Opportunistic File Transfer over a Fading Channel under Energy and Delay Constraints *

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Abstract

We consider binary (on/off) power control strategies for transferring a fixed size file (finite number of packets) over fading channels under constraints on both transmit energy and transmission delay. The goal is to maximize the probability of successfully transferring the entire file over a time-varying wireless channel modeled as a finite state Markov process. We consider two scenarios for the delay constraints: an average delay constraint and a strict delay constraint. The resulting optimal policies are shown to be a function of the channel state information (CSI), the residual battery energy at the transmitter and also the number of residual packets in the transmit buffer. It is observed that the probability of successful file transfer increases significantly when the CSI is exploited opportunistically.

Keywords: power control, file transfer, energy constraint, delay constraint, fading channels.

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1 Introduction

With the development of personal communication services, portable terminals such as mobile telephones and notebook computers are expected to be used more frequently and for longer times. Hence power consumption will become even more important than it is now. One of the major concerns in supporting such mobile applications is the energy conservation and management in mobile devices. There is considerable work on the energy saving approaches in the literature. These range from energy management in communication devices to various layers of the protocol stack by intelligently using knowledge about the traffic, the channel and the network. For example, from network architecture design [1] to software strategies [2], from media access and logical link control protocols [3] to data compression and source coding [4], various sorts of energy efficient strategies have been discussed.

In second generation wireless communication networks, power control is mainly intended to provide each user an acceptable connection by eliminating unnecessary interference in the predominantly voice only network [5]. In next generation wireless systems, satisfying the Quality-of-Service (QoS) requirements of heterogeneous services is more complicated. There has been recent work on exploring both adaptive link level and system level strategies for wireless communication over fading channels. Reference [6] gives an overview of the coding and modulation schemes under power constraints. Reference [7] analyzes the information-theoretic limits of fading channels from a single-user system point of view. A packet traffic scheduling problem is described by Collins and Cruz in [8], with a delay constraint, where an optimal policy is found to minimize the average transmit power.

All of the above works discuss the energy conservation control problem by assuming there is an infinite supply of information to be transferred. In this work, however, we analyze the power control problem in transferring a finite-size data file under a total energy constraint as well as delay constraints. In this paper the channel model considered is that of a slow fading channel. Two performance metrics are taken into consideration:

- **Probability of Success**: the probability of successfully transferring the entire file within the given energy budget.
- **File Transfer Delay**: the total time spent in transferring the file.

In this work, to simplify the problem, we allow only two transmit power levels: a constant power or zero power (i.e., no transmission). A randomized power control scheme is discussed, i.e., the transmitter can select either power level with a certain probability. The power control problem is (given the total energy and the file size) to find the optimal policy that maximizes the probability of success under either an average delay constraint or a strict delay constraint. The paper is organized as follows. Section 2 reviews the channel model. The communication system model is described in section 3. In section 4 and 5, we present results for the average delay constrained problem and the strict delay constrained problem, respectively.

2 Finite State Markov Channel (FSMC) Model

In many wireless communication situations, changes in the propagation environment occur on a very slow time scale with respect to the signaling rate. Thus, in high rate packet data systems, it is reasonable to assume that the channel symbols in one packet experience the same "channel state". Thus the channel state is random but constant over one packet duration. For such a slow fading channel, we assume that, the received signal-to-noise ratio (SNR) remains at a constant level for the entire packet duration. Therefore, in each packet duration $\triangle t$, the channel can be modeled as an AWGN channel, i.e.,

$$y = \sqrt{h}\,x + n \tag{1}$$

where x and y are input and output signals respectively. n is the AWGN noise, and h is the fading attenuation factor. For the Rayleigh fading channel, h is distributed exponentially with probability density function $f_h(x) = \frac{1}{\bar{h}} \exp\{-\frac{x}{\bar{h}}\}$ for $x \geq 0$. If the power of the background noise is normalized to 1, h characterizes the average received SNR. In this work, we assume that the packet rate is constant and the packet duration $\triangle t$ is given as $\triangle t \triangleq \frac{1}{R_p}$, where R_p is the packet rate.

A Markov channel model can be built to characterize the time-varying behavior of the Rayleigh fading channel as follows [9]. Select a sequence of fading thresholds: $0 = h^{(0)} < h^{(1)} < \cdots < h^{(K)} = \infty$, by which we partition h into a finite number of SNR intervals. Then the channel is said to be in state $s^{(k)}$ if $h \in [h^{(k)}, h^{(k+1)})$, $k = 0, 1, \cdots, K-1$. The steady state probabilities of the channel state are given by

$$q_k = \int_{h^{(k)}}^{h^{(k+1)}} f_h(x)\, dx, \quad k = 0, 1, \cdots, K-1 \tag{2}$$

where we assume that the transitions occur only into neighboring (or same) states and the channel stays in each state for a specified packet duration $\triangle t$. As in [9], the transition probabilities are approximated by the ratio of the expected number of level crossings of the state SNR boundary to the average number of blocks per second in that state. They are given by

$$p_{k,k+1} = \frac{\Gamma(h^{(k+1)})}{q_k R_p}, \quad k = 0, \cdots, K-2 \qquad p_{k,k-1} = \frac{\Gamma(h^{(k)})}{q_k R_p}, \quad k = 1, \cdots, K-1. \tag{3}$$

where $p_{i,j}$ is the state transition probability from $s^{(i)}$ to $s^{(j)}$. R_p is the block rate and $\Gamma(\cdot)$ is the expected number of level crossings [10] given as $\Gamma(x) = \sqrt{\frac{2\pi x}{h}}\, f_d \exp\{-\frac{x}{h}\}$, where f_d is the maximum Doppler frequency defined as $f_d = \frac{v}{\lambda}$ with v being the speed of the vehicle and λ being the wavelength. This fading channel model is verified to be precise when the fading process is slow (i.e., the transition probability $p_{i,j} \ll 1$) [9].

Given a channel state $s^{(k)}$, considering a fixed size data packet, we assume that the probability of correct reception of the packet is μ_k and it is monotonically increasing in the channel states. For a given modulation scheme and packet size, we evaluate μ_k as a function of the average SNR in that state. We now present in the following how μ_k is evaluated for the example of BPSK modulation. Let B denote the packet size in the number of bits. Given the received SNR h, the probability of correct reception of one packet is given as $\left[1 - Q(\sqrt{2h})\right]^B$, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}}\, dt$. In the above we have assumed that a packet is successfully transferred only if all the bits in it are correctly received since no channel coding is employed here. Hence, in state $s^{(k)}$, the probability of correct reception of one packet can be obtained by averaging over the corresponding channel state interval[1]:

$$\mu_k = \frac{1}{q_k} \int_{h^{(k)}}^{h^{(k+1)}} \left[1 - Q(\sqrt{2h})\right]^B f_h(x)\, dx \quad k = 0, \cdots, K-1 \tag{4}$$

3 System Model

The system model is shown in Figure 1. The size of the data file is L packets and the total energy budget is given as E Joules. Consider a time-slotted system where one time slot corresponds to the constant packet duration ($\triangle t$). By storing all the L packets in a buffer temporarily, the transmitter tries to transfer the data packets consecutively through a wireless link. In each time slot, $[t_i, t_{i+1})$, $i = 0, 1, \cdots$, one packet can be transmitted with a certain transmit power P_i. As far as the ARQ (Automatic Repeat Request) mechanism [11] is concerned, the simplest "stop and wait" scheme is used here to overcome transmission errors. We also assume there is an instantaneous error-free feedback channel that informs the transmitter of the reception status ("success" or "failure"). Upon the immediate notification of a packet reception failure, the mobile will arrange to re-send the same packet until it is correctly received. Note that there may be a feedback delay in notifying the transmitter of the CSI. In Figure 1, $D_C = 0$ denotes that the transmitter has the perfect CSI of the current slot before transmission. $D_C = n$ implies that the transmitter only knows the CSI from n slots before. Further, let $D_C = \infty$ denote the case that the transmitter has no CSI. In this work, we will show that the system performance can be improved significantly by exploiting the CSI in comparison with the transmission strategy without using CSI. Hence, we will focus on the analysis in the scenarios with $D_C < \infty$ in the remainder of this paper. Before we formulate the power control problem, we present the following preliminaries

[1] Here, we assume that the total transmission time is long enough such that in any channel state, the SNR process $h(t)$ is ergodic.

Definition 1 *Let E_1 denote the event that all L packets are successfully transmitted within the energy budget of E Joules and let E_0 denote the event that some of the L packets are not transferred correctly when the communication is terminated, i.e.,*

$$E_1 \triangleq \text{"File transfer success"} \quad \text{and} \quad E_0 \triangleq \text{"File transfer failure"}.$$

Note that event E_0 occurs when there is neither enough energy nor enough time to transmit the residual packets in the buffer. We assume the communication will be terminated once we find that it is impossible to transmit all the residual packets with the residual energy or in the remaining time. Hence, we define the communication window as follows:

Definition 2 *The communication window of the file transfer (denoted by N) is defined as the time duration from the starting time point to the time point when either event E_1 or E_0 occurs.*

N can be used to characterize the file transfer delay. Note that N is a random variable which varies with the channel as well as the energy constraint. At each time instant t_i, $i = 0, 1, \cdots, N-1$, we will determine the transmit power P_i based on the current system state. To simplify the mathematical analysis in this work, we only admit two power levels for the transmit power: $P_i = P$ where P is a fixed power; and P_i=0. If $P_i = P$, the transmitter attempts to transmit with power P. If $P_i = 0$, the transmitter remains silent for one packet slot. The transmission action is defined as follows:

Definition 3 *At time instant t_i, for $i = 0, 1, \cdots, N-1$, let "$a_i = \lambda$" ($0 \leq \lambda \leq 1$) denote the following randomized transmission action:*

$$P_i = \begin{cases} P, & \text{with probability } \lambda \\ 0, & \text{with probability } 1-\lambda \end{cases} \tag{5}$$

Further, we assume that the energy consumed for one packet transmission is $\triangle E$ Joules, where $\triangle E = P \cdot \triangle t$. Without loss of generality, we assume the total energy budget E is an integer multiple of $\triangle E$, i.e., $E = M_E \cdot \triangle E$ for some integer M_E. Then we define the system state space as below.

Definition 4 *The system state space* $\mathcal{S} \triangleq \{\mathbf{v} = (s, e, l)\}$*, where* $s \in \{s^{(0)}, s^{(1)}, \cdots, s^{(K-1)}\}$ *is the CSI,* $e \in \{0, \triangle E, \cdots, M_E \triangle E\}$ *is the residual energy in Joules, and* $l \in \{0, 1, \cdots, L\}$ *is the number of residual packets in the buffer.*

The initial state $\mathbf{v}_0 = (s, E, L)$. Note that the CSI "s" is the obtained channel state information from n slots before if $D_C = n$. If applying a transmission action, the state transition probabilities between any two states $\mathbf{v} = (s^{(k)}, e, l)$ and $\mathbf{w} = (s^{(k')}, e', l')$ are given by $p_{\mathbf{vw}}(a) = \text{Prob}\{\mathbf{w}|\mathbf{v}, a\}$. It follows

$$p_{\mathbf{vw}}(a = \lambda) = \begin{cases} (1-\lambda) \cdot p_{kk'}, & e' = e, l' = l \\ \lambda \cdot p_{k,k'} \tilde{\mu}_k, & e' = e - \triangle E,\ l' = l - 1,\ \text{for } e > \triangle E \text{ and } l > 0 \\ \lambda \cdot p_{k,k'}(1 - \tilde{\mu}_k), & e' = e - \triangle E,\ l' = l,\ \text{for } e > \triangle E \text{ and } l > 0 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where all the probabilities $p_{k,k'}$ are the transition probabilities given by equation (3) and $\tilde{\mu}_k$ is probability of correct reception of one packet when the CSI is $s^{(k)}$. Since the CSI depends on D_C, we can write $\tilde{\mu}_k = \mu_k$ for $D_C = 0$, or $\tilde{\mu}_k = \sum_{j=0}^{K-1} p_{kj}^{(n)} \mu_j$ for $D_C = n > 0$, where $p_{k,j}^{(n)}$ is the n-step channel state transition probability and μ_k being the correct probability of packet reception in (4). The n-step transition probability of the channel states can be derived from the one-step transition probabilities given in equation (3).

Definition 5 *The sequence of transmission actions in the communication window is called a transmission policy* π*, i.e.,* $\pi = (a_0, a_1, \cdots, a_i \cdots, a_{N-1})$.

Note that if the probability λ is either 1 or 0 for all the actions in a policy, this policy is called a *deterministic* policy. In other words, only two actions are allowed: $a_i = 1$ which denotes $P_i = P$ with probability 1, and $a_i = 0$ which represents $P_i = 0$ with probability 1. Further, if all the actions are independent of the system time t_i, the transmission policy is called a *stationary* policy. Given a transmission policy π, the probability of successful file transfer is denoted as

$$R(\pi) = \Pr\{E_1 \,|\, \pi\} \tag{7}$$

For any π, it follows that $R(\pi) = 0$, if $M_E < L$. To avoid this degenerate situation, in this paper, we assume that at the initial time t_0, there is enough energy to transfer the entire file, i.e., $M_E \geq L$.

4 Power Control with Average Delay Constraints

The goal is to find the optimal power control policy that maximizes the probability of successful file transfer while guaranteeing that the average communication window is less than a pre-specified value. To formulate this problem mathematically, we need to define the following sets.

Definition 6 *At initial time t_0, the system is on any channel state with residual energy E Joules and L remaining packets in the buffer. The initial state set is given as $U_0 = \{(s, E, L) \,|\, s = s^{(k)},\ k = 0, 1, \cdots, K-1\}$. At time t_N, the communication is terminated. Let U_e denote the set of final states that correspond to the situation that there is not enough energy for transmitting the residual packets, i.e., $U_e = \{(s, e, l) \,|\, e < l\triangle E\,\}$. Let U_l denote the set of final states that correspond to the situation that all the packets have been transmitted correctly, i.e., $U_l = \{(s, e, l) \,|\, l = 0\,\}$.*

Note that when $D_C = n > 0$, the transmission starts at time t_0 and the CSI at time t_{-n} is assumed to be known by the transmitter. If the system state at t_N belongs to U_l, the file transfer has been completed successfully. If the final state is in U_e, then the file transfer has failed. Hence, it follows that the events E_1 and E_0 can be represented in terms of the final state $\mathbf{v}_N$ as $E_1 \Longleftrightarrow \mathbf{v}_N \in U_l$ and $E_0 \Longleftrightarrow \mathbf{v}_N \in U_e$. Given a transmission policy π, the probability of successful file transfer is given as

$$R(\pi) = \mathbf{E}_{U_0}\{R(\pi, \mathbf{v}_0)\} \tag{8}$$

where $R(\pi, \mathbf{v}_0) = \text{Prob}\,\{\mathbf{v}_N \in U_l\}$ if starting from $\mathbf{v}_0$ and $\mathbf{E}_{U_0}$ denotes the expectation over the set of initial states U_0. The average value of the communication window N is given as

$$N_{avg}(\pi) = \mathbf{E}_{U_0}\{N_{avg}(\pi, \mathbf{v}_0)\} \tag{9}$$

where $N_{avg}(\pi, \mathbf{v}_0) = \mathbf{E}_{\mathbf{v}}^{\pi}\{N(\pi, \mathbf{v}_0)\}$. $\mathbf{E}_{\mathbf{v}}^{\pi}$ is the expectation over the system state $\mathbf{v}$ when using policy π, and $N(\pi, \mathbf{v}_0)$ denotes the corresponding window size.

Note that $N_{avg}(\pi)$ and $R(\pi)$ are two conflicting performance metrics. To increase the probability of success, the transmitter will wait for some high SNR channel states for transmission, which lowers the retransmission probability at the expense of increased delay. On the contrary, to decrease the delay, the transmitter can transmit as soon as possible regardless of the channel states, which certainly lowers the probability of success. Then we define two extreme policies as follows.

Definition 7 *For a finite channel state model with $s^{(k)}$, $k = 0, 1, \cdots, K-1$, assume $s^{(K-1)}$ is the best state, i.e., $\mu_{K-1} > \mu_k$, $k = 0, 1, \cdots, K-2$ and the steady state probability of every state $q_k > 0$, $k = 0, 1, \cdots, K-1$. Policy π_s is defined as*

$$a(\mathbf{v} = (s, e)) = \begin{cases} \lambda, & s = s^{(k^*)} \text{ and } \mathbf{v} \notin U_l \cup U_e; \\ 0, & \text{otherwise}. \end{cases} \tag{10}$$

where $\lambda > 0$. $k^* = K-1$ for $D_C = 0$, or $k^* = \arg\max_k \{p_{k,K-1}^{(n)}\}$ for $D_C = n > 0$ with $p_{k,K-1}^{(n)}$ being the n-step channel state transition probability.

When $D_C = 0$, π_s is such a policy that the transmitter chooses to transmit with power P only on the best state; otherwise, the transmitter remains silent and waits. For $D_C = n > 0$, π_s is such a policy where the transmitter chooses to transmit on the state which has the highest transition probability to the best channel state after n successive transitions.

Definition 8 *Policy π_d is defined as*

$$a(\mathbf{v}) = \begin{cases} 1, & \mathbf{v} \notin U_l \cup U_e; \\ 0, & \text{otherwise}. \end{cases} \tag{11}$$

π_d is the policy where the transmitter chooses to transmit on every channel state. We now state the following claim.

Claim 1

$$\pi_s = \arg\max_{\pi} R(\pi); \quad \textit{and} \quad \pi_d = \arg\min_{\pi} N_{avg}(\pi). \tag{12}$$

The above claim is fairly intuitive and straightforward to prove and a formal proof is omitted here. We instead focus on the qualitative implications of the Claim in the following. Note that the maximization in the 1st equation in Claim 1 has no delay constraint and the policy π_s waits for either the best channel state (when $D_c = 0$) or the best CSI k^* (when $D_C = n > 0$). As a result, the resulting $R(\pi_s)$ is independent of the value of probability $\lambda > 0$. Instead, λ only affects the communication time for completion of the file transfer. Considering the 2nd equation, transmitting everywhere is the fastest strategy to transmit all the packets or to exhaust all the energy. Hence π_d results in the smallest average communication window. These two policies give two extreme solutions in the tradeoff between the probability of success and the file transfer time. In practice, for some delay sensitive services, we would like to maximize the probability of success while keeping the average delay below an acceptable range. The mathematical formulation of the average delay constrained problem is given as

Problem A: Given a certain $D_C < \infty$,

$$\max_{\pi} R(\pi) \quad \text{subject to} \quad N_{avg}(\pi) \leq N_D \tag{A}$$

The delay constraint N_D has to satisfy $N_D \geq N_{avg}(\pi_d)$. Otherwise, Problem A has no solution. In order to solve Problem A, we will take the approach of solving its Lagrangian dual problem (see [12], p. 176). The Lagrangian dual problem can be written as:

Problem A'

$$\min_{\beta \geq 0} \left\{ \max_{\pi} \Big[R(\pi) - \beta N_{avg}(\pi) \Big] + \beta N_D \right\} \tag{13}$$

Theorem 1 *Problem A' has the same solution as Problem A.*

The proof is given in Appendix A. In addition, the following corollary can be inferred to illustrate qualitative properties of the optimal solution.

Corollary 1 *Let π^* denote a solution of Problem A. We have*

$$R(\pi^*) = \begin{cases} 0, & N_D < D_1; \\ R_1 + \frac{R_2 - R_1}{D_2 - D_1}(N_D - D_1), & D_1 \leq N_D < D_2 \\ R_2, & N_D \geq D_2 \end{cases} \quad (14)$$

where $D_1 = N_{avg}(\pi_d)$, $D_2 = N_{avg}(\pi_s)$, $R_1 = R(\pi_d)$ and $R_2 = R(\pi_s)$. Further the delay constraint shall be met with equality, i.e., $N_{avg}(\pi^*) = N_D$.

The above corollary shows that the probability of success increases as the delay constraint is relaxed and it is proved in Appendix B.

Note that Problem A' can be formulated into a constrained Markov decision problem (MDP) and solved by an infinite horizon dynamic programming (DP) approach [13,14] (Appendix C). The achieved optimal transmission policy is stationary and is only a function of the channel state, the residual energy level and the number of remaining packets in the buffer.

4.1 Numerical Results

In real file transfer systems, deterministic policies are generally employed. As an illustration, we use deterministic policies in the following numerical examples (i.e., $a = 1$ or $a = 0$). Consider a FSMC model with $K = 8$ channel states by partitioning the channel into SNR intervals with identical steady state probabilities. The corresponding SNR intervals and the probability of correct reception of one packet are shown in Table 1. Assume the transmission bit rate is 2 Mbps, the file size is 10 Kbits, and the frame size is 500 bits. Then we have $\triangle t$=0.25 ms and L=20. The energy budget is assumed to be M_E=50 and the maximum Doppler frequency f_d is 75 Hz.

Assume the transmitter knows the perfect CSI ($D_C = 0$) and the average delay constraint N_D=45 slots. Figure 2(a) and Figure 2(b) show 2 slices of the optimal policy when $l = 18$ and $l = 5$, respectively. The shaded regions signify $a = 1$ (i.e., to transmit one packet with probability 1) and the white regions represent $a = 0$ (i.e., to remain silent with probability 1). Note that when the residual energy is not enough to transmit the residual packets (i.e., the region with $e < l\triangle E$), the communication will be terminated (as shown in figures by $a = 0$ as well). By observing the numerical results, we find that, at any instant, if there is more residual energy or less packets in the buffer, the optimal policy is less selective (i.e., the transmitter can transmit on some low SNR channel states as well). This is because the looser constraints result in a less selective transmission strategy. A similar behavior (not shown here) is observed with $D_C > 0$ when the fading process is slow (i.e., the transition probability of channel state $p_{ij} \ll 1$).

Figure 3 shows the minimum probability of failure $1 - R(\pi^*)$ corresponding to different values of D_C varying with the Doppler frequency f_d. The curve labeled with $D_C = \infty$ corresponds to the channel-independent policy. When the CSI is unavailable, the probability of success is independent of the residual energy and packets. Thus, the "always transmit" policy (i.e., choose $a = 1$ on any state) is used here for the case of $D_C = \infty$. For all other cases with $D_C < \infty$, the optimal policy π^* is channel dependent. It is found that the probability of success increases significantly when D_C decreases.

When $D_C = 0$, it is observed that the maximum probability of success increases monotonically with the Doppler frequency. Since the faster fading (i.e. higher Doppler frequency) causes higher transition probabilities between channel states, it implies that the system may visit the high SNR states more frequently. Though it also increases the probability of visiting low SNR states, the transmitter does not transmit on those states according to the optimal policy shown in Figure 2. Therefore, under constraints on both energy and delay, the increase of transition probability results in a higher probability of success. It suggests that the channel variation can benefit the system when using a channel dependent transmission strategy. However, when $D_C > 0$, it is not always true. This is because there is a tradeoff between exploiting the channel variation and estimating the channel precisely. Since the FSMC model is valid only for slow fading situations, this phenomenon cannot be observed clearly in the given numerical results. We present a more detailed discussion of this tradeoff in Appendix D.

5 Power Control with Strict Delay Constraints

The goal is to find the optimal binary power control policy that maximizes the probability of successful file transfer when the communication window is strictly constrained to be $N \leq N_D$. For any given transmission policy π, $R(\pi) = \text{Prob}\{E_1 \,|\, \pi\} = 0$, if $N_D < L$. To avoid this degenerate case, we assume the delay constraint $N_D \geq L$. The problem can be formally stated as

<u>Problem B</u>: Given a certain $D_C < \infty$,

$$\max_{\pi} R(\pi) \quad \text{subject to} \quad N(\pi) \leq N_D \tag{B}$$

Besides the sets of final states U_l and U_e in Definition 6, we now introduce for this scenario an additional set of final states.

Definition 9 *Let U_t denote the set of final states that correspond to the situation that there is not enough time for transmitting the residual packets, i.e., $U_t = \{\mathbf{v}_i = (s, e, l) \,|\, N_D - i < l\}$, where i is time index.*

We have the relationship between the events E_0, E_1 and the sets of final states as $E_1 \Longleftrightarrow \mathbf{v}_N \in U_l$ and $E_0 \Longleftrightarrow \mathbf{v}_N \in U_e \cup U_t$. Given N_D, the definition of the probability of success is

$$R(\pi^*) = \mathbf{E}_{U_0}\{R(\pi^*, \mathbf{v}_0)\} \tag{15}$$

where $R(\pi, \mathbf{v}_0) = \text{Prob}\,\{\mathbf{v}_N \in U_l \,|\, \pi, \mathbf{v}_0\}$. Problem B can be formulated into a Markov decision problem and solved by a finite horizon DP algorithm (Appendix C). The achieved optimal transmission policy is non-stationary, which means the optimal decision rules depend both on the state information and on the time index i.

5.1 Numerical Results

As an illustration, we consider the same FSMC model considered before (in Table 1), and choose $\triangle t$=0.25ms, f_d=100Hz and N_D=100. For $D_C = 0$ and $N_D = 100$, Figures 4(a) and 4(b) depict the snapshots of optimal policies at time $i = 0$ and $i = 82$. When $i = 0$, the system has a communication window of 100 slots. Thus it can afford to wait for possible future better channel states. Therefore, when $i = 0$, the optimal action is to transmit only when the channel is good enough. However, when $i = 82$ and there are still 18 packets in the buffer, the optimal policy stipulates transmission everywhere in order to try to send the residual 18 packets during the remaining 18 time slots. In other words, the optimal policy gets less selective (almost desperate in this example) as the residual communication window shrinks.

Figure 5 shows the minimum probability of failure $1-R(\pi^*)$ varying with f_d. The curve labeled with $D_C = \infty$ corresponds to the case using the channel independent policy. When $N_D = 100$, there is not much difference between $D_C = 0$, 1 and 2. When $N_D = 500$, it is apparent that the probability of success reaches its highest value with $D_C = 0$. For the same reasons discussed in the previous section, $R(\pi^*)$ for $D_C = 0$ is found to be increasing as f_d increases.

6 Conclusion

In this paper, we considered a finite-size data file transfer problem over a fading channel. Randomized binary power control policies were employed with a total energy budget. The goal was to maximize the probability of successfully transferring the entire file under either an average delay constraint or a strict delay constraint. The results show that exploiting the channel state information opportunistically can substantially increase the probability of successful file transfer. Extensions to multilevel (continuous) power control yield qualitatively similar results but a detailed analysis and description is a topic of future study.

A Proof of Theorem 1

If $R(\pi)$ is a concave function of the policy π and $N_{avg}(\pi)$ is a convex function of π, then by the strong duality theorem (see [12], p. 183), Problem A' will have the same solution as Problem A. Therefore, the remainder of this proof is to show the concavity of $R(\pi)$ and the convexity of $N_{avg}(\pi)$.

We first present the following preliminaries.

Definition 10 *Given a policy π, assume the transmitter is on state $\mathbf{v}_i$ at time t_i. Then define the residual communication window M as the time duration from t_i to the point in time when either event E_1 or E_0 occurs.*

Lemma 1 *For $\forall\pi$ and any state $\mathbf{v}_i$ at time i, we have*

$$R(\pi, \mathbf{v}_i) = \sum_{\mathbf{v}_{i+1}} p_{\mathbf{v}_i\mathbf{v}_{i+1}}(a_i) R(\pi, \mathbf{v}_{i+1}); \tag{16}$$

$$R(\pi, \mathbf{u}) = 0 \quad \forall\, \mathbf{u} \in U_e; \quad R(\pi, \mathbf{u}) = 1 \quad \forall\, \mathbf{u} \in U_l; \tag{17}$$

Let $\Pr(M = m \,|\, \pi, \mathbf{v}_i)$ denote the distribution of the residual communication window. Then it follows that

$$\Pr(M = m \,|\, \pi, \mathbf{v}_i) = \sum_{\mathbf{v}_{i+1}} p_{\mathbf{v}_i\mathbf{v}_{i+1}}(a_i) \Pr(M = m-1 \,|\, \pi, \mathbf{v}_{i+1}); \tag{18}$$

$$\Pr(M = 0 \,|\, \pi, \mathbf{u}) = 1 \quad \forall\, \mathbf{u} \in U_e \cup U_l; \quad R(M = 0 \,|\, \pi, \mathbf{u}) = 0 \quad \forall\, \mathbf{u} \notin U_e \cup U_l; \tag{19}$$

Proof: Consider events A = $\{\mathbf{v}_N \in U_l \,|\, \mathbf{v}_i\}$ and B=$\{\ \mathbf{v}_N \in U_l \,|\, \mathbf{v}_{i+1}\}$. It follows that

$$R(\pi, \mathbf{v}_i) = \sum_B \text{Prob}\{A|B\} \cdot \text{Prob}\{B\} = \sum_{\mathbf{v}_{i+1}} \text{Prob}\{A|B\} R(\pi, \mathbf{v}_{i+1}) \tag{20}$$

Prob$\{A|B\}$ is equal to the probability of transition from $\mathbf{v}_i$ to $\mathbf{v}_{i+1}$ under policy π, i.e., action a_i. Thus Prob$\{A|B\}$ = $p_{\mathbf{v}_i\mathbf{v}_{i+1}}(a_i)$ and equation (16) follows. Equation (17) follows trivially from the definitions of the sets U_e and U_l, respectively. Equations (18) and (19) can be proved by following similar arguments. $\square$

Let π_1 and π_2 be two policies such that on each state $\mathbf{v}$, $a(\mathbf{v}) = \lambda^{(1)}(\mathbf{v})$ and $a(\mathbf{v}) = \lambda^{(2)}(\mathbf{v})$ respectively, i.e., $\pi_1 = (\lambda_0^{(1)}, \lambda_1^{(1)}, \cdots)$ and $\pi_2 = (\lambda_0^{(2)}, \lambda_1^{(2)}, \cdots)$. Then we define a new policy π_0 such that $a_i = \gamma\lambda_i^{(1)} + (1-\gamma)\lambda_i^{(2)}$ for any $\gamma \in [0, 1]$. Consider any $\mathbf{v}_0 \in U_0$. According to Lemma 1,

$R(\pi_0, \mathbf{v}_0)$ can be expanded as follows.

$$R(\pi_0, \mathbf{v}_0) = \sum_{\mathbf{v}_1} p_{\mathbf{v}_0\mathbf{v}_1}(a_0) \sum_{\mathbf{v}_2} p_{\mathbf{v}_1\mathbf{v}_2}(a_1) \cdots \sum_{\mathbf{v}_N} p_{\mathbf{v}_{N-1}\mathbf{v}_N}(a_{N-1}) R(\pi_0, \mathbf{v}_N) \tag{21}$$

where $\mathbf{v}_N$ is assumed to be in final state set $U_e \cup U_l$. Equation (17) shows that $R(\pi_0, \mathbf{v}_N)$ depends only on the final state $\mathbf{v}_N$ and does not depend on the policy π_0, i.e., $R(\pi_0, \mathbf{v}_N) = R(\pi_1, \mathbf{v}_N) = R(\pi_2, \mathbf{v}_N)$. Further, observing that the transition probability $p_{\mathbf{vw}}(a_i)$ in equation (6) is linear in the action a, it follows that

$$p_{\mathbf{vw}}(a_i) = p_{\mathbf{vw}}(\gamma\lambda_i^{(1)} + (1-\gamma)(\lambda_i^{(2)})) = \gamma p_{\mathbf{vw}}(\lambda_i^{(1)}) + (1-\gamma) p_{\mathbf{vw}}(\lambda_i^{(2)}) \tag{22}$$

Using (22) and replacing $R(\pi_0, \mathbf{v}_N)$ by $R(\pi_1, \mathbf{v}_N)$ and $R(\pi_2, \mathbf{v}_N)$, the probability of success obtained by applying π_0 in (21) can be rewritten as

$$\begin{aligned} R(\pi_0, \mathbf{v}_0) \quad = \quad & \sum_{\mathbf{v}_1} p_{\mathbf{v}_0\mathbf{v}_1}(a_0) \sum_{\mathbf{v}_2} p_{\mathbf{v}_1\mathbf{v}_2}(a_1) \cdots \sum_{\mathbf{v}_N} \Big[\gamma p_{\mathbf{v}_{N-1}\mathbf{v}_N}(\lambda_{N-1}^{(1)}) R(\pi_1, \mathbf{v}_N) \\ & + (1-\gamma) p_{\mathbf{v}_{N-1}\mathbf{v}_N}(\lambda_{N-1}^{(2)}) R(\pi_2, \mathbf{v}_N) \Big] \overset{(*)}{=} \gamma R(\pi_1, \mathbf{v}_0) + (1-\gamma) R(\pi_2, \mathbf{v}_0) \end{aligned} \tag{23}$$

Step (*) is obtained by applying equation (16) backwards from $i = N-1$ to $i = 0$. Therefore, $R(\pi_0) = \gamma R(\pi_1) + (1-\gamma) R(\pi_2)$. Similarly, we have

$$\text{Prob}\{N = m \,|\, \pi_0\} = \gamma \, \text{Prob}\{N = m \,|\, \pi_1\} + (1-\gamma) \, \text{Prob}\{N = m \,|\, \pi_2\}. \tag{24}$$

Then, $N_{avg}(\pi_0) = \gamma N_{avg}(\pi_1) + (1-\gamma) N_{avg}(\pi_2)$. Hence, the concavity of $R(\pi)$ and convexity of $N_{avg}(\pi)$ are proved. In fact, $R(\pi)$ and $N_{avg}(\pi)$ are linear functions of π. □

B Proof of Corollary 1

Define the set G as $G = \{(x, y) : x = N_{avg}(\pi),\ y = R(\pi),\ \forall \pi\}$. Due to the linearity of $R(\pi)$ and $N_{avg}(\pi)$, any point on the line segment joining any two points $(R(\pi_1), N_{avg}(\pi_1))$ and $(R(\pi_2), N_{avg}(\pi_2))$ also belongs to G (as shown in Figure 6) because a corresponding policy π_0 can be found. Thus,

G is convex. From Claim 1, it is clear that the policies π_s and π_d are at the extremes of the set of all policies. As shown in Figure 6, D_1 (=$N_{avg}(\pi_d)$) gives the minimum average delay that can be achieved for all policies. If the average delay constraint N_D is less than D_1, the probability of success is zero because no admissible policy can be found. Further, R_2 (=$R(\pi_s)$) is the maximum probability of success that can be achieved. If N_D is bigger than D_2 (=$N_{avg}(\pi_s)$), π_s is the solution and R_2 can be obtained. When N_D is between D_1 and D_2, the boundary of the set G is linear and is given as

$$R_1 + \frac{R_2 - R_1}{D_2 - D_1}(N_D - D_1) \quad D_1 \leq N_D < D_2 \tag{25}$$

This boundary also gives the maximum achievable probability of success when $D_1 \leq N_D < D_2$. From Figure 6, it is seen that $R(\pi^*)$ is a continuous non-decreasing function of N_D. Therefore, $N_{avg}(\pi^*) = N_D$. □

C Solution Methodology for Problem A' and Problem B

Problem A'

Note that Problem A' can be rewritten into two subproblems as follows.

$$\min_{\beta \geq 0}\{f_p(\beta) + \beta N_D\}, \quad \beta \geq 0 \tag{26}$$

$$f_p(\beta) = \max_{\pi}\{R(\pi) - \beta N_{avg}(\pi)\} \tag{27}$$

The above two problems together form a constrained MDP. In particular the problem in (27) can be solved via an infinite horizon DP algorithm and the problem in (26) can be solved using any standard search technique. The two optimization problems are iteratively solved until convergence [14]. In order to introduce the DP algorithm to solve problem in (27), we need the following preliminaries.

Let us define a function $r(\mathbf{v})$ such that

$$r(\mathbf{v}) = \begin{cases} 1, & \mathbf{v} \in U_l; \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

Theorem 2 *Assuming that the communication window is N under policy π, the probability of successful file transfer from initial state $\mathbf{v}_0$ is*

$$R(\pi, \mathbf{v}_0) = \mathbf{E}_{\mathbf{v}}^{\pi} \left\{ \sum_{i=0}^{N-1} r(\mathbf{v}_i) \right\} \tag{29}$$

Proof: Since the communication window is N, it implies that $\mathbf{v}_N \in U_e$ or $\mathbf{v}_N \in U_l$, and $\mathbf{v}_i \notin U_l$ for $i = 0, 1, \cdots, N-1$. By the definition of $r(\mathbf{v})$, $r(\mathbf{v}_i) = 0$ for $i = 0, 1, \cdots, N-1$. We can now write the expectation in equation (29) (actually the conditional probability of success from initial sate $\mathbf{v}_0$) as a sequential sum of conditional expectations. It follows that

$$\begin{aligned} \mathbf{E}_{\mathbf{v}}^{\pi} \left\{ \sum_{i=0}^{N-1} r(\mathbf{v}_i) \right\} &= r(\mathbf{v}_0) + \sum_{\mathbf{v}_1} \left\{ p_{\mathbf{v}_0\mathbf{v}_1}(a_0) \cdot \mathbf{E}_{\mathbf{v}|\mathbf{v}_1}^{\pi} \left[\sum_{i=1}^{N-1} r(\mathbf{v}_i) \right] \right\} \\ &= \sum_{\mathbf{v}_1} p_{\mathbf{v}_0\mathbf{v}_1}(a_0) \sum_{\mathbf{v}_2} p_{\mathbf{v}_1\mathbf{v}_2}(a_1) \cdots \sum_{\mathbf{v}_N} p_{\mathbf{v}_{N-1}\mathbf{v}_N}(a_{N-1}) r(\mathbf{v}_N) \\ &\overset{(*)}{=} \sum_{\mathbf{v}_1} p_{\mathbf{v}_0\mathbf{v}_1}(a_0) \sum_{\mathbf{v}_2} p_{\mathbf{v}_1\mathbf{v}_2}(a_1) \cdots \sum_{\mathbf{v}_N} p_{\mathbf{v}_{N-1}\mathbf{v}_N}(a_{N-1}) R(\pi, \mathbf{v}_N) = R(\pi, \mathbf{v}_0) \end{aligned}$$

Step (*) follows because $r(\mathbf{v}_N) = R(\pi, \mathbf{v}_N)$ according to the definition of $r(\mathbf{v})$ and from (17). □

To calculate the average value of the communication window N, we define a one stage delay cost as follows.

$$c_d(\mathbf{v}) = \begin{cases} 1, & \mathbf{v} \notin U_l \cup U_e \\ 0, & \mathbf{v} \in U_l \cup U_e \end{cases} \tag{30}$$

The above definition imposes a unit delay cost on every interval until the communication is terminated. Once the final state is in $U_l \cup U_e$, which means the communication has been terminated, then we do not count the delay cost any more. Then we can rewrite

$$N_{avg}(\pi, \mathbf{v}_0) = \mathbf{E}_{\mathbf{v}}^{\pi} \left\{ \sum_{i=0}^{\infty} c_d(\mathbf{v}_i) \right\} \tag{31}$$

From equations (29) and (31), we can define $f_p(\mathbf{v}_0, \beta)$ as

$$f_p(\mathbf{v}_0, \beta) \triangleq \max_{\pi} \left\{ \mathbf{E}_{\mathbf{v}}^{\pi} \Big\{ \sum_{i=0}^{\infty} \Big[r(\mathbf{v}_i) - c_d\left(\mathbf{v}_i\right) \Big] \Big\} \right\} \tag{32}$$

where $\mathbf{v}_0$ denotes the initial state. Note that $f_p(\beta) = \mathbf{E}_{U_0}\{f_p(\beta, \mathbf{v}_0)\}$. The problem defined in (32) can be solved using a standard DP algorithm [13].

Problem B

The probability of success can be written as

$$R(\pi, \mathbf{v}_0) = \mathbf{E}_{\mathbf{v}}^{\pi} \left\{ \sum_{i=0}^{N_D} r(\mathbf{v}_i) \right\} \tag{33}$$

which can be solved by a finite horizon DP algorithm [13]. □

D Tradeoff between Channel Variation and Estimation

In both sections 4 and 5, we pointed out that the maximum probability of success $R(\pi^*)$ monotonically increases with the Doppler frequency f_d when the CSI feedback delay $D_C = 0$. However, when $D_C > 0$, this is not always true. We show this using the following illustrative example. Define only two channel states, $s^{(0)}$ and $s^{(1)}$. Assume that the transition probability between these two states p_{01}, p_{10} are equal, i.e., $p_{01} = p_{10} = \alpha$. Note that the channel variation is characterized by α. The channel fluctuates faster with bigger α. If $\alpha = 0.5$, the CSI is memoryless. Consider a strict delay constrained problem with $E = 50$ units and $L = 20$ packets. Assume the correct probabilities of packet reception are $\mu_0 = 0.2$ and $\mu_1 = 0.8$. Figure 7 shows the probability of failure $1 - R(\pi^*)$ varying with the transition probability α.

When $\alpha = 0$, it means that the channel is invariant. Irrespective of whether the CSI is known or not, the same probability of success $R(\pi^*)$ will be obtained ($D_C = 0, 1, \infty$) because the channel is time invariant. When the perfect channel state information is known ($D_C = 0$), $R(\pi^*)$ monotonically increases with α since the optimal policy exploits the current CSI. However, when $D_C > 0$, there exists a tradeoff between exploiting the CSI and estimating the CSI. For slow fading channels, $R(\pi^*)$ increases as α increases. It implies that it is accurate to estimate the current channel state using the CSI from D_C slots before, when the fading process varies slowly. Thus, the optimal policy can exploit the estimated CSI to achieve higher $R(\pi^*)$. As the channel varies faster, on the contrary, $R(\pi^*)$ decreases as α increases. This is because now it is difficult to exploit the CSI based on the inaccurate estimation due to the fast variation. When $\alpha = 0.5$, the CSI is unpredictable. It is equivalent to the case when the channel is unknown. □

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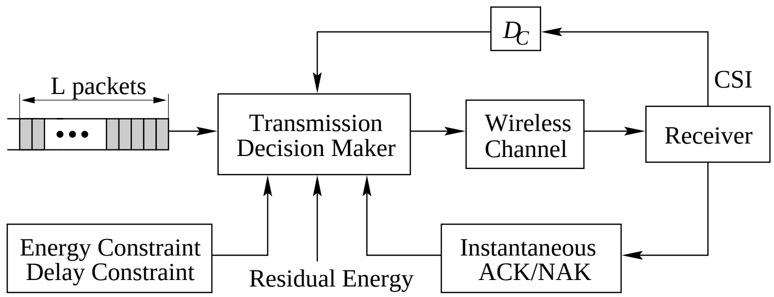
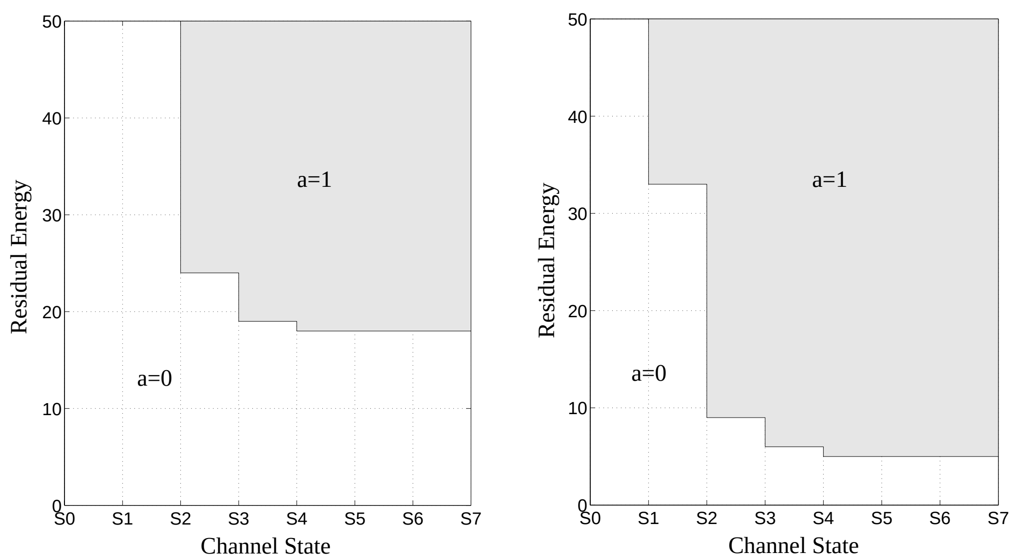


Figure 1: System Diagram

State Index	SNR Interval	Packet Correct Probability μ_k
0	$[-\infty$, 1.26dB)	0
1	[1.26dB, 4.59dB)	0.0017
2	[4.59dB, 6.72dB)	0.2433
3	[6.72dB, 8.41dB)	0.8156
4	[8.41dB, 9.92dB)	0.9839
5	[9.92dB, 11.42dB)	0.9994
6	[11.42dB, 13.18dB)	1.0
7	[13.18dB, ∞)	1.0

Table 1: FSMC states with $\bar{h}$=10dB



(a) l=18

(b) l=5

Figure 2: Optimal Policy with Average Delay Constraints

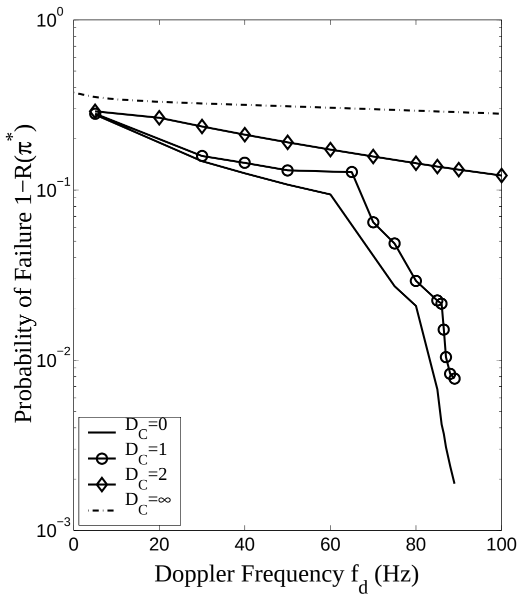
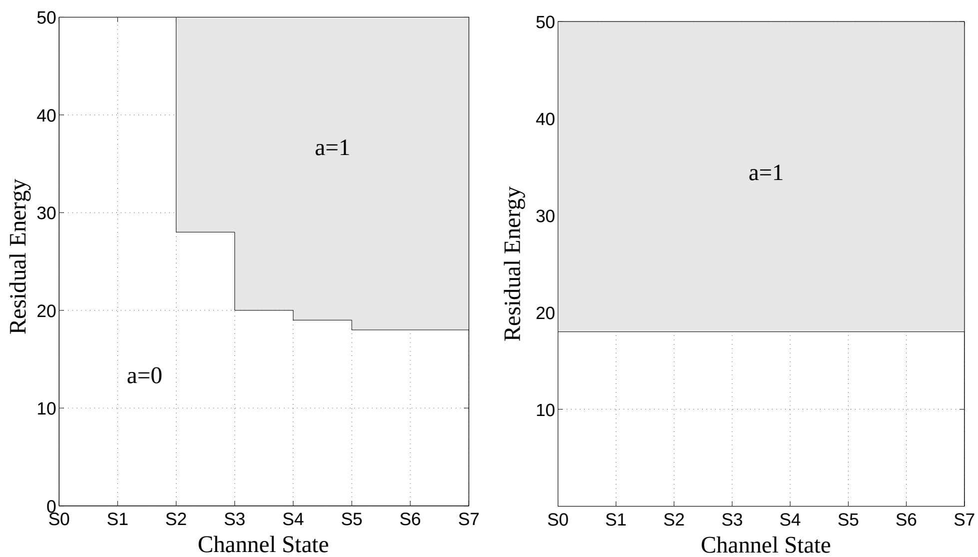


Figure 3: Probability of Success vs. Doppler Frequency



(a) i=0, l=18

(b) i=82, l=18

Figure 4: Optimal Policy with Strict Delay Constraints

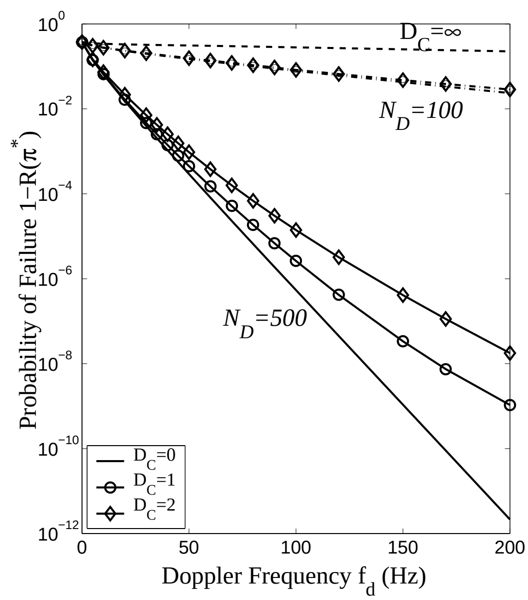


Figure 5: Probability of Success vs. Doppler Frequency

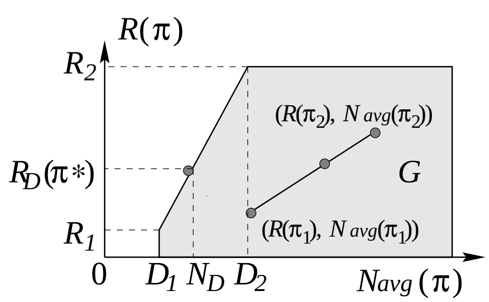


Figure 6: Geometric interpretation of $R(\pi)$ and $N_{avg}(\pi)$

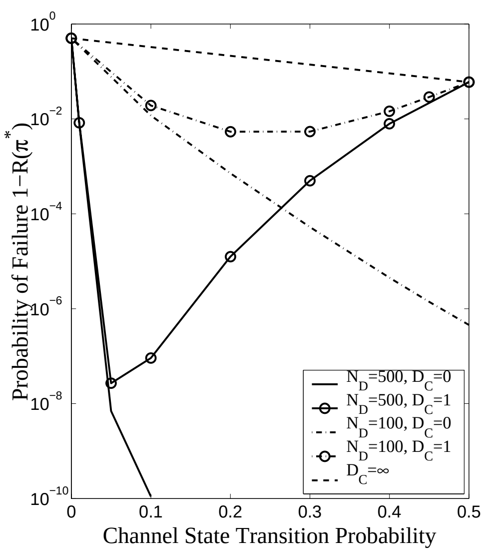


Figure 7: $R(\pi^*)$ vs. α