Efficient Emulation of Wideband Channels with Interference

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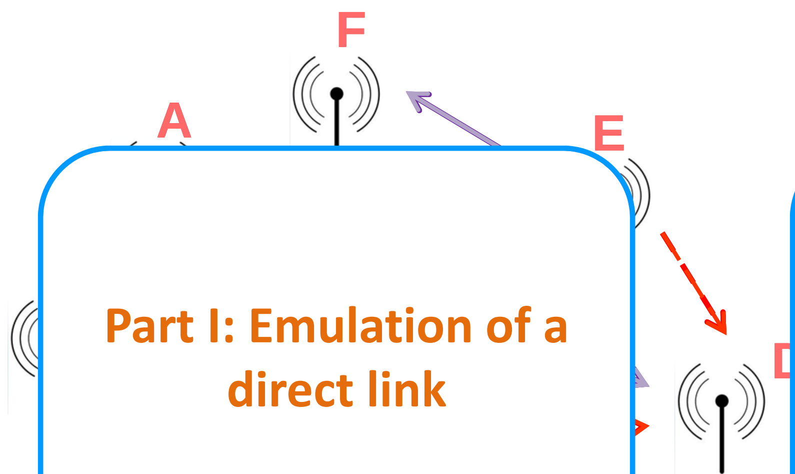
Multi-node Wireless Network

Assuming dispersive, multipath fading channel



Part I: Emulation of a direct link

Part II: Emulation of interfering links

For commu… …ation link between A and D

$$\underbrace{r_D(t) = h_{AD}(t) * X_A(t)}_{\text{Signal of interest}} + \underbrace{h_{ED}(t) * X_E(t) + h_{CD}(t) * X_C(t)}_{\text{Interference}} + \eta_E$$

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Network Emulation

Representation of Multipath Channel Response

- Multi-tapped delay line with Rayleigh-fading tap gains and taps spacing of $1/W$, where W is signal bandwidth
- Assuming RMS delay spread τ_{rms}, number of n significant taps = multiplication of $W\tau_{rms}$
- For wideband channel, value of n can be very high

Complexity of Emulation

- For a single communication link
  - Convolution with n-tap filter
  - Computation complexity = $O(n)$
- For K node network
  - Number of possible communication links = $K(K - 1)$
  - Overall computational complexity = $O(n K(K - 1))$

Thus, need to reduce channel emulation complexity while maintaining important characteristics!!

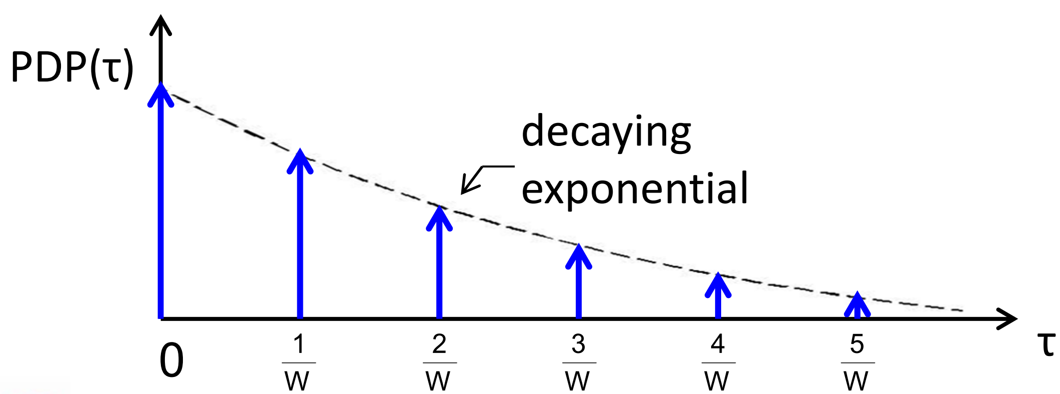
Baseline: “True” Channel

- Assumed power delay profile (PDP) shape for multipath channel: decaying exponential, exp(-τ/ τ_{rms}), $\tau \geq 0$.
- The PDP is a sum of impulses sampling this function, with uniform spacing of 1/*W*, where *W* is signal bandwidth, and the sum of powers is normalized to 1:

$$\mathrm{PDP}(\tau) = \sum_k [A \exp\left(\tau/\tau_{rms}\right)]\delta(\tau - k/W) \ ;$$

where *A is a normalizing constant.*

- Why exponential PDP?
  - Easy to use in analysis and simulation, and is widely reported by several measurement programs



Reduced Tap Direct Links

$$r_D(t) = \underbrace{h_{AD}(t) * X_A(t)} + h_{ED}(t) * X_E(t) + h_{CD}(t) * X_C(t) + \eta_E$$

- **Goal**: To find a 3-tap version that predicts link performance close to that for the 'true' channel
- **Approach**: To match the power *delay profile* (PDP) of the 'true' channel and the PDP of 3-tap channel.
- **Power Delay Profile**: The PDP of any multipath channel is a sum of multipath echoes,

$$\mathrm{PDP}(\tau) = \sum_{k=0}^{2} P_k \delta(\tau - T_k)$$

where P_k is the power in the k^{th} echo, and T_k is its delay (relative to the first echo).

- At the kth tap, the signal gain $g_k(t) = [P_k]^{1/2} u_k(t)$, where $u_k(t)$ is a random variable or time-varying process with zero mean and unit standard deviation.
- By convention, the P_k's are normalized so as to sum to 1.

3-tap Channel Approximation

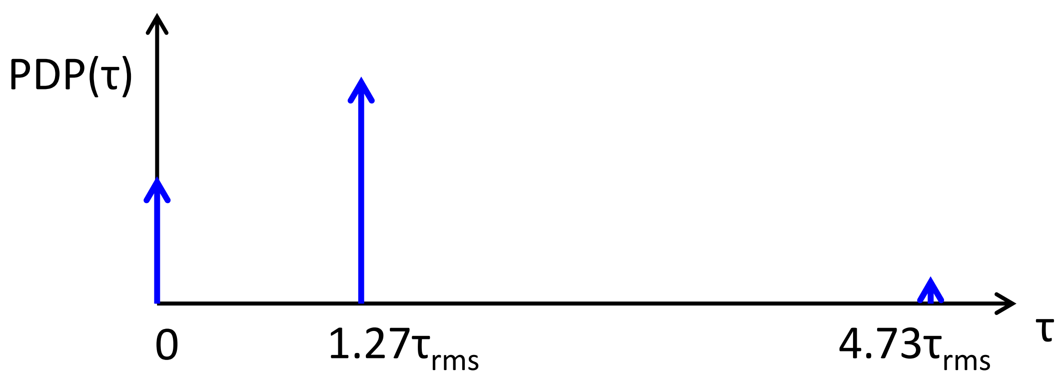
A Moment-Matching Approach: For low $W\tau_{rms}$($\leq$ 2)

- **Goal**: To match first four moments of 3-tap approximated channel to those for 'true' channel i.e. τ_{rms}, $2(\tau_{rms})^2$, $6(\tau_{rms})^3$ and $24(\tau_{rms})^4$ respectively

- Subject to $P_0 + P_1 + P_2 = 1$ and $T_0 = 0$, the PDP of the 3-tap filter is

$$\mathrm{PDP}(\tau) = [1 - P_1 - P_2]\,\delta(0) + P_1\delta(\tau - T_1) + P_2\delta(\tau - T_2)$$

- P_0, P_1, P_2, T_1, T_2 can be solved so the first 4 moments of the two channels match

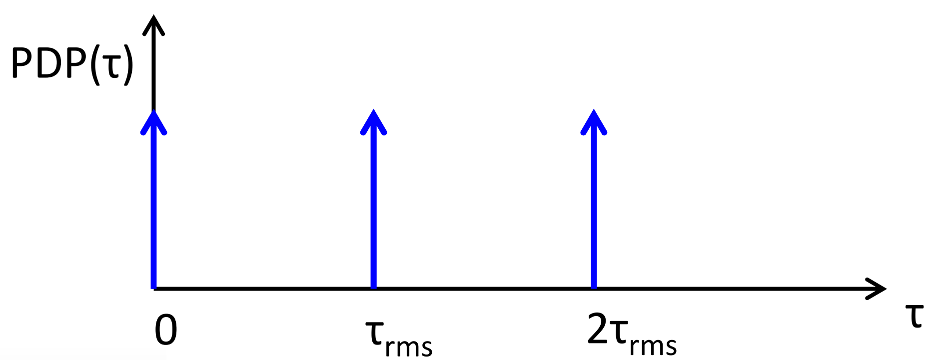


$(P_0, T_0) = (0.3333, 0)$
$(P_1, T_1) = (0.6220, 1.2679\tau_{rms})$
$(P_2, T_2) = (0.0447, 4.7318\tau_{rms})$

3-tap Channel Approximation

An Alternative 3-tap Channel – An Ad Hoc Approach: For high $W\tau_{rms}$($\geq$ 2)

- As $W\tau_{rms}$ increases beyond the order of 1 or 2, power series modeling of $H(f)$ is no longer useful; too much variation of $H(f)$ over the bandwidth, W.
- To better capture the variation, more energy is needed in the gain of the 3rd tap.
- A proposed alternative 3-tap design closely matches the first two PDP moments
- This design is intuitive, but *ad hoc*; a more rigorous design method is needed.

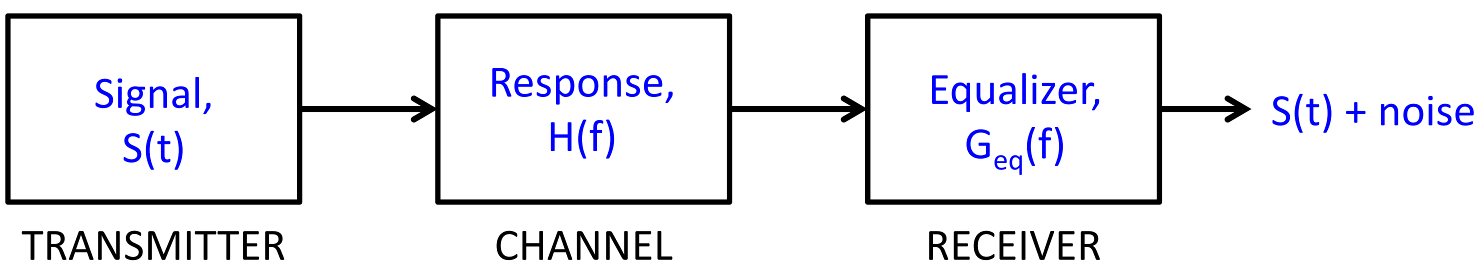


$(P_0, T_0) = (1/3, 0)$
$(P_1, T_1) = (1/3, \tau_{rms})$
$(P_2, T_2) = (1/3, 2\tau_{rms})$

Method of Evaluation

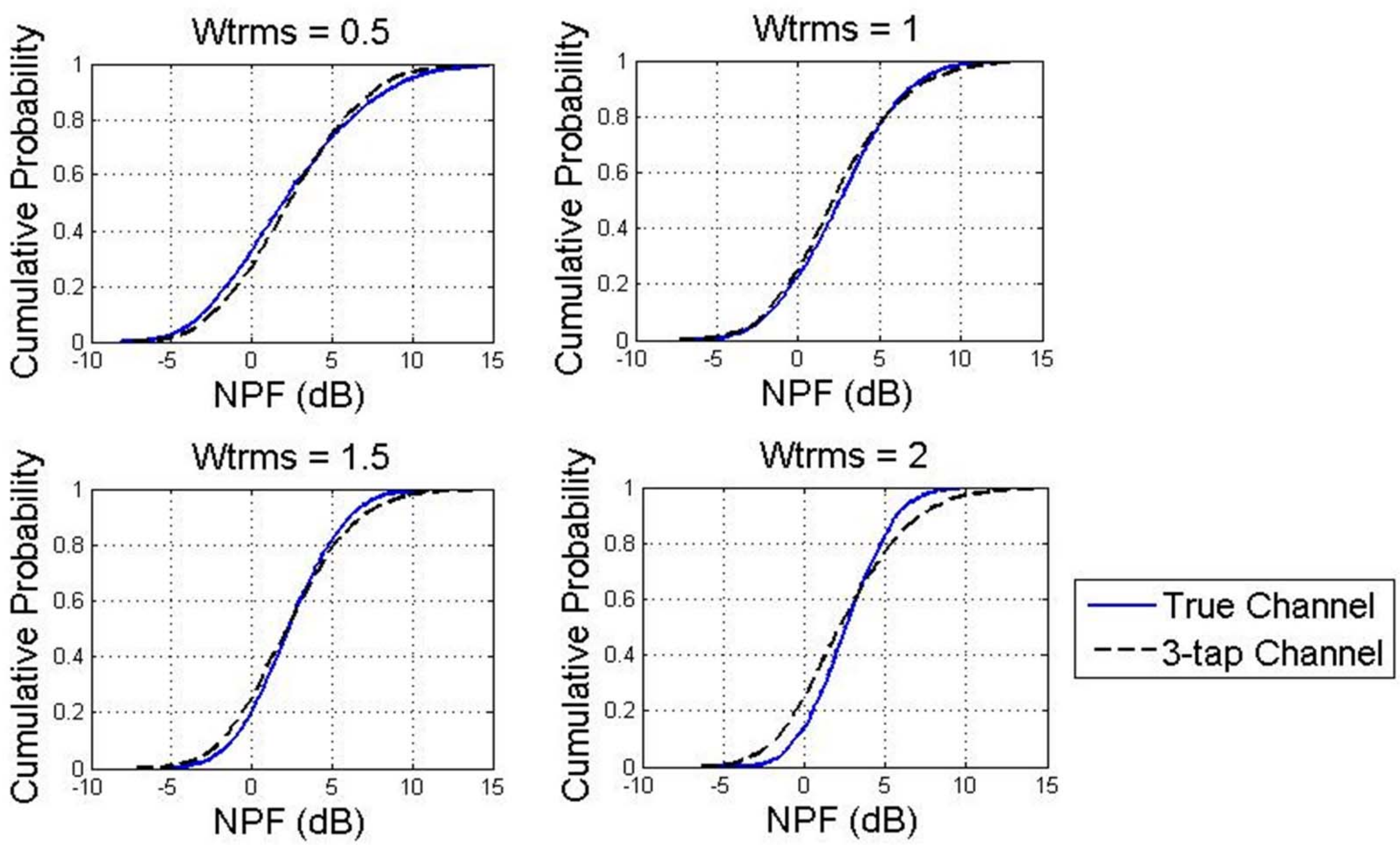
- Performance of a radio link is evaluated based on
  - We assume a single-carrier transmission and a ZF-equalized receiver.
  - If the channel response is $H(f)$, the receiver equalizer response is $G_{eq}(f) = 1/H(f)$.
  - The receiver output has no inter-symbol interference, but increased noise power.
  - The noise increase is the integral, over the signal band, of $1/|H(f)|^2$.
  - This is the *noise peaking factor* (NPF), random over the random realizations of $H(f)$.

- Given 1,000's of values of the NPF, we can compute <*BER*> vs. *SNR,* where <*BER*> is the bit error rate averaged over 1000's of fades, *SNR* is at the receiver input.



Sample results

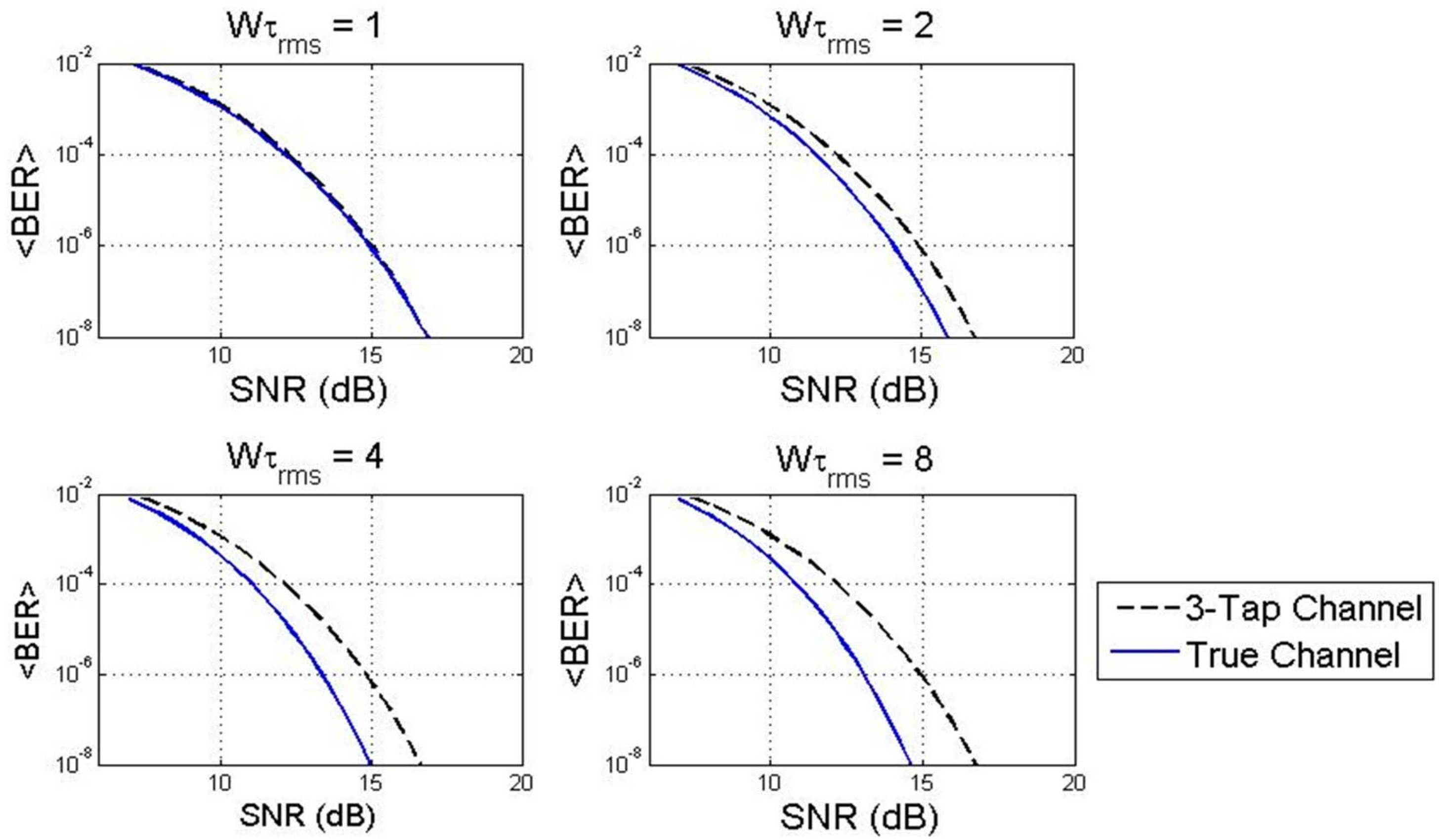
CDFs of Noise Peaking Factor



Results are based on moment matching 3-tap channel. Note that the CDF's for the true and 3-tap channels start to separate at $W\tau_{rms}$ ~ 2.

Sample results

<BER> vs. SNR



Results are based on moment matching 3-tap channel. Note that the separation between curves increases with $W\tau_{rms}$, leveling off at high values.

Sample results

Comparison of moment-matching and ad hoc approach

Discrepancies in SNR @ <*BER*> = 10^{-8}

$W\tau_{rms}$	Moment-Matching 3-tap Channel	Ad Hoc 3-tap Channel
1	**0.22**	0.67
2	0.91	**0.22**
4	1.76	**0.95**
8	2.06	**1.42**
16	2.46	**1.75**
32	2.69	**1.94**
64	2.64	**2.04**
128	2.61	**1.91**

NOTE: Discrepancies in *SNR* @ <*BER*> = 10^{-4} are roughly half those at 10^{-8}.

Reduced Tap Interfering Links

$$r_D(t) = h_{AD}(t) * X_A(t) + \underbrace{h_{ED}(t) * X_E(t) + h_{CD}(t) * X_C(t)} + \eta_E$$

- **Goal**: To determine reduced number of per-interference-link filter taps, *n*, to model aggregating effect of *M* interfering links
- **Approach**: To match CDF of total instantaneous received interference power taken over M channel fades to that for the 'true'
- **Assumptions**: *M* interfering channels have independent fading, and the same values of $W\tau_{rms}$, transmitted power spectral density, *S(f)*, and mean-squared path loss, *G* =1. Also, *S*(*f*) is uniform over bandwidth *W* with value 1/*W*.
- **Total Instantaneous Interference Power**: For a single interfering link, the received power spectral density ($S_{rec}(f)$) and received instantaneous power (*y*) are given respectively as

$$S_{\text{rec}}(f) = S(f)|H(f)|^2; 0 \leq f \leq W \quad \text{and}$$

$$y = \int_f (1/W)\,|H(f)|^2\,df$$

The total instantaneous interference power is *M*-fold sum of i.i.d. variables *y*

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n-tap Channel Approximation

- **Considering PDP of approximated channel as**

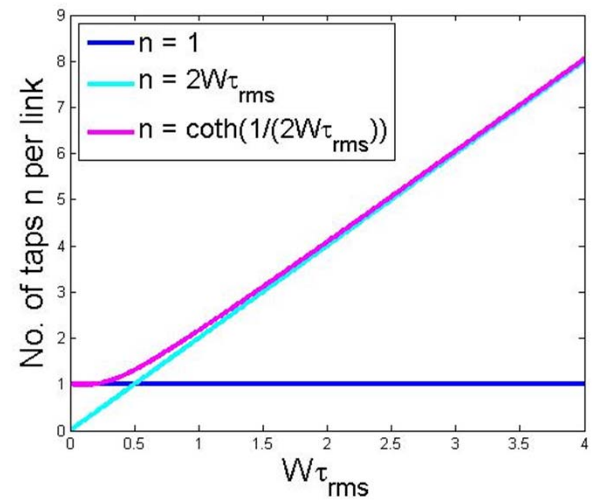
$$\mathrm{PDP}(\tau) = \sum_{k=0}^{n-1} P_k \delta(\tau - T_k)$$

with set of values of (P_k, T_k) as

$$P_k = 1/n, T_k = 2k/W .$$

- **Required number of taps per-link, n**:
  - Matching the mean and variance of total instantaneous interference power, we obtain

$$n = \frac{1 + \exp\left(-1/W\tau_{rms}\right)}{1 - \exp\left(-1/W\tau_{rms}\right)} = \coth\left(1/2W\tau_{rms}\right)$$



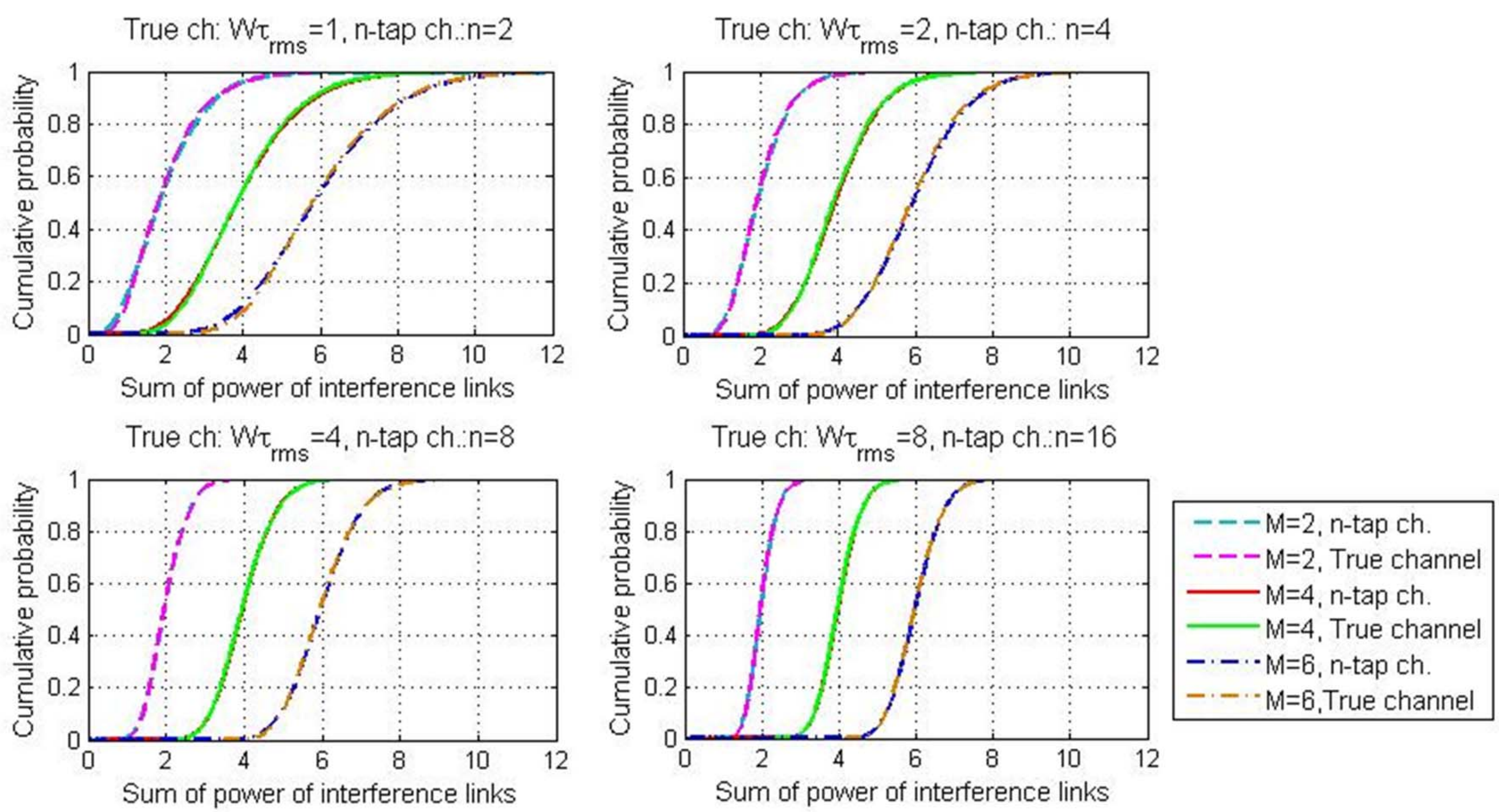
  - Further approximation for *n*

$$n = Q\left(\max\left(1, 2W\tau_{rms}\right)\right)$$

where *Q(x)* means quantization of x to nearest integer

Sample results

CDF of total instantaneous received power



Comparison of CDFs for *n*-tap channel and true (exponential) channel, when number of interfering links M = 2,4,6 and $n = 2W\tau_{rms}$

Complexity Conclusion

- Two methods proposed for reducing number of channel taps
  1) 3-tap Direct links
  2) Reduced-tap interference links
- Direct Links: 3-tap channel approximations appear to be feasible for single-carrier transmissions and equalized receivers.
- Interference Links: The CDF of the total received interference power can be made virtually the same as the CDF for the actual links.
- These approaches simplify the design and reduces the complexity with tap reduction up to 50 to 80% and cost of hardware emulators.

Future Work:

- Optimization of 3-tap design; consideration of more cases , e.g., OFDM signalling;
- Consideration of PDP of other shapes, e.g., channel with sparse multipath, and for unequal conditions (PDP and average power) among the interfering links

Thank You!

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