that consumes the minimum energy per unit flow of packet, MSL uses a maximum residual energy path routing algorithm to maximize the time until any node fails. These routing protocols are discussed in more detail in the next section.

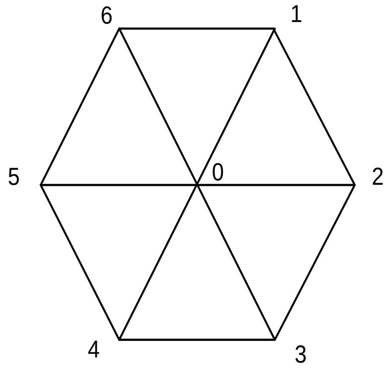


Figure 2.1: A simple network illustrating the problem with shortest hop or minimum energy as the cost metric— both metrics would result in early death of node 0 due to overuse.

2.5 Power Conservation

Most nodes in an ad-hoc network are battery-powered and therefore power conservation issues in MANETs have seen a rapid growth of interest in recent years. Shortest hop algorithms, while causing minimum delay, often result in early death for some nodes since they may be overused and their energy resources quickly exhausted [26]. For instance, in the network illustrated in Figure 2.1, node 0 will be selected as the relay node for best routes between 1–4, 2–5 and 3–6. As a result, node 0 will expend its battery resources at a faster rate than the other nodes in the network and will be the first to die. It thus follows that a serious drawback of the shortest hop or minimum delay routing algorithms is that the nodes may have widely differing energy consumption profiles which would ultimately reduce the system lifetime.

There are several protocols addressing efficient energy consumption in an ad-hoc wireless network:

- *Lazy Packet Scheduling (LPS)* [23]: This protocol considers the problem of minimising the energy used by a node on a point-to-point link to transmit packetized information within a given amount of time. It differs from common routing schemes in trying to minimize power subject to a fixed amount of information being successfully transmitted, in contrast to maximizing the information throughput for a given average power constraint. To minimize energy consumption, the transmission power is lowered and the packet is transmitted over a longer period of time, and hence it is called lazy scheduling. However, the energy is minimized subject to a deadline or a delay constraint.

- *Minimum Energy Routing* [24]: The main idea of this protocol is that a node need not consider all nodes in a network to find the global minimum power path. By defining and updating geometric relay regions corresponding to every node, the search for the next-hop node is restricted to a localized search, which leads to the lowest total energy consumption. Although implementation of this approach would require sophisticated devices relying on the Global Positioning System (GPS) to define the relay regions, simulations suggest that it results in significantly lower average power consumption per node [15].

- *Power-Aware Routing* [26]: This network protocol works in conjunction with a MAC–layer protocol called Power-Aware Multiple Access protocol with Signaling (PAMAS) [25]. It aims at increasing node and network life by using load-balancing concepts that attempt to evenly distribute routing through critical nodes, and particularly, restrict routing through nodes with depleted energy reserves. The protocol relies on special signals like RTS (request to send) and CTS (clear to send) to trigger transmission, and saves energy by turning off nodes when they are incapable of transmitting or receiving. To determine how long a node should remain switched off in an ad-hoc network scenario, a probe protocol would have to be defined which would cause the node to power back on at certain intervals.

- *Maximum System Lifetime (MSL)* [7]: This algorithm maximizes battery lifetime by routing traffic such that the energy consumption is balanced among the nodes in proportion to their energy reserves, instead of routing to minimize the absolute consumed power. The idea can be reduced to a *max-min* linear programming problem since the lifetime of a system, under a particular flow rate, is defined as the *minimum* battery lifetime over all nodes, and this lifetime is *maximized.* However, the work is limited to the analysis of static node classes— wireless static nodes (source) and static gateway nodes (destination).

- *Broadcast Incremental Power* [28]: This algorithm exploits the *wireless multicast advantage* of a broadcast environment, by which only those nodes are selected for transmission that can cover the maximum number of recipients with the minimum power. This algorithm is similar in principle to Prim–Dijkstra's algorithm for the Minimum Weight Spanning Tree [5].

- *Distributed Mobility-Aware Routing Selection (DMARS)* [1]: This algorithm uses mobility predictions to improve the performance of existing routing protocols. The DMARS algorithm is a link-state based route selection scheme that suggests modifications of exisiting unicast routing protocols by having each node maintain, update and periodically broadcast a neighbor vector that stores the unique ID, functional status and mobility vector of all its neighbors. This helps select more stable routes and reduces the routing overhead caused by user mobility.

Chapter 3

Problem Formulation

3.1 Introduction

This thesis aims at proposing a number of relay transmission schemes that guarantee delivery of information from a source to a destination with minimum consumption of energy. Each source is concerned with conserving its limited battery power as far as possible, and to this end, may choose to transmit via a mobile node should that lead to a lower consumption of its energy. If the lifetime of the system is defined as the time until any one of the source nodes runs out of power, then the algorithms formulated in the thesis will result in the system lifetime being extended.

A typical scenario would involve the following classes of nodes:

1. *Fixed gateway*, which is the final destination and has no power constraints.
2. *Large number of fixed sensors*, which transmit information signals periodically but are

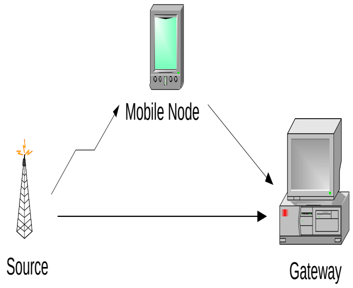


Figure 3.1: Layout of model for analysis comprising a source node, a destination and a mobile relay node. The two possible routes are a direct transmission from the source to the gateway, or a relay transmission via the mobile node.

powered by energy-limited batteries.

3. *Small number of mobile nodes*, which are capable of transmission and reception and have lower power constraints than the source nodes.

3.2 Model

For simplicity, the analysis concentrates on the power consumption behavior of a system comprising a mobile node in the presence of a single source node. Since the transmission algorithm thus obtained is executed by the source node, the results may be applied to any number of source nodes by taking interference between source nodes into consideration, although that is beyond th scope of this work. This situation is illustrated in Figure 3.1

Some of the assumptions made in the course of this analysis are as follows:

- *Collision between data packets is not considered.* This might not appear to carry much significance in the case of a single source node, but even in the general case of multiple sources, the protocol does not include any collision detection or collision avoidance schemes.

- *The propagation delay is negligible.* Since the data signals are electromagnetic signals travelling with a velocity of 3×10^8 m/s the propagation of the signals may be considered to be instantaneous.
- *Mobile nodes move with constant speed.* While the change of direction of the mobile node is considered, the speed of the node (which is calculated in real-time) is assumed to be finite and unchanged.
- *Power is consumed only during transmission.* It is assumed that energy is expended only during transmission of data (given by the inverse square law) and reception of data is cost–free. Most analyses make use of this assumption, although some authors [11] consider this to be 'unrealisitic'.

3.3 Cost Metric

As was explained before, the aim of the thesis is to design an algorithm to transmit data from a source to a destination (*gateway*) within a time constraint, using the minimum amount of energy. Accordingly, the power cost metric is being defined below as a function of transmission energy. The aim of the algorithm is to minimize the cost function given a particular time constraint.

Assuming power loss to be inversely proportional to the square of the propagation distance, the cost function is defined as:

$$
\begin{aligned}
C &= \alpha_s.P_s + \alpha_m.P_m \\
&= \alpha_s.d_{sm}^n + \alpha_m.d_{mg}^n \\
&= \alpha_s.[d_{sm}^n + (\tfrac{\alpha_m}{\alpha_s}).d_{mg}^n] \\
&= \alpha_s.[d_{sm}^n + \eta.d_{mg}^n]
\end{aligned}
$$

where P_s: power needed to transmit from source to mobile

P_m: power needed to transmit from mobile to gateway

α_s: proportionality constant for source power

α_m: proportionality constant for mobile power

d_{sm}: distance between source and mobile nodes

d_{mg}: distance between mobile and gateway nodes

and $\eta = \frac{\alpha_m}{\alpha_s}$

Depending on the value of η, the following four cases can be enumerated straightaway:

1. $\eta = 0$: Mobile node has infinite power (since $\alpha_m = 0$).

2. $0 \leq \eta \leq 1$: Source node is more power constrained (since $0 \leq \alpha_m \leq \alpha_s$).

3. $\eta = 1$: Source and mobile nodes are equally weighted (since $\alpha_m = \alpha_s$).

4. $\eta \geq 1$: Mobile node is more power constrained (since $\alpha_m \geq \alpha_s$).

Of these cases, the analysis will concentrate on case 2, *i.e.*, the source nodes are more power constrained than the mobile nodes.

Chapter 4

Analysis of Deterministic Motion

4.1 Introduction

Most work on power conservation in ad-hoc networks [6, 8, 7] consider a set of static nodes randomly distributed in an area and minimize the energy consumption through a multihop routing algorithm. In contrast, this work introduces a third class of nodes, mobile nodes, and obtains the minimum energy relay path through these nodes.

There are several real-life applications in which the trajectory of the mobile node is known *a priori*. For instance, if sensors nodes are distributed in a battlefield, security and other ground realities would limit the path selected by a reconnaisance aircraft (which acts a the mobile node) to collect data from the field nodes. In a similar manner, for sensor nodes spread over a mountainous region to collect environmental data, the mobile node (in the form of a jeep, for instance) would choose the least inhospitable route for data collection.

This section considers deterministic motion only, and mobility is initially restricted to straight line motion. This will be followed by the analysis for piece-wise linear motion, which may be considered as a sequence of straight lines in different directions at rapid succession. In both the cases, the speed of the mobile node is assumed to be constant.

The time constraint is introduced into the analysis as follows: since the speed and directions of the mobile node are known ahead of time, the point in space at which the transmission time expires is also known deterministically. Thus, the algorithms ensure that the transmission is completed (either through the mobile node or directly from the source to the gateway) before the mobile node reaches this critical point. In the following notation this critical point, corresponding to the expiration time, is represented by P^*.

4.2 Straight Line Motion

4.2.1 Model

As the name suggests, the straight line motion algorithm is applicable when the mobile node moves in a straight line with constant velocity. The points of interest in the trajectory of the mobile node are the starting point P_0 and the point by which the transmission must be completed P^*. The starting point may be defined as the time at which data becomes available at the source for transmission to the gateway. This scenario is illustrated in Figure 4.1.

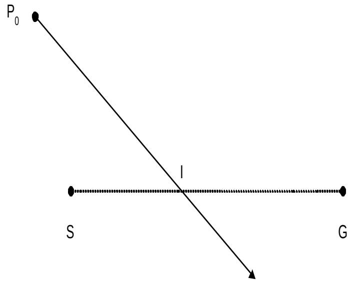


Figure 4.1: Layout of model for analysis comprising source node S, gateway G and trajectory of mobile node starting from P_0.

The model comprises a source S, gateway (or destination) G, and a mobile node moving in a straight line with constant velocity v. Furthermore, the analysis is carried out only when the mobile node enters the circle with center at S and having a radius equal to the segment SG. This follows from the fact that the cost metric is a function of distance and it is less costly for the source to transmit *directly* to the gateway if it takes more power to transmit from the source to the mobile node alone (which is the case when the mobile node is outside the SG circle).

The notation being used in the following analysis is summarized below:

S: source

G: gateway (or destination)

I: point of intersection of mobile node with SG axis; it also defines the left-half and right-half of

P_0: starting point for mobile node

P^*: position of mobile node at the termination of maximum allowable delay

$I \in (S, G)$: point I lies between the points S and G

$I \notin (S, G)$: point I does not lie between the points S and G

4.2.2 Optimal Transmission Points

Depending on the relative positions of the starting point P_0 and the point of intersection of the mobile trajectory with the SG-axis, I, the results for the optimum relay transmission schemes can be divided into four categories. The results are based on the fact that only one point on a straight line (in this case, the mobile trajectory) is at the shortest distance from any other point (in this case, the source node) and the distance increases monotonically on either side of that minimum point [Theorem 1 in Appendix A].

CASE I: $I \in (S, G)$

P_0 in left-half plane

Referring to Figure 4.2,

1. $P_0 \in (-\infty, P_1)$
   (a) $P^* \in (P_0, P_1)$: direct transmission
   (b) $P^* \in (P_1, P_2)$: receive and transmit at P^*
   (c) $P^* \in (P_2, P_3)$: receive at P_2 and transmit at P^*
   (d) $P^* \in (P_3, \infty)$: receive at P_2 and transmit at P_3
2. $P_0 \in (P_1, P_2)$

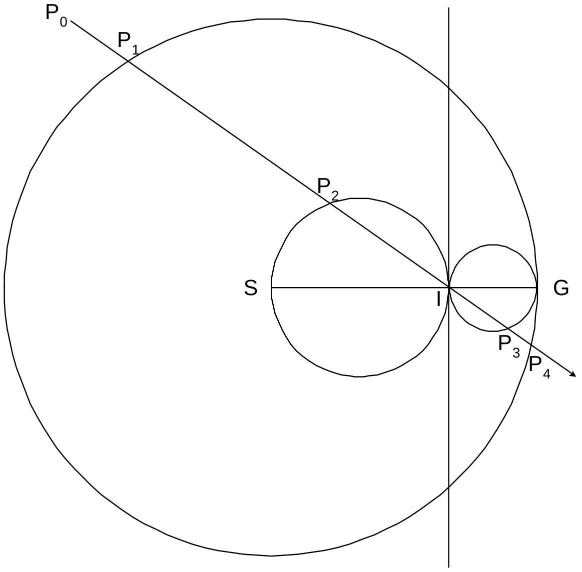


Figure 4.2: Deterministic straight line motion with P_0 on the left-side of I and $I \in (S, G)$.

   (a) $P^* \in (P_0, P_2)$: receive and transmit at P^*
   (b) $P^* \in (P_2, P_3)$: receive at P_2 and transmit at P^*
   (c) $P^* \in (P_3, \infty)$: receive at P_2 and transmit at P_3

3. $P_0 \in (P_2, P_3)$
   (a) $P^* \in (P_0, P_3)$: receive at P_0 and transmit at P^*
   (b) $P^* \in (P_3, \infty)$: receive at P_0 and transmit at P_3

4. $P_0 \in (P_3, P_4)$
   (a) $P^* \in (P_0, \infty)$: receive and transmit at P_0

5. $P_0 \in (P_4, \infty)$

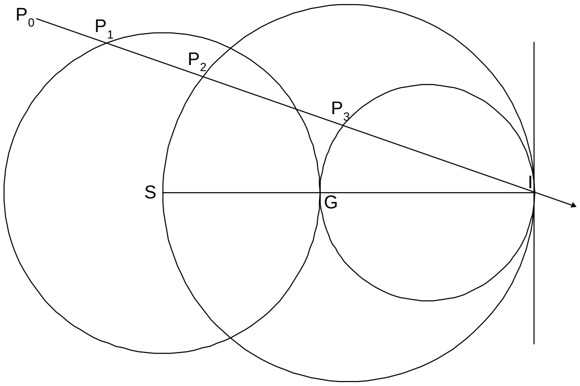


Figure 4.3: Deterministic straight line motion with P_0 on the left-side of I and $I \notin (S, G)$.

   (a) $P^* \in (P_0, \infty)$: direct transmission

CASE II: $I \notin (S, G)$

P_0 in left-half plane

Referring to Figure 4.3,

1. $P_0 \in (-\infty, P_1)$
   (a) $P^* \in (P_0, P_1)$: direct transmission
   (b) $P^* \in (P_1, P_2)$: receive and transmit at P^*
   (c) $P^* \in (P_2, P_3)$: receive at P_2 and transmit at P^*
   (d) $P^* \in (P_3, \infty)$: receive at P_2 and transmit at P_3
2. $P_0 \in (P_1, P_2)$
   (a) $P^* \in (P_0, P_2)$: receive and transmit at P^*

    (b) $P^* \in (P_2, P_3)$: receive at P_2 and transmit at P^*
    (c) $P^* \in (P_3, \infty)$: receive at P_2 and transmit at P_3

3. $P_0 \in (P_2, P_3)$
    (a) $P^* \in (P_0, P_3)$: receive at P_0 and transmit at P^*
    (b) $P^* \in (P_3, \infty)$: receive at P_0 and transmit at P_3
4. $P_0 \in (P_3, \infty)$
    (a) $P^* \in (P_0, \infty)$: direct transmission

CASE III: $I \in (S, G)$

P_0 in right-half plane

Referring to Figure 4.4, where l is defined as the distance between the points P_2 and P_3 along the mobile trajectory,

1. $P_0 \in (-\infty, P_1)$
    (a) $P^* \in (P_0, P_1)$: direct transmission
    (b) $P^* \in (P_1, P_2)$: receive and transmit at P^*
    (c) $P^* \in (P_2, \infty)$: receive and transmit at a distance of $x = \frac{l}{1+\eta}$ from P_2 [Theorem II in Appendix A]
2. $P_0 \in (P_1, P_2)$
    (a) $P^* \in (P_0, P_2)$: receive and transmit at P^*

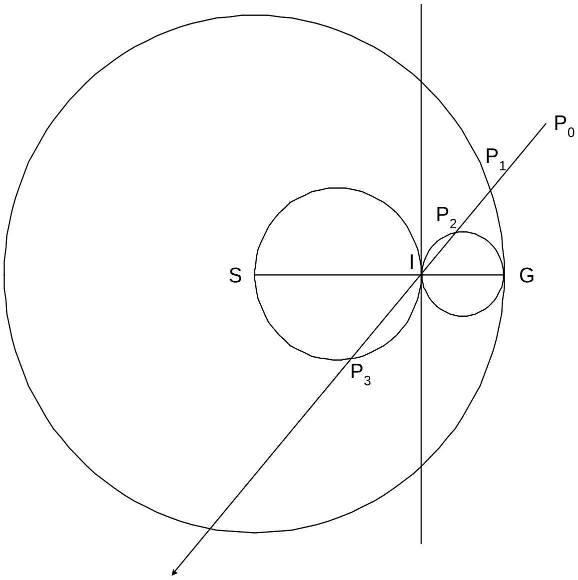


Figure 4.4: Deterministic straight line motion with P_0 on the right-side of I and $I \in (S, G)$.

(b) $P^* \in (P_2, \infty)$: receive and transmit at a distance of $x = \frac{l}{1+\eta}$ from P_2 [Theorem II in Appendix A]

3. $P_0 \in (P_2, P_3)$

   (a) $P^* \in (P_0, \infty)$: receive and transmit at a distance of $x = \frac{l}{1+\eta}$ from P_2 [Theorem II in Appendix A]

4. $P_0 \in (P_3, \infty)$

   (a) $P^* \in (P_0, \infty)$: receive and transmit at P_0

CASE IV: $I \notin (S, G)$

P_0 in right-half plane

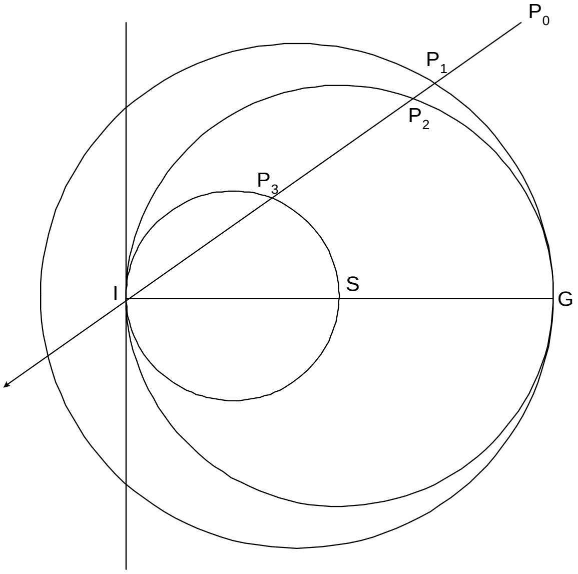


Figure 4.5: Deterministic straight line motion with P_0 on the right-side of I and $I \notin (S, G)$.

Referring to Figure 4.5, where l is defined as the distance between the points P_2 and P_3 along the mobile trajectory,

1. $P_0 \in (-\infty, P_1)$
   (a) $P^* \in (P_0, P_1)$: direct transmission
   (b) $P^* \in (P_1, P_2)$: receive and transmit at P^*
   (c) $P^* \in (P_2, \infty)$: receive and transmit at a distance of $x = \frac{l}{1+\eta}$ from P_2 [Theorem III in Appendix A]
2. $P_0 \in (P_1, P_2)$
   (a) $P^* \in (P_0, P_2)$: receive and transmit at P^*

    (b) $P^* \in (P_2, \infty)$: receive and transmit at a distance of $x = \frac{l}{1+\eta}$ from P_2 [Proof in Appendix]

3. $P_0 \in (P_2, P_3)$

    (a) $P^* \in (P_0, \infty)$: receive and transmit at a distance of $x = \frac{l}{1+\eta}$ from P_2 [Theorem III in Appendix A]

4. $P_0 \in (P_3, \infty)$

    (a) $P^* \in (P_0, \infty)$: receive and transmit at P_0

4.2.3 Protocol

The thirty-five cases enumerated in Section 4.2.2 exhaustively cover all possibilities arising from relay transmission via a mobile node that moves in a straight line. If the source and gateway nodes make transmission and reception decisions in accordance with these results, then a minimum energy transmission is guaranteed. The results are applied in the minimum energy transmission protocol in the following manner.

1. Mobile node broadcasts beacon (with absolute direction information) at regular intervals.

2. Source (and gateway) narrow down the trajectory of the mobile node to one of two possible trajectories and calculates speed of mobile node [Algorithms in Appendix].

3. Source (and gateway) calculate the time at which the transmission is to be made. Let θ be the angle made by the mobile trajectory with the positive x-axis (*i.e.*, $\theta = \tan^{-1}(\text{slope})$). Then,

    (a) transmission logic follows Case I and Case IV if $-90^o < \theta < 90^o$
    (b) transmission logic follows Case II and Case III if $90^o \leq \theta \leq 270^o$

4. Source transmits at calculated time.
5. Gateway requests data from the mobile node at scheduled time.
6. Mobile node transmits data upon receiving the request.

Although the protocol requires the source *and* the gateway to monitor the mobile node's beacons, it is general in every respect. The results of the analysis have been extended to pre-determined piece-wise linear motion in the following section.

4.3 Piece-Wise Linear Motion

4.3.1 Model

The aim of this section is to obtain the optimum reception and transmission points along the trajectory of a mobile node that is approximated by piece-wise linear segments as illustrated in Figure 4.6. Since deterministic motion is being considered, *i.e.*, the complete trajectory is known *a priori*, the results obtained earlier for stright line motion may be applied to each segment and the optimal point (or pair of points) selected.

The following subsection looks at the problem of obtaining the optimum points of transmission and reception for such a piece-wise linear motion. For ease of analysis, the motion is classified into two categories—

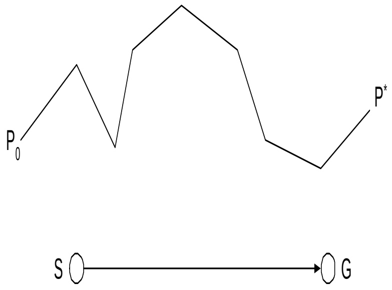


Figure 4.6: Layout of model for analysis comprising source node S, gateway G and piece-wise linear trajectory of mobile node starting at P_0 and ending at P^*.

1. When the mobile node is moving from the direction of the source to the gateway.
2. When the mobile node is moving from the direction of the gateway to the source.

The former corresponds to Cases I and IV and the latter to Cases II and III in the analysis of straight line motions in Section 4.2.2.

4.3.2 Optimal Transmission Points

Depending on the direction of motion, the analysis is divided into two categories. The proofs of the results are given in Appendix A.

Case I: Mobile node moves from the direction of source to gateway

Application of the results obtained for Cases I and IV in Section 4.2.2 leads to two unique points for the mobile node, one each for receiving data from the source and then transmitting to the gateway, on each piece-wise linear segment. In Figure 4.7, these are named R_i and T_i, where i refers to the sequence number of the line segment.

For an n-segment piece-wise linear path, there will be n pairs of R_i and T_i points. The overall optimum reception and transmission points, R_j and T_k, are obtained by sorting the R_i and T_i points according to distance (*i.e.*, distance of R_i from the source S and distance of T_i from the gateway G) and a table is formed with R_i as the columns and T_i as the rows.

The best relay path (*i.e.*, the path that consumes the minimum energy) is given by the valid (*i.e.*, causal) R_i–T_j pair closest to the upper left-hand cell of the table. This is proved in the Appendix.

CASE II: Mobile node moves from the direction of gateway to source

Application of the results obtained for Cases II and III in Section 4.2.2 leads to a single point for reception and transmission on each piece-wise linear segment [Proof in Appendix]. This is illustrated in Figure 4.8, where R_i and T_i coincide for each of the line segments.

Once again, the R_i and T_i points are sorted according to distance (*i.e.*, distance of R_i from the source S and distance of T_i from the gateway G) and a table is formed with R_i as the columns and T_i as the rows. The best relay path (*i.e.*, the path that consumes the minimum energy) is given by the valid (*i.e.*, causal) R_i–T_i pair closest to the upper left-hand cell of the table. This is proved in the Appendix.

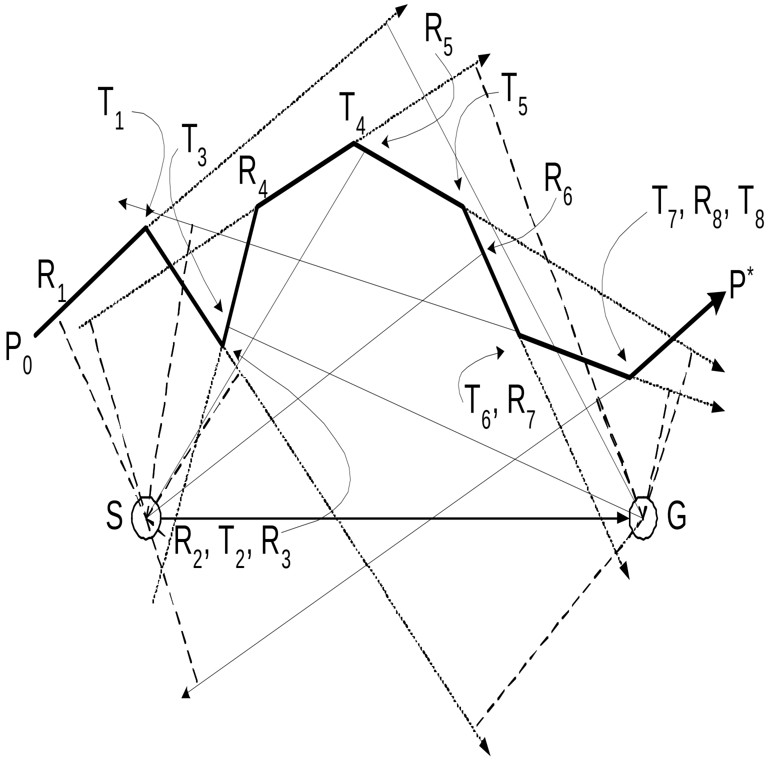


Figure 4.7: Layout of model for analysis comprising source node S, gateway G and eight piece-wise linear segments on the mobile trajectory. There is a pair of optimum reception and transmission points corresponding to each segment.

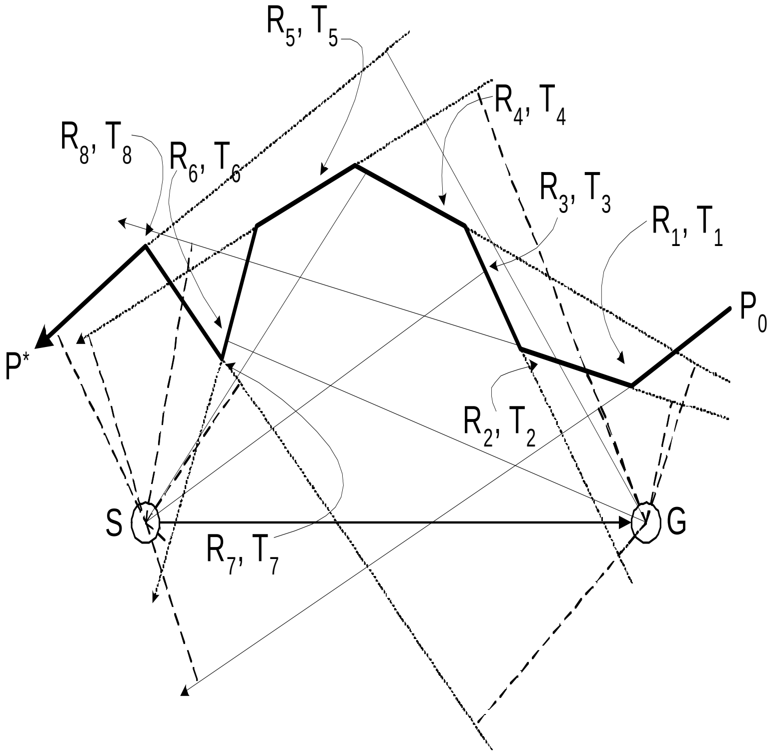


Figure 4.8: Layout of model for analysis comprising source node S, gateway G and eight piece-wise linear segments on the mobile trajectory. There is a single optimum point for reception and transmission corresponding to each segment.

4.3.3 Protocol

The scheme proposed in this section identifies the optimum relay points in an ad-hoc network provided that the trajectory of the mobile node is known *a priori* to itself. The success of the protocol depends on the ability of the mobile node to make its future positions available to the source (and the gateway) so that they may calculate the time at which the transmission (or transmission request) should be made.

Again, since several sources in an ad-hoc network want to transmit data to a common gateway, the address of the gateway should be made known to each of them. However, since the network is formed in an ad-hoc manner, the only way this can be done is by having the gateway advertise its absolute position by periodic broadcasts (the broadcasts may be made at long intervals since the gateway does not change its position often). The distance of the gateway from each individual source node can be calculated from the strength of the received signal. In a similar manner, the distance from the mobile node can also be calculated from the strength of the received signal. However, the source node requires at least two broadcasts from the mobile node to *triangulate* its position in a two-dimensional plane. Thus, the protocol comprises the following main steps.

1. The gateway broadcasts its position at regular intervals at a known transmission power.

2. The mobile node broadcasts its absolute position and future positions at regular intervals at a known transmission power.

3. The source node receives the transmitted signals immediately and accurately estimates the distance on the basis of the received signal strength.

4. The source determines its position from the gateway broadcast and two successive mobile broadcasts.
5. The source calculates its distance to the various piece-wise linear segments of the mobile trajectory from the mobile information broadcast.
6. The source applies the results obtained in Section 4.2.2 to calculate the optimum points for reception and transmission.
7. Relay transmission occurs.

A drawback of this protocol is that the computational overhead increases considerably at the source. However, this is justified since computational cost is typically *less expensive* than transmission cost. Moreover, while the mobile node and the gateway advertise their positions and other information periodically, the source node transmits only when data becomes available to be relayed. Since the source nodes are immobile (after their initial ad-hoc arrangement), the calculation of their positions may be performed only once and then placed in memory— this would reduce some of the computational overhead.

Chapter 5

Analysis of Random Motion

5.1 Introduction

A relay transmission algorithm for ad-hoc networks does not serve any practical purpose unless it can be applied to a real-life scenario where the motion of the mobile node is usually random. In a stochastic process, the position of the mobile node at any time is not known for certain; instead, there is a probability distribution associated with its position corresponding to every time instant. Thus, the algorithms discussed earlier have to be modified to take this uncertainty into account. For instance, the transmission from source to mobile may now be either successful (*hit*) or unsuccessful (*miss*), since the mobile node may not be present within the layer it is *expected* to. In order to fit this probabilistic scenario, the optimum points for reception and transmission are now estimated to maximize the *probability* of making a hit, while minimizing the *expected* value of energy that is consumed in the transmission.

In additon, the space over which the location probabilities are calculated may also be classified into two categories. The former, called closed or *bounded* space, assumes that the motion of the mobile node is restricted to a closed area. This model is particularly appropriate for scenarios in which the transmitted signal cannot penetrate through the walls of a room without considerable degradation, and is discussed in greater detail in Section 5.2.2. For motion in *unbounded* space however, the probability of the mobile node reaching its neighbor is independent of its position. Thus, this model does not place any restriction on the position of the mobile node and is discussed in Section 5.2.3.

An important idea that is used implicitly to obtain the results in this section takes advantage of the omni-directional behavior of the transmission antennas. Since the transmission power of the source can be controlled, it effectively varies a broadcast *region*, rather than a particular layer or cell, where the signal can be received. Thus, the transmission will be considered successful if the mobile node lies anywhere inside a circle centered on the source and having a radius that depends on the signal's transmission power.

The other issue central to the idea of an optimum transmission algorithm is the time delay. Typically, a data packet available for transmission also has a deadline (or *critical* time) within which the transmission has to be completed. Accordingly, the analysis is modified in Section 5.3 to ensure that the transmission, while maximizing the probability of hit and minimizing the expected energy, is also completed before the critical time expires.

Different authors have tackled the issue of randomness in ad-hoc networks in different ways as have been discussed in Section 2.5. In this work, uncertainty in the mobile node is formulated as a random walk model in which, for simplicity, the mobile node's movements are restricted to the four cardinal directions— north, south, east and west— with equal probability of $\frac{1}{4}$. However, the results obtained are perfectly general and arbitrary motion (*i.e.*, movement in any number of directions) can be accomodated by using the corresponding transition probability density function.

The thesis proposes several protocols that could be applied to meet the transmission requirements discussed above– namely, maximize the hit probability, while minimizing the expected energy consumption. In each case, the different regions of the location grid are defined as states in a Markov chain, and the transition probabilities from one state to another are obtained. The results are obtained either as a closed–form solution or as a recursive algorithm.

5.2 Most Probable Location Without Time Constraint

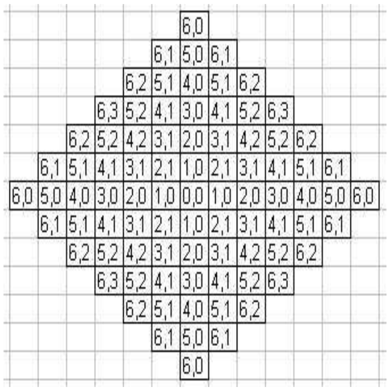
This section suggests two different approaches to calculating location probabilities of the mobile node– one each for bounded and unbounded regions– without explicitly taking delay into consideration. As will be shown in Section 5.3, introducing time constraint into the analysis follows logically from these results.

The source and gateway nodes are responsible for calculating the optimum points for data transmission and reception. This is logical since a typical ad-hoc network would have a large number of source nodes being served by a single mobile node, and it would be unwieldy, if not downright impossible, for the mobile node to keep track of the data reception times from all these nodes. Thus, the analyses that follow focus on the source node and propose algorithms for computing the optimum transmission points that lead to the minimum expected value of the transmission cost. A similar algorithm exists for the gateway node to trigger the relay transmission from the mobile node.

5.2.1 Model

Viewed from a high level of abstraction, the problem is concerned with minimizing the cost involved in transmitting information from a source node to a gateway node with the aid of a mobile node moving along a random trajectory, subject to the four-direction restriction. The mobile node is assumed to move with a constant speed and advertises its absolute direction through periodic broadcasts. It has already been established that the speed of the mobile node may be calculated from this information.

Since the relay transmission algorithm aims at minimizing the total cost of data transmission, a direct transmission would be preferred if the minimum distance reached by the mobile node from the source within the stipulated time exceeds the source–gateway distance. In other words, the mobile node is considered for relay transmission only if it enters an imaginary circle centered at the source node and having a radius equal to the distance between the source and the gateway. The logic behind this is that if the mobile node is outside this circle, it is less *costly* for the source to transmit directly to the gateway rather than to make a relay transmission.



						6,0						
					6,1	5,0	6,1					
				6,2	5,1	4,0	5,1	6,2				
			6,3	5,2	4,1	3,0	4,1	5,2	6,3			
		6,2	5,2	4,2	3,1	2,0	3,1	4,2	5,2	6,2		
	6,1	5,1	4,1	3,1	2,1	1,0	2,1	3,1	4,1	5,1	6,1	
6,0	5,0	4,0	3,0	2,0	1,0	0,0	1,0	2,0	3,0	4,0	5,0	6,0
	6,1	5,1	4,1	3,1	2,1	1,0	2,1	3,1	4,1	5,1	6,1	
		6,2	5,2	4,2	3,1	2,0	3,1	4,2	5,2	6,2		
			6,3	5,2	4,1	3,0	4,1	5,2	6,3			
				6,2	5,1	4,0	5,1	6,2				
					6,1	5,0	6,1					
						6,0						

Figure 5.1: Part of the mesh grid that is used to estimate the position and the *most likely* trajectory of the mobile node.

Once the mobile node comes within a radial enters this circle, the position of the mobile node is obtained according to a mesh grid [Figure 5.1] centered on the source node which is assigned the coordinates (0,0). This grid forms the core of the algorithm since the coordinates of each cells represent a state in a Markov chain and cells having the same neighbors are assigned the same state. The numbering of the cells follows an algorithm modified from the one proposed by Akyildiz, Lin, Lai and Chen [3] and is described below.

The numbering of the grid is done according to the following algorithm:

1. The source is assigned the state (0,0).
2. Cells in each of the four perpendicular directions from (0,0) are numbered (1,0), (2,0), (3,0), etc. depending on the number of hops it takes to reach (0,0).
3. (a) For even i, starting from (i,0), $i > 1$, cells are numbered diagonally as (i,1), (i,2), etc. until (i,$\lfloor \frac{i}{2} \rfloor$) is reached and then the remaining cells are marked in a

decreasing order as (i, $\lfloor \frac{i}{2} \rfloor - 1$), (i, $\lfloor \frac{i}{2} \rfloor - 2$), etc. until the next (i,0) is reached. This is continued until all four sides of each diamond-shaped region are numbered.

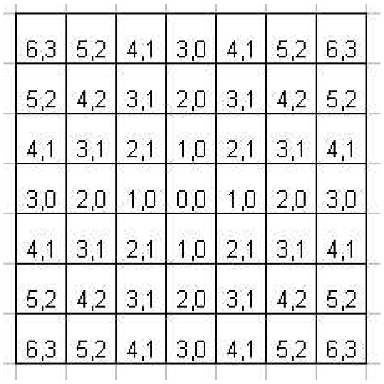
(b) For odd i, starting from (i,0), $i > 1$, cells are numbered diagonally as (i,1), (i,2), etc. until (i,$\lfloor \frac{i}{2} \rfloor$) is reached, then (i,$\lfloor \frac{i}{2} \rfloor$) is repeated one more time, and finally the remaining cells are marked in a decreasing order as (i, $\lfloor \frac{i}{2} \rfloor - 1$), (i, $\lfloor \frac{i}{2} \rfloor - 2$), etc. until the next (i,0) is reached. This is continued until all four sides of each diamond-shaped region are numbered.

In the case of only four directions of motion, it is evident that the grid-numbering algorithm results in a simple transformation of coordinate systems according to the following rule:

$$X = |x| + |y|$$
$$Y = \min(|x|, |y|)$$

where (x, y) are the coordinates of the Cartesian system and (X, Y) are the positions in the new transformed grid system.

Moreover, the numbering aids in the calculation of the *most probable* radial distance from the source (which is considered at the origin (0,0) of the numbering system). Since the movement of the mobile node is restricted to only four directions, the different layers are obtained as diamonds [Figure 5.1] instead of circles. However, with increased degrees of freedom of the mobile node, the shape of the layers will gradually approximate that of a circle. It is important to note that the exact directional position of the mobile node *vis-a-vis* the source node is not required as all transmissions are assumed to be omni-directional broadcasts.



6,3	5,2	4,1	3,0	4,1	5,2	6,3
5,2	4,2	3,1	2,0	3,1	4,2	5,2
4,1	3,1	2,1	1,0	2,1	3,1	4,1
3,0	2,0	1,0	0,0	1,0	2,0	3,0
4,1	3,1	2,1	1,0	2,1	3,1	4,1
5,2	4,2	3,1	2,0	3,1	4,2	5,2
6,3	5,2	4,1	3,0	4,1	5,2	6,3

Figure 5.2: Example of a bounded (or closed) space mesh grid that is used to estimate the position of the mobile node.

The same numbering system is followed even if the location of the mobile node is limited to a restricted area (for instance, a square of dimension 7 × 7 as shown in Figure 5.2).

5.2.2 Bounded Space Analysis

With reference to the bounded space illustrated in Figure 5.2, there are three categories of cells corresponding to their single-hop probabilities:

1. *Corner cells* that can move to either of its two neighboring cells with equal probability $\frac{1}{2}$.
2. *Edge cells* that can move to any of its three neighboring cells with equal probability $\frac{1}{3}$.
3. *Body cells* that are neither corner nor edge cells, and can move to any of its neighboring cells with probability $\frac{1}{4}$.

The Markov chain formulation corresponding to the cells in Figure 5.2 is illustrated in Figure 5.3.

The transition probabilities of this bounded random walk space can be represented as a *transition matrix* as follows:

$$\mathbf{P} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

where the elements in each column and row are listed in the following (x, y) order: (0,0), (1,0), (2,0), (2,1), (3,0), (3,1), (4,1), (4,2), (5,2) and (6,3).

Finally, the the Chapman–Kolmogorov forward equation [16] may be used to compute the probability of moving from one cell to another in k number of steps.

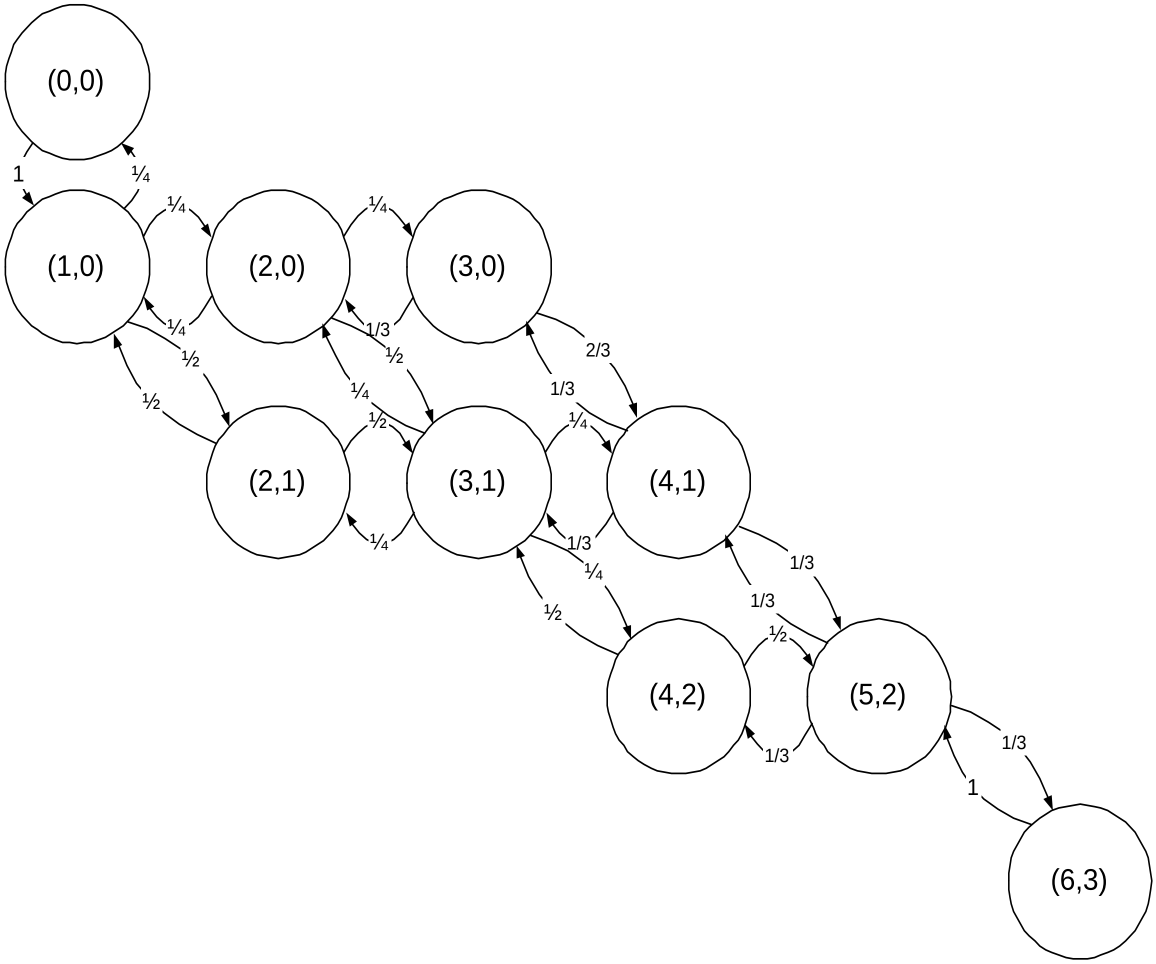


Figure 5.3: Markov chain representation of the different states of the mesh grid in Figure 5.2. The numbers represent the one-hop probabilities from a node to its neigboring node.

$$\mathbf{P}^{(k)} = \begin{cases} \mathbf{P}, & \text{if } k = 1 \\ \mathbf{P}.\mathbf{P}^{(k-1)}, & \text{if } k > 1. \end{cases}$$

Thus, given the number of steps, the probability of moving from any point to any other point is obtained by recursively mulitplying the transition probability matrix. An interesting observation is that since the mobile node is assumed to move with a constant speed, the number of steps (or hops) is an indication of the time index. This idea is used to incorporate the time deadline in Section 5.3.

The protocol for the bounded space algorithm would allow for transmission to the layer that has the maximum location probability corresponding to a a given starting point and number of allowable hops. Although the location probabilities corresponding to a starting point were obtained as a closed form solution, the disadvantage of this scheme is that they are limited only to the region that lies between the starting point and the source.

5.2.3 Unbounded Space Analysis

The analysis for an unbounded space analysis becomes more tricky because there are an infinite number of states in the Markov chain. For instance, only sixteen states corresponding to the mesh in Figure 5.1 is illustrated in Figure 5.4– all the neighboring nodes for the boundary cells are not considered. Thus, a different approach is needed to obtain the location probabilities of the mobile node– to this end, it is assumed that only the shortest route to the destination will be considered. Thus, the algorithm in this section is based on finding the probability of moving from an arbitrary starting point to any other cell in the grid using the least number of hops. In this way it tries to minimize the time until the transmission occurs.

To obtain the probability of moving from a given starting point (x_0, y_0) to any arbitrary point (x, y) using the shortest route, first the probability of moving from (x_0, y_0) to the central cell (0, 0) is evaluated:

$$P^{(x_0)}_{(x_0,y_0)\rightarrow(0,0)} = \tfrac{1}{4^{x_0}}.\tbinom{x_0}{y_0}$$

where $\binom{x_0}{y_0}$ represents x *combination* y and the superscript of the probability P indicates the minimum number of hops x it takes to move from (x_0, y_0) to (0,0). The combination factor ensures that all possible minimum–hop paths are considered in the calculation of the probability [Theorem 10 in Appendix A].

By changing the origin, this result can be extended to movement between any two arbitrary cells (x_0, y_0) and (x,y) as follows:

$$P^{(x_0-x)}_{(x_0,y_0)\rightarrow(x,y)} = \tfrac{1}{4^{(x_0-x)}}.\tbinom{x_0-x}{y_0-y}$$
$$\text{for } x < x_0$$

As evident from the ranges of (x, y), this result is valid only when the motion results in moving inwards through the layers towards the center. As an illustration, the Markov Chain representation [Figure 5.2] of the grid in Figure 5.1 clearly shows that while the probability of moving from (1,0) to (0,0) is $\frac{1}{4}$ (as predicted), the probability of moving from (0,0) to (1,0) is 1.

This algorithm provides a closed form expression to obtain the probability of the mobile node starting at a given point (x_0, y_0) and reaching a particular point (i, j), provided $i < x_0$. Once the location probabilities are obtained, the transmission from the source is made with enough power to reach the layer that has the maximum location probability. It is important to note that the two parameters of each point represent its state and not its unique position in space (as a coordinate would). The exact directional position of the mobile node *vis-a-vis* the source node is not required because all transmissions are assumed to be omni-directional broadcasts.

5.3 Most Probable Location With Time Constraint

This section offers a more rigorous analysis of unbounded space random motion. It makes time deadline and the expected energy consumption a more central part of the analysis. The analysis also introduces two new parameters, k^* and p^*. Since the mobile node moves with constant speed, the critical time for transmission can be represented by the number of discrete hops k^* within which the transmission must be completed. It will be observed later that k^* can be used interchangeably to represent the critical number of hops and the critical time. On the other hand, p^* is a probability threshold that needs to be met to be assured of a successful transmission or hit. If the probability threshold is not met, then the source goes ahead and transmits directly to the gateway. The expected energy introduced in the section takes the probability of *hit* or *miss* into account.

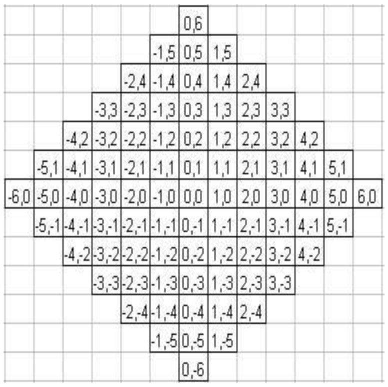


Figure 5.4: Part of the mesh grid that is used to estimate the *most likely* position of the mobile node at a particular time.

5.3.1 Model

The grid numbering for the model in this algorithm is according to the Cartesian coordinate system with the source node as the origin (0,0). This is illustrated in Figure 5.4.

Since the mobile node has equal probability ($\frac{1}{4}$) of moving to an adjacent neighbor in any of the four perpendicular directions, given a starting point (x_0, y_0), the one-hop probabilities may be written down as:

$$\begin{aligned}
P^{(1)}_{(x_0,y_0)\to(x_0+1,y_0)} &= P^{(1)}_{(x_0+1,y_0)|(x_0,y_0))} = \tfrac{1}{4}\\
P^{(1)}_{(x_0,y_0)\to(x_0-1,y_0)} &= P^{(1)}_{(x_0-1,y_0)|(x_0,y_0))} = \tfrac{1}{4}\\
P^{(1)}_{(x_0,y_0)\to(x_0,y_0+1)} &= P^{(1)}_{(x_0,y_0+1)|(x_0,y_0))} = \tfrac{1}{4}\\
P^{(1)}_{(x_0,y_0)\to(x_0,y_0-1)} &= P^{(1)}_{(x_0,y_0-1)|(x_0,y_0))} = \tfrac{1}{4}
\end{aligned}$$

These results may be summarized as follows:

$$P^{(1)}_{(x_0,y_0)\to(x_0+i,y_0+j)} = \begin{cases} \frac{1}{4}, & \text{if } |i| + |j| = 1 \\ 0, & otherwise \end{cases}$$

This result may now be generalized to k hops using the Chapman–Kolmogorov equation, and the final result is as follows:

$$P^{(k)}_{(x_0,y_0)\to(x,y)} = \begin{cases} 0 & \text{if } |x - x_0| + |y - y_0| > k \\ \frac{1}{4} & \text{if } k = 1 \\ \frac{1}{4}.P^{(k-1)}_{(x_0+i,y_0+j)\to(x,y)} & \text{for all other } k\text{, where } |i| + |j| = 1 \end{cases}$$

5.3.2 Protocol

The recursive location probability result obtained in Section 5.3.1 will now be combined with minization of the expected energy to obtain a single transmission protocol. The name indicates that the source will be allowed only one transmission, either to a region that meets the probability threshold p^*, or to the gateway.

The parameters p^* and k^* defined previously play an important role in the definition of this protocol. p^* is the probability threshold for success. In other words, if the source transmits to a region (comprising a certain number of cells depending on the transmission power) having an aggregate location probability greater than p^*, it will be considered a success. However, this transmission must also be completed within $k \leq k^*$ hops since that is an indicator of the expiration time. Should there be no cell having a location probability greater than p^* within the stipulated time k^*, the source must transmit directly to the gateway at

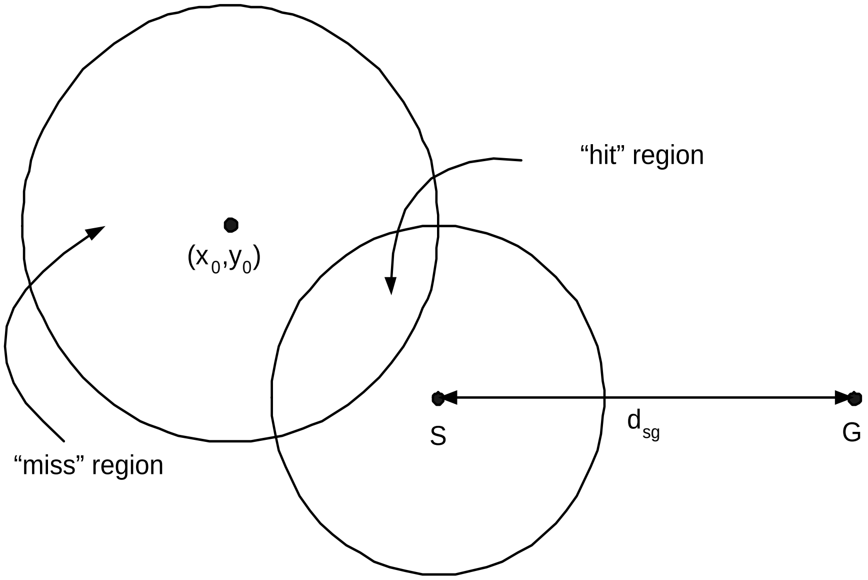


Figure 5.5: Regions of "hit" and "miss' corresponding to a starting point (x_0, y_0) after an arbitrary number of steps. The circle centered at (x_0, y_0) corresponds to the possible position of the mobile node after a certain number of steps. The circle centered at S is the area covered by the source in a certain transmission. For random motion in four directions, the circles are approximated by diamonds.

any instant before k^*.

Moreover, since transmission power is a function of the radius of coverage, the aim of the source is to transmit to the smallest area that satisfies the probability threshold p^*, as has been illustrated in Figure 5.5. This minimum radius corresponding to the kth time interval is denoted by $R_{min}^{(k)}$. Accordingly, the source transmission protocol is defined as follows.

1. Given the maximum allowable number of hops k^* and the starting point of the mobile node (x_0, y_0), the location probabilities of all cells that can be reached for each $k \leq k^*$ hops is calculated. As defined earlier, these are written as:

   $$P^{(k)}_{(x_0,y_0) \to (x,y)}$$

   where (x, y) lies anywhere in space subject to the condition $|x - x_0| + |y - y_0| \leq k^*$.

2. Corresponding to every value of time (or hop) index k, the minimum value of transmission radius $R_i^{(k)}$ is found that satisfies the following condition:

   $$\sum_{(x,y) \in R_i} P^{(k)}_{(x_0,y_0) \to (x,y)} \geq p^*$$

   The minimum distance for the kth time interval is denoted by $R^{(k)}_{min}$.

3. The minimum value of $R^{(k)}_{min}$ over all values of $k \leq k^*$ is denoted by R_{min}.

   $$i.e., \ R_{min} = \min \{R^k_{min}\} \text{ for } 0 \leq k \leq k^*.$$

   Let the time corresponding to R_{min} be k'.

4. If $R_{min} < d_{sg}$, then the source transmits at time k' with transmission power $\alpha_s.R^n_{min}$, otherwise it transmits directly to the gateway with power $\alpha_s.d^n_{sg}$ at any time $k \leq k^*$.

The data request protocol for the gateway may be defined in a similar manner.

5.3.3 Transmission Energy

The energy consumed in the case of stochastic motion must account for the probabilites of hit and miss. As discussed earlier, the probability threshold p^* must be met to guarantee success. Thus, the individual location probabilities of the cells that lie within the region covered by a particular broadcast must add up to p^* for a successful transmission. Should there be no region having a location probability greater than p^*, a direct transmission to the gateway is made.

It is important to note that once the source knows the starting point of the mobile node (x_0, y_0), it can execute the algorithm presented in the last section and deterministically calculate the time at which the transmission must be made. The energy consumed in the transmission, given that the starting position of the mobile node is at (x_0, y_0), is as follows:

$$\varepsilon(x_0, y_0) = \begin{cases} \alpha_s.R_{min}^n, & \text{if } R_{min} \leq d_{sg} \\ \alpha_s.d_{sg}^n, & \text{otherwise} \end{cases}$$

Thus, the single transmission scheme provides a practical approach to guarantee successful transmission (either relayed or direct), while minimizing the transmission energy. If, however, the starting position of the mobile node were given by a probability distribution function $p(x_0, y_0)$ over all space {x,y}, then the *expected* value of energy could be calculated by summing over all space defined by the distribution function as shown below:

$$E\{\varepsilon\} = \sum \sum_{(x_0, y_0)} p(x_0, y_0).\varepsilon(x_0, y_0)$$

Chapter 6

Conclusion

This thesis proposed several novel relay transmission schemes that lead to minimum energy consumption in mobile ad-hoc networks. Unlike most work that confine themselves to examining the effect of energy conservation in terms of a fixed set of nodes, the thesis introduced mobility into the analysis, in the form of a mobile node that is capable of collecting data from the source and transmitting it to the gateway. After defining an energy cost function based on transmission power, the work explored different ways of minimizing this cost for a host of network configurations and mobile trajectories. For ease of analysis, the motion of the mobile node was classified into deterministic and random motion.

The analysis began by considering deterministic straight line motion and obtaining an algorithm for all possible cases. While straight line motion for a mobile node is quite restrictive in reality, any random motion can be approximated by piece-wise linear motion and the same results applied. In the case of deterministic motion, the piece-wise linear segments can be formed by sampling as is discussed in Appendix C.

Random motion was analyzed by allowing the mobile node to move in one of four perpendicular directions at every step with equal probability (in unbounded space). The same results may be applied when the mobile node is allowed to move in more than four directions, as long as the probabilities are uniform. Thus, the random walk model was considered for discrete space and discrete time, and future work may consider extending the results to continuous space. While the continuous space results may be more useful in real–world scenarios, they would invariably be accompanied by a considerable increase in processing overhead.

All results in this thesis have been conclusively proved. However, there is scope to simulate the algorithms that have been proposed and compare them to existing algorithms to get a measure of the improvement in energy efficiency. This would also enable a good estimate for the probability threshold p^* that was introduced in Section 5.3. Moreover, random motion with time constraints was analyzed for single transmission only. It might be worthwhile to study if there are any advantages associated with a similar scheme for multiple transmissions. Finally, an interesting complementary problem would be to determine the most efficient trajectory of the mobile node if the positions of several source nodes are given.

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Appendix A

Theorems and Proofs

Theorem 1 *When moving in a straight line, the transmission power from the mobile node to another node continue to decrease until it reaches a minimum and then continues to increase.*

Since transmission power is assumed to be proportional to the square of the distance between the two nodes, minimum energy is expended when the distance between the two nodes is minimum. This occurs when the line joining the fixed node is perpendicular to the trajectory of the mobile node. Thus the energy is higher on both sides of this minimum point, and a moving node would find the energy decreasing until it reaches a minimum value (corresponding to the shortest distance, *i.e.*, the perpendicular line) and then it begins to increase.

Theorem 2 *The optimum transmission distance for the source in Case III of Section 4.2.2 is* $x = \frac{l}{1+\eta}$.

Transmission cost was defined in Section 3.3 as follows:

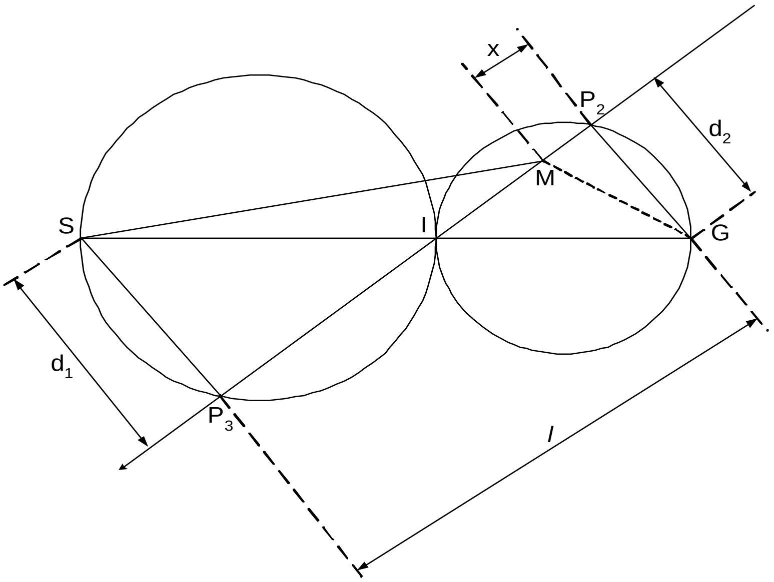

Figure A.1: Close-up view of Figure 4.4 for determination of x.

$$
\begin{aligned}
C &= \alpha_s.d_{SM}^2 + \alpha_m.d_{SG}^2 \\
&= \alpha_s.[d_{SM}^2 + \eta.d_{MG}^2] \\
&= \alpha_s.[d_1^2 + (l-x)^2 + \eta.(d_2^2 + x^2)] \\
&= \alpha_s.[d_1^2 + l^2 + x^2 - 2lx + \eta.d_2^2 + \eta.x]
\end{aligned}
$$

To minimize C, the following condition must be satisfied: $\frac{dC}{dx} = 0$

i.e., $2.\alpha_s.x - 2.l.\alpha_s + 2.\alpha_s.\eta.x = 0$

Hence, $x = \frac{l}{1+\eta}$.

Theorem 3 *The optimum transmission distance for the source in Case IV of Section 4.2.2 is* $x = \frac{l}{1+\eta}$.

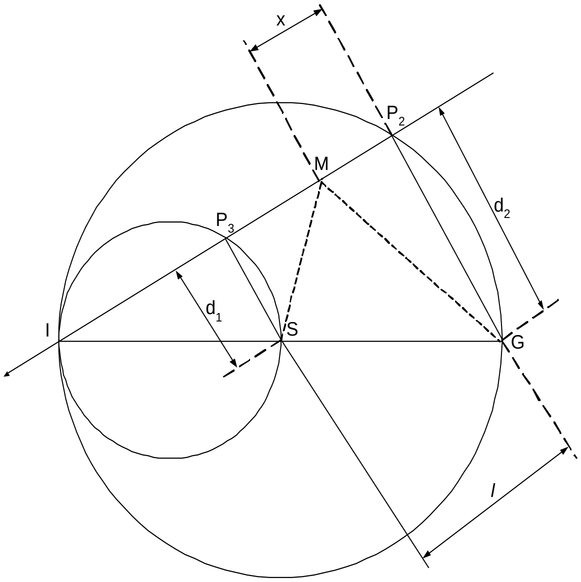

Figure A.2: Close-up view of Figure 4.5 for determination of x.

Transmission cost was defined in Section 3.3 as follows:

$$C = \alpha_s.d_{SM}^2 + \alpha_m.d_{SG}^2$$

$$= \alpha_s.[d_{SM}^2 + \eta.d_{MG}^2]$$

$$= \alpha_s.[d_1^2 + (l - x)^2 + \eta.(d_2^2 + x^2)]$$

$$= \alpha_s.[d_1^2 + l^2 + x^2 - 2lx + \eta.d_2^2 + \eta.x]$$

To minimize C, the following condition must be satisfied: $\frac{dC}{dx} = 0$

$i.e.$, $2.\alpha_s.x - 2.l.\alpha_s + 2.\alpha_s.\eta.x = 0$

Hence, $x = \frac{l}{1+\eta}$.

Theorem 4 *In Section 4.3.2, R_i and T_i are respectively the most efficient reception and* *transmission points on each line segment i of the piece-wise linear path.*

Since on each line segment i, the shortest distance (*i.e.*, a perpendicular path), or a path closest to a perpendicular path (depending on constraints of starting and ending points) are chosen, the resulting R_i and T_i points are the most efficient points for transmission and reception.

Theorem 5 *The table described in Section 4.3.2 combines the most efficient reception point with the most efficient transmission point as long as the resulting combination is causal.*

The table sorts receiver efficiency in one direction and transmitter efficiency in the other direction. The best valid (*i.e.*, causal) combination is the one that is nearest to the upper left-hand corner. This may be proved by contradiction.

1. Assume that there exists a more efficient receiver point for the same transmission point. In that case, that receiver would be sorted to the left of the given point making it closer to the upper left-hand corner.

2. Assume that there exists a more efficient transmission point for the same receiver point. In that case, that transmitter would be sorted above the given point making it closer to the upper left-hand corner.

3. Assume that there exists a more efficient receiver–transmitter combination. In that case, either the transmitter or the receiver or both would be more efficient than the current point and would therefore be either closer to the upper left-hand corner or

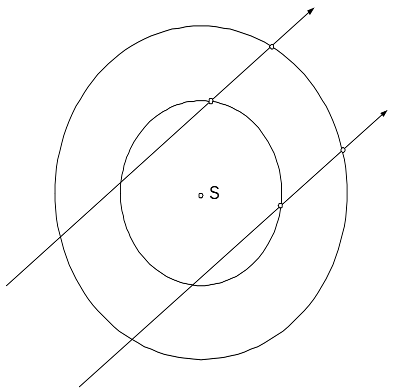

Figure A.3: Source narrows the trajectory of the mobile node to one of two possibilities.

along a line parallel to the table's diagonal from the top-right to the bottom-left corners.

Theorem 6 *The global efficient point for piece-wise linear motion must be one of the local minima.*

This may be proved by contradiction.
Assume that there exists a point on a line segment that is the global minimum but is different from the "efficient" point for that line. In that case, the global minimum point would consume more energy than the "efficient" point for that segment, which contradicts the definition of an efficient point. Hence the requirement.

Theorem 7 *If the mobile node moves at a constant speed and broadcasts its absolute direction at regular intervals, the trajectory of the mobile node can be narrowed down to one of two possibilities.*

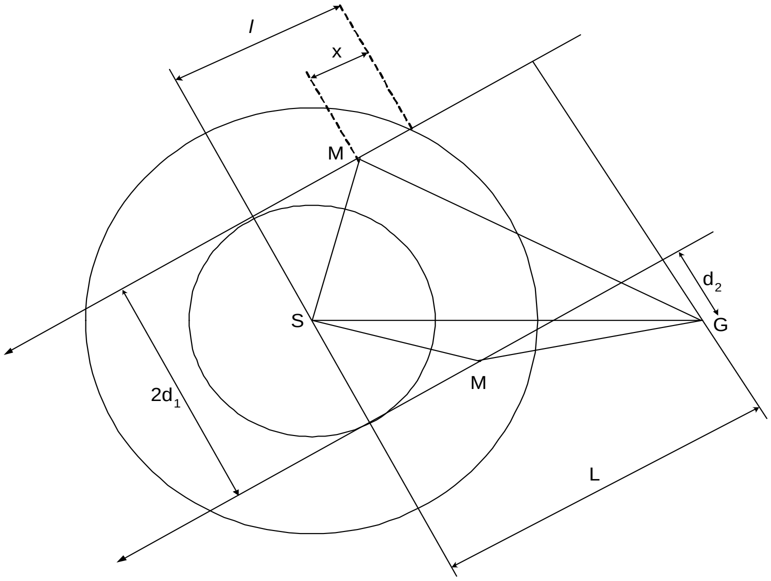

Figure A.4: Reception or transmission occur simultaneously for the two paths illustrated in Figure A.3.

Since the node moves at a constant speed, it will cover equal distances in each of its regular intervals. As a result, of the infinitesimally many chords [see Figure A.3] that can be drawn through the two circles with the same direction, only two of them will have covered the distance $d = s \times \tau$ where s is the speed of the mobile and τ is the interval between successive broadcasts.

Theorem 8 *For both the paths identified in* Theorem 7*, the time for optimum reception or transmission occur simultaneously.*

Referring to Figure A.4, the following results may be obtained.

For path 1:

$C = \alpha_s.[d_{SM}^2 + \eta \times d_{MG}^2] = \alpha_s.[d_1^2 + (l - x)^2 + \eta \times \{(L - l + x)^2 + d_2^2\}]$

To minimize the cost function, we must have: $\frac{\partial C}{\partial x} = 0$

Thus, $2x - 2l + \eta \times (2x - 2l + 2L) = 0$

For path 2:

$C = \alpha_s.[d_{SM}^2 + \eta \times d_{MG}^2] = \alpha_s.[d_1^2 + (l - x)^2 + \eta \times \{d_2^2 + (L - l + x)^2\}]$

To minimize the cost function, we must have: $\frac{\partial C}{\partial x} = 0$

Thus, $2x - 2l + \eta \times (2x - 2l + 2L) = 0$

In both cases, $x = \frac{l+\eta\times(l-L)}{1+\eta} = \frac{l-\eta\times(L-l)}{1+\eta}$ which proves the theorem.

Theorem 9 *The speed of the mobile node may be calculated if at least three broadcasts* at regular intervals *are received by the source.*

As usual, it is assumed that the mobile node moves with a constant, but unknown, speed and sends broadcasts with limited power at regular intervals. It is also assumed that the beacons (electromagnetic signals) broadcast by the mobile node reach the source immediately.

The problem reduces to a purely geometrical problem as illustrated in Figure A.5.

Given:$M_1M_2 = M_2M_3, M_1S, M_2S, M_3S, \theta$

To find: M_1M_2 or M_2M_3

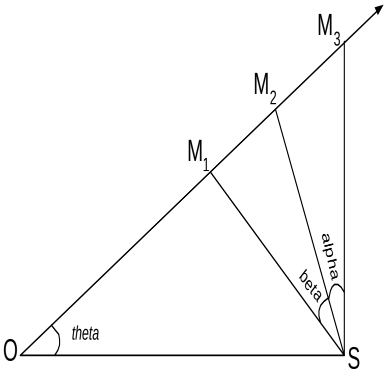

Figure A.5: Calculation of speed of mobile node based on three broadcasts made at regular intervals.

$$M_2M_3^2 = M_3S^2 + M_2S^2 - 2 \times M_3S \times M_2S \times \cos(\alpha)$$
$$\Rightarrow \cos(\alpha) = \frac{M_3S^2+M_2S^2-M_2M_3^2}{2\times M_3S\times M_2S}$$
$$M_2M_1^2 = M_2S^2 + M_1S^2 - 2 \times M_2S \times M_1S \times \cos(\beta)$$
$$\Rightarrow \cos(\beta) = \frac{M_2S^2+M_1S^2-M_2M_1^2}{2\times M_2S\times M_1S}$$
$$M_3M_1^2 = M_3S^2 + M_1S^2 - 2 \times M_3S \times M_1S \times \cos(\alpha+\beta)$$
$$\Rightarrow \cos(\alpha+\beta) = \frac{M_3S^2+M_1S^2-M_3M_1^2}{2\times M_3S\times M_1S}$$

But, $\cos(\alpha+\beta) = \cos(\alpha).\cos(\beta) - \sin(\alpha).\sin(\beta)$

Therefore, $\frac{M_3S^2+M_1S^2-M_3M_1^2}{2\times M_3S\times M_1S} = \frac{M_3S^2+M_2S^2-M_2M_3^2}{2\times M_3S\times M_2S}.\frac{M_2S^2+M_1S^2-M_2M_1^2}{2\times M_2S\times M_1S} - \sqrt{[1-\{\frac{M_3S^2+M_2S^2-M_2M_3^2}{2\times M_3S\times M_2S}\}^2].[1-\{\frac{M_2S^2+M_1S^2-M_2M_1^2}{2\times M_2S\times M_1S}\}^2]}$

The only unknown here is M_1M_2 (which is equal to M_2M_3). Thus, the distance traveled can be obtained and hence the speed may be calculated. Note that the angle θ plays no role in the calculation of the distance or the speed. However, the direction information may be combined with the speed to obtain the velocity.

Theorem 10 *The number of possible shortest routes in moving from* (x, y) *to* $(0, 0)$ *is given by* $\binom{x}{y}$.

The grid numbering system introduced in Section 5.3.1 carries complete information about the number of hops needed to reach the origin $(0, 0)$.
For instance, in the case of cell (x, y), it takes a total of x hops to reach the origin, of which y steps are taken in one direction and $x - y$ steps in its perpendicular direction.
Since the steps may be taken in any order, there is more than one possible route to start from (x, y) and reach $(0, 0)$ in x steps.
Thus, the problem reduces to finding out the number of ways in which y steps may be chosen out of a total of x steps, and this combination is mathematically defined as $\binom{x}{y}$.

Appendix B

Conditions for Relay Transmission

Most of the previous analyses provide the position (in time and space) for the most optimum relay transmission, provided that a direct transmission is more costly. While it is intuitively obvious that a direct transmission is preferrable when the distance between the source and the mobile node is more than the source–gateway distance, there are other cases when a direct transmission should be made. The following analysis obtains the conditions for making a direct transmission in the case of straight line motion with index of propagation $n = 2$. A similar analysis can be carried out for other value of the index of propagation.

Let the direction of the mobile node be θ with respect to the positive x-axis.
Then, the slope of the mobile trajectory is $m = \tan\theta$
and, equation of trajectory is: $y - y_0 = \tan\theta.(x - x_0)$

Recalling that the cost function for relay transmission is given by:
$C = \alpha_s.[d_{sm}^2 + \eta.d_{mg}^2]$,

the following results can be obtained for the given mobile trajectory:

Cost for direct transmission, $C_{DT}= \alpha_s$

Cost for relay transmission, $C_{RT}= \alpha s.[x'^2 + y_0^2 + (x' - x_0)^2. \tan^2\theta + 2.(x' - x_0).y_0. \tan\theta$

$+\eta.\{x'^2 + 1 - 2.x' + y_0^2 + (x' - x_0)^2. \tan^2\theta + 2.(x' - x_0).y_0. \tan\theta\}]$

Thus, $\frac{C_{RT}}{C_{DT}} = x'^2 + y_0^2 + (x' - x_0)^2. \tan^2\theta + 2.(x' - x_0).y_0. \tan\theta$

$+\eta.\{x'^2 + 1 - 2.x' + y_0^2 + (x' - x_0)^2. \tan^2\theta + 2.(x' - x_0).y_0. \tan\theta\}$

A direct transmission will be preferrable for the range of x' that causes $C_{RT} > C_{DT}$.

i.e., $\frac{C_{RT}}{C_{DT}} > 1$

i.e., $\frac{C_{RT}}{C_{DT}} - 1 > 0$

But, $\frac{C_{RT}}{C_{DT}} - 1 = x'^2.[1 + \tan^2\theta + \eta + \eta. \tan^2\theta]$

$+x'.[-2.x_0. \tan^2\theta - 2.\eta - 2.\eta.x_0. \tan^2\theta + 2.\eta.y_0. \tan\theta]$

$+[y_0^2 + x_0^2. \tan^2\theta - 2.x_0.y_0. \tan\theta - 1 + \eta + \eta.y_0^2 + \eta.x_0^2. \tan^2\theta - 2.\eta.x_0.y_0. \tan\theta]$

This is of the form $f(x')= a.x'_2 + b.x' + c$

where, $a = 1 + \tan^2\theta + \eta + \eta. \tan^2\theta$

$= (1 + \eta).1 + \tan^2\theta)$

$= (1 + \eta). \sec^2\theta$

$b = -2.x_0. \tan^2\theta - 2.\eta - 2.\eta.x_0. \tan^2\theta + 2.\eta.y_0. \tan\theta$

$c = y_0^2 + x_0^2. \tan^2\theta - 2.x_0.y_0. \tan\theta - 1 + \eta + \eta.y_0^2 + \eta.x_0^2. \tan^2\theta - 2.\eta.x_0.y_0. \tan\theta$

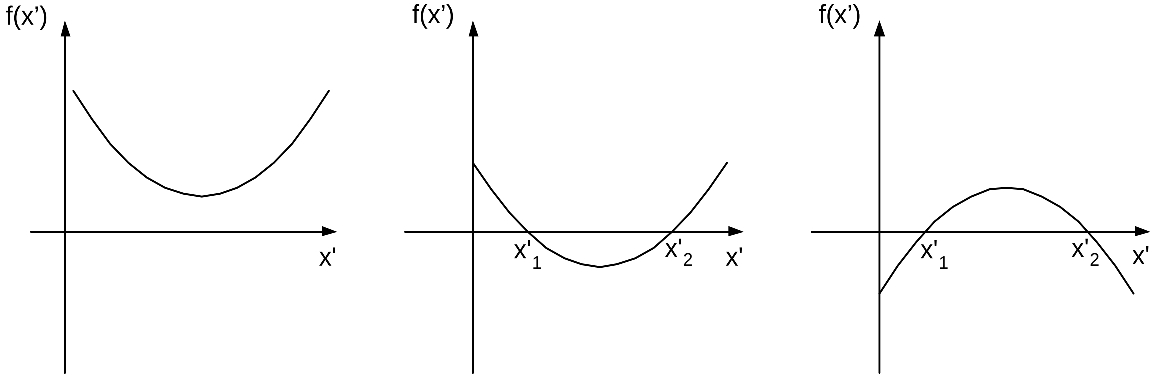

Figure B.1: The three conditions that lead to direct transmission being preferred: (*left*) $a > 0$ and $\Delta \leq 0$; (*center*) $a > 0$ and $\Delta > 0$; (*right*) $a, 0$ and $\Delta \geq 0$.

This will be positive under the following circumstances [Figure]:

1. $a > 0$

   $\Delta \leq 0$

2. $a > 0$

   $\Delta > 0$

3. $a < 0$

   $\Delta \geq 0$

where, Δ=discriminant

$$= b^2 - 4.a.c$$

Thus, a direct transmission occurs when:

1. $1 + \eta > 0$

   $\Delta \leq 0$

2. $1 + \eta > 0$

   $\Delta > 0$

   $i.e.$, at $(-\infty, x'_1)$ and (x'_2, ∞)

   where, $x'_1, x'_2 = \frac{-b \pm \sqrt{b^2 - 4.a.c}}{2.a}$

3. $1 + \eta < 0$

   $\Delta \geq 0$

   $i.e.$, at 4(x'$_1$, x'_2) where, $x'_1, x'_2 = \frac{-b \pm \sqrt{b^2 - 4.a.c}}{2.a}$

It may be noted that since $a = (1 + \eta). \sec^2 \theta$, the third possibility may be eliminated.

A relay transmission is made for all other cases.

Appendix C

Piece-Wise Linearization

The term *pre-determined* motion was used liberally in Chapter 4, for the analysis of deterministic motion. In this context, the term “pre-defined motion” begs further explanation. It is assumed that the trajectory of the mobile node is defined by a set of coordinates of points that the mobile node traces. Obviously, recording all the points the mobile node passes through is neither straightforward nor desirable, so an algorithm is applied to select (or sample) a set of points from all the possible points. While sophisticated methods of sampling or real-time tracking is an active field of research [4], the following simple methods may be applied to obtain a smaller set of candidate points from a larger or global set of points through which the mobile node travels.

Method I

The simplest method to choose a smaller subset of coordinates from a given set of points is to sample the points at a certain rate. This rate should be able to reduce the number of points being considered, while maintaining enough information about the shape of the trajectory. Standard results in Signal Processing [19] suggest that the minimum number of points that meets these requirements is that which satisfies the Nyquist rate of sampling. However, since human motion (and hence the motion of the mobile node) may change quite frequently, the sampled data may still be generated at a rate that quickly clogs up the system. The alternative, reducing the sampling frequency, would compare a fewer number of points and thus runs the risk of *overlooking* the optimum points for transmission.

Method II

While Method I calculates the Nyquist rate and hence the number of sample points before proceeding to obtain the sample set, the method being discussed determines the number of points recursively. The strategy rests on repeatedly calculating the cost function for each iteration until successive iterations do not result in a significant difference (according to a pre-defined threshold) of the minimum cost functions. The initial relay transmission considers the median of the starting and ending coordinate points and calculates the cost function corresponding to that. Subsequently the number of points is increased geometrically in each iteration, *i.e.*, N=2, 4, 8, 16, etc., by considering the median point in each segment. This results in the length of the segments between successive points being halved in each iteration. While the number of calculations is still very large, this method would stabilize to the optimum points quite quickly as it is based on the principle of a binary search. Another advantage to this technique is that the existing information can be reused— since the distances are being halved in every iteration, the cost functions corresponding to the new points that are being introduced complement the points already present. Since the aim is to minimize the cost function, the cycle repeats itself recursively till the minimum value of the cost function in successive iterations lies within the threshold.

Method III

The previous two methods rely on a table of coordinates being available ahead of the motion of the mobile node. Thus the scope of those techniques is limited only to deterministic motion of the mobile node. The method currently under discussion may be applied to both deterministic and real-time random motion— its application to the latter scenario is briefly touched upon. Another major point of difference is that this technique does not sample points on the trajectory at regular intervals— this follows from the observation that any number of points lying on the *same straight line* is completely defined by the starting and ending points of the line. Thus, sampling should be less frequent when motion is linear and more when it is non-linear. This method determines the degree of linearity by calculating the slopes for every pair of points while Method IV uses curvature to resolve the same.

The algorithm begins by recording the starting point and then recording subsequent coordinates only if the slope between successive pairs of points differs from the last calculated slope by an amount greater than a pre-defined threshold. Once the "slope difference" exceeds the threshold, the previous (*i.e.*, the last point on the straight line) and the current point are noted. Since the starting point of each segment and the slope are both known (and the motion is assumed to be at constant speed), the subset of coordinates guarantees that the position of the mobile node is known within a small margin of error (which depends on the "slope difference" threshold) at every instant. It also follows that points will be noted down more frequently when the motion is "more" non-linear.

In the case of real-time data gathering, the mobile node can broadcast its position every time it changes its direction by an angle greater than the defined threshold. For all other cases, the static nodes can assume that the mobile node is moving in a straight line.

Method IV

This method employs an alternative definition of the “non-linearity” of trajectory. Instead of calculating sloped for every pair of points, it calculates the curvature at every instant of time and notes down the point if it exceeds the acceptable curvature value. In the Cartesian coordinate system this is given by:

$$\kappa = \frac{x'.y'' - y'.x''}{(x'^2 + y'^2)^{\frac{3}{2}}}$$

The definiton of curvature, however, requires the motion to be defined in terms of a set of parametric equations, which limits the usefulness of the result.