Prospect Theoretic Analysis of Energy Exchange Among Microgrids

Liang Xiao, *Senior Member, IEEE,* Narayan B. Mandayam, *Fellow, IEEE,* and H. Vincent Poor, *Fellow, IEEE*

***Abstract*—The energy exchange between microgrids (MGs) that are capable of generating power from renewable energy sources in smart grids is investigated. As MGs are autonomous and have control over their energy exchange, prospect theory is a useful tool to provide a user-centric view on MG power trading. More specifically, in this paper, the energy exchange among MGs that are also connected to a power plant as a backup energy supply is formulated as a prospect theory-based static game and Nash equilibria are provided under various scenarios. The impact of user objective weight is evaluated during the outcome evaluation on the performance of the game. Simulation results show that user subjectivity tends to exaggerate selling and buying probabilities when battery levels are high (and low), and thus decreases the overall utility and increases the amount of the energy bought at either low battery levels or low MG selling prices. Conditions on the pricing system to ensure that the energy exchange system is not impacted by the subjective view of MGs are also provided.**

***Index Terms*—Energy exchange, game theory, microgrids (MGs), prospect theory (PT).**

I. Introduction

MICROGRIDS (MGs) are significant entities in the development of smart grids. A MG consists of energy components including the active loads such as air conditioning, renewable power generators such as solar panels and wind turbines, smart meters and energy storage devices such as batteries [1]. Due to the intermittent production of renewable power and the time varying power demand, the MG energy surplus, either positive (i.e., the MG has extra power) or negative (i.e., the MG needs energy), has been shown to vary over time and MGs [1]. By constituting a local energy trading market where the MGs with extra energy trade with the MGs with insufficient power, MGs reduce their power demand from power plants and consequently reduce the losses due to the long-distance power transportation and conversion in the substation's transformer [2].

Located at the consumers' premises, MGs usually have autonomy and control in their energy consumption and transactions, making game theory a powerful tool to investigate the MG energy performance [2]–[9]. Traditional game theory assumes that all the players in the game are rational and uninfluenced by real-life perception. Consequently, most existing game theoretic studies of smart grids are based on the assumption that MGs make decisions according to their expected utilities. However, this assumption deviates from real-life decision-making and the traditional expected utility theory (EUT) cannot explain the deviations due to end-user subjectivity such as illustrated by Allais's paradox [10]. For example, we consider a two-option experiment with A: to win \$10, and B: to win \$0 and \$20 each with a probability 0.5. Due to the subjective nature of human decision-making, experimental results have shown that most people choose option A, although B leads to a higher expected utility [10], [11].

Therefore, prospect theory (PT) has been proposed to provide a user-centric view to address this issue, which applies a probability weighting effect to transform the objective probabilities into subjective probabilities [10], [11]. This Nobel prize winning theory was proposed to explain the fact that people usually over-weigh the low probability bad outcomes and under-weigh their favorite outcomes with high probabilities, which is ignored by EUT. This theory also explains the framing effect, i.e., players take into account the relative gains regarding a reference point rather than the final asset position, and the fact that losses loom larger than gains. Originally developed for monetary transactions, prospect theory has since then been used in social sciences [12], [13] as well as recently to study influence of end-users in wireless networks [14]–[18].

In this paper, we apply prospect theory to analyze MG energy exchange in smart grids from a user-centric viewpoint. More specifically, we formulate the energy exchange between seller and buyer MGs that are also connected to a power plant as a backup energy supply into a static game. The Prelec's [19] probability weight function is used to model the MG subjectivity in the PT-based energy exchange game. If controlled by a subjective owner, the MG emphasizes the small-chance case that the selling payoff does not compensate for the energy loss at lower battery levels or the buying price is high compared with the energy gain at high battery levels and chooses its action to avoid it.

Manuscript received December 13, 2013; revised May 1, 2014 and July 15, 2014; accepted August 22, 2014. Date of publication September 11, 2014; date of current version December 17, 2014. The work of L. Xiao was supported in part by the National Natural Science Foundation of China (NSFC) under Grant 61271242 and Grant 61001072, in part by the New Century Excellent Talents of Fujian Province University (NCETFJ), and in part by the Fundamental Research Funds for the Central Universities under Grant 2012121028. The work of N. B. Mandayam was supported by the U.S. National Science Foundation through the NeTS Program under Grant 1421961. Paper no. TSG-00913-2013.

L. Xiao is with the Department of Communication Engineering, Xiamen University, Xiamen 361005, China (e-mail: lxiao@xmu.edu.cn).

N. B. Mandayam is with Wireless Information Network Laboratory, Department of Electrical and Computer Engineering, Rutgers University, New Brunswick, NJ 08816 USA (e-mail: narayan@winlab.rutgers.edu).

H. V. Poor is with the Department of Electrical Engineering, Princeton University, Princeton, NJ 08544 USA (e-mail: poor@princeton.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSG.2014.2352335

We present the Nash equilibria (NE) of the game between two connecting MGs that can also trade with the power plant under various scenarios, whose performance depends on the MG states, energy trading prices and the energy gain function of the MGs. We provide conditions for the existence of a unique NE in the game, and evaluate the impact of user subjectivity on the performance including the utility and the energy bought from the power plant. Finally, pricing design criteria to eliminate the user subjective impact for all MG states are provided.

The remainder of the paper is organized as follows. We briefly review some related work in Section II and present the system model in Section III. We formulate the MG energy exchange as a PT-based game in Section IV. We study the NE strategy in the PT-based game with one energy quantization level in Section V and extend the results to a generic energy quantization scenario in Section VI. We next provide simulation results in Section VII. Finally, we conclude the paper in Section VIII.

II. Related Work

In recent years, the energy control of MGs has attracted significant research attention and the autonomy of MG makes game theory a powerful tool to this end [3]–[8], [20]. For instance, the interactions among the loads and sources within a MG have been formulated as a noncooperative game with the load modeled as variable resistances [3]. In [5], the demand and supply in a solar-powered electrical MG was formulated as a Potluck problem [21], where market bidding techniques were applied for the pricing in the electricity market. In addition, in the cooperative game model for the power system consisting of several renewable energy sources with different properties, the energy quantities for the generators are optimized for both system operating costs and emissions [4]. In [8], the risky and firm power contract offering is introduced for the wind power producers to trade risky power in the forward market.

Game theory has been used to model the MG energy exchange. For example, an MG coalition algorithm has been proposed in [6] to match buyer and seller MGs, which has been shown to reduce the loss during the distribution of power between the MG sellers and buyers in the same coalition. Our work is focused on the interactions of MGs after an MG coalition or pairing decision is made. Matamoros *et al*. [7] analyzed the energy trading between two MGs that are disconnected from the power plant, and provided a centralized solution to minimize the total cost as well as a distributed strategy which converges after enough iterations.

While prospect theory was originally developed to model and explain decision-making in monetary transactions, it has recently found widespread use in many contexts: social sciences [12], [13], [22], communication networks [14], [15], [17], [23]–[25] and smart energy management [20]. In this paper, we introduce user subjectivity and extend the discussion to the multiple-MG case for a more generalized and common case where the MGs are also connected to a power plant.

III. System Model

In this paper, we consider a smart grid consisting of N MGs, denoted as MG_x, with $1 \leq x \leq N$, which are connected to a power plant[1] (PP) in the main grid via a substation. Equipped with one or several active loads, renewable power generators, batteries and a local controller, each MG can not only consume power, but can also generate energy and deliver its extra energy to the other MGs or the power plant. Located at a consumer's premises, each MG has autonomy and holds a subjective view when making decisions in energy transactions. We apply the Prelec's [19] probability weight function to model user subjectivity, as will be presented in detail in Section IV.

In a slotted-time energy market where MGs exchange energy with other MGs and the power plant, the amount of the energy in the trade is quantized into several levels. We consider a static game and denote the action of MG_x by a_x, with $M_s \leq a_x \leq M$, which is the amount of energy that MG_x buys, where M (or the absolute value of the negative integer M_s) is the maximum amount of energy that an MG can buy (or sell) in each trade. For example, MG_x does not trade with $a_x = 0$; it buys in the local power market with $a_x > 0$, and sells its extra energy with $a_x < 0$.

Considering the time varying power generation by the renewable power generator in the MG and power demand by the consumers, without loss of generality, we denote the state of MG_x by s_x, with $0 \leq s_x \leq S$, which represents the battery level of the MG, where S is the maximum battery storage and $s_x = 0$ indicates zero power. The maximum amount of the energy exchange (M) and the maximum battery capability (S) characterize the capabilities of the MGs.

In the local energy exchange, a buyer MG pays ρ^+ per unit energy (i.e., the MG buying price) and a seller MG obtains ρ^+ per unit energy (i.e., the MG selling price). On the other hand, if trading with the opponent MG is infeasible, a MG trades with the power plant: pays ρ_p^+ (i.e., the power plant selling price) per unit energy to the power plant and obtains ρ_p^- per unit as the power plant buying price. All the prices are positive by nature.

In the energy exchange process, if $\rho^+ = \rho^-$, all the payment of the buyer MG is sent to the seller; the local energy market pays extra credits to the selling MG to promote MG cooperation, if $\rho^+ < \rho^-$; otherwise if $\rho^+ > \rho^-$, the local market takes maintenance fee from the trade.

Note that the energy trading depends on the MG battery levels, energy prices, energy production and demand forecast. Let $g(s)$ denote the energy gain function of the MG, representing the benefit that a MG obtains at state s. The energy gain function $g(s)$ is determined by the specific MG system. Without loss of generality, $g(s)$ is an increasing function of s, with $g(s = 0) = 0$. However, the rising rate of $g(s)$ decreases with s. For instance, a buyer MG at state $s = 1$ benefits much more from receiving unit energy than the buyer at high battery levels with $s = S$.

The overall utility that an MG obtains depends on the energy trading price, trading quantity, the MG state, the action of its opponent MG and the energy gain function $g(s)$. Let $u(m, s)$ denote the utility that a MG at state s obtains by exchanging

[1] Note that the power plant is sometimes a generator hidden behind some balancing party, the ISO balancing services, aggregator, supplier, etc. The proposed scheme can be easily extended to any mechanism between the MGs and a backup unit.

TABLE I
SUMMARY OF SYMBOLS AND NOTATIONS

$M/\lvert M_s\rvert$	Max amount of buying/selling energy in a trade
S	Max amount of energy that can be stored in an MG
a_x	Action of X
s_x	State of X (battery level)
$\rho^{+/-}$	MG selling/buying price
$\rho_p^{+/-}$	PP selling/buying price
$g(s)$	Energy gain of an MG at state s
$\Delta(s)$	Benefits to an MG at s when receiving unit energy
$w_x(p)$	subjective probability weight function of X
α_x	objective weight of X
$C_{xy}^{+/-}(s,m)$	Payoff to X buying/selling m units energy from/to Y
$C_{xp}^{+/-}(s,m)$	Payoff to X buying/selling m units energy from/to PP
$u_x(m,n)$	Instantaneous payoff to X
Ξ	Payoff matrix to X
Θ	Payoff matrix to Y
$U_x^{EUT/PT}$	Average objective/subjective utility to X

m units of energy, which can be written as

$$u(m,s) = \begin{cases} g(s+m) - g(s) - m\rho^{+}, & \text{buy } m \text{ units from MG} \\ g(s+m) - g(s) - m\rho_p^{+}, & \text{buy } m \text{ units from PP} \\ g(s-m) - g(s) + m\rho^{-}, & \text{sell } m \text{ units to MG} \\ g(s-m) - g(s) + m\rho_p^{-}, & \text{sell } m \text{ units to PP.} \end{cases} \tag{1}$$

Clearly, a MG obtains zero payoff during the trading moment if it does not trade with $m = 0$. We will investigate dynamic trading processes in which an MG loses from a reduced battery level over time in the future.

For simplicity, we assume constant pricing in the energy exchange with $\rho^+ < \rho_p^+$ and $\rho^- > \rho_p^-$, i.e., the local market provides a lower selling price and a higher buying price than the power plant to encourage the local energy exchange. Equation (1) indicates that a MG obtains a higher utility by trading with a MG instead of the power plant. This condition is assumed in this paper. Consequently, each MG prefers to exchange energy with the pairing MG to its own interests and trades with the power plant only if its pairing MG is unavailable. For ease of reference, we summarize the commonly used notation in Table I.

IV. PT-Based MG Energy Exchange Game

Prospect theory has been developed to provide models and explanations for the decision-making of human beings (when faced with uncertainty) that is shown to deviate from the assumptions of expected utility theory, specifically in relation to behaviors such as risk seeking, loss aversion and the nonlinear and nonuniform weighting of gains and losses. Therefore, we formulate the energy exchange among subjective MGs and the power plant as a noncooperative static game and apply prospect theory to provide a user-centric view of decision making of MGs in trading under uncertainty.

Without loss of generality, we consider the energy exchange for MG_x and MG_y, with $1 \le x \ne y \le N$. Note that this game consists of two players, MG_x and MG_y. In a smart grid consisting of more than two MGs, we first apply the MG coalition or pairing algorithm such as [6] to group them into several MG-pairs according to locations and energy consumption and generation histories of the MGs. In this paper, we focus on the energy exchange within each MG pair.

The actions taken by MG_x and MG_y, denoted with a_x and a_y respectively, represent the amounts of energy in the energy trading. More specifically, a positive action represents the amount of energy that the MG buys; the MG does not trade with $a = 0$; and the MG sells $-a$ units of energy if $a < 0$. The MG can neither sell more energy than its saving nor can it buy more than its battery space allowed, i.e., $S \ge a_x + s_x > 0$. The feasible action set for MG_x is $A_x \triangleq \{M_s, \ldots, M\}$ if $S - M \ge s_x > -M_s$. In general, the feasible action set for MG_x is given by $\max(M_s, -s_x) \le a_x \le \min(M, S - s_x)$.

Let $u_x(a_x, a_y)$ denote the utility that MG_x at state s_x obtains by taking action a_x toward MG_y with action a_y. As both players prefer trading with each other to the power plant, according to (1), the utility to MG_x is given by

$$u_x(a_x, a_y) = g(s_x + a_x) - g(s_x) + Ct$$
$$Ct = \begin{cases} -a_x\rho^{+}, & \text{if } a_x \ge 0, a_y \le -a_x \\ a_y\rho^{+} - (a_x + a_y)\rho_p^{+}, & \text{if } a_x \ge 0, 0 > a_y > -a_x \\ -a_x\rho_p^{+}, & \text{if } a_x \ge 0, a_y \ge 0 \\ -a_x\rho^{-}, & \text{if } a_x < 0, a_y \ge -a_x \\ a_y\rho^{-} - (a_x + a_y)\rho_p^{-}, & \text{if } a_x < 0, 0 < a_y < -a_x \\ -a_x\rho_p^{-}, & \text{if } a_x < 0, a_y \le 0. \end{cases} \tag{2}$$

According to (2), we present the payoffs to MG_x and MG_y in the pure-strategy static game in Table II for the case with $M = 2$, $M_s = -2$, $S - 2 \ge \max(s_x, s_y)$ and $\min(s_x, s_y) > 2$. For example, if both MG_x and MG_y buy unit energy with $a_x = a_y = 1$, they have to trade with the power plant and obtain the instant payoffs $u_x = g(s_x + 1) - g(s_x) - \rho_p^+$ and $u_y = g(s_y + 1) - g(s_y) - \rho_p^+$, respectively. In this game, if $g(s_x + 2) - g(s_x) > 2\rho^+$, $g(s_x + 2) - g(s_x - 2) > 2\rho^+ - 2\rho_p^+$, $g(s_y - 2) - g(s_y) + 2\rho^- > 0$, and $g(s_y - 2) - g(s_y + 2) + 2\rho^- + 2\rho_p^+ > 0$, an MG cannot obtain higher utility by choosing the actions unilateral from ($a_x = 2$, $a_y = -2$), which is an NE in this case. This condition holds if MG_x has a low battery level (s_x) while MG_y has a high battery level (s_y).

As each MG has autonomy and can be controlled by its owner, we apply prospect theory to reflect the subjective view of the MGs in the energy exchange. As users might have uncertainty in their action selections, we use the mixed strategy to take into account this randomness and formulate the one-shot energy exchange of subjective MGs into a mixed-strategy static game. In this game, MG_x and MG_y take actions over the action set A_x according to the probabilities $\mathbf{p}_x \triangleq [p_{x,m}]_{M_s \le m \le M}$, $\mathbf{p}_y \triangleq [p_{y,m}]_{M_s \le m \le M}$, where $p_{x,m} \triangleq \Pr(a_x = m)$ and $p_{y,m} \triangleq \Pr(a_y = m)$.

Let $U_x^{\mathrm{EUT}}(\mathbf{p}_x, \mathbf{p}_y)$ denote the expected utility to MG_x averaged over all the action realizations of the two players following $\mathbf{p}_x$ and $\mathbf{p}_y$. Assuming independent s_x and s_y, we can write the expected utility to MG_x with

$$U_x^{\mathrm{EUT}}\left(\mathbf{p}_x, \mathbf{p}_y\right) = \sum_{m=M_s}^{M}\sum_{n=M_s}^{M} u_x(m,n)\Pr(a_x = m)\Pr(a_y = n) \tag{3}$$

where the instantaneous payoff $u_x(m,n)$ is given by (2).

TABLE II
PAYOFFS TO MG_x AND MG_y IN THE ENERGY EXCHANGE GAME WITH PURE STRATEGY, WHERE EACH ROW CORRESPONDS TO AN ACTION BY MG_x, WITH $M = 2$, $M_s = -2$, $S - 2 \geq \max(s_x, s_y)$, AND $\min(s_x, s_y) > 2$

ax	$ay = -2$	$ay = -1$	$ay = 0$	$ay = 1$	$ay = 2$
-2	$g(sx-2) - g(sx) + 2\rho_p^-$, $g(sy-2) - g(sy) + 2\rho_p^-$	$g(sx-2) - g(sx) + 2\rho_p^-$ $g(sy-1) - g(sy) + \rho_p^-$	$g(sx-2) - g(sx) + 2\rho_p^-$ 0	$g(sx-2) - g(sx) + \rho^- + \rho_p^-$ $g(sy+1) - g(sy) - \rho^+$	$g(sx-2) - g(sx) + 2\rho^-$ $g(sy+2) - g(sy) - 2\rho^+$
-1	$g(sx-1) - g(sx) + \rho_p^-$, $g(sy-2) - g(sy) + 2\rho_p^-$	$g(sx-1) - g(sx) + \rho_p^-$, $g(sy-1) - g(sy) + \rho_p^-$	$g(sx-1) - g(sx) + \rho_p^-$ 0	$g(sx-1) - g(sx) + \rho^-$, $g(sy+1) - g(sy) - \rho^+$	$g(sx-1) - g(sx) + \rho^-$, $g(sy+2) - g(sy) - \rho^+ - \rho_p^+$
0	0, $g(sy-2) - g(sy) + 2\rho_p^-$	0, $g(sy-1) - g(sy) + \rho_p^-$	0, 0	0, $g(sy+1) - g(sy) - \rho_p^+$	0, $g(sy+2) - g(sy) - 2\rho_p^+$
1	$g(sx+1) - g(sx) - \rho^+$, $g(sy-2) - g(sy) + \rho^- + \rho_p^-$	$g(sx+1) - g(sx) - \rho^+$ $g(sy-1) - g(sy) + \rho^-$	$g(sx+1) - g(sx) - \rho_p^+$ 0	$g(sx+1) - g(sx) - \rho_p^+$ $g(sy+1) - g(sy) - \rho_p^+$	$g(sx+1) - g(sx) - \rho_p^+$ $g(sy+2) - g(sy) - 2\rho_p^+$
2	$g(sx+2) - g(sx) - 2\rho^+$, $g(sy-2) - g(sy) + 2\rho^-$	$g(sx+2) - g(sx) - \rho^+ - \rho_p^+$ $g(sy-1) - g(sy) + \rho^-$	$g(sx+2) - g(sx) - 2\rho_p^+$, 0	$g(sx+2) - g(sx) - 2\rho_p^+$ $g(sy+1) - g(sy) - \rho_p^+$	$g(sx+2) - g(sx) - 2\rho_p^+$ $g(sy+2) - g(sy) - 2\rho_p^+$

Instead of relying on the expected utility function in (3), a subjective player in prospect theory makes decisions according to the prospect theory-based utility resulting from a probability weighting function denoted by $w(\cdot)$. Let $U_x^{\text{PT}}(\mathbf{p}_x, \mathbf{p}_y)$ denote the prospect theory-based utility to the subjective player MG_x

$$U_x^{\text{PT}}\left(\mathbf{p}_x, \mathbf{p}_y\right) = \sum_{m=M_s}^{M} \sum_{n=M_s}^{M} u_x(m,n) \Pr(a_x = m) w_x\left(\Pr(a_y = n)\right) \quad (4)$$

where $w_x(\cdot)$ is the weighting function for MG_x. According to Prelec's work [19], we use the weighting function as follows:

$$w_x(p) = \exp\left(-(-\ln p)^{\alpha_x}\right) \quad (5)$$

where the objective weight $\alpha_x \in (0, 1]$ decreases with MG_x's subjective evaluation distortion. With an inverse-S shape, i.e., being concave near 0 and convex near 1, the weighting function reflects the fact that the impact of a change to a subjective MG diminishes with the distance from certainty (i.e., $p = 1$) and impossibility (i.e., $p = 0$). As an extreme case, an objective user has $\alpha = 1$ and $w(p) = p$ and thus $U_x^{\text{PT}}(\mathbf{p}_x, \mathbf{p}_y; \alpha_x = 1) = U_x^{\text{EUT}}(\mathbf{p}_x, \mathbf{p}_y)$.

V. PERFORMANCE OF THE PT-BASED ENERGY EXCHANGE WITH $M = 1$

We first consider the PT-based energy exchange game between MG_x and MG_y with unit trading energy (i.e., $M = 1$ and $M_s = -1$) and deep battery storage (i.e., $S \gg M$). For simplicity of notation, we use in this section $\mathbf{p}_x = [p_x^+, p_x^-]$ and $\mathbf{p}_y = [p_y^+, p_y^-]$ to denote the mixed strategies of MG_x and MG_y, respectively, with the MG_x buying and selling probabilities $p_x^+ = p_{x,1}$, and $p_x^- = p_{x,-1}$. By definition, $p_{x,0} = 1 - p_x^+ - p_x^-$. According to (1), we use $C_{xj}^{+/-}$ to denote the instant payoff to MG_x in different scenarios

$$\begin{cases} C_{xy}^+ = \Delta_{s_x+1} - \rho^+ \\ C_{xp}^+ = \Delta_{s_x+1} - \rho_p^+ \\ C_{xy}^- = -\Delta_{s_x} + \rho^- \\ C_{xp}^- = -\Delta_{s_x} + \rho_p^- \end{cases} \quad (6)$$

where the energy gain difference $\Delta(s) = g(s) - g(s-1)$, $s = 1, \ldots, S$. As an MG obtains zero payoff by taking action $a = 0$, the subjective utility in (4) can be simplified into

$$U_x^{\text{PT}}\left(\mathbf{p}_x, \mathbf{p}_y\right) = p_x^+ \left(C_{xy}^+ w_x\left(p_y^-\right) + C_{xp}^+ w_x\left(1 - p_y^-\right)\right) + p_x^- \left(C_{xy}^- w_x\left(p_y^+\right) + C_{xp}^- w_x\left(1 - p_y^+\right)\right) \quad (7)$$

$$U_y^{\text{PT}}\left(\mathbf{p}_y, \mathbf{p}_x\right) = p_y^+ \left(C_{yx}^+ w_y\left(p_x^-\right) + C_{yp}^+ w_y\left(1 - p_x^-\right)\right) + p_y^- \left(C_{yx}^- w_y\left(p_x^+\right) + C_{yp}^- w_y\left(1 - p_x^+\right)\right). \quad (8)$$

The expected utility to MG_x, denoted with U_x^{EUT}, is a special case of U_x^{PT}

$$U_x^{\text{EUT}}\left(\mathbf{p}_x, \mathbf{p}_y\right) = p_x^+ p_y^- \left(C_{xy}^+ - C_{xp}^+\right) + p_x^- p_y^+ \left(C_{xy}^- - C_{xp}^-\right) + p_x^+ C_{xp}^+ + p_x^- C_{xp}^- \quad (9)$$

$$U_y^{\text{EUT}}\left(\mathbf{p}_y, \mathbf{p}_x\right) = p_y^+ p_x^- \left(C_{yx}^+ - C_{yp}^+\right) + p_y^- p_x^+ \left(C_{yx}^- - C_{yp}^-\right) + p_y^+ C_{yp}^+ + p_y^- C_{yp}^-. \quad (10)$$

According to (7) and (8), the overall expected utility that MG_x and MG_y obtain is given by

$$\begin{aligned} U_{xy}\left(\mathbf{p}_x, \mathbf{p}_y\right) &= U_x^{\text{EUT}}\left(\mathbf{p}_x, \mathbf{p}_y\right) + U_y^{\text{EUT}}\left(\mathbf{p}_y, \mathbf{p}_x\right) \\ &= p_x^+ p_y^- \left(C_{xy}^+ - C_{xp}^+\right) + p_x^- p_y^+ \left(C_{xy}^- - C_{xp}^-\right) \\ &\quad + p_x^+ C_{xp}^+ + p_x^- C_{xp}^- + p_y^+ p_x^- \left(C_{yx}^+ - C_{yp}^+\right) \\ &\quad + p_y^- p_x^+ \left(C_{yx}^- - C_{yp}^-\right) + p_y^+ C_{yp}^+ + p_y^- C_{yp}^-. \end{aligned} \quad (11)$$

As the local energy price is cheaper than the supply from the power plant in this system, MGs reduce their reliance on the power plant, which also decreases the overall carbon emissions. To this end, let $E_p(\mathbf{p}_x, \mathbf{p}_y)$ denote the total energy that MG_x and MG_y buy from the power plant, which is given by

$$E_p\left(\mathbf{p}_x, \mathbf{p}_y\right) = p_x^+\left(1 - p_y^-\right) + p_y^+\left(1 - p_x^-\right). \quad (12)$$

We denote the NE in this case by $\mathbf{p}_x^* = [p_x^{+*}, p_x^{-*}]$ and $\mathbf{p}_y^* = [p_y^{+*}, p_y^{-*}]$, where each MG chooses the action strategy to maximize its own subjective utility function with the other MG choosing the NE strategy. As each strategy in a Nash equilibrium is a best response to all other strategies in that equilibrium, we obtain the NE by

$$\begin{aligned} \mathbf{p}_x^* &= \arg\max_{\mathbf{p}_x} U_x^{\text{PT}}\left(\mathbf{p}_x, \mathbf{p}_y^*\right) \\ \mathbf{p}_y^* &= \arg\max_{\mathbf{p}_y} U_y^{\text{PT}}\left(\mathbf{p}_y, \mathbf{p}_x^*\right) \\ \text{s.t. } & 0 \leq p_x^+, p_x^-, p_y^+, p_y^- \\ & p_x^+ + p_x^- \leq 1, \qquad p_y^+ + p_y^- \leq 1. \end{aligned} \quad (13)$$

Theorem 1: If Condition A1 holds

$$\begin{aligned} A1\colon \min\left(C_{xy}^+, C_{xy}^-, C_{yx}^+, C_{yx}^-\right) > 0, \qquad & C_{yx}^+ > C_{yp}^- \\ C_{yx}^- > C_{yp}^+, \qquad C_{xy}^+ > C_{xp}^-, \qquad & C_{xy}^- > C_{xp}^+ \end{aligned} \quad (14)$$

the PT-based energy exchange game is affected by user subjective effects and has a unique NE given by $\mathbf{p}_x^* = [p_x^{+*}, 1-p_x^{+*}]$ and $\mathbf{p}_y^* = [p_y^{+*}, 1-p_y^{+*}]$, where

$$\left(C_{yx}^+ - C_{yp}^-\right)\exp\left(-\left(-\ln\left(1-p_x^{+*}\right)\right)^{\alpha_y}\right) = \left(C_{yx}^- - C_{yp}^+\right)\exp\left(-\left(-\ln\left(p_x^{+*}\right)\right)^{\alpha_y}\right) \quad (15)$$

$$\left(C_{xy}^+ - C_{xp}^-\right)\exp\left(-\left(-\ln\left(1-p_y^{+*}\right)\right)^{\alpha_x}\right) = \left(C_{xy}^- - C_{xp}^+\right)\exp\left(-\left(-\ln(p_y^{+*})\right)^{\alpha_x}\right). \quad (16)$$

Proof: As $\min(C_{xy}^+, C_{xy}^-, C_{yx}^+, C_{yx}^-) > 0 = u_x(a_x = 0) = u_y(a_y = 0)$, we have $p^*(x, m = 0) = p^*(y, m = 0) = 0$, yielding $p_x^{+*} + p_x^{-*} = 1$ and $p_y^{+*} + p_y^{-*} = 1$. By (8), we define F_y as

$$\begin{aligned} F_y &= U_y^{\mathrm{PT}}\left(\mathbf{p}_y, \mathbf{p}_x^*\right) - \lambda_y\left(p_y^+ + p_y^-\right) \\ &= p_y^+\left(C_{yx}^+ w_y\left(p_x^{-*}\right) + C_{yp}^+ w_y\left(1-p_x^{-*}\right)\right) - \lambda_y\left(p_y^+ + p_y^-\right) \\ &\quad + p_y^-\left(C_{yx}^- w_y\left(p_x^{+*}\right) + C_{yp}^- w_y\left(1-p_x^{+*}\right)\right). \end{aligned} \quad (17)$$

According to the Karush–Kuhn–Tucker (KKT) optimality condition for (13)

$$\begin{cases} \partial F_y/\partial p_y^+ = C_{yx}^+ w_y(p_x^{-*}) + C_{yp}^+ w_y(1-p_x^{-*}) - \lambda_y = 0 \\ \partial F_y/\partial p_y^- = C_{yx}^- w_y(p_x^{+*}) + C_{yp}^- w_y(1-p_x^{+*}) - \lambda_y = 0 \\ p_x^{+*} + p_x^{-*} = 1, \quad p_x^{+*} \geq 0, \quad p_x^{-*} \geq 0, \quad \lambda_y > 0. \end{cases} \quad (18)$$

Thus,

$$\left(C_{yx}^+ - C_{yp}^-\right) w_y\left(1-p_x^{+*}\right) = \left(C_{yx}^- - C_{yp}^+\right) w_y\left(p_x^{+*}\right). \quad (19)$$

By (5), (19) becomes

$$\left(C_{yx}^+ - C_{yp}^-\right)\exp\left(-\left(-\ln\left(1-p_x^{+*}\right)\right)^{\alpha_y}\right) = \left(C_{yx}^- - C_{yp}^+\right)\exp\left(-\left(-\ln\left(p_x^{+*}\right)\right)^{\alpha_y}\right) \quad (20)$$

which has a positive solution if $C_{yx}^+ > C_{yp}^-$ and $C_{yx}^- > C_{yp}^+$.

Similarly,

$$\left(C_{xy}^+ - C_{xp}^-\right) w_x\left(1-p_y^{+*}\right) = \left(C_{xy}^- - C_{xp}^+\right) w_x\left(p_y^{+*}\right) \quad (21)$$

which can be rewritten as

$$\left(C_{xy}^+ - C_{xp}^-\right)\exp\left(-\left(-\ln(1-p_y^{+*})\right)^{\alpha_x}\right) = \left(C_{xy}^- - C_{xp}^+\right)\exp\left(-\left(-\ln(p_y^{+*})\right)^{\alpha_x}\right) \quad (22)$$

which has a positive solution if $C_{xy}^+ > C_{xp}^-$ and $C_{xy}^- > C_{xp}^+$. ■

Remark: When condition A1 holds, each MG benefits from both buying and selling. In this case, with a large battery space to store more energy and energy saving to sell, each MG is motivated to take part in the trade. In addition, as $C_{yx}^+ > C_{yp}^-$ and $C_{yx}^- > C_{yp}^+$, the payoff from trading with the opponent MG exceeds that with the power plant. During the interaction with a subjective MG, the trading strategy of an MG is impacted by the subjective view of the opponent.

Corollary 1: Given an objective MG_y with $\alpha_y = 1$, if condition A1 holds, the PT-based energy exchange game has a mixed strategy NE given by

$$\mathbf{p}_x^* = \left(\frac{C_{yx}^+ - C_{yp}^-}{C_{yx}^- + C_{yx}^+ - C_{yp}^- - C_{yp}^+}, \frac{C_{yx}^- - C_{yp}^+}{C_{yx}^- + C_{yx}^+ - C_{yp}^- - C_{yp}^+}\right). \quad (23)$$

Proof: According to Theorem 5, when MG_y is rational with $\alpha_y = 1$ and $w_y(p) = p$, (15) becomes

$$\left(C_{yx}^+ - C_{yp}^-\right)\left(1-p_x^{+*}\right) = \left(C_{yx}^- - C_{yp}^+\right)p_x^{+*}. \quad (24)$$

The solution is (23) after simplification. ■

Theorem 2: For given $\min(C_{xy}^+, C_{xy}^-) < 0$, the PT-based energy exchange game is not impacted by user subjectivity and has a unique NE $\mathbf{p}_x^* = (0,0)$, (0, 1) or (1, 0) for unique and nonzero payoffs $C_{ij}^{+/-}$.

Proof: See the Appendix. ■

Remark: If $C_{xy}^- < 0$, the energy loss for MG_x exceeds the selling price, which happens with either a low battery level or a high selling price. Consequently, the MG does not sell for its own interests, i.e., $p_x^{-*} = 0$, $w_y(p_x^{-*}) = 0$ and $w_y(1 - p_x^{-*}) = 1$. Similarly, if $C_{xy}^+ < 0$, MG_x does not buy with $p_x^{+*} = 0$. Therefore, for $\min(C_{xy}^+, C_{xy}^-) < 0$, MG_x takes action $a_x = 0$ with probability 1.

Theorem 3: If $\min(C_{xy}^+, C_{xy}^-) > 0$ and condition A1 does not hold, the PT-based energy exchange game is not impacted by user subjectivity and has a unique NE $\mathbf{p}_x^* = (0,0)$, (0, 1) or (1, 0) for unique and nonzero payoffs $C_{ij}^{+/-}$.

Proof: Similar to the proof to Theorem 2. Due to the space limitation, the proof is omitted. ■

Theorem 4: The condition for the PT-based energy exchange game is not impacted by user subjectivity for any MG states is given by

$$\begin{aligned} &\rho^+ \geq g(2) - g(1), \text{ or} \\ &\rho^- \leq g(S) - g(S-1), \text{ or} \\ &\rho^+ + \rho_p^- \geq g(2) - g(0), \text{ or} \\ &\rho^- + \rho_p^+ \leq g(S) - g(S-2). \end{aligned} \quad (25)$$

Proof: According to Theorems 2, 3, and 5, the performance is not impacted by user subjectivity unless condition A1 holds. By (6), $C_{xy}^+ > 0$ in A1 yields $C_{xy}^+ = \Delta_{s_x+1} - \rho^+ > 0$, i.e., $\rho^+ < \Delta_{s_x+1}$. Similarly, $C_{xy}^+ > C_{xp}^-$ in condition A1 yields $\Delta_{s_x+1} - \rho^+ > -\Delta_{s_x} + \rho_p^-$, i.e., $\rho^+ + \rho_p^- < \Delta_{s_x+1} + \Delta_{s_x}$. In this way, condition A1 can be rewritten as

$$\begin{aligned} &\rho^+ < \min\left(\Delta_{s_x+1}, \Delta_{s_y+1}\right) \\ &\rho^- > \max\left(\Delta_{s_x}, \Delta_{s_y}\right) \\ &\rho^+ + \rho_p^- < \min\left(\Delta_{s_y} + \Delta_{s_y+1}, \Delta_{s_x} + \Delta_{s_x+1}\right) \\ &\rho^- + \rho_p^+ > \max\left(\Delta_{s_x} + \Delta_{s_x+1}, \Delta_{s_y} + \Delta_{s_y+1}\right). \end{aligned} \quad (26)$$

As $\Delta_s = g(s) - g(s-1)$ is a decreasing function of s, with $0 \leq s \leq S$, we have $\max(\Delta_{s+1}) = g(2) - g(1)$. Thus, condition

A1 in (26) does not hold, if

$$\begin{aligned}&\rho^{+} \geq \max\left(\Delta_{s+1}\right) = g(2) - g(1), \text{ or}\\ &\rho^{-} \leq \min\left(\Delta_{s}\right) = g(S) - g\left(S-1\right), \text{ or}\\ &\rho^{+} + \rho_p^{-} \geq \max\left(\Delta_s + \Delta_{s+1}\right) = g(2) - g(0), \text{ or}\\ &\rho^{-} + \rho_p^{+} \leq \min\left(\Delta_s + \Delta_{s+1}\right) = g(S) - g(S-2).\end{aligned} \quad (27)$$

■

Remark: An MG has no motivation to buy from another MG with a high MG buying price ($\rho^{+} \geq g(1)-g(0)$), or to sell to another MG with a low selling price ($\rho^{-} \leq g(S)-g(S-1)$). Similarly, an MG buys if the power plant provides a very low price (i.e., $\rho_p^{+} \leq g(S)-g(S-2)-\rho^{-}$), and sells if the power plant provides a high payment ($\rho_p^{-} \geq g(2)-g(0)-\rho^{+}$). Thus, for given (25), an MG is motivated to trade with the power plant with certainty and is not impacted by the random action of the opponent MG.

VI. Performance of Game With Generic Energy Quantization

Now we extend the results for $M = 1$ in Section V into a generalized energy exchange game between MG_x and MG_y with mixed strategies $\mathbf{p}_x$ and $\mathbf{p}_y$. Similar to (13), the NE in this game denoted with $(\mathbf{p}_x^*, \mathbf{p}_y^*)$ can be written as

$$\begin{aligned}&\mathbf{p}_x^* = \arg\max_{\mathbf{p}_x} U_x^{\mathrm{PT}}\left(\mathbf{p}_x, \mathbf{p}_y^*\right)\\ &\mathbf{p}_y^* = \arg\max_{\mathbf{p}_y} U_y^{\mathrm{PT}}\left(\mathbf{p}_y, \mathbf{p}_x^*\right)\\ &\text{s.t. } \mathbf{p}_x \succeq 0, \mathbf{p}_y \succeq 0\\ &\quad \sum_{k=M_s}^{M} p_{x,k} = 1, \quad \sum_{k=M_s}^{M} p_{y,k} = 1\end{aligned} \quad (28)$$

where $U_{\cdot}^{\mathrm{PT}}(\cdot,\cdot)$ is the expected utility by (4). For convenience of denotation, we define the payoff matrixes for MG_x and MG_y with $\Xi \triangleq \{u_x(m,n)\}_{M_s \leq m,n \leq M}$ and $\Theta \triangleq \{u_y(m,n)\}_{M_s \leq m,n \leq M}$, respectively, where each element is a parameter known by both players according to (2).

Theorem 5: If the payoff matrixes Ξ and Θ are positive definite and have a rank of $M-M_s+1$, the NE of the PT-based energy exchange game is given by

$$\begin{cases} p_{x,k}^* = \exp\left(-\left(-\ln\left(\lambda_y \sum_{m=M_s}^{M}\left[\Theta^{-1}\right]_{m,k}\right)\right)^{\frac{1}{\alpha_y}}\right)\\ p_{y,k}^* = \exp\left(-\left(-\ln\left(\lambda_x \sum_{m=M_s}^{M}\left[\Xi^{-1}\right]_{m,k}\right)\right)^{\frac{1}{\alpha_x}}\right)\\ \forall k = M_s, \ldots, M\\ \sum_{m=M_s}^{M} p_{x,m}^* = 1, \quad \sum_{m=M_s}^{M} p_{y,m}^* = 1\end{cases} \quad (29)$$

where $[A]_{i,j}$ is the (i,j)th element of matrix A.

Proof: First, we define

$$\begin{aligned}F_x &= U_x^{\mathrm{PT}}(\mathbf{p}_x, \mathbf{p}_y^*) - \lambda_x \sum_{m=M_s}^{M} p_{x,m}\\ &= \sum_{m=M_s}^{M}\sum_{n=M_s}^{M} u_x(m,n) p_{x,m} w_x\left(p_{y,n}^*\right) - \lambda_x \sum_{m=M_s}^{M} p_{x,m}.\end{aligned} \quad (30)$$

Similar to the proof to Theorem 5, according to the KKT optimality condition of (28), we have

$$\frac{\partial F_x}{\partial p_{x,k}} = \sum_{n=M_s}^{M} u_x(k,n) w_x\left(p_{y,n}^*\right) - \lambda_x = 0 \quad (31)$$

with $M_s \leq k \leq M$, which can be rewritten as

$$\begin{bmatrix} u_x(M_s, M_s) & u_x(M_s, 1+M_s) & \cdot\, u_x(M_s, M)\\ u_x(1+M_s, M_s) & u_x(1+M_s, 1+M_s) & \cdot\, u_x(1+M_s, M)\\ \cdots & \cdots & \cdots\cdot\\ u_x(M, M_s) & u_x(M, 1+M_s) & \cdot\, u_x(M, M)\end{bmatrix} \times \begin{bmatrix} w_x\left(p_{y,M_s}^*\right)\\ w_x\left(p_{y,1+M_s}^*\right)\\ \cdots\\ w_x\left(p_{y,M}^*\right)\end{bmatrix} = \Xi \begin{bmatrix} w_x\left(p_{y,M_s}^*\right)\\ w_x\left(p_{y,1+M_s}^*\right)\\ \cdots\\ w_x\left(p_{y,M}^*\right)\end{bmatrix} = \lambda_x \begin{bmatrix}1\\1\\\cdots\\1\end{bmatrix}. \quad (32)$$

Since $\lambda_x > 0$ and $w_x(\cdot) \geq 0$, if Ξ is positive definite and has a rank $M - M_s + 1$, Ξ has an inverse matrix that is also positive definite. In this case, the linear equations (32) can be solved and we can simplify the first line in (31) into

$$w_x(p_{y,k}^*) = \lambda_x \sum_{m=M_s}^{M}\left[\Xi^{-1}\right]_{m,k}, k = M_s, \ldots, M. \quad (33)$$

By integrating (5) into (33), we have

$$p_{y,k}^* = \exp\left(-\left(-\ln\left(\lambda_x \sum_{m=M_s}^{M}\left[\Xi^{-1}\right]_{m,k}\right)\right)^{\frac{1}{\alpha_x}}\right). \quad (34)$$

Similarly, we have $p_{x,k}^*$ and obtain (29). ■

Next, we consider the EUT-based energy exchange game with rational MGs, which is a special case of the PT-based game.

Corollary 2: If the payoff matrixes Ξ and Θ are positive definite and have a rank of $M - M_s + 1$, the NE of the EUT-based energy exchange game is given by

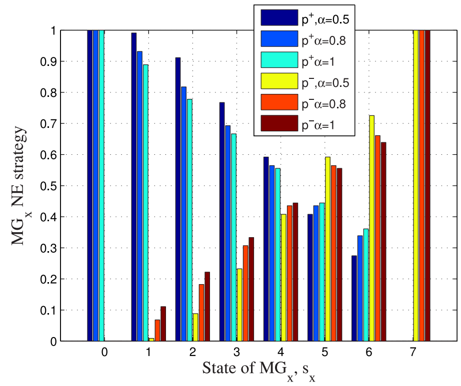
$$\begin{cases} p_{x,k}^* = \dfrac{\sum_{m=M_s}^{M}\left[\Theta^{-1}\right]_{m,k}}{\sum_{m=M_s}^{M}\sum_{n=M_s}^{M}\left[\Theta^{-1}\right]_{m,n}}\\ p_{y,k}^* = \dfrac{\sum_{m=M_s}^{M}\left[\Xi^{-1}\right]_{m,k}}{\sum_{m=M_s}^{M}\sum_{n=M_s}^{M}\left[\Xi^{-1}\right]_{m,n}}, k = M_s, \ldots, M.\end{cases} \quad (35)$$

Proof: By taking $\alpha_x = \alpha_y = 1$ into (29), we have (35) after simplification. ■

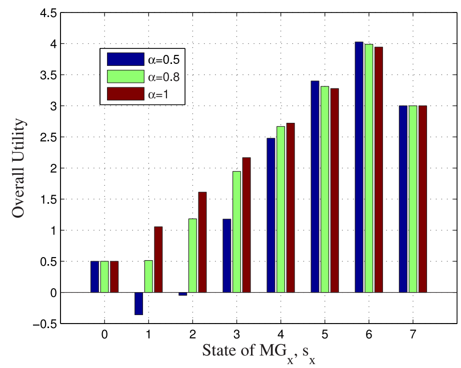
VII. Simulation Results

We performed simulations to evaluate the impact of the user's objective weight α in (5) on the performance of the NE strategy in the PT-based energy exchange game, including the overall utility to both MGs in (11) and the total amount of the energy bought from the power plant in (12). In the simulations, we set $M = -M_s = 1$, $S = 7$ and the unit amount of trading energy was 100 kWh.

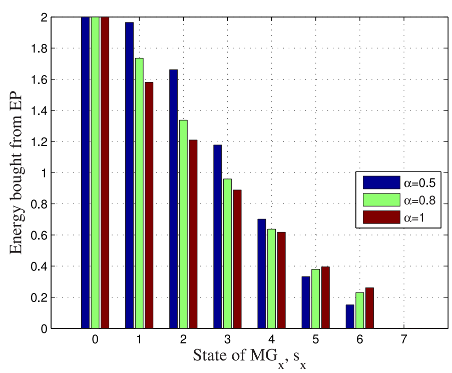
First, the performance for the energy exchange between two MGs with the same states is presented in Fig. 1, with



(a)

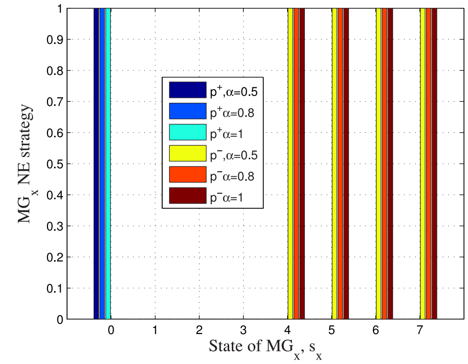


(b)

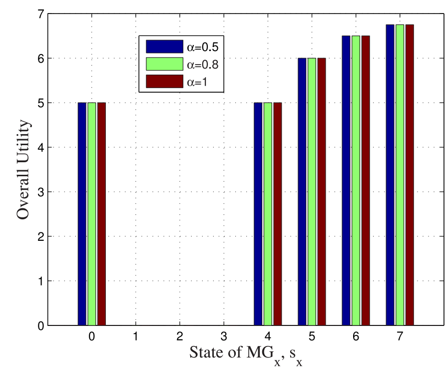


(c)

Fig. 1. Performance versus MG state in the PT-based energy exchange game with $\rho^+ = \$0.01/\text{kWh}$, $\rho_p^+ = \$0.08/\text{kWh}$, $\rho^- = \$0.09/\text{kWh}$, $\rho_p^- = \$0.07/\text{kWh}$, $g(s) = [0, 8.25, 16, 23.25, 30, 36.25, 42]$, and $s_x = s_y$. (a) Mixed NE strategy for MG_x, $\mathbf{p}_x^* = [p_x^+, p_x^-]$. (b) Overall utility, $U_{xy}(\mathbf{p}_x^*, \mathbf{p}_y^*)$. (c) Overall energy bought from EP, $E_p(\mathbf{p}_x^*, \mathbf{p}_y^*)$.



(a)



(b)

Fig. 2. Performance versus MG state in the PT-based energy exchange game, with $\rho^+ = \$0.01/\text{kWh}$, $\rho_p^+ = \$0.08/\text{kWh}$, $\rho^- = \$0.04/\text{kWh}$, $\rho_p^- = \$0.01/\text{kWh}$, $g(s) = [0, 8.25, 14.5, 18.75, 21, 22.25, 23, 23.5]$, and $s_y = 2$. (a) Mixed NE strategy for MG_x, $\mathbf{p}_x^* = [p_x^+, p_x^-]$. (b) Overall utility, $U_{xy}(\mathbf{p}_x^*, \mathbf{p}_y^*)$.

$\rho^+ = \$0.01/\text{kWh}$, $\rho_p^+ = \$0.08/\text{kWh}$, $\rho^- = \$0.09/\text{kWh}$, $\rho_p^- = \$0.07/\text{kWh}$, $g(s) = [0, 8.25, 16, 23.25, 30, 36.25, 42]$, and $s_x = s_y$. As shown in Fig. 1(a), subjective MGs are more likely to sell at high battery levels and to buy at low battery levels than objective users, i.e., user subjectivity exaggerates the selling and buying probabilities when the battery level is high or low. More specifically, a subjective user (e.g., $\alpha = 0.5$) is more likely to buy at a low battery level than an objective user (i.e., $\alpha = 1$), e.g., $p_x^{+*}(\alpha = 0.5, s = 1) > p_x^{+*}(\alpha = 1, s = 1)$, and is more likely to sell at a high battery level, e.g., $p_x^{-*}(\alpha = 0.5, s = 6) > p_x^{-*}(\alpha = 1, s = 6)$. Consequently, subjective users have a lower overall utility and request more energy from the power plant at low battery levels (e.g., $s = 1$), as shown in Fig. 1(b) and (c), resulting from a higher probability of buying. On the other hand, subjective users have a slightly higher overall utility and buy less from the power plant at high battery levels (e.g., $s = 6$) due to a higher selling rate.

Fig. 2 presents six (s_x, s_y) cases with $\rho^+ = \$0.01/\text{kWh}$, $\rho_p^+ = \$0.08/\text{kWh}$, $\rho^- = \$0.04/\text{kWh}$, $\rho_p^- = \$0.01/\text{kWh}$, $g(s) = [0, 8.25, 14.5, 18.75, 21, 22.25, 23, 23.5]$, and $s_y = 2$. In these cases, the MGs choose deterministic actions and the trading performance is not impacted by user subjectivity. For example, for given $s_y = 2$, MG_x does not trade at low battery levels with $s_x = 1, 2, 3$ and sells with probability one at high battery levels with $s_x = 4, 5, 6$.

Finally, Fig. 3 shows the performance versus the selling MG price ρ^-, with $\rho^+ = \$0.01/\text{kWh}$, $\rho_p^+ = \$0.08/\text{kWh}$, $\rho_p^- = \$0.07/\text{kWh}$, and the two MGs are in the same state with $\Delta = [7.25, 6.75]$. As indicated by Fig. 3, an MG with extra energy is more likely to sell and obtains higher utility with a higher MG selling price ρ^-. The energy bought from the power plant decreases with ρ^- due to lower buying probability. Moreover, by exaggerating the buying and selling probabilities at low and high selling prices, a subjective user buys more energy from the power plant at low MG selling prices and less energy from the power plant at high ρ^-.

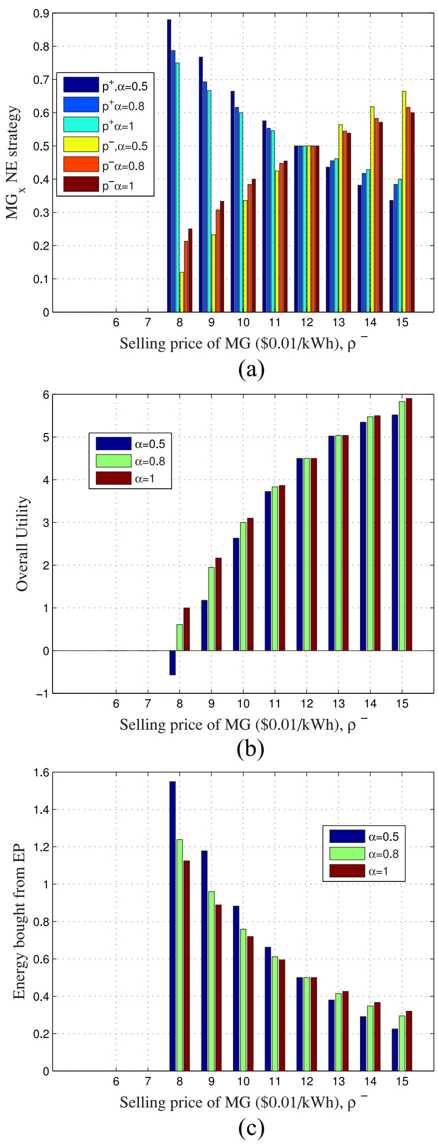


Fig. 3. Performance versus selling price between MGs in the PT-based energy exchange game, with $\rho^+ = \$0.01/\text{kWh}$, $\rho_p^+ = \$0.08/\text{kWh}$, and $\rho_p^- = \$0.07/\text{kWh}$. X and Y are in the same state with $\Delta = [7.25, 6.75]$. (a) Mixed NE strategy, $\mathbf{p}_x^* = [p_x^+, p_x^-]$. (b) Overall utility, $U_{xy}(\mathbf{p}_x^*, \mathbf{p}_y^*)$. (c) Overall energy bought from EP, $E_p(\mathbf{p}_x^*, \mathbf{p}_y^*)$.

VIII. Conclusion

We have formulated an energy exchange game among MGs and a power plant as a prospect theory-based static game, which takes into account the subjective view of the MGs that has been omitted by conventional game theory. We have derived the NE in the PT-based energy exchange game and investigated the impact of MG subjectivity on MG energy exchange. Simulation results show that by exaggerating the buying and selling behavior at low and high battery levels, subjective MGs at low battery levels decrease the system utility and request more energy from the power plant and vice versa. We have also provided the criteria on the energy price in the local energy market in order to avoid the impact of user subjectivity in the trade.

In the future, we plan to formulate the MG energy exchange during a short (e.g., 15 min), medium (e.g., 2 h) or long term (a day) as a dynamic game, in which MGs hold subjective views on the random MG states, which depend on the energy trades and the forecast accuracies of the renewable energy generation, demand and prices. In addition, we will investigate the continuous energy exchange instead of the quantized energy levels and consider the power transfer rate for given systemic constraints such as the battery/source ratings and limits on the power lines. Finally, an interesting related issue is to study how the fast change of the MG pairing impacts the MGs' decision making.

Appendix
Proof of Theorem 2

If $C_{xy}^+ < 0$, we have $C_{xp}^+ < C_{xy}^+ < 0$. It is clear by $0 \le w(p) \le 1$ that $C_{xy}^+ w_x(p_y^{-*}) + C_{xp}^+ w_x(1 - p_y^{-*}) < 0$. Since $p_x^+, p_x^- \ge 0$, by (7) and (13), we have $p_x^{+*} = 0$. Similarly, $p_x^{-*} = 0$ if $C_{xy}^- < 0$ and $p_y^{+/-*} = 0$ if $C_{yx}^{+/-} < 0$.

Case (1) with $C_{xy}^+ < 0$ and $C_{xy}^- < 0$: there is a unique NE with $\mathbf{p}_x^* = [0, 0]$.

Case (2) with $C_{xy}^+ < 0$, $C_{xy}^- > 0$, and $C_{yx}^+ < 0$: As $C_{xy}^+ < 0$ and $C_{yx}^+ < 0$, we have $p_x^{+*} = 0$ and $p_y^{+*} = 0$. Thus, $U_x^{\text{PT}}([0, p_x^-], [0, p_y^{-*}]) = p_x^- C_{xp}^-$. If $C_{xp}^- < 0$, we have $\mathbf{p}_x^* = [0, 0]$; otherwise, $\mathbf{p}_x^* = [0, 1]$ for $C_{xp}^- > 0$.

Case (3) with $C_{xy}^+ > 0$, $C_{xy}^- < 0$, and $C_{yx}^- < 0$: Similar to case (2), we have $p_x^{-*} = 0$ and $p_y^{-*} = 0$. Thus, $U_x^{\text{PT}}([p_x^+, 0], [p_y^{+*}, 0]) = p_x^+ C_{xp}^+$. If $C_{xp}^+ < 0$, we have $\mathbf{p}_x^* = [0, 0]$; otherwise, $\mathbf{p}_x^* = [1, 0]$ for $C_{xp}^+ > 0$.

Case (4) with $C_{xy}^+ < 0$, $C_{xy}^- > 0$, $C_{yx}^+ > 0$, and $C_{yp}^- < 0$: It is clear from $C_{xy}^+ < 0$ that $p_x^{+*} = 0$. By (7) and (8), we have

$$U_x^{\text{PT}}\left([0, p_x^-], \mathbf{p}_y^*\right) = p_x^- \left(C_{xy}^- w_x\left(p_y^{+*}\right) + C_{xp}^- w_x\left(1 - p_y^{+*}\right)\right) \tag{36}$$

$$U_y^{\text{PT}}\left(\mathbf{p}_y, [0, p_x^{-*}]\right) = p_y^+ \left(C_{yx}^+ w_y\left(p_x^{-*}\right) + C_{yp}^+ w_y\left(1 - p_x^{-*}\right)\right) + p_y^- C_{yp}^-. \tag{37}$$

As $C_{yp}^- < 0$, by (37) we have $p_y^{-*} = 0$ and (37) becomes

$$U_y^{\text{PT}}\left(\left[p_y^+, 0\right], \left[0, p_x^{-*}\right]\right) = p_y^+ \left(C_{yx}^+ w_y\left(p_x^{-*}\right) + C_{yp}^+ w_y\left(1 - p_x^{-*}\right)\right). \tag{38}$$

Clearly, U_x^{PT} in (36) [or U_y^{PT} in (38)] is a linear function of p_x^- (or p_y^+). As $C_{xy}^- > 0$, by (36), we have $U_x^{\text{PT}}(\mathbf{p}_x = [0, 1], \mathbf{p}_y^* = [1, 0]) = C_{xy}^- > U_x^{\text{PT}}([0, p_x^-], \mathbf{p}_y^* = [1, 0]) = p_x^- C_{xy}^-$. Similarly, by (38) and $C_{yx}^+ > 0$ $U_y^{\text{PT}}(\mathbf{p}_y = [1, 0], \mathbf{p}_x^* = [0, 1]) = C_{yx}^+ > U_y^{\text{PT}}([p_y^+, 0], \mathbf{p}_x^* = [0, 1]) = p_y^+ C_{yx}^+$. Thus, $\mathbf{p}_x^* = [0, 1]$ and $\mathbf{p}_y^* = [1, 0]$ is the NE for this case.

Case (5) with $C_{xy}^- < 0$, $C_{xy}^+ > 0$, $C_{yx}^- > 0$, and $C_{yp}^+ < 0$: Similar to Case (1.4), $\mathbf{p}_x^* = [1, 0]$ and $\mathbf{p}_y^* = [0, 1]$ is the NE of the game.

Case (6) with $C_{xy}^+ < 0$, $C_{xy}^- > 0$, $C_{yx}^+ > 0$, and $C_{yp}^- > 0$: Similar to Case (4), we have $p_x^{+*} = 0$ and

$$U_x^{\text{PT}}\left([0, p_x^-], \mathbf{p}_y^*\right) = p_x^- \left(C_{xy}^- w_x\left(p_y^{+*}\right) + C_{xp}^- w_x\left(1 - p_y^{+*}\right)\right) \tag{39}$$

$$U_y^{\mathrm{PT}}\left(\mathbf{p}_y, \left[0, p_x^{-*}\right]\right) = p_y^+\left(C_{yx}^+ w_y\left(p_x^{-*}\right) + C_{yp}^+ w_y\left(1 - p_x^{-*}\right)\right) + p_y^- C_{yp}^-. \tag{40}$$

If $C_{xp}^- > 0$, we have $C_{xy}^- w_x(p_y^{+*}) + C_{xp}^- w_x(1 - p_y^{+*}) > 0$ and thus $\mathbf{p}_x^* = [0, 1]$. Otherwise, if $C_{xp}^- < 0$, by (39), if $\ell_1 : C_{xy}^- w_x(p_y^{+*}) + C_{xp}^- w_x(1 - p_y^{+*}) > 0$, we have $p_x^{-*} = 1$ and $\mathbf{p}_x^* = [0, 1]$. Thus, (40) becomes $U_y^{\mathrm{PT}}(\mathbf{p}_y, [0, 1]) = p_y^+ C_{yx}^+ + p_y^- C_{yp}^-$. If $C_{yx}^+ > C_{yp}^-$, we have $\mathbf{p}_y^* = [1, 0]$. As $p_y^{+*} = 1$ and $C_{xy}^- > 0$, it is clear that the condition $\ell_1 : C_{xy}^- w_x(p_y^{+*}) + C_{xp}^- w_x(1 - p_y^{+*}) = C_{xy}^- > 0$ holds. On the other hand, if $C_{yx}^+ < C_{yp}^-$, we have $\mathbf{p}_y^* = [0, 1]$ and thus $C_{xy}^- w_x(p_y^{+*}) + C_{xp}^- w_x(1 - p_y^{+*}) = C_{xp}^- < 0$, i.e., ℓ_1 holds.

Similarly, if ℓ_2:$C_{xy}^- w_x(p_y^{+*}) + C_{xp}^- w_x(1 - p_y^{+*}) < 0$, we have $\mathbf{p}_x^* = [0, 0]$, and $U_y^{\mathrm{PT}}(\mathbf{p}_y, [0, 0]) = p_y^+ C_{yp}^+ + p_y^- C_{yp}^-$. If $C_{yp}^+ > C_{yp}^-$, we have $\mathbf{p}_y^* = [1, 0]$. As $p_y^{+*} = 1$, it is clear that $C_{xy}^- w_x(p_y^{+*}) + C_{xp}^- w_x(1 - p_y^{+*}) = C_{xy}^- < 0$, contradicting to the condition ℓ_2. On the other hand, if $C_{yp}^+ < C_{yp}^-$, we have $\mathbf{p}_y^* = [0, 1]$ and thus $C_{xy}^- w_x(p_y^{+*}) + C_{xp}^- w_x(1 - p_y^{+*}) = C_{xp}^- < 0$, matching with the condition ℓ_2. Thus, $\mathbf{p}_x^* = [0, 1]$ is the NE if $C_{xp}^- > 0$ or $C_{yx}^+ > C_{yp}^-$; and $\mathbf{p}_x^* = [0, 0]$ is the NE if $C_{xp}^- < 0$ and $C_{yp}^+ < C_{yp}^-$.

Case (7) with $C_{xy}^- < 0$, $C_{xy}^+ > 0$, $C_{yx}^- > 0$, and $C_{yp}^+ > 0$: Similar to Case (1.6), if $C_{xp}^+ > 0$, we have $\mathbf{p}_x^* = [1, 0]$. Otherwise, if $C_{xp}^+ < 0$, $\mathbf{p}_x^* = [1, 0]$ is the NE for $C_{yx}^- > C_{yp}^+$, and $\mathbf{p}_x^* = [0, 0]$ for $C_{yp}^- < C_{yp}^+$.

In summary, we have $\mathbf{p}_x^* = (0, 0)$, for condition Ω_0; $\mathbf{p}_x^* = (0, 1)$ for Ω_1; and $\mathbf{p}_x^* = (1, 0)$ for Ω_2 with

$$\begin{aligned}
\Omega_0 : &\left(C_{xy}^+ < 0, C_{xy}^- < 0\right)\\
&\text{or } \left(C_{xy}^+ < 0, C_{xy}^- > 0, C_{yx}^+ < 0, C_{xp}^- < 0\right)\\
&\text{or } \left(C_{xy}^+ > 0, C_{xy}^- < 0, C_{yx}^- < 0, C_{xp}^+ < 0\right)\\
&\text{or } \left(C_{xy}^+ < 0, C_{xy}^- > 0, C_{xp}^- < 0, C_{yx}^+ > 0, C_{yp}^- > 0, C_{yp}^+ < C_{yp}^-\right)\\
&\text{or } \left(C_{xy}^- < 0, C_{xy}^+ > 0, C_{xp}^+ < 0, C_{yx}^- > 0, C_{yp}^+ > 0, C_{yp}^- < C_{yp}^+\right)\\
\Omega_1 : &\left(C_{xy}^+ < 0, C_{xy}^- > 0, C_{yx}^+ < 0, C_{xp}^- > 0\right)\\
&\text{or } \left(C_{xy}^+ < 0, C_{xy}^- > 0, C_{yx}^+ > 0, C_{yp}^- < 0\right)\\
&\text{or } \Big(C_{xy}^+ < 0, C_{xy}^- > 0, C_{yx}^+ > 0, C_{yp}^- > 0, \Big(C_{xp}^- > 0\\
&\quad\text{or } C_{yx}^+ < C_{yp}^-\Big)\Big)\\
\Omega_2 : &\left(C_{xy}^+ > 0, C_{xy}^- < 0, C_{yx}^- < 0, C_{xp}^+ > 0\right)\\
&\text{or } \left(C_{xy}^+ > 0, C_{xy}^- < 0, C_{yx}^- > 0, C_{yp}^+ < 0\right) \text{ or}\\
&\Big(C_{xy}^+ > 0, C_{xy}^- < 0, C_{yx}^- > 0, C_{yp}^+ > 0\\
&\Big(C_{xp}^+ < 0 \text{ or } C_{yx}^- < C_{yp}^+\Big)\Big).
\end{aligned}$$

References

[1] S. M. Amin and B. F. Wollenberg, "Toward a smart grid: Power delivery for the 21st century," *IEEE Power Energy Mag.*, vol. 3, no. 5, pp. 34–41, Sep./Oct. 2005.

[2] W. Saad, Z. Han, H. V. Poor, and T. Basar, "Game-theoretic methods for the smart grid: An overview of microgrid systems, demand-side management, and smart grid communications," *IEEE Signal Process. Mag.*, vol. 29, no. 5, pp. 86–105, Sep. 2012.

[3] W. W. Weaver and P. T. Krein, "Game-theoretic control of small-scale power systems," *IEEE Trans. Power Del.*, vol. 24, no. 3, pp. 1560–1567, Jul. 2009.

[4] F. A. Mohamed and H. N. Koivo, "Multiobjective optimization using modified game theory for online management of microgrid," *Eur. Trans. Elect. Power*, vol. 21, no. 1, pp. 839–854, Jan. 2011.

[5] I. Maity and S. Rao, "Simulation and pricing mechanism analysis of a solarpowered electrical microgrid," *IEEE Syst. J.*, vol. 4, no. 3, pp. 275–284, Sep. 2010.

[6] W. Saad, Z. Han, and H. V. Poor, "Coalitional game theory for cooperative microgrid distribution networks," in *Proc. IEEE Int. Conf. Commun. Workshop (ICC)*, Kyoto, Japan, Jun. 2011, pp. 1–5.

[7] J. Matamoros, D. Gregoratti, and M. Dohler, "Microgrids energy trading in islanding mode," in *Proc. IEEE 3rd Int. Conf. Smart Grid Commun. (SmartGridComm)*, Tainan, Taiwan, Nov. 2012, pp. 49–54.

[8] Y. Zhao, J. Qin, R. Rajagopal, A. Goldsmith, and H. V. Poor, "Risky power forward contracts for wind aggregation," in *Proc. 51st Annu. Allerton Conf. Commun. Control Comput.*, Monticello, IL, USA, Oct. 2013, pp. 54–61.

[9] L. Xiao, Y. Chen, and K. J. R. Liu, "Anti-cheating prosumer energy exchange based on indirect reciprocity," in *Proc. IEEE Int. Conf. Commun. (ICC)*, 2014, pp. 599–604.

[10] D. Kahneman and A. Tversky, "Prospect theory: An analysis of decision under risk," *Econometrica*, vol. 47, no. 2, pp. 263–291, 1979.

[11] A. Tversky and D. Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," *J. Risk Uncertainty*, vol. 5, no. 4, pp. 297–323, Oct. 1992.

[12] S. Gao, E. Frejinger, and M. Ben-Akiva, "Adaptive route choices in risky traffic networks: A prospect theory approach," *Transp. Res. C, Emerg. Technol.*, vol. 18, no. 5, pp. 727–740, Oct. 2010.

[13] G. W. Harrison and E. E. Rutstrom, "Expected utility theory and prospect theory: One wedding and a decent funeral," *Exp. Econ.*, vol. 12, no. 2, pp. 133–158, Feb. 2009.

[14] T. Li and N. B. Mandayam, "Prospects in a wireless random access game," in *Proc. 46th Annu. Conf. Inf. Sci. Syst.*, Princeton, NJ, USA, Mar. 2012, pp. 1–6.

[15] D. D. Clark, J. Wroclawski, K. R. Sollins, and R. Braden, "Tussle in cyberspace: Defining tomorrow's internet," *IEEE/ACM Trans. Netw.*, vol. 13, no. 3, pp. 462–475, Jun. 2005.

[16] Y. Wang, A. V. Vasilakos, Q. Jin, and J. Ma, "On studying relationship between altruism and the psychological phenomenon of self-deception in rational and autonomous networks," in *Proc. Int. Conf. Distrib. Comput. Syst. Workshops (ICDCSW)*, Macau, China, 2012, pp. 336–341.

[17] T. Okuda, T. Ichikawa, T. Ideguchi, and X. Tian, "A design method of local community network service systems with ad-hoc network technology," in *Proc. IEEE 70th Veh. Technol. Conf. Fall (VTC)*, Anchorage, AK, USA, 2009, pp. 1–3.

[18] L. Xiao, J. Liu, Y. Li, N. B. Mandayam, and H. V. Poor, "Prospect theoretic analysis of anti-jamming communications in cognitive radio networks," in *Proc. IEEE Global Commun. Conf.*, 2014, pp. 1–6.

[19] D. Prelec, "The probability weighting function," *Econometrica*, vol. 66, no. 3, pp. 497–528, May 1998.

[20] Y. Wang, W. Saad, N. B. Mandayam, and H. V. Poor, "Integrating energy storage into the smart grid: A prospect theoretic approach," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, 2014, pp. 7779–7783.

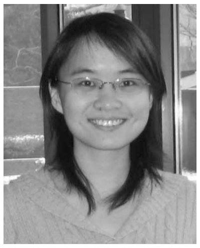
[21] P. K. Enumula and S. Rao, "The potluck problem," *Econ. Lett.*, vol. 107, no. 1, pp. 10–12, Apr. 2010.

[22] A. Tversky and D. Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," *J. Risk Uncertainty*, vol. 5, no. 4, pp. 297–323, Apr. 1992.

[23] T. Li and N. B. Mandayam, "When users interfere with protocols: Prospect theory in wireless networks using random access and data pricing as an example," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 1888–1907, Apr. 2014.

[24] J. Yu, M. H. Cheung, and J. Huang, "Spectrum investment with uncertainty based on prospect theory," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Sydney, NSW, Australia, 2014, pp. 1620–1625.

[25] Y. Wang, A. V. Vasilakos, Q. Jin, and J. Ma, "On studying relationship between altruism and the psychological phenomenon of self-deception in rational and autonomous networks," in *Proc. 32nd Int. Conf. Distrib. Comput. Syst. Workshops (ICDCSW)*, Macau, China, 2012, pp. 336–341.



Liang Xiao (M'09–SM'13) received the B.S. degree in communication engineering from the Nanjing University of Posts and Telecommunications, Nanjing, China; the M.S. degree in electrical engineering from Tsinghua University, Beijing, China; and the Ph.D. degree in electrical engineering from Rutgers University, New Brunswick, NJ, USA, in 2000, 2003, and 2009, respectively.

She is currently a Professor with the Department of Communication Engineering, Xiamen University, Fujian, China. Her current research interests include smart grids, network security, and wireless communications.

Narayan B. Mandayam (S'89–M'94–SM'99–F'09) received the B.Tech (Hons.) degree from the Indian Institute of Technology, Kharagpur, India, in 1989, and the M.S. and Ph.D. degrees from Rice University, Houston, TX, USA, in 1991 and 1994, respectively, all in electrical engineering.

From 1994 to 1996, he was a Research Associate at the Wireless Information Network Laboratory (WINLAB), Rutgers University, New Brunswick, NJ, USA, before joining the faculty of the Electrical and Computer Engineering Department at Rutgers University, where he is currently a Distinguished Professor. He also serves as an Associate Director at WINLAB. He was a Visiting Faculty Fellow at the Department of Electrical Engineering, Princeton University, Princeton, NJ, USA, in 2002 and a Visiting Faculty at the Indian Institute of Science, Bengaluru, India, in 2003. His current research interests include various aspects of wireless data transmission with an emphasis on techniques for cognitive radio networks including their implications for spectrum policy, and also in modeling social knowledge creation on the internet. Using constructs from game theory, communications, and networking, his work focusses on radio resource management as well as signal processing for enabling wireless technologies to support various applications. He has co-authored the books *Principles of Cognitive Radio* (Cambridge University Press, 2012) and *Wireless Networks: Multiuser Detection in Cross-Layer Design* (Springer, 2004).

Dr. Mandayam is a co-recipient of the 2014 IEEE Donald G. Fink Award for his IEEE Proceedings paper entitled "Frontiers of Wireless and Mobile Communications" and the 2009 Fred W. Ellersick Prize from the IEEE Communications Society for his work on dynamic spectrum access models and spectrum policy. He is also the recipient of the Peter D. Cherasia Faculty Scholar Award from Rutgers University in 2010, the National Science Foundation CAREER Award in 1998, and the Institute Silver Medal from the Indian Institute of Technology in 1989. He has served as an Editor for the IEEE COMMUNICATION LETTERS and the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He has also served as a Guest Editor of the IEEE JSAC Special Issues on Adaptive, Spectrum Agile and Cognitive Radio Networks in 2007 and Game Theory in Communication Systems in 2008. He currently serves as a Distinguished Lecturer of the IEEE.

H. Vincent Poor (S'72–M'77–SM'82–F'87) received the Ph.D. degree in electrical engineering and computer science from Princeton University, Princeton, NJ, USA, in 1977.

From 1977 to 1990, he was on the faculty of the University of Illinois at Urbana-Champaign, Champaign, IL, USA. Since 1990, he has been on the faculty at Princeton, where he is the Michael Henry Strater University Professor of Electrical Engineering, and the Dean of the School of Engineering and Applied Science. His current research interests include stochastic analysis, statistical signal processing, and information theory, and their applications in wireless networks and related fields such as social networks and smart grid. Among his publications in these areas are the recent books *Principles of Cognitive Radio* (Cambridge, 2013) and *Mechanisms and Games for Dynamic Spectrum Allocation* (Cambridge University Press, 2014).

Dr. Poor received a Guggenheim Fellowship in 2002 and the IEEE Education Medal in 2005. Recent recognition of his work includes the 2014 URSI Booker Gold Medal and honorary doctorates from Aalborg University, Aalborg, Denmark; the Hong Kong University of Science and Technology, Hong Kong; and the University of Edinburgh, Edinburgh, U.K. He is a Member of the National Academy of Engineering and the National Academy of Sciences, and a Foreign Member of Academia Europaea and the Royal Society. He is also a Fellow of the American Academy of Arts and Sciences, Cambridge, MA, USA; the Royal Academy of Engineering, London, U.K.; and the Royal Society of Edinburgh, Edinburgh, Scotland. In 1990, he served as the President of the IEEE Information Theory Society, and from 2004 to 2007 he served as Editor-in-Chief of the IEEE TRANSACTIONS ON INFORMATION THEORY.