Radio Scene Analysis using Trilinear Decomposition

Goran Ivkovic, Predrag Spasojevic,
and Ivan Seskar

Introduction

- Radio sources (802.11b/g, Bluetooth, cordless phones, microwave ovens, etc.) operate in the 2.4 GHz ISM band
- Transmitted signals may overlap in time and frequency
- Each source is characterized by its power spectral density (PSD) and on/off activity sequence

Introduction

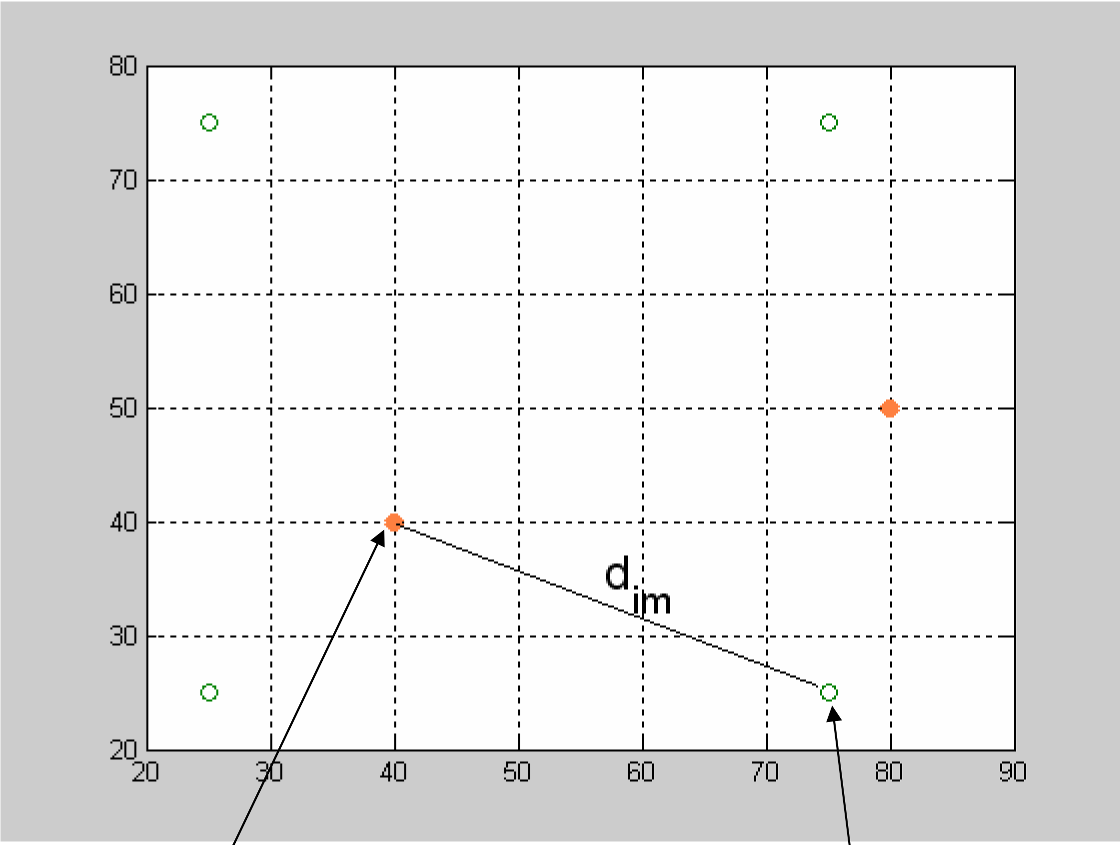
- A network of sensors performs measurements
- Each sensor computes spectrogram with some time and frequency resolution
- Spectrograms from different sensors are collected and arraigned in a three way array

Introduction

- ***Key idea***: recover channel gain coefficients, source PSDs and activity sequences by performing low-rank decomposition of the three way array

- ***An application***: monitoring spectrum usage in a frequency band

Spectrum Sensing Network



source m

sensor i

- I sensors, M sources

- Channel gain

  path loss coefficient n:

$$g_{im} = \frac{c}{d_{im}^{n}}$$

Power Spectral Density Measurement

- PSD at sensor *i*, over the time interval [*kT*, *(k+1)T*]:

$$S(i,k,f) = \sum_{m=1}^{M} g_{im} S_m(f) b_{km}$$

channel gain

source PSD

source on/off activity sequence

Three-way Array

- PSD estimation with frequency resolution Δf results in a three way array:

$$\mathbf{X}(i,k,j) = \int_{(j-1)\Delta f}^{j\Delta f} S(i,k,f)\,df = \sum_{m=1}^{M} g_{im} c_{jm} b_{km}$$

$$c_{jm} = \int_{(j-1)\Delta f}^{j\Delta f} S_m(f)\,df$$

Source PSD Description

- A source can transmit signals with different PSDs at different time intervals
  - (examples: 802.11b/g radio or frequency hopping devices)

- Different sources can use signals with same PSD
  - (example: two 802.11b sources)

Three-way Array Representation

X can be expressed in terms of three matrices:

Channel gains $\mathbf{G} = \begin{bmatrix} \mathbf{g}_1 & \cdots & \mathbf{g}_1 & \cdots & \mathbf{g}_M & \cdots & \mathbf{g}_M \end{bmatrix}$ $I \times R$

Source PSDs $\mathbf{C} = \begin{bmatrix} \mathbf{C}_1 & \cdots & \mathbf{C}_M \end{bmatrix}$ $J \times R$

On/off activity sequences $\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \cdots & \mathbf{B}_M \end{bmatrix}$ $K \times R$

Def: Rank(**X**) is the minimum number of rank-one terms needed to decompose **X.**

Rank(**X**)=R is the total number of distinct PSDs used by the sources

Objective:

find **G**, **C**, and **B** by performing low-rank decomposition of **X**

Low-rank decomposition

- Decomposition of a matrix into rank-one terms is ***not unique*** without imposing additional constraints
- Decomposition of a three-way array into rank-one terms can be ***unique*** under certain conditions
- When the uniqueness condition holds **G**, **C**, and **B** can be recovered from **X** up to permutation and scaling of columns

Basic uniqueness condition

- *K*-rank of a matrix is the maximum number *k* such that every subset of *k* columns is linearly independent
- Matrices **G**, **C**, and **B** can be recovered uniquely by decomposing **X** into rank-one components if (Kruskal'77):

$$k_{\mathbf{G}} + k_{\mathbf{C}} + k_{\mathbf{B}} \geq 2R + 2$$

Rank of **X**

Satisfying the basic uniqueness condition

- If all sources have distinct time invariant PSDs, the uniqueness condition is satisfied with 2 sensors (I=2)

$$k_{\mathbf{G}} + k_{\mathbf{C}} + k_{\mathbf{B}} \geq 2R + 2 \quad (k_{\mathbf{G}} = 2, k_{\mathbf{C}} = k_{\mathbf{B}} = R)$$

- When **G** or **C** contain proportional columns ($k_{\mathbf{G}}$=1 or $k_{\mathbf{C}}$=1), the uniqueness condition cannot be satisfied

$$k_{\mathbf{G}} + k_{\mathbf{C}} + k_{\mathbf{B}} < 2R + 2$$

$$(\text{example}: k_{\mathbf{G}} = 1, k_{\mathbf{C}} = R, k_{\mathbf{B}} = R)$$

Block Decomposition-Source Grouping

- When **G** and **C** contain proportional columns
  - **X** can be decomposed into Q block components
  - Rank of each block component is larger or equal to one
- Block decomposition of **X** corresponds to groups of sources
- Q source groups use signals with different PSDs
  - Q heterogeneous source groups

Block Decomposition: Uniqueness

- ***Key problem***: when is block decomposition unique?
- Generalize the basic uniqueness condition
- Linear independence of subspaces spanned by blocks of columns
  - instead of the linear independence of the column vectors of matrices

Linearly independent subspaces

- Definition (Cardoso' 98):
  - a set subspaces $S_1,\ldots S_Q$ is linearly independent if any vector **x** from the space spanned by $S_1,\ldots S_Q$ admits a unique decomposition as:

$$\mathbf{x} = \sum_{p=1}^{Q} \mathbf{x}_p$$

$$\mathbf{x}_p \in S_p$$

Block *k*-rank

- For a given partition of columns of a matrix we have a set of linear subspaces

- Block *k*-rank: maximum number q such that every subset of q subspaces is linearly independent

Uniqueness condition for Q-block decomposition

- If the number of blocks is Q and **B** is of full column rank then the block decomposition is unique if:

$$q_{\mathbf{G}} + q_{\mathbf{C}} + q_{\mathbf{B}} \geq 2Q + 2$$

•Subspaces corresponding to column blocks of **G**, **C**, and **B** are unique

Uniqueness condition: an example

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_1 & \mathbf{g}_2 & \mathbf{g}_3 & \mathbf{g}_3 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \underbrace{\mathbf{b}_{\mathbf{1}} \quad \mathbf{b}_2} & \underbrace{\mathbf{b}_3 \quad \mathbf{b}_4} \end{bmatrix}$$

- **X** can be decomposed into two rank-two blocks (Q=2)

Uniqueness condition: an example

- For two sensors (I=2), block decomposition is not unique since:

$$q_{\mathbf{G}} = 1, \quad q_{\mathbf{C}} = 2, \quad q_{\mathbf{B}} = 2, \quad Q = 2$$

$$(\mathbf{g}_3 \in span\{\mathbf{g}_1, \mathbf{g}_2\}) \qquad (q_{\mathbf{G}} + q_{\mathbf{C}} + q_{\mathbf{B}} < 2Q + 2)$$

- For three sensors (I=3), block decomposition is unique since:

$$q_{\mathbf{G}} = 2, \quad q_{\mathbf{C}} = 2, \quad q_{\mathbf{B}} = 2, \quad Q = 2$$

$$(\mathbf{g}_3 \notin span\{\mathbf{g}_1, \mathbf{g}_2\}) \qquad (q_{\mathbf{G}} + q_{\mathbf{C}} + q_{\mathbf{B}} \geq 2Q + 2)$$

Alternating Least Squares for Block Decomposition

- Alternating least squares (ALS) algorithm computes the block decomposition of **X**

- Block decomposition of **X** separates heterogeneous groups of sources

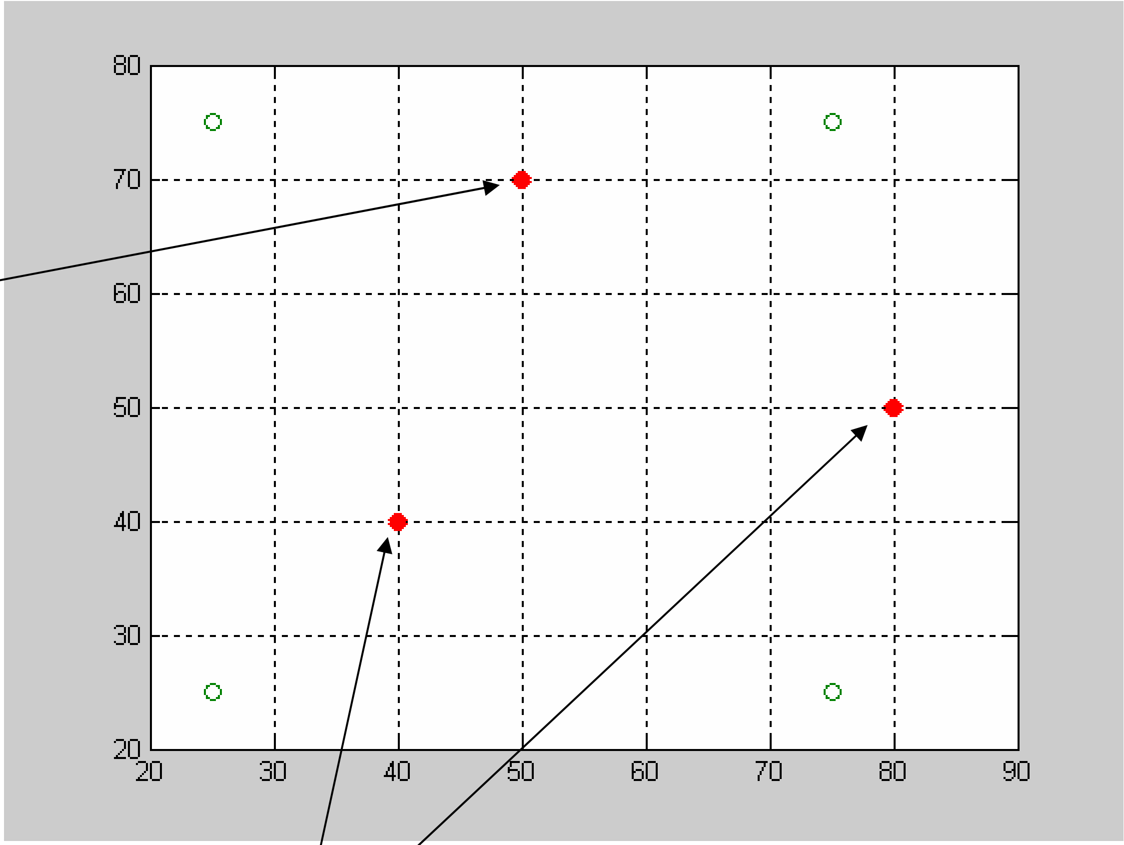
Source Clustering

- In the second step we separate the sources from each group

- We assume that sources from the same group transmit non-overlapping signals in time

- This step reduces to a clustering problem

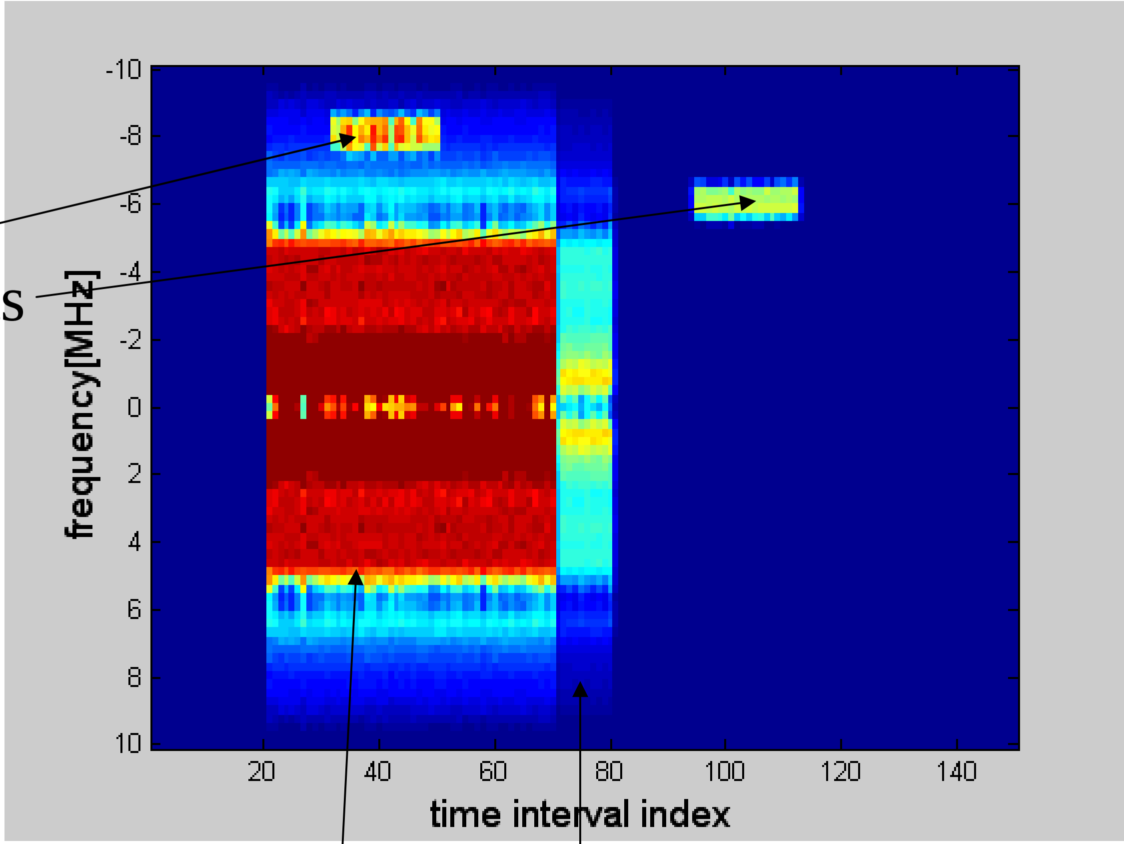
Example



BT node

WLAN nodes

Example: spectrogram



BT packets

WLAN packets

Model:

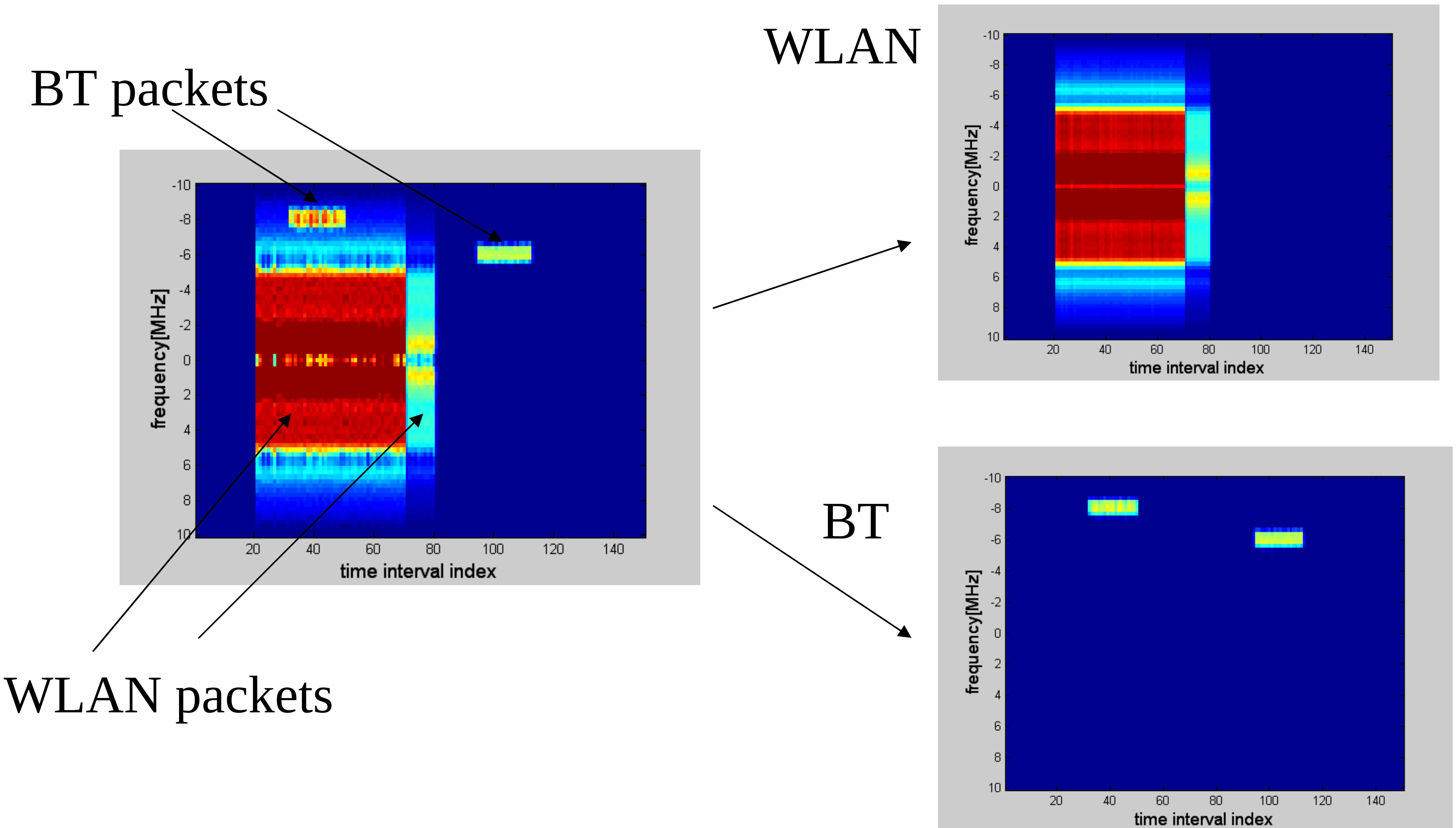
$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_1 & \mathbf{g}_2 & \mathbf{g}_3 & \mathbf{g}_3 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{bmatrix}$$

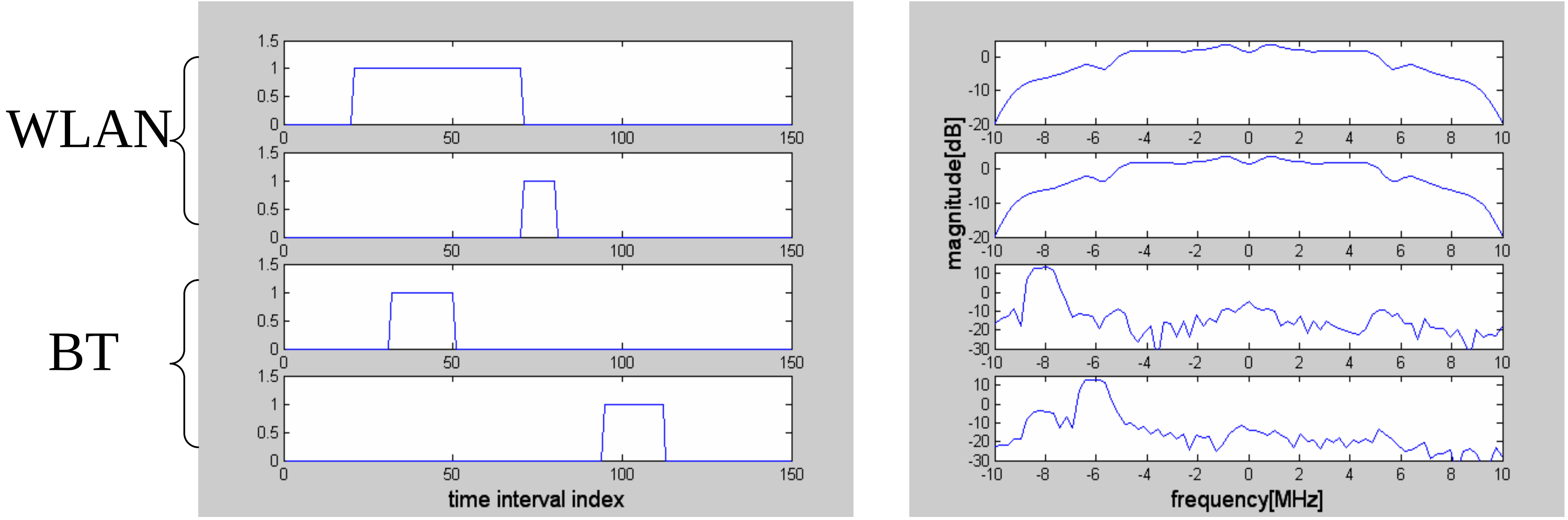
$$\mathbf{B} = \begin{bmatrix} \underbrace{\mathbf{b}_1 \quad \mathbf{b}_2} & \underbrace{\mathbf{b}_3 \quad \mathbf{b}_4} \end{bmatrix}$$

Two rank-two blocks

Block Decomposition



Recovered activity sequences and PSDs



Summary of the Approach

- Three-way array is formed of spectrogram from different sensors
- Block decomposition of the array recovers heterogeneous groups of sources
- Sources from each group are recovered by clustering

Conclusions and future work

- We have established identifiability conditions for this problem

- Extension to frequency selective channels

- Performance evaluation