Coalitions in Cooperative Radio Networks

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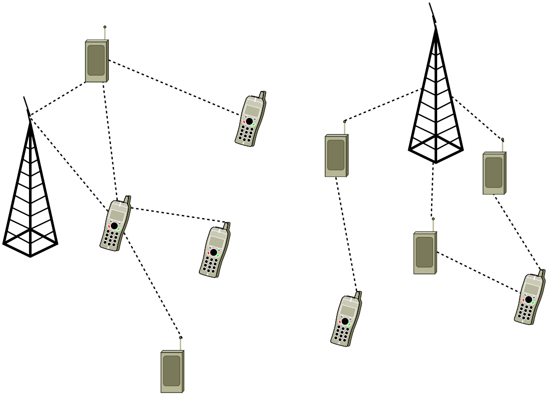
Joint work with L. Sankar and N. Mandayam

Cooperative Communications



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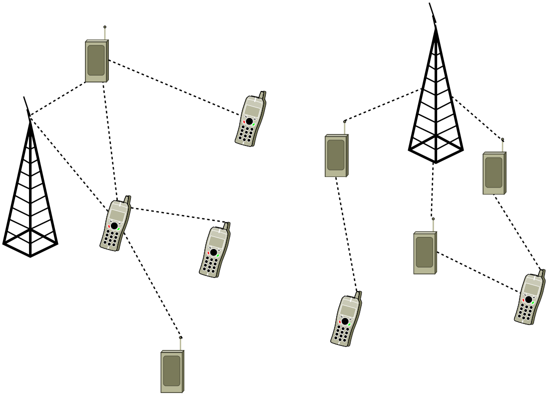
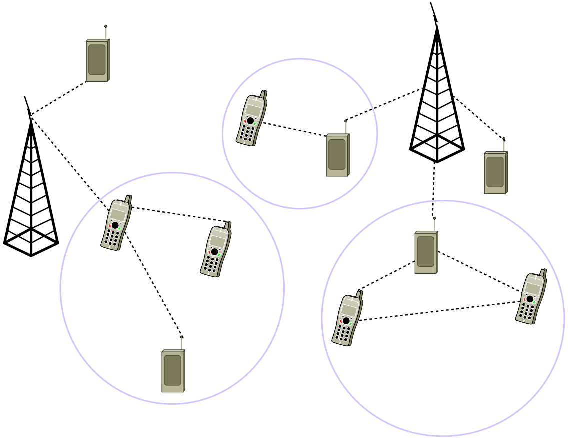


But why should self-interested rational users cooperate?

Cooperative Communications



Users may prefer to form coalitions

Pros

- 'Adapt' to the channel
- Better utilization of power
- Better utilization of bandwidth
- Higher rates overall

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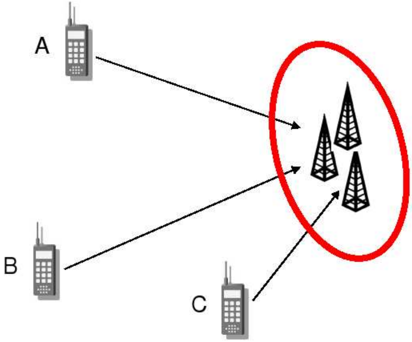
Cons

- Is the gain from cooperation worth it?
- No guaranteed future incentive
- Can I do better by myself (not cooperating) ?
- Can I do better cooperating only selectively (coalition) ?

A simple example

- 3 users A, B and C communicating with their receivers (assume co-located)
- Receivers can cooperate by jointly decoding their received signals.
- Suppose sum-rate achieved by a coalition is apportioned equally.
  - What cooperative behavior emerges?
  - What coalitions are formed?



Equal Rate Splitting

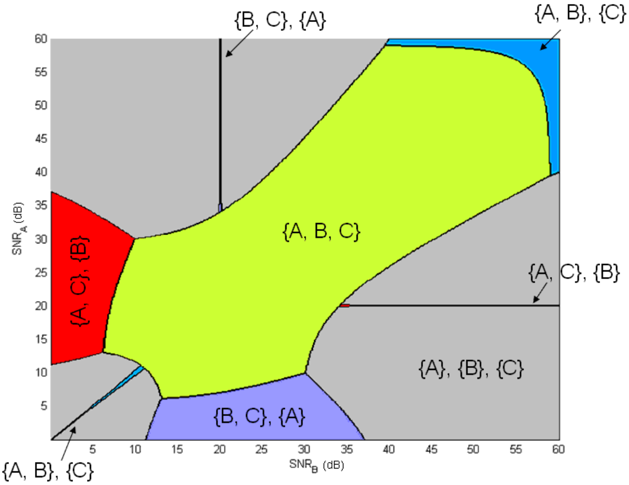


Figure: Stable coalition structures when recd. SNR of user 3 is fixed while those of A and B are varied.

Coalitional Game Theory: Overview

A coalitional game with transferable utility $\langle \mathcal{S}, v \rangle$

- finite set of players $\mathcal{S}$
- value function $v : \mathcal{G} \rightarrow \mathbb{R} \quad \forall \mathcal{G} \subseteq \mathcal{S}$

Payoff: Share of the value $v(\mathcal{G})$ to each player.

Characteristic function form game: $v(\mathcal{G})$ is unaffected by the "strategy" of users not in $\mathcal{G}$.

When $v(\mathcal{G})$ can be flexibly apportioned between cooperating players, the game is said to have transferable utility (TU).

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Properties of Coalitions under TU

A coalitional game with TU is cohesive if $\sum_{i=1}^{K} v(\mathcal{G}_k) \leq v(\mathcal{S})$ for every partition $\{\mathcal{G}_1, \ldots, \mathcal{G}_K\}$ of $\mathcal{S}$.

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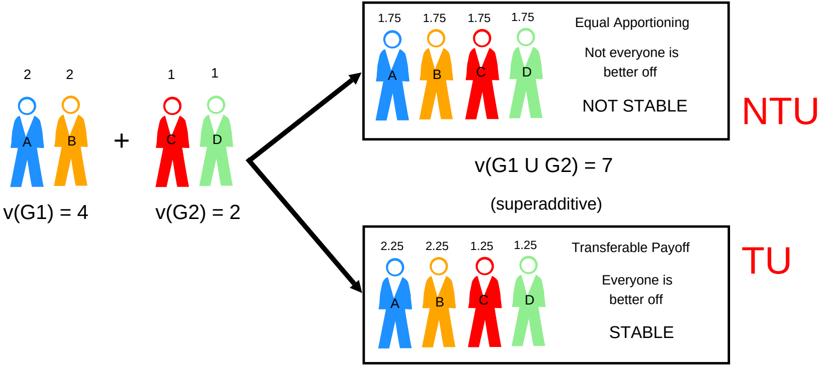

Core & Stability

The core $C(v)$ of a coalitional game is the set of feasible payoffs for which no coalition $\mathcal{G}$ has incentive to defect by achieving a greater payoff for all its members.

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The core can be empty!

Empty core $\Rightarrow$ No stable form of cooperation.

An example with an empty core

A simple 3-player game:

$$
\begin{aligned}
\mathcal{S} &= \{1,2,3\} \\
v(\{i\}) &= 0, i = 1,2,3. \\
v(\mathcal{G}) &= \alpha, \forall |\mathcal{G}| = 2 \\
0 < &\ \alpha < 1 \\
v(\mathcal{S}) &= 1
\end{aligned}
$$

Any feasible payoff profile in the core must satisfy:

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x_1 &\geq v(\{1\}) = 0 \\
x_2 &\geq v(\{2\}) = 0 \\
x_3 &\geq v(\{3\}) = 0 \\
x_1 + x_2 &\geq v(\{1,2\}) = \alpha \\
x_2 + x_3 &\geq v(\{2,3\}) = \alpha \\
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- Existence of a non-empty core $\iff$ feasibility of an LP.

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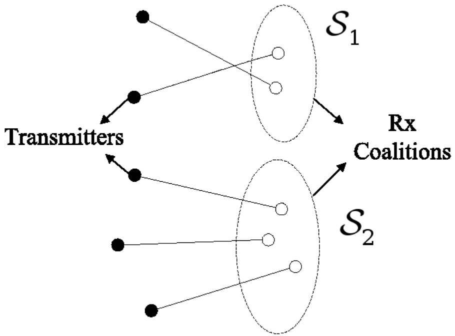
- Existence of a non-empty core $\iff$ feasibility of an LP.
- Core is non-empty only if $\alpha \leq \frac{2}{3}$.
  - Game is cohesive. Cohesiveness does not guarantee non-empty core.

Receiver cooperation



1. Coop. rcvrs jointly decode RX signals.
2. Signals from links outside a coalition are treated as Gaussian noise.
3. A coalition $\mathcal{S}$ forms a SIMO MAC
4. Capacity region of a SIMO-MAC with $|\mathcal{S}|$ links: $C_{\mathcal{S}} = \{\underline{R}_{\mathcal{S}} : \sum_{k \in \mathcal{S}} R_k \leq I(X_{\mathcal{A}}; Y_{\mathcal{S}} | X_{\mathcal{S} \setminus \mathcal{A}}); \forall \mathcal{S} \subseteq \mathcal{S}\}$
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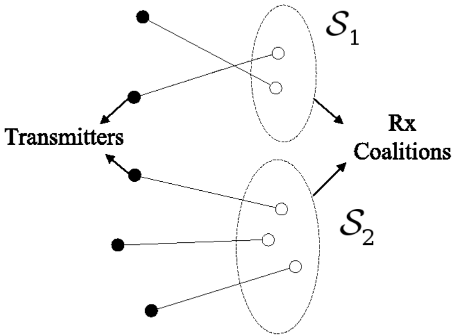

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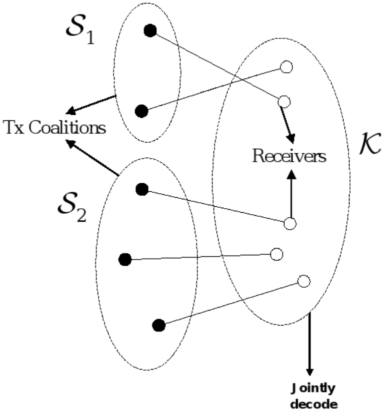
The grand coalition is sum-rate optimal.
The grand coalition always forms when receivers cooperate!

Transmitter Cooperation - A simple model



1. Non-causal info. of TX messages
2. All RX jointly decode signals
3. TXs optimally choose covariance matrix.
4. Results in a MIMO-MAC model.
5. Value of a coalition $\mathcal{S}$:
   $v(\mathcal{S}) = \max_{Q_{\mathcal{S}}:EX_k^2 \leq P_k} I(X_{\mathcal{S}} : Y_{\mathcal{K}})$
6. Value of a coalition depends on the actions of users outside the coalition (interference)
7. Jamming game: Value $v(\mathcal{S})$ is based on worst case interference from placers in $\mathcal{S}^c$

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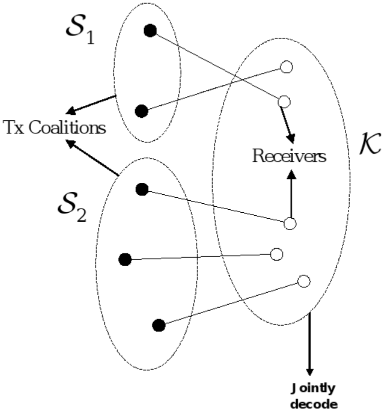

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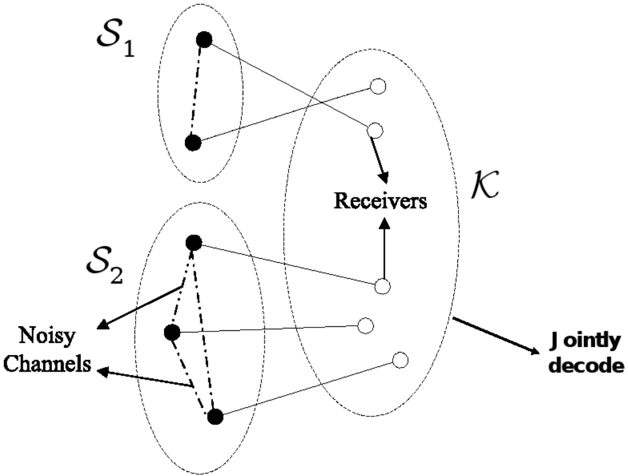
TX cooperation game is a cohesive game.
The grand coalition does not always form.

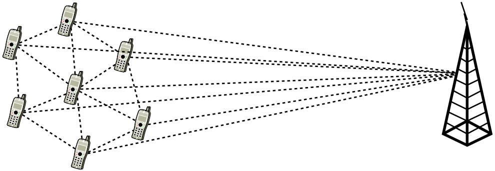
User Cooperation



Each user is a transceiver and must decode other users' messages in order to cooperate

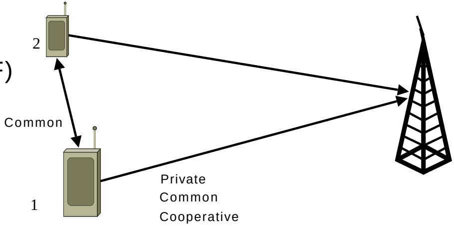
User Cooperation

1. Will relaxing the assumption of noise-free links change the stability of the core?
   - Consider a K-user clustered model.



   - Inter-user channels stronger than user-destination channel $(h_{i,j} > hi, d, \forall i \neq j)$.
   - User decode and fwd. msgs. over noisy links.
   - Simplest models where we expect users to cooperate
     - If users don't decode for one another, then they must face greater interference.
     - In the absence of means like spread spectrum & power control, clustered users would interfere strongly with one another at the destination.

Partial Decode
and Forward (PDF)



1. Cooperating Txmitter k splits power between private, common and cooperative messages as $P_{k,d}, P_{k,c}, P_{k,u}$.
2. Users decode common messages for one another.
3. Destination decodes all messages.
4. Rates achieved by a coalition are not independent of the transmit strategies of the users outside $\rightarrow$ resort to a jamming game interpretation.
5. $v(\mathcal{S})$ is now a set, given by the convex hull of regions due to different power allocations on $(P_{k,d}, P_{k,c}, P_{k,u})$.
6. What is the optimal power allocation strategy ?

Theorem

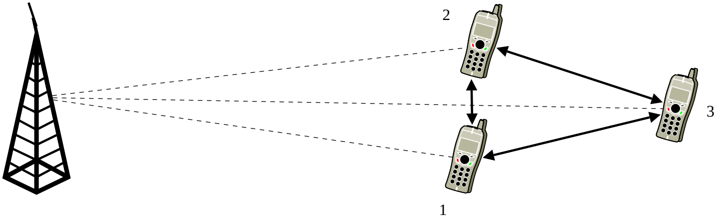
The rate-region is maximized when each user does not transmit any messages directly to the destination.

1. All power is placed into the inter-user channel. Hence: $P_{k,d} = 0$ and $P_{k,c} = P_k - P_{k,u}$
2. Is the NTU - PDF game cohesive?
3. The game is cohesive if

$$\bigcap_{n=1}^{N} v(\mathcal{S}_n) \subseteq v(\mathcal{K})$$

4. Proving cohesiveness is not straightforward
   - Provide a counter-example

A non-cohesive example



$$h_{d,1} = h_{d,2} = 0.05$$
$$h_{d,3} = 0.025$$
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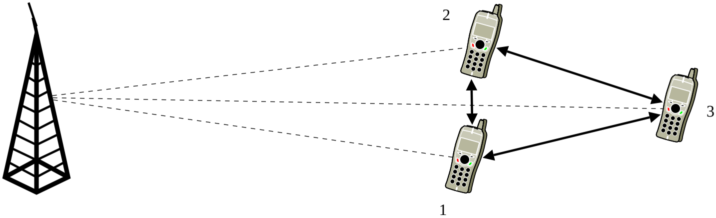

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User 1 and 2 can **always** do better by breaking away from 3 $\rightarrow$ game is not cohesive.

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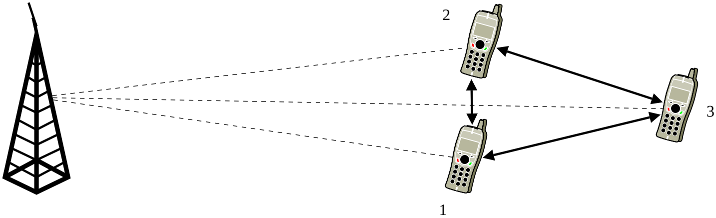

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User 1 and 2 can **always** do better by breaking away from 3 → game is not cohesive.

Users 1 and 2 can do with not having to have their messages decoded by user 3 and live with the worst case jamming from user 3.

Related Papers

1. Suhas Mathur, Lalitha Sankar and Narayan B. Mandayam, "Coalitions in Cooperative Wireless Networks", Submitted to IEEE JSAC, 2007.
2. Suhas Mathur, Lalitha Sankar and Narayan B. Mandayam, "Coalitional Games in Cooperative Radio Networks", Asilomar SSC 2006.
3. Suhas Mathur, Lalitha Sankar and Narayan B. Mandayam , "Coalitional Games in Gaussian Interference Channels", ISIT 2006.
4. Suhas Mathur, Lalitha Sankar and Narayan B. Mandayam, "Coalitional Games in Receiver Cooperation for Spectrum Sharing", CISS 2006
5. Suhas Mathur, "Coalitional Games in Cooperative Networks", M.S. Thesis, Rutgers University, October 2006.

Conclusions

- Receiver cooperation:
  - The grand coalition is always stable.
  - Grand coalition maximizes sum-rate.
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  - Bargaining theory can be used to guarantee fair allocations.
- Transmitter cooperation as a jamming game:
  - Grand coalition is the only candidate for the core (cohesive)
  - Grand coalition does not always form.
- User Cooperation through PDF
  - Even in situations where we would expect users to cooperate, cooperation of all users cannot be guaranteed.
  - Outcome depends upon the strength of inter-user links and the power available to individual users.