Sparse Sensing in Colocated MIMO Radar: A Matrix Completion Approach

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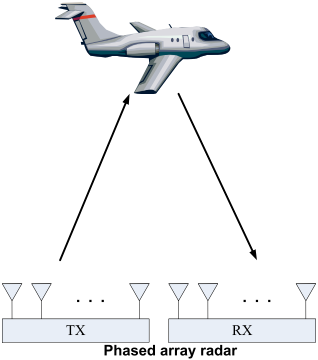
Motivation

- There is increasing interest in networked radars that are inexpensive and enable reliable surveillance. Unfortunately, these requirements are competing in nature.
- In a networked radar the processing can be done at a fusion center, which collects the measurements of all receive antennas. Reliable surveillance requires collection, communication and fusion of vast amounts of data from various antennas, which is a bandwidth and power intensive task.
- The communication with the fusion center could occur via a wireless link (radar on a wireless sensor network).
- MIMO radars have received considerable recent attention as they can achieve superior resolution.
- The talk presents new results on networked MIMO radars that rely on advanced signal processing, and in particular, sparse sensing and matrix completion, in order to achieve an optimal tradeoff between reliability and cost (bandwidth, power).
- These techniques will enable the radar to meet the same operational objectives with traditional MIMO radars while involving significantly fewer samples, be robust, and operate on mobile platforms.

Phased Array Radars



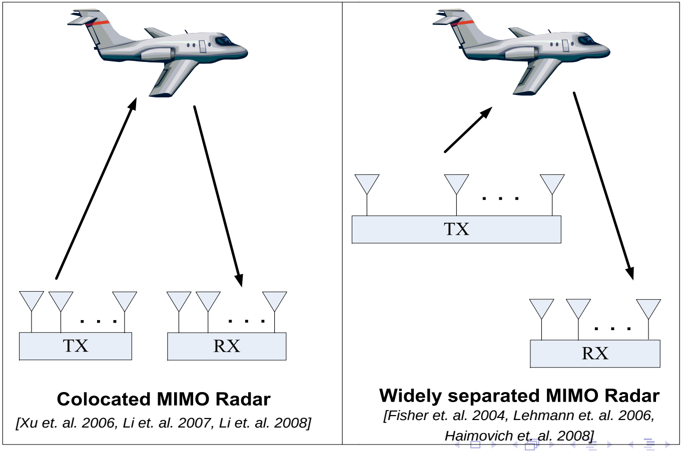
A phased array radar

- is composed of many closely spaced antennas
- all antennas transmit the same waveform
- is capable of cohering and steering the transmit energy

MIMO Radars

Multiple input multiple output (MIMO) radar

- employs colocated TX/RX antennas or widely separated TX/RX antennas;
- uses multiple waveforms:
  - Independent waveforms ⇒ omnidirectional beampattern
  - Correlated waveforms ⇒ desired beampattern



Matrix Completion

[Candes & Recht, 2009],[Candes & Tao, 2010],[Candes & Plan, 2010]

- Matrix completion is done by solving a relaxed nuclear norm optimization problem

$$\begin{aligned} \min \ & \|\mathbf{X}\|_* \\ \text{s.t.} \ & \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M}) \end{aligned} \tag{1}$$

where where Ω is the set of indices of observed entries with cardinality m, and the observation operation is defined as

$$[\mathbf{Y}]_{ij} = \begin{cases} [\mathbf{M}]_{ij}, & (i,j) \in \Omega \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

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- For noisy observations: $[\mathbf{Y}]_{ij} = [\mathbf{M}]_{ij} + [\mathbf{E}]_{ij}, (i,j) \in \Omega$

$$\begin{aligned} \min \ & \|\mathbf{X}\|_* \\ \text{s.t.} \ & \|\mathcal{P}_\Omega(\mathbf{X} - \mathbf{Y})\|_F \leq \delta, \end{aligned} \tag{3}$$

Matrix Coherence and Recovery Guarantee

Definition

Let U be a subspace of $\mathbb{C}^{n_1}$ of dimension r that is spanned by the set of orthogonal vectors $\{\mathbf{u}_i \in \mathbb{C}^{n_1}\}_{i=1,\ldots,r}$, P_U the orthogonal projection onto U, i.e., $P_U = \sum_{1\le i\le r} \mathbf{u}_i\mathbf{u}_i^H$, and $\mathbf{e}_i$ the standard basis vector whose i-th element is 1. The coherence of U is defined as

$$\mu(U) = \frac{n_1}{r} \max_{1\le i\le n_1} \|P_U\mathbf{e}_i\|^2 \quad \in \left[1, \frac{n_1}{r}\right]$$

$$\equiv \frac{n_1}{r} \max_{1\le i\le n_1} \left\|\mathbf{U}^{(i)}\right\|^2; \qquad \mathbf{U} = [\mathbf{u}_1, ..., \mathbf{u}_r] \tag{4}$$

$\mathbf{U}^{(i)}$: *$i-th$ row of* $\mathbf{U}$

-
- Consider the compact SVD of $\mathbf{M}$, i.e., $\mathbf{M} = \sum_{k=1}^{r} \rho_k\mathbf{u}_k\mathbf{v}_k^H = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$

  Matrix $\mathbf{M}$ has coherence with parameters μ_0 and μ_1 if
  (**A0**) $\max(\mu(U), \mu(V)) \le \mu_0$ for some positive μ_0.
  (**A1**) The maximum element of $\sum_{1\le i\le r} \mathbf{u}_i\mathbf{v}_i^H$ is bounded by $\mu_1\sqrt{r/(n_1n_2)}$ in absolute

Matrix Coherence and Recovery Guarantee, Continued

- Suppose that matrix $\mathbf{M} \in \mathbb{C}^{n_1 \times n_2}$ satisfies (**A0**) and (**A1**). The following lemma gives a probabilistic bound for the number of entries, m, needed to estimate $\mathbf{M}$.

Theorem

[Candès & Recht 2009] Suppose that we observe m entries of the rank$-r$ matrix $M \in \mathbb{C}^{n_1 \times n_2}$, with matrix coordinates sampled uniformly at random. Let $n = \max\{n_1, n_2\}$. There exist constants C and c such that if

$$m \geq C \max\left\{\mu_1^2, \mu_0^{1/2}\mu_1, \mu_0 n^{1/4}\right\} nr\beta \log n$$

for some $\beta > 2$, the minimizer of the nuclear norm problem is unique and equal to $\mathbf{M}$ with probability at least $1 - cn^{-\beta}$.
For $r \leq \mu_0^{-1} n^{1/5}$ the bound can be improved to

$$m \geq C\mu_0 n^{6/5} r\beta \log n,$$

without affecting the probability of success.

Generic Assumptions for Colocated MIMO Radar

- Transmission antennas transmit **narrowband and orthogonal** waveforms, that is,

$$\frac{1}{T_p} \ll \frac{c}{\lambda}, \tag{5}$$

where $T_p \in \mathbb{R}$, $\lambda \in \mathbb{R}$ and $c \equiv 3 \cdot 10^8$ m/s denotes the waveform duration, the communication wavelength and the speed of light, respectively.

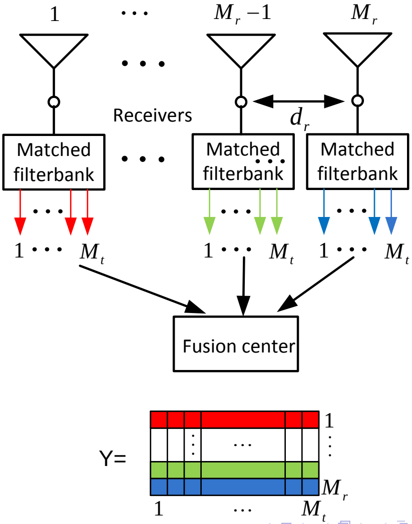
- The target reflection coefficients $\{\beta_i \in \mathbb{C}\}_{i \in \mathbb{N}_K^+}$ (K is the number of targets in the far field) remain constant during a number of pulses Q.
- The delay spread in the received signals is smaller that the temporal support of each waveform T_p.
- The Doppler spread of the received signals is much smaller than the bandwidth of the pulse, that is,

$$\frac{2\vartheta_i}{\lambda} \ll \frac{1}{T_p}, \quad \forall i \in \mathbb{N}_K^+ \tag{6}$$

where $\vartheta_i \in \mathbb{R}$ denotes the speed of the respective target.

MIMO Radar with Matric Completion (MC-MIMO)

- orthogonal transmit waveforms,
- K targets



MC-MIMO Radar (2)

[Kalogerias, Petropulu, IEEE TSP 2014, GLOBALSIP 2013], [Sun, Bajwa, Petropulu, IEEE AES 2014, IEEE ICASSP 2013]

It can be shown that the fully observed version of the data matrix formulated at the fusion center can be expressed as

$$\mathbf{Y} \triangleq \mathbf{\Delta} + \mathbf{Z} \in \mathbb{C}^{M_r \times M_t}, \tag{7}$$

where $\mathbf{Z}$ is an interference/observation noise matrix that may also describe model mismatch due to weak correlations among the transmit waveforms and

$$\mathbf{\Delta} \triangleq \mathbf{X}_r \mathbf{D} \mathbf{X}_t^{\boldsymbol{T}}, \tag{8}$$

where $\mathbf{X}_r \in \mathbb{C}^{M_r \times K}$ (respectively for $\mathbf{X}_t \in \mathbb{C}^{M_t \times K}$) constitutes an *alternant* matrix defined as

$$\mathbf{X}_r \triangleq \begin{bmatrix} \gamma_0^0 & \gamma_1^0 & \cdots & \gamma_{K-1}^0 \\ \gamma_0^1 & \gamma_1^1 & \cdots & \gamma_{K-1}^1 \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_0^{M_r-1} & \gamma_1^{M_r-1} & \cdots & \gamma_{K-1}^{M_r-1} \end{bmatrix} \in \mathbb{C}^{M_r \times K}, \tag{9}$$

...

MC-MIMO Radar (3)

...with

$$\gamma_k^l \triangleq e^{j2\pi \mathbf{r}_r^{\boldsymbol{T}}(l)\mathcal{T}(\theta_k)}, \quad (l,k) \in \mathbb{N}_{M_r-1} \times \mathbb{N}_{K-1} \tag{10}$$

$$\mathbf{r}_r(l) \triangleq \frac{1}{\lambda}\left[x_l^r \; y_l^r\right]^{\boldsymbol{T}} \in \mathbb{R}^{2\times 1}, \quad l \in \mathbb{N}_{M_r-1} \quad \text{and} \tag{11}$$

$$\mathcal{T}(\theta_k) \triangleq \begin{bmatrix} \cos(\theta_k) \\ \sin(\theta_k) \end{bmatrix} \in \mathbb{R}^{2\times 1}, \quad k \in \mathbb{N}_{K-1}. \tag{12}$$

- The sets $\left\{\left[x_l^r \; y_l^r\right]^{\boldsymbol{T}}\right\}_{l\in\mathbb{N}_{M_r-1}}$ and $\{\theta_k\}_{k\in\mathbb{N}_{K-1}}$ contain the 2-dimensional antenna coordinates of the reception array and the target angles, respectively,
- $\lambda \in \mathbb{R}_{++}$ denotes the carrier wavelength, and
- $\mathbf{D} \in \mathbb{C}^{K\times K}$ is a non-zero diagonal matrix whose elements depend on the target reflection properties and the speeds.

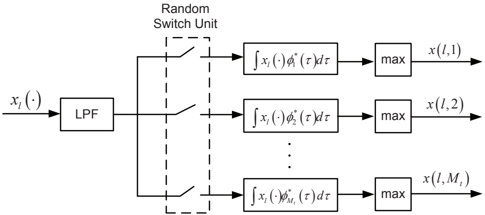
For the simplest ULA case,

$$\left[x_l^{r(t)} \; y_l^{r(t)}\right]^{\boldsymbol{T}} \equiv \left[0 \; l d_{r(t)}\right]^{\boldsymbol{T}}, \quad l \in \mathbb{N}_{M_{r(t)}-1}. \tag{13}$$

and $\mathbf{X}_r$ and $\mathbf{X}_t$ degenerate to Vandermonde matrices.

MC-MIMO Radar: Sampling Scheme I

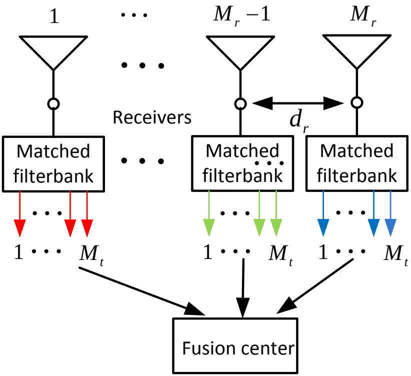
Sparse Sensing is implemented through the following **Random Matched Filter Bank** (RMFB) architecture.

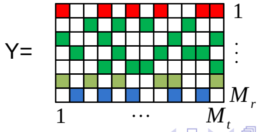


- Simple power saving Bernoulli switching with selection probability p.
- RMFBs are implemented in receiver, constructing the Bernoulli subsampled version of $\mathbf{\Delta}$, $\mathcal{P}(\mathbf{\Delta})$.
- Matrix Completion is applied for the stable recovery of $\mathbf{\Delta}$.
- The recovered matrix is fed into standard array processing methods (e.g. MUSIC) for extracting target information.

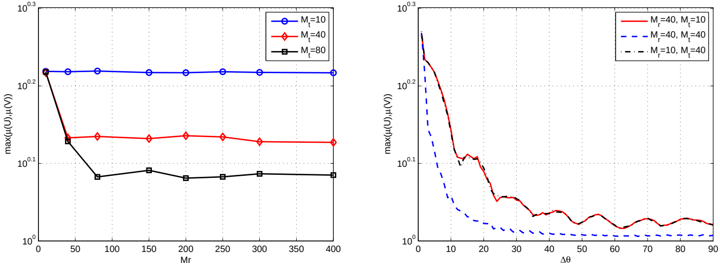
MC-MIMO Radar: Sampling Scheme I

[Kalogerias, Petropulu, IEEE TSP 2014, GLOBALSIP 2013], [Sun, Bajwa, Petropulu, IEEE AES 2014, IEEE ICASSP 2013]



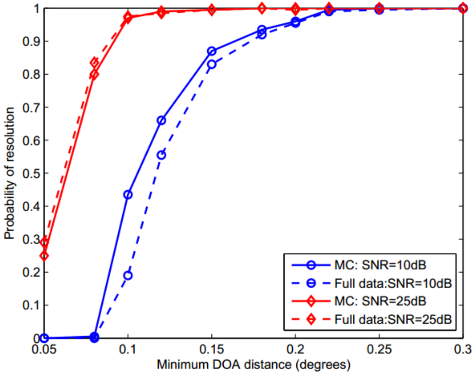


Coherence bounds - Simulations results



Scheme I, $K = 2$ targets: (a) the average $\max\left(\mu\left(U\right), \mu\left(V\right)\right)$ of $\mathbf{Z}_q^{MF}$ as function of number of transmit and receive antennas, and for $\Delta\theta = 5^\circ$; (b) the average $\max\left(\mu\left(U\right), \mu\left(V\right)\right)$ of $\mathbf{Z}_q^{MF}$ as function of DOA separation.

DOA resolution - Simulations results



Scheme I: DOA resolution. The parameter are set as $M_r = M_t = 20$, $p_1 = 0.5$ and $\mathrm{SNR} = 10, 25\mathrm{dB}$.

Recoverability & Performance Guarantees: Scheme I [Kalogerias & Petropulu, 2013, 2014]

Useful results:

Theorem

[Wolkowicz 1980] *Let* $\mathbf{M} \in \mathbb{C}^{N \times N}$ *be a matrix with real eigenvalues. Define*

$$\tau \triangleq \frac{\mathrm{tr}\left(\mathbf{M}\right)}{N} \quad \textit{and} \quad s^2 \triangleq \frac{\mathrm{tr}\left(\mathbf{M}^2\right)}{N} - \tau^2. \tag{14}$$

Then, it is true that

$$\tau - s\sqrt{N-1} \leq \lambda_{min}\left(\mathbf{M}\right) \leq \tau - \frac{s}{\sqrt{N-1}} \quad \textit{and} \tag{15}$$

$$\tau + \frac{s}{\sqrt{N-1}} \leq \lambda_{max}\left(\mathbf{M}\right) \leq \tau + s\sqrt{N-1}. \tag{16}$$

Further, equality holds on the left (right) of (15) *if and only if equality holds on the left (right) of* (16) *if and only if the* $N-1$ *largest (smallest) eigenvalues are equal.*

Recoverability & Performance Guarantees: Scheme I (2)

We can show that if $\mathbf{\Delta} = \mathbf{X}_r \mathbf{D} \mathbf{X}_t^{\boldsymbol{T}} \in \mathbb{C}^{M_r \times M_t}$,

- $\mu(U) \leq \dfrac{M_r}{\lambda_{min}\left(\mathbf{X}_r^{\mathbf{H}} \mathbf{X}_r\right)}$ and $\mu(V) \leq \dfrac{M_t}{\lambda_{min}\left(\mathbf{X}_t^{\mathbf{H}} \mathbf{X}_t\right)}$.
- For a ULA, the elements of $\mathbf{X}_t^{\mathbf{H}} \mathbf{X}_t$ (respectively for $\mathbf{X}_r^{\mathbf{H}} \mathbf{X}_r$) are of the form

$$\delta_{i,j} \triangleq \sum_{m=0}^{M_t-1} e^{j2\pi m\left(\alpha_i^t - \alpha_j^t\right)}, \ \forall \ (i,j) \in \mathbb{N}_{K-1} \times \mathbb{N}_{K-1}. \tag{17}$$

- The trace of $\mathbf{X}_t^{H} \mathbf{X}_t$ is $M_t K$.

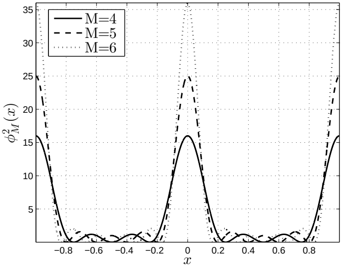
$$\operatorname{tr}\left(\left(\mathbf{X}_t^H \mathbf{X}_t\right)^2\right) = \sum_{k_1=0}^{K-1} \left\{ M_t^2 + \sum_{\substack{k_2=0 \\ k_1 \neq k_2}}^{K-1} \frac{\sin^2\left(\pi M_t \left(\alpha_{k_1}^t - \alpha_{k_2}^t\right)\right)}{\sin^2\left(\pi\left(\alpha_{k_1}^t - \alpha_{k_2}^t\right)\right)} \right\}$$

$$\equiv \sum_{k_1=0}^{K-1} \left\{ M_t^2 + \sum_{\substack{k_2=0 \\ k_1 \neq k_2}}^{K-1} \phi_{M_t}^2 \left(\alpha_{k_1}^t - \alpha_{k_2}^t\right) \right\} \leq \sum_{k_1=0}^{K-1} \left\{ M_t^2 + (K-1) \sup_{x \in \left[\xi_t, \frac{1}{2}\right]} \phi_{M_t}^2 (x) \right\}$$

$$\triangleq K M_t^2 + K(K-1)\beta_{\xi_t}(M_t).$$

$$\alpha_k^r \triangleq \frac{d_r \sin(\theta_k)}{\lambda}$$

ξ_t : smallest $\alpha_i^t - \alpha_j^t$ folded in$[0, \frac{1}{2}]$



Recoverability & Performance Guarantees: Scheme I (4)

Theorem

(Brief Version) **(Coherence for ULAs)** *Consider a Uniform Linear Array (ULA) transmission - reception pair and assume that the set of target angles* $\{\theta_k\}_{k\in\mathbb{N}_{K-1}}$ *consists of almost surely distinct members. Then, for any fixed* M_t *and* M_r*, as long as*

$$K \le \min_{i\in\{t,r\}} \left\{ \frac{M_i}{\sqrt{\beta_{\xi_i}(M_i)}} \right\}, \tag{18}$$

the associated matrix $\boldsymbol{\Delta}$ *obeys the assumptions* **A0** *and* **A1** *with*

$$\mu_0 \triangleq \max_{i\in\{t,r\}} \left\{ \frac{M_i}{M_i - (K-1)\sqrt{\beta_{\xi_i}(M_i)}} \right\} \quad \text{and} \quad \mu_1 \triangleq \mu_0\sqrt{K}.$$

In the above $\sqrt{\beta_{\xi_i}(M_i)}$ *denotes a constant dependent on* M_i*.* ξ_i*,* $i \in \{t,r\}$*, mostly depends on the pairwise differences* $|\sin(\theta_i) - \sin(\theta_j)|, (i,j) \in \mathbb{N}_{K-1} \times \mathbb{N}_{K-1}, i \neq j$.

Conclusions

- We have investigated the problem of reducing the volume of data typically required for accurate target detection and estimation in colocated MIMO radars.
- We have presented a sparse sensing scheme for information acquisition, leading to the natural formulation of a low rank matrix completion problem, which can be efficiently solved using convex optimization.
- Numerical simulations have justified the effectiveness of our approach.
- We have presented theoretical results, guaranteeing near optimal performance of the respective matrix completion problem, for the case where ULAs are employed for transmission and reception.

Relevant Publications

- D. Kalogerias and A. Petropulu, "Matrix Completion in Colocated MIMO Radar: Recoverability, Bounds & Theoretical Guarantees," *IEEE Transactions on Signal Processing*, Volume 62, Issue: 2, Page(s): 309 - 321, 2014.
- S. Sun, W. Bajwa and A. Petropulu, "MIMO-MC Radar: A MIMO Radar Approach Based on Matrix Completion," *IEEE Trans. on Aerospace and Electronic Systems*, under review in 2014.
- S. Sun, A. P. Petropulu and W. U. Bajwa, "High-resolution networked MIMO radar based on sub-Nyquist observations," *Signal Processing with Adaptive Sparse Structured Representations Workshop (SPARSE)*, EPFL, Lausanne, Switzerland, July 8-11, 2013.
  item S. Sun, A. P. Petropulu and W. U. Bajwa, "Target estimation in colocated MIMO radar via matrix completion," in Proc. of *International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vancouver, Canada, May 26-31, 2013.