Inter-Network Dynamic Spectrum Allocation Via a Colonel Blotto Game

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***Abstract*—This paper investigates a scenario where multiple network service providers (NSPs) compete to provide wireless connectivity to a set of users. The users could either be a single mobile device, a set of localized Internet-of-Things (IoT) devices, or even a campus-wide network requiring wireless backhaul. The NSPs compete with one another to provide wireless service to the users by strategically allocating the available bandwidth so as to maximize their total payoff. The NSPs present each user with an offer to provide wireless connectivity using a certain amount of bandwidth. Users then decide to connect to that NSP whose offered bandwidth maximizes their utility function. Under such an architecture, this paper focuses on the optimal bandwidth allocation strategies for the NSPs. Such a problem is best modeled using a classical problem in game theory called the Colonel Blotto game—a multidimensional strategic resource allocation game. We show that the problem of spectrum allocation can be reframed as a Colonel Blotto game and analyze the mixed strategies that achieve Nash equilibrium. Depending on whether spectrum is treated a discrete or a continuous resource, we take recourse to either existing theoretical results or rely on numerical techniques to establish the equilibrium-achieving mixed strategies. We finally discuss interesting aspects about these mixed strategies, including an intrinsic user-association mechanism that emerges when spectral efficiency is taken into consideration.**

***Keywords*—spectrum allocation, noncooperative game theory, Colonel Blotto game, mixed strategies.**

I. Introduction

Traditional wireless networks operate as parallel, independent infrastructures with little to no inter-network coordination [1]. This can lead to poor utilization of the licensed spectrum, inability to address load imbalance, and high interference in the case of unlicensed spectrum. Prime examples of such scenarios include multiple co-located Wi-Fi hotspots, and cellular networks with mismatched data demand and spectrum availability. To address better usage of licensed and unlicensed spectrum, several new notions of spectrum usage are being promoted, including licensed assisted access (LAA) [2], licensed shared access [3], co-primary sharing [4], etc. While these solutions are aimed at a more harmonious use of the available spectrum through coordination and cooperation among the network service providers (NSPs), better spectrum utilization can also be realized through competition among the NSPs. Such an environment can be created by allowing users to choose their NSPs on an on-demand basis, without them committing to monthly subscription plans. NSPs are then required to compete with one another to provide service to these untethered users by competitively allocating the available radio resources. While such a model enables users to choose the best available service without committing to a single NSP, it also forces the NSPs to constantly employ all available resources, thus promoting better spectrum utilization.

As an example of such an architecture, consider the concept behind Google's Project Fi [5], where mobile users have no dedicated NSPs, and instead opportunistically connect to the cellular service or WiFi hotspot offering them the best service. Such a setup unlocks spectrum to be used in an opportunistic manner, with the ancillary benefits of better interference management and higher spectral efficiency. A key component in such a system is the competitive bidding by the multiple NSPs to be chosen as service providers to the different users in the pool. In such a setting, it is of interest to analyze the competitive allocation of spectrum by multiple NSPs among a group of users. Naturally, to analyze such competitive scenarios, game-theoretic tools [6] are known to be a suitable framework.

In particular, we consider two NSPs, each with a fixed amount of non-overlapping bandwidth, competing over a common pool of users. The users can either be of equal or different value to the NSPs. Depending on the total available bandwidth and the value of the user, each user receives an offer of service from the two NSPs, quantified by the amount of bandwidth the NSP is willing to support that user with. The user then picks the offer that maximizes its utility. The NSPs aim to serve as many users as possible so as to maximize their revenue. This process is constantly repeated as users move in and out of the system and when more bandwidth becomes available.

Each NSP must deal with the challenging task of strategically allocating spectrum so as to out-bid the other NSP while adhering to the constraints on the total available bandwidth. The competitive allocation of spectrum by the NSPs is closely related to the Colonel Blotto game (CBG) [7], [8]—a multidimensional problem on strategic resource allocation. The classical CBG is a two-person constant-sum game in which two players (colonels) are tasked with allocating a limited resource (troops) over multiple fronts (battlefields), with the player allocating the most resources to a front being declared the winner, and the overall payoff being proportional to the number fronts won. The classical CBG and its variants are known to be

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challenging problems due to the complex strategy space. Yet, recent progress by Roberson [7] has provided valuable insight on equilibrium-achieving mixed strategies in such problems. Characterizing equilibrium-achieving mixed strategies of other variants of the CBG is an active area of research [9], [10].

The main contribution of this paper is to introduce a novel approach to the inter-network spectrum allocation problem using the framework of CBG in both the discrete and continuous domains. In the continuous case, we assume spectrum to be an infinitely divisible resource and establish parameter settings under which equilibrium achieving mixed strategies to inter-network spectrum allocation are known. We then proceed to consider spectrum as a quantized resource and shift focus to the discrete CBG. Since the discrete CBG is a 2-player constant-sum matrix game, we propose a learning algorithm based on fictitious play [11] to numerically compute the mixed strategies that achieve Nash equilibrium (NE). We compare the numerically obtained strategies to those predicted by the theoretical results in the continuous case and further proceed to consider parameter settings for which no theoretical results are available.

The rest of the paper is organized as follows. In Section II, inter-network spectrum allocation is introduced in a formal setting. Section III discusses important results on the continuous CBG and adapts them to inter-network spectrum allocation. Sections IV and V consider spectrum as a quantized resource and use numerical methods to compute the equilibrium mixed strategies. Conclusions are drawn in Section VI.

II. Inter-Network Spectrum Allocation

Consider two independent NSPs R_1 and R_2 with non-overlapping bandwidths W_1 and W_2, respectively. R_1 and R_2 compete to provide service to a pool of N users labeled U_1, U_2, ..., U_N. We let p_i denote the payoff/revenue to the NSP that is chosen to provide service to user U_i. The two NSPs strategically divide the available bandwidth W_i among the pool of N users so as to maximize their payoff. Each user U_i thus receives a *bid* of w_{1i} and w_{2i} from the two NSPs, indicating an intention to provide service using an amount, w_{ik} of bandwidth. Using an estimate of the spectral efficiency σ_{ik} that can be achieved when served by NSP R_i, User U_k chooses the NSP maximizing the total rate achieved, i.e., users choose the NSP that maximizes $\sigma_{ik}w_{ik}$[1]. It is assumed that information regarding the spectral efficiencies is relayed to both the NSPs. Spectral efficiency for the link between R_i and U_k is obtained by measuring the signal-to-noise ratio (SNR_{ik}) and setting $\sigma_{ik} = \log(1 + SNR_{ik})$. If spectral efficiencies are not estimated, they are assumed to be 1 and such a scenario is termed SNR-agnostic spectrum allocation. Note that since the payoff does not incentivize bandwidth conservation, it can be assumed without loss of generality that all available bandwidth is used in the bidding process, i.e., the N bids by R_i satisfy $\sum_{k=1}^{N} w_{ik} = W_i$. The total payoff to NSP R_1 from such a process, assuming no ties, is given by

$$c_1 = \sum_{k:\sigma_{1k}w_{1k} > \sigma_{2k}w_{2k}} p_k. \tag{1}$$

After this process, the users get served by their NSP of choice using the promised amount of bandwidth. Since it is unlikely that all users choose to associate with an NSP, the unused bandwidth at each NSP is rolled over to the next session or time slot when the NSPs compete again to serve a new pool of users. This paper restricts focus to a single instance of such a bidding process, with the design and behavior of the repetitive bidding process being of interest in the future. Note that not all the N bids made by an NSP are likely to be accepted and this results in some unallocated bandwidth. We assume that this residual bandwidth is not reassigned as the users have already agreed to be saved and further, there is an economic incentive to save this bandwidth for a subsequent bidding session.

For our model, one important consideration is whether or not bandwidth can be treated as an infinitely divisible resource. In theory, while it is indeed possible to treat bandwidth as an infinitely divisible resource, in practice, bandwidth is typically assigned in certain preset quantized values. This subtle but important distinction in how this resource is treated leads to two different problem formulations. Treating bandwidth as an infinitely divisible resource allows us to take recourse to well-known results in the context of continuous CBG.

In particular, we view inter-network spectrum allocation as a strategic game $\mathcal{G}(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\})$ between two non-cooperating NSPs and aim to characterize strategies for bandwidth allocation that achieve NE. As we note in the next section, pure strategies, i.e., strategies that allocate a predetermined amount of bandwidth to each of N users, achieve NE only under rare circumstances. This turns our attention to mixed strategies where bandwidth allocation to the N users is governed by an underlying probability distribution. Let the set of all possible mixed strategies of NSP R_i for the game $\mathcal{G}(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\})$ be denoted by $\mathcal{S}_i$, where $\mathcal{S}_i$ consists of all N-variate probability density functions $f_i(w_{i1}, w_{i2}, \cdots, w_{iN})$ with support $\Delta_i = \{\{w_{ik}\}_{k=1}^{N} : \sum_{k=1}^{N} w_{ik} = W_i\}$. Characterizing equilibrium-achieving mixed strategies for $\mathcal{G}(N, \{W_i\}, \{\sigma_{ik}, \{p_k\}\})$ requires establishing a pair of N-variate probability density functions $f_i(\cdot) \in \mathcal{S}_i$ and $f_j \in \mathcal{S}_j$ that satisfy

$$c_i\left(f_i^*, f_j^*\right) \geq c_i\left(f_i, f_j^*\right) \ \forall f_i \in \mathcal{S}_i, \ i \neq j, \tag{2}$$

where $c_i(f_i, f_j)$ denotes the expected payoff to R_i when f_i^* and f_j^* are chosen as the strategies by R_i and R_j respectively. Since the game $\mathcal{G}(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\})$ is a constant-sum game with compact pure strategy spaces and has a semicontinuous payoff function, we can apply a result by Dasgupta and Maskin [12] to establish the following proposition.

Proposition 1: For the spectrum allocation game $\mathcal{G}(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\})$, there always exists a pair of mixed strategies that achieve the NE.

Since this is a constant-sum game, due to the minimax theorem [13], it is not necessary to specify optimal strategy profiles (f_1^*, f_2^*) as a pair, and instead it suffices to establish equilibrium-achieving mixed strategies for each individual NSP which can then be paired in any manner to obtain an optimal strategy profile (f_1^*, f_2^*). Optimal mixed strategies for certain parameter settings of $\mathcal{G}(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\})$ are described in the next section.

From a practical standpoint, it is of interest to also study

[1] Tie resolution depends on whether bandwidth is treated as a continuous or a discrete parameter. In the continuous case, ties are always resolved in favor of NSP R_1, while in the discrete case they are resolved using a coin toss.

the discrete version of the spectrum allocation problem. Although analytical results are difficult to obtain when treating bandwidth as a quantized resource, such a formulation is more amenable to well-known numerical techniques such as fictitious play [11]. Section IV discusses the 2-player constant-sum matrix game that results when bandwidth is treated as a quantized resource and numerically computes the optimal mixed strategies.

III. Inter-Network Spectrum Allocation as a Continuous Colonel Blotto Game

Proposed as early as 1921 by Borel, CBG is one of the best examples of resource allocation in a competitive environment. It closely mirrors the spectrum allocation problem that is of interest here, but is presented in the context of a war between two colonels over multiple battlefields. The canonical CBG involves two colonels (players) B_1 and B_2 engaged in a war over N battlefields with a total of T_1 and T_2 troops (assume $T_1 \leq T_2$) at their disposal. The colonels strategically assign the available troops among the N battlefields, with the winner of each battlefield determined to be the colonel assigning the greater number of troops to that battlefield. Assuming the kth battlefield to have a payoff of q_i, the goal for each colonel is to assign troops in such a manner that the total payoff is maximized. Denoting t_{ik} as the troops assigned by B_i to the kth battlefield, the troop assignments must satisfy $\sum_{k=1}^{N} t_{ik} \leq T_i$. CBG is typically studied as a continuous game with the troops T_1 and T_2 being treated as infinitely divisible. This is a constant-sum game and a NE in mixed strategies exists due to the result by Dasgupta and Maskin [12]. Early studies on CBG [8] assumed symmetric colonels ($T_1 = T_2$) and symmetric battlefields ($q_i = q_j \ \forall i, j$), a setup called doubly-symmetric CBG. More recently, CBG with symmetric colonels but asymmetric battlefields is studied in [14], while CBG with asymmetric colonels but symmetric battlefields is studied in [7]. To the best of our knowledge, there are no known results when both symmetries are broken. While other variants of the CBG have also been studied, they are not immediately relevant to the spectrum allocation problem.

The analogy between CBG and inter-network spectrum allocation is immediate once we note that the NSPs play the role of colonels, with bandwidth as their constrained resource (troops), and users serving as the N-battlefields. Thus, denoting the CBG as $\mathcal{B}\left(N, \{T_i\}, \{q_i\}\right)$, it is straightforward to establish the following proposition.

Proposition 2: When spectral efficiencies $\{\sigma_{ik}\}$ satisfy $\sigma_{1k} = \sigma_{2k} \ \forall k$, the inter-network spectrum allocation game $\mathcal{G}\left(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\}\right)$ is equivalent to the colonel Blotto game $\mathcal{B}\left(N, \{W_i\}, \{p_i\}\right)$.

This equivalence allows us to reframe equilibrium strategies for the CBG in the context of inter-network spectrum allocation. Despite its relatively simple formulation, equilibrium-achieving mixed strategies for the CBG are only known for certain parameter settings. Similar to the spectrum allocation game, a mixed strategy for the CBG is an N-variate density function with support contained in the set of feasible allocations of the troops. Typically, characterizing the mixed strategies that achieve NE is split into two parts, one focused on specifying the N univariate marginal distributions of the N-variable equilibrium distribution, and the other on constructing an N-variate distribution that has the appropriate univariate marginal distributions.

Proposition 2 is most relevant to a SNR-agnostic spectrum allocation game where the spectral efficiencies are not immediately available and assumed to be 1. In such a setting, the results on equilibrium mixed strategies in [7], [14] can be immediately adapted to establish results of the following form:

Theorem 3 (based on Proposition 1 in [14]): For a spectrum allocation game $\mathcal{G}\left(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\}\right)$ that satisfies (a) $W_1 = W_2$ (symmetric colonels), (b) $\sigma_{1k} = \sigma_{2k} \ \forall k$ (SNR-agnostic) and (c) $p_k < \sum_{j \neq k} p_j \ \forall \ k$ (no dominant user), any N-variate probability density function with support Δ_i where the kth univariate marginal density function is uniformly distributed on $\left[0, \frac{2W_i p_i}{\sum p_i}\right]$ constitutes an equilibrium-achieving mixed strategy providing equal payoffs to both the NSPs.

Theorem 4 (based on Theorem 2 in [7]): For a spectrum allocation game $\mathcal{G}\left(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\}\right)$ that satisfies (a) $\frac{2}{N} \leq \frac{W_1}{W_2} \leq 1$ (asymmetric colonels), (b) $\sigma_{1k} = \sigma_{2k} \ \forall k$ (SNR-agnostic) and (c) $p_i = p_j \ \forall \ i, j$ (symmetric users), the equilibrium univariate marginal density functions $f_{1i}()$ and $f_{2i}()$ are given by

$$f_{1i}(w_{1i}) \sim (1 - \tfrac{W_1}{W_2})\delta(w_{1i}) + \tfrac{W_1}{W_2}\mathcal{U}([0, \tfrac{2W_2}{N}]) \quad (3)$$

$$f_{2i}(w_{2i}) \sim \mathcal{U}([0, \tfrac{2W_2}{N}]) \quad (4)$$

where $\delta(\cdot)$ denotes the unit impulse function and $\mathcal{U}(\cdot)$ denoted the uniform density function over a specified interval. The equilibrium payoff to NSP 1 is $W_1/2W_2$.

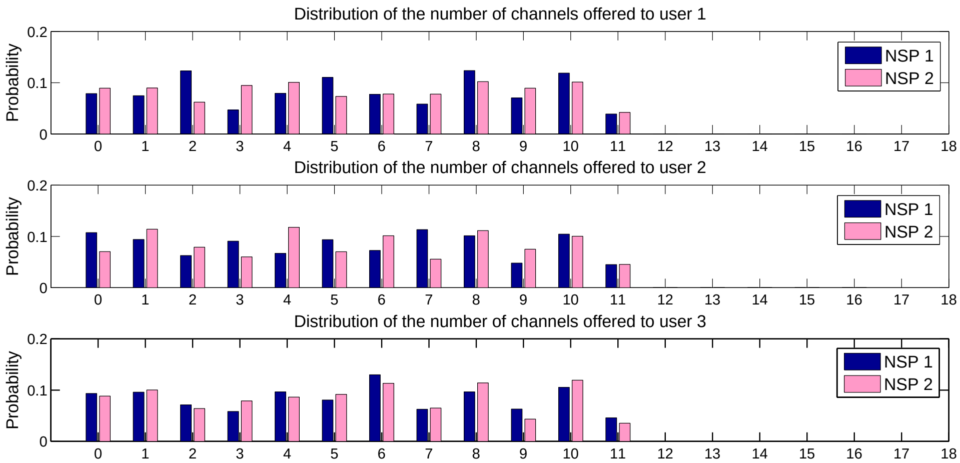
Constructing N-variate distributions that satisfy the above univariate marginals is non-trivial. Geometric methods of construction are proposed in [8], [14]–[16], while other approaches are suggested in [7]. For brevity, we omit the exact details.

As an illustration of these results, suppose two NSPs with 10 MHz each compete to serve a set of 5 users, each offering the same payoff, then according to Theorem 3, the optimal strategy is to offer each user a bandwidth chosen at random from 0 to 4 MHz, while satisfying the bandwidth constraint. Suppose instead, the first NSP only has 5 MHz of bandwidth, then by Theorem 4, a user is allocated non-zero bandwidth only 50% of the time. This in effect halves the total number of users for whom NSP 1 allocates a non-zero bandwidth. Thus, it can be observed that bandwidth-constrained NSPs tend to adopt a strategy whereby they only compete over a random subset of users while bandwidth-rich NSPs tend to spread out the available bandwidth among all the users in the pool.

Extending these results to incorporate spectral efficiencies is a challenging problem and requires further research. However, spectral efficiencies can be naturally factored in when treating bandwidth as a quantized resource, where we rely on computational methods to design equilibrium strategies. This is discussed further in the next section.

IV. Inter-Network Spectrum Allocation: The Discrete Case

The discrete spectrum allocation game is the same as its continuous version, except that the total bandwidth is now specified in terms of the number of orthogonal channels owned by the NSP and the bandwidth allocation is a nonnegative-integer vector specifying the number of channels allocated to



Parameters	
N	3
W_1	16
W_2	16
σ_{1k}	$[1,1,1]$
σ_{2k}	$[1,1,1]$
p_i	$[4,4,4]$

Payoffs	
c_1	6
c_2	6

Fig. 1. Optimal mixed strategy marginal distribution in a 2-NSP 3-user inter-network spectrum allocation problem with symmetric bandwidth availability and equal payoffs across users

the N users in the pool. Due to the integer constraints on bandwidth allocation, the resulting game is a constant-sum two-player matrix game with finite number of strategies. Such a game is known to have NE in mixed strategies, with all such strategies yielding the same payoff. As before, the equilibrium-achieving strategies of the two NSPs can be paired in any manner to obtain an equilibrium strategy profile.

The discrete CBG is an immediate analogue of such a game and Proposition 2 also applies here. However, due to the combinatorial nature of the strategy space, the discrete CBG is not as extensively studied as the continuous CBG. The best result in this context is by Hart [17], who studied the discrete CBG with a primary focus on the doubly symmetric case.

Rather than pursuing analytical results, this section focuses on numerical techniques for computing equilibrium mixed strategies under general parameter settings. In particular, we adopt fictitious play [11], a well-known learning algorithm, to compute the equilibrium mixed strategies. Fictitious play is a belief based learning rule that is commonly used in the context of 2-player matrix games. Fictitious play for two player game simulates a repeated game where the two players play an action/strategy in each round and try to learn the best strategy from the cumulative outcome of all the previous rounds.

Let $\mathcal{G}_D\left(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\}\right)$ denote the discrete spectrum allocation game where $W_i, w_{ik} \in \mathbb{Z}^+ \; \forall i, k$. Denote the set of all possible bandwidth allocations of NSP R_i as $\mathcal{A}_i = \{\mathbf{a}_{i1}, \mathbf{a}_{i2}, ..., \mathbf{a}_{iH_i}\}$ where the allocation $\mathbf{a}_{ik}$ represents an N-integer tuple $\{w_{i1}, w_{i2}, \ldots, w_{iN}\}$ satisfying $\sum_{k=1}^{N} w_{ik} = W_i$. The size of the set A_i, denoted as H_i, is equal to $\binom{W_i+N-1}{W_i}$.

Fictitious play for the game $\mathcal{G}_D\left(N, \{W_i\}, \{\sigma_{ik}\}, \{p_k\}\right)$ simulates an iterated spectrum allocation game between the two NSPs, where after the kth iteration, NSP R_i holds the belief that its opponent is playing this iterative game using a stationary (possibly mixed) strategy that is characterized by the belief vector $\mathbf{q}_i^{(k)} = \left[q_{i1}^{(k)}, q_{i2}^{(k)}, \ldots, q_{iH_i}^{(k)}\right]$, where $q_{il}^{(k)}$ represents the belief held by NSP R_i, after the kth iteration, that R_j's ($j \neq i$) mixed strategy plays the lth action with probability $q_{il}^{(k)}$. In every iteration of this game, R_i updates this belief based on the strategy played by R_j. Thus, if R_j plays the lth strategy at the $(k+1)$th iteration, R_i updates the belief vector as follows:

$$q_{im}^{(k+1)} = \begin{cases} \frac{k}{k+1} q_{im}^{(k)} + \frac{1}{k+1} & m = l \\ \frac{k}{k+1} q_{im}^{(k)} & m \neq l \end{cases} \tag{5}$$

Now, the strategy chosen by R_i at the $(k+1)$th iteration is based on its beliefs at the end of the kth iteration. In particular, R_i chooses the action that maximizes its payoff in response to a mixed strategy of R_j governed by $\mathbf{q}_i^{(k)}$, i.e.,

$$\underset{a_{il} \in A_i}{\arg\max} \quad C_i\left(a_{il}, \mathbf{q}_i^{(k)}\right), \tag{6}$$

where $C_i(\cdot)$ denotes the expected payoff, i.e., $E_{a_{il}, \mathbf{q}_i^{(k)}}[c_i]$.

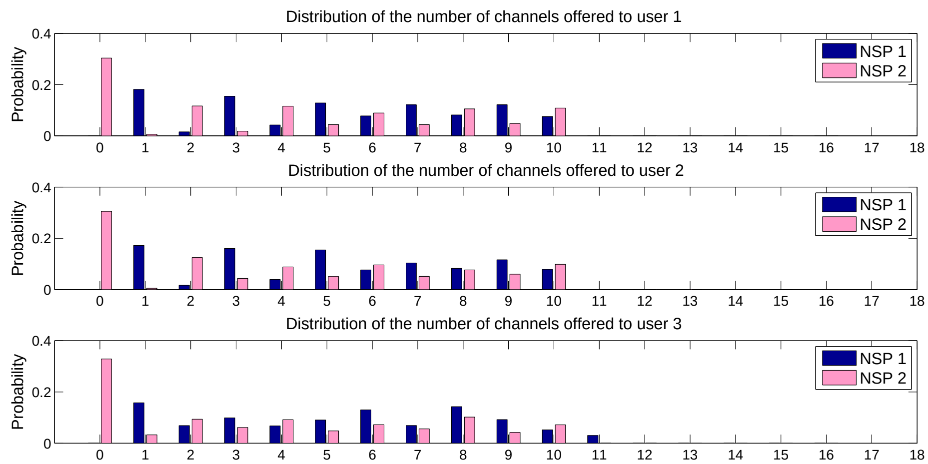
Starting from a random initialization of the belief vectors, this process is repeated until convergence (Convergence to a mixed strategy equilibrium is guaranteed in constant sum games [18]). At convergence, $\mathbf{q}_i^{(k)}$ represents an equilibrium strategy of R_j.

We use this process to numerically compute the equilibrium mixed strategies to the discrete inter-network spectrum allocation problem under general parameter settings.

V. Numerical Results

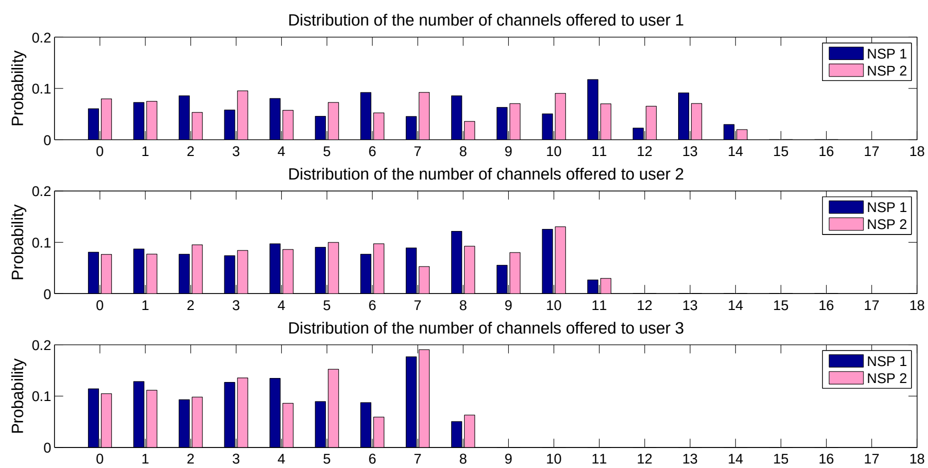
We use fictitious play on a network with two NSPs competing to provide service to three users. We consider four different parameter settings and highlight the important features of the resulting equilibrium mixed strategies.

Case (i): This case considers SNR-agnostic spectrum allocation with symmetric NSPs and equal payoffs for all users. The two NSPs are assumed to have a total of 10 MHz of bandwidth that can only be assigned in multiples of 1.25 MHz (16 channels to be assigned). The mixed strategy obtained for such a scenario is presented in Fig. 1, where it is seen that the resulting univariate marginals randomly allocate up to 11 channels to a user. Interestingly, the marginal distributions are not uniform distributions as predicted by theory in the continuous case [7], [8]. However, the support of the marginal



Parameters	
N	3
W_1	16
W_2	12
σ_{1k}	$[1,1,1]$
σ_{2k}	$[1,1,1]$
p_i	$[4,4,4]$
Payoffs	
c_1	7.5
c_2	4.5

Fig. 2. Optimal mixed strategy marginal distribution in a 2-NSP 3-user inter-network spectrum allocation problem with asymmetric bandwidth availability and equal payoffs across users



Parameters	
N	3
W_1	16
W_2	16
σ_{1k}	$[1,1,1]$
σ_{2k}	$[1,1,1]$
p_i	$[5,4,3]$
Payoffs	
c_1	6
c_2	6

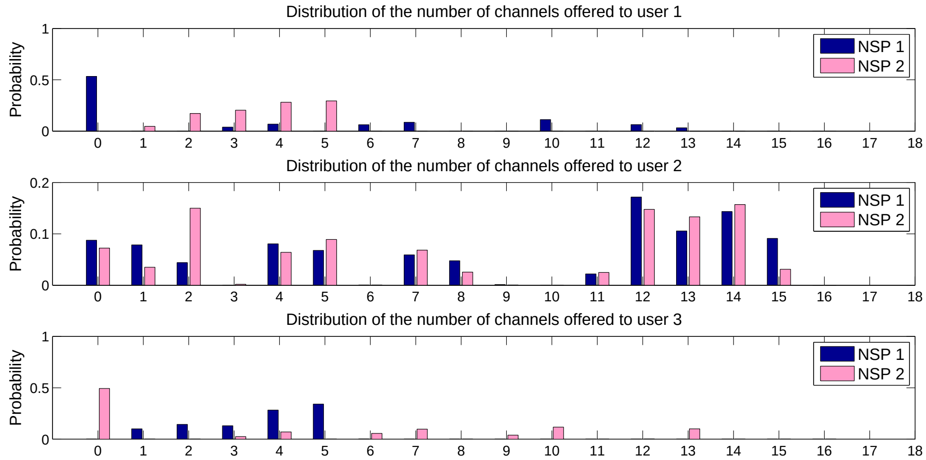
Fig. 3. Optimal mixed strategy marginal distribution in a 2-NSP 3-user inter-network spectrum allocation problem with symmetric bandwidth availability and unequal payoffs across users

distributions is in line with that predicted by theory ($2W_i/N$). Since all three users are identical from an NSP's perspective, the marginal distributions suggest that the NSPs compete to provide service to all users, with no preference given to any of them. By symmetry the two NSPs receive equal payoffs.

Case (ii): This case is similar to the previous case, except that the second NSP is now assumed to have only 12 channels. It is seen from Fig. 2 that the bandwidth-constrained NSP now tries to compete only over a random subset of users. This is inferred by noting that NSP 2 chooses to allocate no channels to a user i with a probability ≈ 0.5. Interestingly, the bandwidth-rich NSP is cognizant of this behavior and ensures that all three users are allocated at least one channel, thus enabling it to win over users that receive no channel allocations from NSP 2, while expending the least amount of channels to win over these users. The resulting payoffs suggest that NSP 2 is likely to serve only one of the three users. These results are in close agreement with those predicted by Theorem 4 for the continuous case, except for non-uniformity of the marginal distributions.

Case (iii): This case uses the same parameters as the first case, except that the users now have unequal payoffs. It can be observed from Fig. 3 that the support of the univariate marginal distributions of the equilibrium mixed strategies is proportional to the user's value. With the NSPs having the same amount of bandwidth at their disposal they compete for all the three users, with a higher interest in winning over the users with larger payoffs. Due to the symmetry among the two NSPs, the net payoff remains equal.

Case (iv): Unlike the previous three cases, this case considers different spectral efficiencies for the user-NSP links, while offering the same payoff for all the users. As seen in Fig. 4 the equilibrium mixed strategies allocate more channels to users with better channel conditions, i.e., higher spectral efficiency. This can be observed by noting NSP 1 (NSP 2) avoids competing for user 1 (user 3) and prefers to not allocate any bandwidth to this user with a probability of 0.5. This feature has a clear practical significance—it shows that such a competitive approach to inter-network spectrum allocation can also capture the salient aspects of user-base-



Parameters	
N	3
W_1	16
W_2	16
σ_{1k}	$[1, 2, 3]$
σ_{2k}	$[3, 2, 1]$
p_i	$[4, 4, 4]$

Payoffs	
c_1	6
c_2	6

Fig. 4. Optimal mixed strategy marginal distribution in a 2-NSP 3-user inter-network spectrum allocation problem with symmetric bandwidth availability and equal payoffs across users but with a different spectral efficiency for each link.

station association in traditional networks, thereby contributing to an increase in the overall throughput across all NSPs. Interestingly, due to similar match-ups between the differences in spectral efficiency, the payoffs get equally divided among the two NSPs.

These results illustrate the broad applicability of fictitious play to compute equilibrium mixed strategies of the inter-network spectrum allocation for any set of parameters. The results further illustrate that the numerically computed strategies are physically meaningful and promote better use of the available spectrum.

VI. Conclusion

This paper considered the problem of spectrum allocation in a network architecture where users are free to choose their network service providers (NSPs) in an opportunistic manner. The NSPs are assumed to compete over a common pool of users by competitively allocating the available bandwidth. Spectrum allocation in such a setup is shown to be closely related to the Colonel Blotto game—a multidimensional resource allocation problem that is well studied in game theory. We cast the inter-network spectrum allocation problem as a CBG and studied it in the case of discrete as well as continuous spectrum (bandwidth) allocation. For the continuous case, we adapted the existing theoretical results for CBG, while a computational approach using fictitious play is used to numerically compute equilibrium mixed strategies in the discrete case. The resulting strategies were analyzed and shown to promote better utilization of available resources across the networks. In summary, the CBG is shown to provide a valuable framework to study competitive spectrum allocation and warrants further investigation.

References

[1] T. Rosowski, J. Kronander, T. Irnich, O. Queseth, M. Tercero, K. Chatzikokolakis, R. Arapoglou, K. Koufos, A. Widaa, J. Markendahl, H. Schotten, M. Uusitalo, L. Campoy, and C. Markwart, "Description of the spectrum needs and usage principles," METIS, Tech. Rep., Sep 2014. [Online]. Available: https://www.metis2020.com/wp-content/uploads/deliverables/METIS_D5.3_v1.pdf

[2] R. Ratasuk, N. Mangalvedhe, and A. Ghosh, "LTE in unlicensed spectrum using licensed-assisted access," *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, pp. 746–751, Dec. 2014.

[3] M. Palola, M. Matinmikko, J. Prokkola, M. Mustonen, M. Heikkila, T. Kippola, S. Yrjola, V. Hartikainen, L. Tudose, A. Kivinen, J. Paavola, and K. Heiska, "Live field trial of licensed shared access (LSA) concept using LTE network in 2.3 ghz band," *Proc. IEEE Symp. on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, pp. 38–47, Apr. 2014.

[4] Y. Teng, Y. Wang, and K. Horneman, "Co-primary spectrum sharing for denser networks in local area," *Proc. EAI Int. Conf. on Cognitive Radio Oriented Wireless Netw. and Commun. (CROWNCOM)*, pp. 120–124, Jun. 2014.

[5] Project Fi. [Online]. Available: https://fi.google.com/

[6] T. Basar, "Control and game-theoretic tools for communication networks," *Appl. Comput. Math*, vol. 6, no. 2, pp. 104–125, 2007.

[7] B. Roberson, "The colonel Blotto game," *Economic Theory*, vol. 29, no. 1, pp. 1–24, 2006.

[8] O. Gross and R. Wagner, "A continuous colonel Blotto game," DTIC Document, Tech. Rep., 1950.

[9] B. Roberson and D. Kvasov, "The non-constant-sum colonel Blotto game," *Economic Theory*, vol. 51, no. 2, pp. 397–433, 2012.

[10] R. Golman and S. E. Page, "General Blotto: games of allocative strategic mismatch," *Public Choice*, vol. 138, no. 3-4, pp. 279–299, 2009.

[11] G. W. Brown, "Iterative solution of games by fictitious play," *Activity analysis of production and allocation*, vol. 13, no. 1, pp. 374–376, 1951.

[12] P. Dasgupta and E. Maskin, "The existence of equilibrium in discontinuous economic games, I: Theory," *The Review of economic studies*, 1986.

[13] M. Sion, "On general minimax theorems," *Pacific J. Math*, vol. 8, no. 1, pp. 171–176, 1958.

[14] C. D. Thomas, "N-dimensional Blotto game with asymmetric battlefield values," Tech. Rep., 2013.

[15] J.-F. Laslier and N. Picard, "Distributive politics and electoral competition," *Journal of Economic Theory*, vol. 103, no. 1, pp. 106–130, 2002.

[16] J. Weinstein, "Two notes on the blotto game," *The BE Journal of Theoretical Economics*, vol. 12, no. 1, 2012.

[17] S. Hart, "Discrete colonel blotto and general lotto games," *International Journal of Game Theory*, vol. 36, no. 3-4, pp. 441–460, 2008.

[18] D. Monderer and L. S. Shapley, "Fictitious play property for games with identical interests," *Journal of Economic Theory*, vol. 68, no. 1, pp. 258–265, 1996.