# Minimum Energy Transmission in Ad-Hoc Networks with Deterministic Motion 

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#### Abstract

This paper proposes several relay transmission schemes that lead to minimum energy consumption in a mobile ad-hoc network. The study introduces mobility in the form of a node that is capable of collecting data from sources randomly distributed in space and delivering it to a fixed gateway. It defines an energy cost function based on transmission powers, and then describes methods for locating points on its predetermined trajectory that would minimize the cost.


## I. INTRODUCTION

An ad-hoc network is a multi-hop wireless network where all nodes cooperatively maintain network connectivity without any centralized infrastructure [5]. If these nodes change their positions dynamically, it is called a mobile adhoc network (MANET). Due to the limited transmission range of wireless nodes, as well as the rapid change in network topology, multiple network hops may be needed for one node to exchange data with another across the network. Thus, each node participates in an ad-hoc routing protocol that allows it to discover multi-hop paths though the network to any other node.

Most work on power conservation in ad-hoc networks [1, $3,2]$ consider a set of static nodes randomly distributed in an area and minimize the energy consumed in transmission from a source to a gateway through a multi-hop routing algorithm. In contrast, this work introduces a third class of nodes, mobile nodes, and obtains the minimum energy relay transmission path through these nodes. The mobile nodes are considered to move along pre-determined trajectories, so that that the position of the mobile node at any point in time is known to all the other nodes.

Apart from minimizing the consumption of energy, a successful transmission must also be completed within a maximum allowable time, usually referred to as the delivery deadline. Since the speed and directions of the mobile node are known ahead of time, the point in space at which the transmission time expires is also known deterministically. Thus, time constraint is introduced into the analysis by ensuring that transmission is completed before the mobile node reaches this critical point on its trajectory.

Mobility of the relay node is initially restricted to straight line motion and results are obtained for all possible combinations of starting point and expiration point. This is followed by the analysis for piece-wise linear motion which

[^0]may be considered as a sequence of straight lines in different directions at rapid succession.

There are several real-life applications in which the trajectory of the mobile node is known a priori. For instance, if sensor nodes are distributed in a battlefield, security and other ground realities would limit the route used by a reconnaissance aircraft (which acts as the mobile node) to collect data from the field nodes. In a similar manner, for sensor nodes spread over a mountainous region to collect environmental data, the mobile node (in the form of a jeep, for instance) would choose the least inhospitable route for data collection.
This paper is arranged in the following manner. Section II defines the energy cost function and lists the assumptions used in the model. Section III analyses deterministic motion of the mobile node along a straight line motion, while Section IV approximates the deterministic motion as piece-wise linear segments. Finally, Section V concludes the paper with a discussion of future areas of work.

## II. MODEL

The relay transmission schemes being proposed guarantee timely delivery of information from a source to a destination with minimum consumption of energy. Each source is concerned with conserving its limited battery power as far as possible, and to this end, may choose to transmit via a mobile node should that lead to a lower cost. If the lifetime of the system is defined as the time until any one of the source nodes runs out of power, then the algorithms formulated in this paper will result in the system lifetime being extended.
A typical scenario would involve the following classes of nodes:

- Fixed gateway, which is the final destination and has no power constraints.
- Large number of fixed sensors, which transmit information signals periodically but are powered by energy-limited batteries.
- Small number of mobile nodes, which are capable of transmission and reception and have lower power constraints than the source nodes.
For simplicity, the analysis concentrates on the power consumption behavior of a system comprising a mobile node in the presence of a single source node. Since the transmission algorithm thus obtained is executed by the


Figure 1: Layout of model for analysis comprising a source node, a destination and a mobile relay node. The two possible routes are a direct transmission from the source to the gateway, or a relay transmission via the mobile node.
source node, the results may be applied to any number of source nodes by taking interference between source nodes into consideration, although that is beyond the scope of this work. This situation is illustrated in Fig. 1.

Some of the assumptions made in the course of this analysis are as follows:

- Collision between data packets is not considered. This might not appear to carry much significance in the case of a single source node, but even in the general case of multiple sources, the algorithm does not include any collision detection or collision avoidance schemes.
- The propagation delay is negligible. The data signals are electromagnetic signals traveling at $3 \times$ $10^{8} \mathrm{~m} / \mathrm{s}$, and the corresponding transmission delay is not considered important in this application.
- Mobile nodes move with constant speed. While the change of direction of the mobile node is considered, the speed of the node is assumed to be finite and unchanged.
- Path loss exponent lies between 2 and 4. The exponent of propagation loss, $n$, typically varies between 2 and 4 , and is close to 2 for small distances.
- Power is consumed only during transmission. It is assumed that energy is expended only during transmission of data and reception of data is cost-free. Most analyses make use of this assumption, although some authors [4] consider this to be 'unrealistic'. The computation cost at the source is also considered negligible.

Since this paper aims at transmitting data from a source to a destination within a deadline and with a minimum amount of energy, based on the above assumptions, the power cost metric is defined below as a function of transmission energy

$$
\begin{aligned}
\mathrm{C} & =\alpha_{\mathrm{s}} \cdot \mathrm{P}_{\mathrm{s}}+\alpha_{\mathrm{m}} \cdot \mathrm{P}_{\mathrm{m}} \\
& =\alpha_{\mathrm{s}} \cdot \mathrm{~d}_{\mathrm{sm}}{ }^{n}+\alpha_{\mathrm{m}} \cdot \mathrm{~d}_{\mathrm{mg}}{ }^{\mathrm{n}} \\
& =\alpha_{\mathrm{s}} \cdot\left[\mathrm{~d}_{\mathrm{sm}}{ }^{n}+\eta \cdot d_{\mathrm{mg}}{ }^{n}\right]
\end{aligned}
$$

where, $\mathrm{P}_{\mathrm{s}}$ : power needed to transmit from source to mobile,
$\mathrm{P}_{\mathrm{m}}$ : power needed to transmit from mobile to gateway,
$\alpha_{s}$ : proportionality constant for source power,
$\alpha_{\mathrm{m}}$ : proportionality constant for mobile power,
$\mathrm{d}_{\mathrm{sm}}$ : distance between source and mobile nodes,
$\mathrm{d}_{\mathrm{mg}}$ : distance between mobile node and gateway,
$\eta=\alpha_{\mathrm{m}} / \alpha_{\mathrm{s}}$
Depending on the value of $\eta$, the following cases may be enumerated:

1. $\eta=0$ : Mobile node has no power constraint.
2. $0 \leq \eta \leq 1$ : Source node is more power constrained.
3. $\eta=1$ : Source and mobile nodes are equally weighted.
4. $\quad \eta \geq 1$ : Mobile node is more power constrained.

The analysis in this paper will primarily consider the case when source nodes are more power constrained than the mobile node.

## III. Straight Line Deterministic Motion

In this section, deterministic straight line motion with constant velocity will be considered. The points of interest in the trajectory of the mobile node are the starting point $P_{0}$ and the point by which the transmission must be completed $P^{*}$. The starting point may be defined as the time at which data becomes available at the source for transmission to the gateway. The model comprises a source $S$, gateway (or destination) $G$, and a mobile node moving in a straight line with constant velocity $v$. Furthermore, the analysis is carried out only when the mobile node enters the circle with center at $S$ and having a radius equal to the segment $S G$. This follows from the fact that the cost metric is a function of distance and it is less costly for the source to transmit directly to the gateway if it takes more power to transmit from the source to the mobile node alone (which is the case when the mobile node is outside the $S G$ circle).

The notation used in the following analysis is summarized below:

- S: source
- G: gateway (or destination)
- I: point of intersection of mobile node with $S G$ axis; it also defines the left-half and right-half of the plane
- $\mathrm{P}_{0}$ : starting point for mobile node
- $P^{*}$ : position of mobile node at the termination of maximum allowable delay
Depending on the relative positions of the starting point $P_{0}$ and the point of intersection of the mobile trajectory with the $S G$-axis, $I$, the results for the optimum relay transmission schemes can be divided into four categories. The results are based on the fact that only one point on a straight line (in this case, the mobile trajectory) is at the shortest distance from any other point (in this case, the source node) and the distance increases monotonically on either side of that minimum point.


Figure 2: Deterministic straight line motion with $P_{0}$ on the left side of $I$ and $I$ lying between $S$ and $G$.

Case A: $\quad P_{0}$ in left-half plane and $I$ lies between $S$ and $G$ [Fig. 2]

1. $\mathrm{P}_{0} \varepsilon\left(-\infty, \mathrm{P}_{1}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{1}\right)$ direct transmission
b) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$ : receive and transmit at $\mathrm{P}^{*}$
c) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{2}, \mathrm{P}_{3}\right)$ : receive at $\mathrm{P}_{2}$ and transmit at $\mathrm{P}^{*}$
d) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{3}, \infty\right)$ : receive at $\mathrm{P}_{2}$ and transmit at $\mathrm{P}_{3}$
2. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{2}\right)$ : receive and transmit at $\mathrm{P}^{*}$
b) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{2}, \mathrm{P}_{3}\right)$ : receive at $\mathrm{P}_{2}$ and transmit at $\mathrm{P}^{*}$
c) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{3}, \infty\right)$ : receive at $\mathrm{P}_{2}$ and transmit at $\mathrm{P}_{3}$
3. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{2}, \mathrm{P}_{3}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{3}\right)$ : receive at $\mathrm{P}_{0}$ and transmit at $\mathrm{P}^{*}$
b) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{3}, \infty\right)$ : receive at $\mathrm{P}_{0}$ and transmit at $\mathrm{P}_{3}$
4. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{3}, \mathrm{P}_{4}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \infty\right)$ : receive and transmit at $\mathrm{P}_{0}$
5. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{4}, \infty\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \infty\right)$ : direct transmission

Case B: $\quad P_{0}$ in left-half plane and $I$ does not lie between $S$ and $G$ [Fig. 3]

1. $\mathrm{P}_{0} \&\left(-\infty, \mathrm{P}_{1}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{1}\right)$ direct transmission
b) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$ : receive and transmit at $\mathrm{P}^{*}$
c) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{2}, \mathrm{P}_{3}\right)$ : receive at $\mathrm{P}_{2}$ and transmit at $\mathrm{P}^{*}$
d) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{3}, \infty\right)$ : receive at $\mathrm{P}_{2}$ and transmit at $\mathrm{P}_{3}$
2. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{2}\right)$ : receive and transmit at $\mathrm{P}^{*}$
b) $\mathrm{P}_{*}^{*} \varepsilon\left(\mathrm{P}_{2}, \mathrm{P}_{3}\right)$ : receive at $\mathrm{P}_{2}$ and transmit at $\mathrm{P}^{*}$
c) $\mathrm{P}^{*} \varepsilon(\mathrm{P} 3, \infty)$ : receive at $\mathrm{P}_{2}$ and transmit at $\mathrm{P}_{3}$
3. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{2}, \mathrm{P}_{3}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{3}\right)$ : receive at $\mathrm{P}_{0}$ and transmit at $\mathrm{P}^{*}$
b) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{3}, \infty\right)$ : receive at $\mathrm{P}_{0}$ and transmit at $\mathrm{P}_{3}$
4. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{3}, \infty\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \infty\right)$ : direct transmission


Figure 3: Deterministic straight line motion with $P_{0}$ on the left side of $I$ and $I$ not lying between $S$ and $G$.


Figure 4: Deterministic straight line motion with $P_{0}$ on the right side of $I$ and $I$ lying between $S$ and $G$. The distance between $P_{2}$ and $P_{3}$ along the mobile trajectory is defined as $l$.
Case C: $\quad P_{0}$ in right-half plane and $I$ lies between $S$ and $G$
[Fig. 4]. The distance between $P_{2}$ and $P_{3}$ is $l$.

1. $\mathrm{P}_{0} \&\left(-\infty, \mathrm{P}_{1}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{1}\right)$ direct transmission
b) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$ : receive and transmit at $\mathrm{P}^{*}$
c) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{2}, \infty\right)$ : receive and transmit at a distance of $x=l \div(1+\eta)$ [Proof in Appendix]
2. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{2}\right)$ : receive and transmit at $\mathrm{P}^{*}$
b) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{2}, \infty\right)$ : receive and transmit at a distance of $x=l \div(1+\eta)$ [Proof in Appendix]
3. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{2}, \mathrm{P}_{3}\right)$


Figure 5: Deterministic straight line motion with $P_{0}$ on the right side of $I$ and $I$ not lying between $S$ and $G$. The distance between $P_{2}$ and $P_{3}$ along the mobile trajectory is defined as $l$.
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \infty\right)$ : receive and transmit at a distance of $x=l \div(1+\eta)$ [Proof in Appendix]
4. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{3}, \infty\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \infty\right)$ : receive and transmit at $\mathrm{P}_{0}$

Case D: $\quad P_{0}$ in right-half plane and $I$ does not lie between $S$ and $G$ [Fig. 5]. The distance between $\mathrm{P}_{2}$ and $\mathrm{P}_{3}$ is $l$.

1. $\mathrm{P}_{0} \&\left(-\infty, \mathrm{P}_{1}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{1}\right)$ : direct transmission
b) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$ : receive and transmit at $\mathrm{P}^{*}$
c) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{2}, \infty\right)$ : receive and transmit at a distance of $x=l \div(1+\eta)$ [Proof in Appendix]
2. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \mathrm{P}_{2}\right)$ : receive and transmit at $\mathrm{P}^{*}$
b) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{2}, \infty\right)$ : receive and transmit at a distance of $x=l \div(1+\eta)$ [Proof in Appendix]
3. $\mathrm{P}_{0} \varepsilon\left(\mathrm{P}_{2}, \mathrm{P}_{3}\right)$
a) $\mathrm{P}^{*} \varepsilon(\mathrm{P} 0, \infty)$ : receive and transmit at a distance of $x=l \div(1+\eta)$ [Proof in Appendix]
4. $\mathrm{P}_{0} \&\left(\mathrm{P}_{3}, \infty\right)$
a) $\mathrm{P}^{*} \varepsilon\left(\mathrm{P}_{0}, \infty\right)$ : receive and transmit at $\mathrm{P}_{0}$

The thirty-five cases enumerated above exhaustively cover all possibilities arising from relay transmission via a mobile node that moves in a straight line. If the source and gateway nodes make transmission and reception decisions in accordance with these results, then a minimum energy transmission is guaranteed.


Figure 6: Layout of model for analysis comprising source node $S$, gateway $G$, and piece-wise linear trajectory of mobile node starting at $P_{0}$ and ending at $P^{*}$.

## IV. Piece-Wise Linear Deterministic Motion

The aim of this section is to obtain the optimum reception and transmission points along the trajectory of a mobile node that is approximated by piece-wise linear segments as illustrated in Fig. 6. Since deterministic motion is being considered, i.e., the complete trajectory is known a priori, the results obtained earlier for straight line motion may be applied to each segment and the optimal point (or pair of points) selected. For ease of analysis, the motion is classified into two categories-
A. When the mobile node is moving from the direction of the source to the gateway.
B. When the mobile node is moving from the direction of the gateway to the source.
The former corresponds to Cases A and D, and the latter to Cases B and C in the analysis of straight line motions discussed in Section III.

Case A: Mobile node moves from the direction of source to gateway

Application of the results obtained for Cases A and D in Section III leads to two unique points for the mobile node, one each for receiving data from the source and then transmitting to the gateway, on each piece-wise linear segment. In Fig. 7, these are named $R_{i}$ and $T_{i}$, where $i$ refers to the sequence number of the line segment.

For an $n$-segment piece-wise linear path, there will be $n$ pairs of $R_{i}$ and $T_{i}$ points. The overall optimum reception and transmission points, $R_{j}$ and $T_{k}$, are obtained by sorting the $R_{i}$ and $T_{i}$ points according to distance (i.e., distance of $R_{i}$ from the source $S$ and distance of $T_{i}$ from the gateway $G$ ) and a table is formed with $R_{i}$ as the columns and $T_{i}$ as the rows.

For $\eta=1$, the best relay path (i.e., the path that consumes the minimum energy) is given by the valid (i.e., causal) $R_{i}-T_{j}$ pair closest to the upper left-hand cell of the table. Causality is guaranteed if $i \leq j$.


Figure 7: Layout of model for analysis comprising source node $S$, gateway $G$, and eight piece-wise linear segments on the mobile trajectory. There is a pair of optimum reception and transmission points corresponding to each segment.

Case B: Mobile node moves from the direction of gateway to source

Application of the results obtained for Cases B and C in Section III leads to a single point for reception and transmission on each piece-wise linear segment. This is illustrated in Fig. 8, where $R_{i}$ and $T_{i}$ coincide for each of the eight line segments.

Once again, the $R_{i}$ and $T_{i}$ points are sorted according to distance (i.e., distance of $R_{i}$ from the source $S$ and distance of $T_{i}$ from the gateway $G$ ) and a table is formed with $R_{i}$ as the columns and $T_{i}$ as the rows. For $\eta=1$, the best relay path (i.e., the path that consumes the minimum energy) is given by the valid (i.e., causal) $R_{i}-T_{i}$ pair closest to the upper left-hand cell of the table.

The scheme proposed in this section identifies the optimum relay points in an ad-hoc network provided that the trajectory of the mobile node is known a priori to itself. The success of the protocol depends on the ability of the mobile node to make its future positions available to the source (and the gateway) so that they may calculate the time at which the transmission (or transmission request) should be made.

The algorithm discussed above is valid for $\eta=1$. For all other values of $\eta$, the cost function (defined in Section II) has to be calculated for the optimum point on each line segment and the point with the minimum cost is selected as the global optimum point.
It may be noted that since the network is formed in an adhoc manner, the gateway must broadcast a beacon periodically so that the source may estimate the sourcegateway distance from the strength of the received signal.


Figure 8: Layout of model for analysis comprising source node $S$, gateway $G$, and eight piece-wise linear segments on the mobile trajectory. There is a single optimum point for reception and transmission corresponding to each segment.

A drawback of this scheme is that the computational overhead increases considerably at the source. However, this is justified since computational cost is typically less expensive than transmission cost.

## V. Conclusion

This paper proposed several novel relay transmission schemes that lead to minimum energy consumption in mobile ad-hoc networks. Unlike most studies that confine themselves to examining the effect of energy conservation in terms of a fixed set of nodes, this paper introduced mobility into the analysis, in the form of a mobile node that is capable of collecting data from the source and transmitting it to the gateway. After defining an energy cost function based on transmission power, the work explored ways of minimizing this cost for a host of network configurations and mobile trajectories assuming deterministic motion.
The work may be expanded to include random motion in which the trajectory of the mobile node is not known a priori. Such an analysis would require the use of Markov chains to define all the possible locations (or states) of the mobile node at any time and obtain the transmission points that minimize the expected energy consumption.

All results in this paper were proved geometrically. However, there is scope to simulate the algorithms that have been proposed and compare them to existing algorithms to get a measure of the improvement in energy efficiency. Finally, an interesting complementary problem would be to determine the most efficient trajectory of the mobile node if the positions of several source nodes are given.

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## Appendix

Proof that the optimum transmission distance for the source in Case C of Section III is $x=l \div(1+\eta)$.

Referring to Fig. A.1, the transmission cost is given as:

$$
\begin{aligned}
\mathrm{C} & =\alpha_{\mathrm{s}} \cdot \mathrm{P}_{\mathrm{s}}+\alpha_{\mathrm{m}} \cdot \mathrm{P}_{\mathrm{m}} \\
& =\alpha_{\mathrm{s}} \cdot \mathrm{~d}_{\mathrm{sm}}{ }^{2}+\alpha_{\mathrm{m}} \cdot \mathrm{~d}_{\mathrm{mg}}{ }^{2} \\
& =\alpha_{\mathrm{s}} \cdot\left[\mathrm{~d}_{\mathrm{sm}}{ }^{2}+\eta \cdot \mathrm{d}_{\mathrm{mg}}{ }^{2}\right] \\
& =\alpha_{\mathrm{s}} \cdot\left[\mathrm{~d}_{1}{ }^{2}+(l-x)^{2}+\eta \cdot\left(\mathrm{d}_{2}{ }^{2}+x^{2}\right)\right] \\
& =\alpha_{\mathrm{s}} \cdot\left[\mathrm{~d}_{1}{ }^{2}+l^{2}+x^{2}-2 \cdot l \cdot x+\eta \cdot \mathrm{d}_{2}{ }^{2}+\eta \cdot x\right]
\end{aligned}
$$

To minimize C , the following condition must be satisfied:

$$
\mathrm{dC} / \mathrm{d} x=0
$$

i.e., $\quad 2 . \alpha_{\mathrm{s}} \cdot x-2 . l . \alpha_{\mathrm{s}}+2 . \alpha_{\mathrm{s}} \cdot \eta \cdot x=0$

Hence, $\quad x=l \div(1+\eta)$
Proof that the optimum transmission distance for the source in Case D of Section III is $x=l \div(1+\eta)$.
Referring to Fig. A.2, the transmission cost is given as:

$$
\begin{aligned}
\mathrm{C} & =\alpha_{\mathrm{s}} \cdot \mathrm{P}_{\mathrm{s}}+\alpha_{\mathrm{m}} \cdot \mathrm{P}_{\mathrm{m}} \\
& =\alpha_{\mathrm{s}} \cdot \mathrm{~d}_{\mathrm{sm}}{ }^{2}+\alpha_{\mathrm{m}} \cdot \mathrm{~d}_{\mathrm{mg}}{ }^{2} \\
& =\alpha_{\mathrm{s}} \cdot\left[\mathrm{~d}_{\mathrm{sm}}{ }^{2}+\eta \cdot \mathrm{d}_{\mathrm{mg}}{ }^{2}\right] \\
& =\alpha_{\mathrm{s}} \cdot\left[\mathrm{~d}_{1}{ }^{2}+(l-x)^{2}+\eta \cdot\left(\mathrm{d}_{2}{ }^{2}+x^{2}\right)\right] \\
& =\alpha_{\mathrm{s}} \cdot\left[\mathrm{~d}_{1}{ }^{2}+l^{2}+x^{2}-2 \cdot l \cdot x+\eta \cdot \mathrm{d}_{2}{ }^{2}+\eta \cdot x\right]
\end{aligned}
$$

To minimize C , the following condition must be satisfied:

$$
\mathrm{dC} / \mathrm{d} x=0
$$

i.e., $\quad 2 . \alpha_{\mathrm{s}} \cdot x-2 . l . \alpha_{\mathrm{s}}+2 . \alpha_{\mathrm{s}} \cdot \eta \cdot x=0$

Hence,

$$
x=l \div(1+\eta)
$$



Figure A. 1: Close-up view of Fig. 4 for determination of $x$.


Figure A. 2: Close-up view of Fig. 5 for estimation of $x$.


[^0]:    This research was funded in part by the generous support of America Online.

