Cooperating to Build a Radio Map to Support Spectrum Agility

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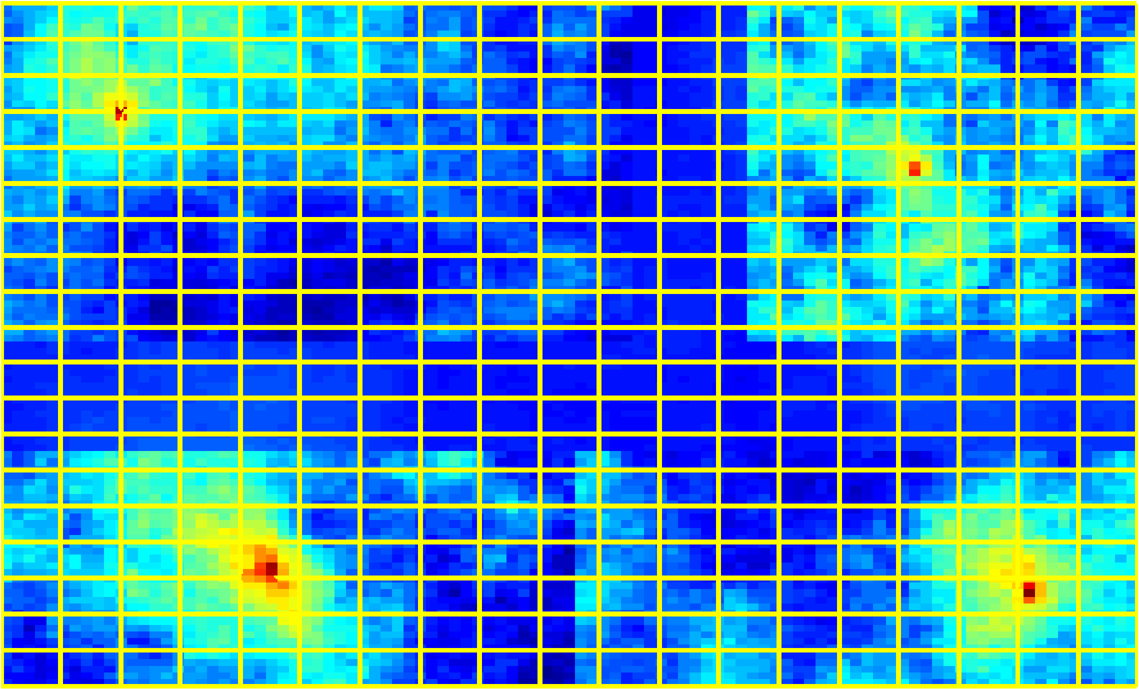
Overview

- Motivation and Background
- Optimal Random Field Reconstruction
- Balanced Spectrum Sampling
- Field Estimation by Hierarchical Interpolation
- Summary

Motivation

- A Typical Spatial Distribution of Spectral Intensity



Challenges:

- Sources with unknown locations
- Random Variations
  - Correlated
  - Non-stationary

Building Radio Maps is harder than it looks.

Physical Facts

- Radio Propagation Model (log-normal)

$$P(\mathbf{x}) = P_0 - 10\gamma \log_{10}\left(\frac{\|\mathbf{x}-\mathbf{x}_0\|}{d_0}\right) + s(\mathbf{x}) \quad \text{(dB)}$$

- Spatially Correlated
  - γ: path loss exponent
  - $s(\mathbf{x})$: shadow fading, *normally* distributed with zero mean and

$$\mathrm{Cov}(s(\mathbf{x}_i)s(\mathbf{x}_j)) = \sigma_{dB}^2 \exp\left(-\frac{\|\mathbf{x}_i-\mathbf{x}_j\|}{X_C}\right)$$

Spectrum Reconstruction

- Reconstruction Criterion for a Random Field:
  - Mean Square Error (MSE)

$$MSE = \frac{1}{M} \sum_{m=N+1}^{N+M} E[(\widehat{P}(\mathbf{x}_m) - P(\mathbf{x}_m))^2 \mid \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}]$$

  - N : the number of sensors (CRs)
  - M : the number of positions of interest
  - $\widehat{P}$: radio power estimate (in dB)

Spectrum Reconstruction (cont'd)

$$MSE = \frac{1}{M}\sum_{m=N+1}^{N+M} E[(\widehat{P}(\mathbf{x}_m) - P(\mathbf{x}_m))^2 \mid \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}]$$

- Sampling
  - Given N sensors, what are the best locations to place them?
- Estimation
  - Given measured data at known locations, how to estimate spectrum level at an unknown location?

A joint process of sampling and estimation.

Optimal Random Field Mapping

- Optimal estimation in a stationary environment
  - MMSE (unbiased): $\hat{P}(\mathbf{x}_m) = E[P(\mathbf{x}_m) \mid P(\mathbf{x}_1), \ldots, P(\mathbf{x}_N)]$
  - Gaussian process:

$$[P(\mathbf{x}_m), P(\mathbf{x}_1), \ldots, P(\mathbf{x}_N)]^T \sim N(\boldsymbol{\mu}, \mathbf{C})$$

$$\boldsymbol{\mu} = \begin{bmatrix} \mu(\mathbf{x}_m) \\ \hdashline \mu(\mathbf{x}_1) \\ \vdots \\ \mu(\mathbf{x}_N) \end{bmatrix} = \begin{bmatrix} \mu(\mathbf{x}_m) \\ \boldsymbol{\mu}_N \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} \sigma_{dB}^2 & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}$$

  - Optimal estimate: $\hat{P}(\mathbf{x}_m) = \mu(\mathbf{x}_m) + \mathbf{C}_{12}\mathbf{C}_{22}^{-1}(\mathbf{P}_N - \boldsymbol{\mu}_N)$
  - Minimum variance: $\sigma_{\mathbf{x}_m|\{\mathbf{x}_N\}}^2 = \sigma_{dB}^2 - \mathbf{C}_{12}\mathbf{C}_{22}^{-1}\mathbf{C}_{21}$

Optimal Sensor Placement

- A set of sensor locations: $A_N = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}$

$$\arg\min_{A_N} \frac{1}{M} \sum_{m=N+1}^{N+M} E[(\widehat{P}(\mathbf{x}_m) - P(\mathbf{x}_m))^2 \mid A_N]$$

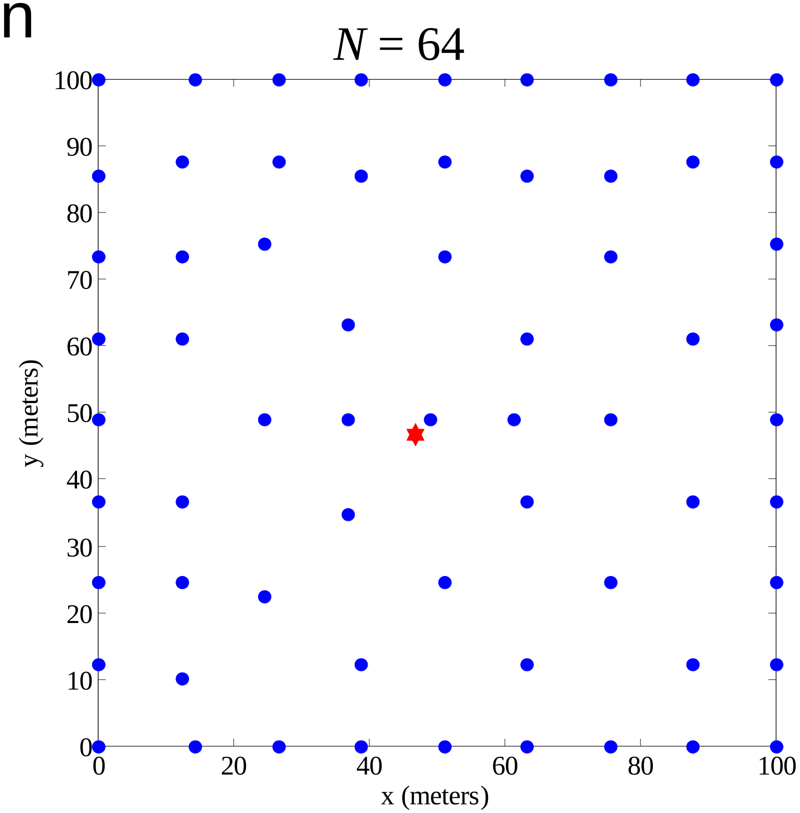
- Given the optimal estimate, MMSE is equivalent to the maximum entropy criterion

$$\arg\min_{\mathbf{A}_N} H(\overline{\mathbf{A}}_N \mid \mathbf{A}_N) \Leftrightarrow \arg\max_{\mathbf{A}_N} H(\mathbf{A}_N) =$$

$$\arg\max_{\mathbf{x}_1} H(\mathbf{x}_1) + \arg\max_{\mathbf{x}_2} H(\mathbf{x}_2 \mid \mathbf{A}_1) + \cdots + \arg\max_{\mathbf{x}_N} H(\mathbf{x}_N \mid \mathbf{A}_{N-1})$$

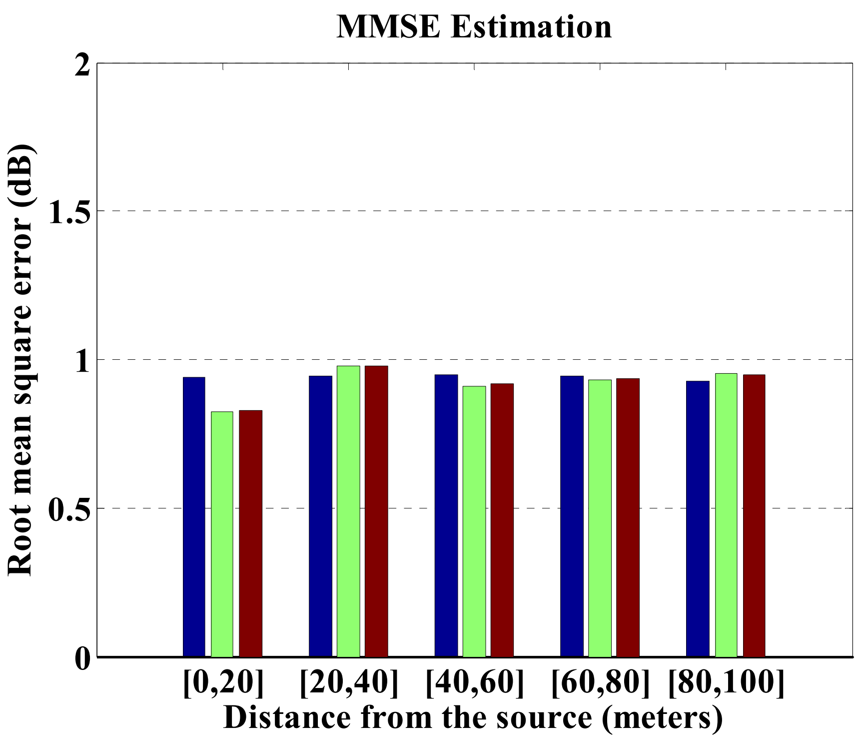
$$H(\mathbf{x}_{n+1} \mid \mathbf{A}_n) = \frac{1}{2} \log(2\pi e \sigma^2_{\mathbf{x}_{n+1}|\mathbf{A}_n})$$

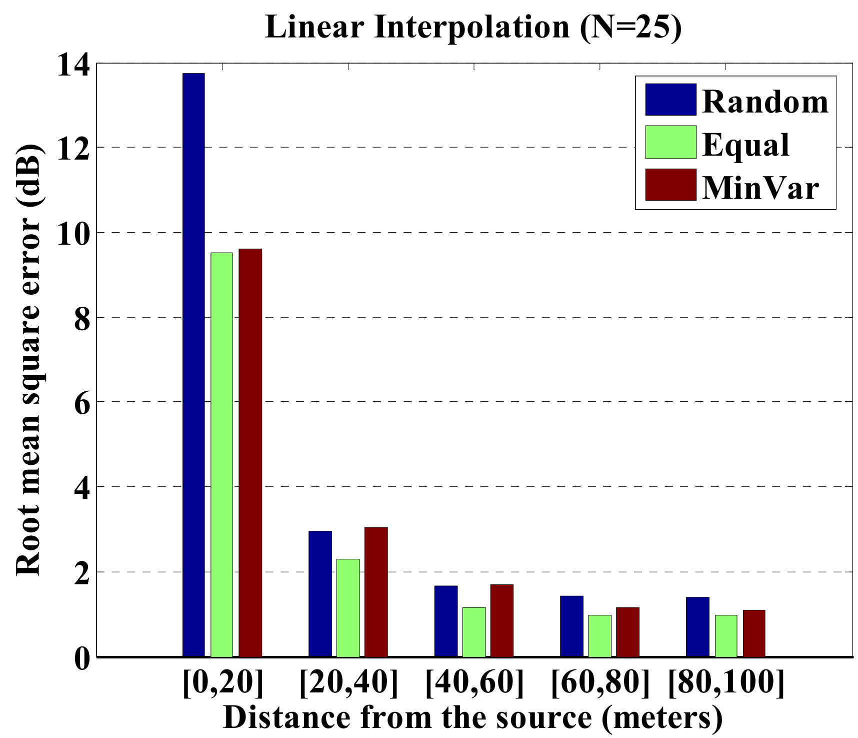
- Optimal ➔ Impractical
  - Optimal estimation has to know ❍ and **C** in prior:
    - ♑, $P_0(d_0)$, $\mathbf{x}_0$, X_C
  - Only work in stationary environments
  - Implicit solution: sub-optimal implementation by discretizing the node position
  - Optimal sampling only depends on **C**



Tend to place sensors uniformly; slightly favor boundaries

- Case study: $N = 25$, ♑ = 2, ♦$_{\mathrm{dB}}$ = 4 dB, $X_C = 200$ m



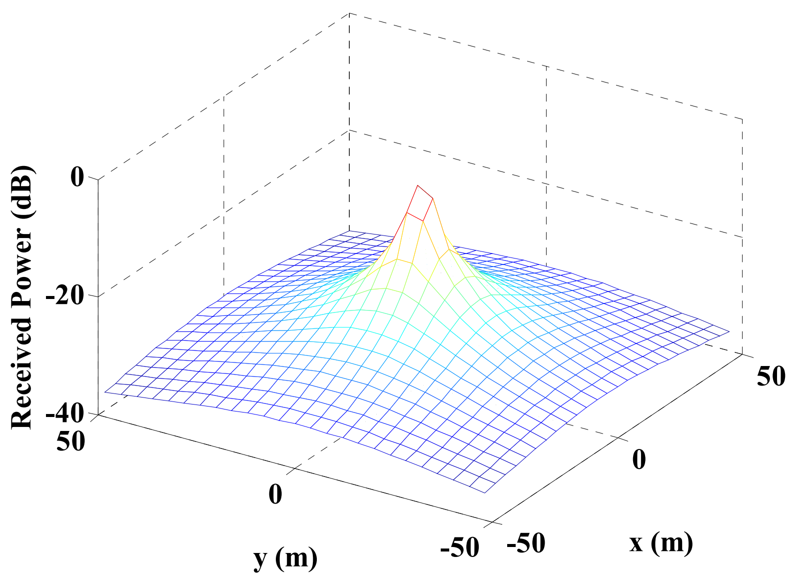


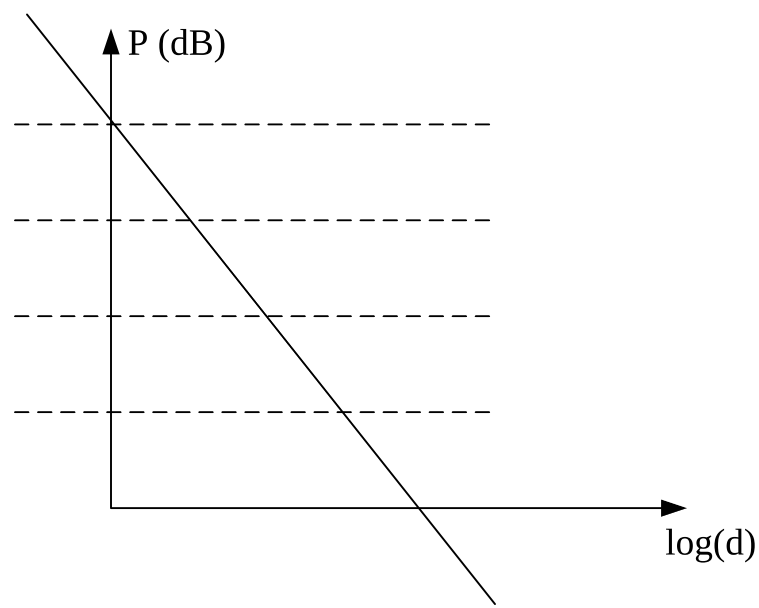
- Sub-optimal placement (MinVar) does not show performance improvement by either the optimal estimation or linear approximation (interpolation)

- In regions close to a radio source, all three placement schemes have high reconstruction errors using the approximation approach

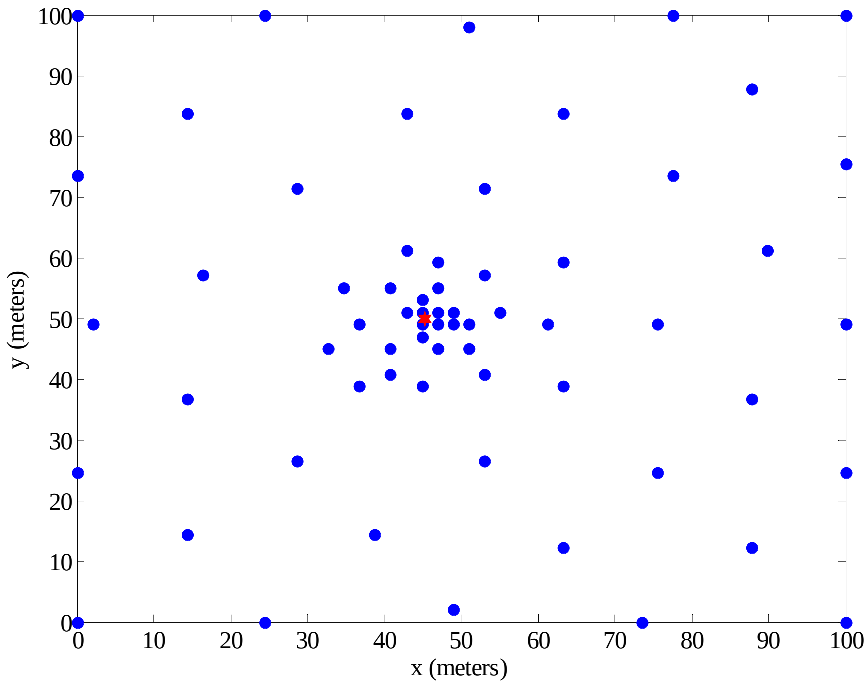
Balanced Spectrum Sampling

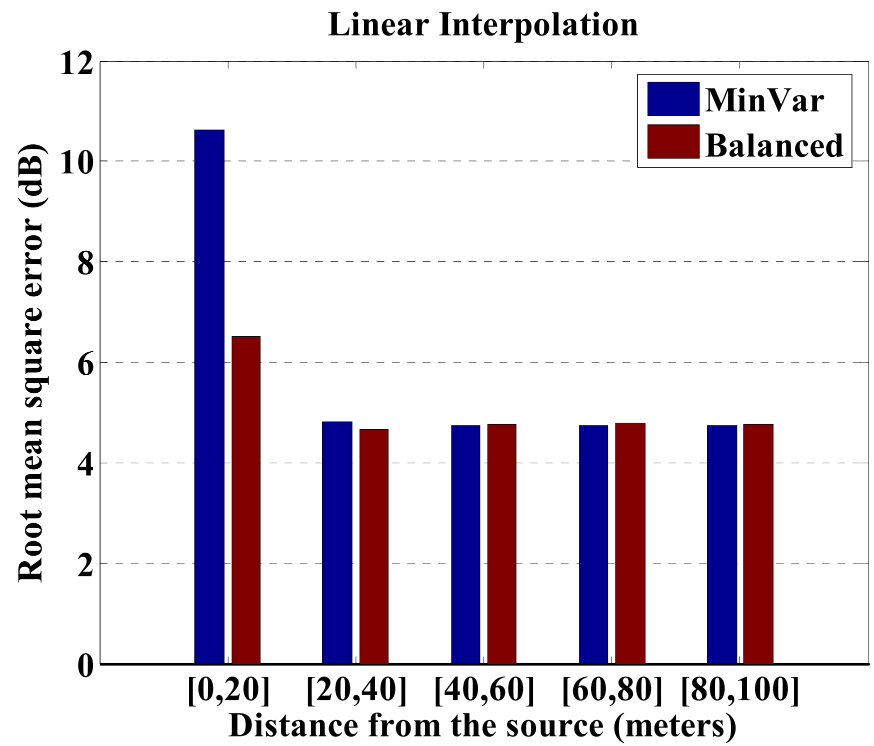
- Balance between the uncertainty and the rapid decreases of spectrum intensity
  - On average, the spectrum intensity changes *logarithmically* along the direction from the source
  - Keep a uniform spectrum resolution across all power levels





- Case study: $N = 64$, ♑ = 2, ♦$_{\text{dB}}$ = 4 dB, $X_C = 125$ m





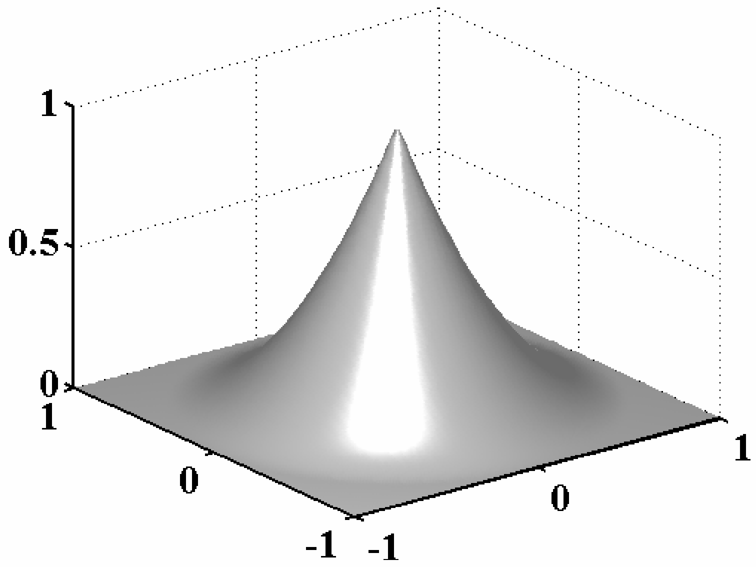
- Reconstruction error can be significantly reduced in regions close to the source without compromising the performance of “outer” regions

Approximate Spectrum Mapping

- Nonparametric Estimation – Interpolation
  - Nearest Neighbor
  - Linear
  - Spline
  - Hierarchical Interpolation using Compact Supported Functions
    - Radial basis functions [Wendland'95]
    - B-splines (Cubic)

Hierarchical Interpolation by 2-D Radial Basis Functions

- 2-D Radial Basis Functions

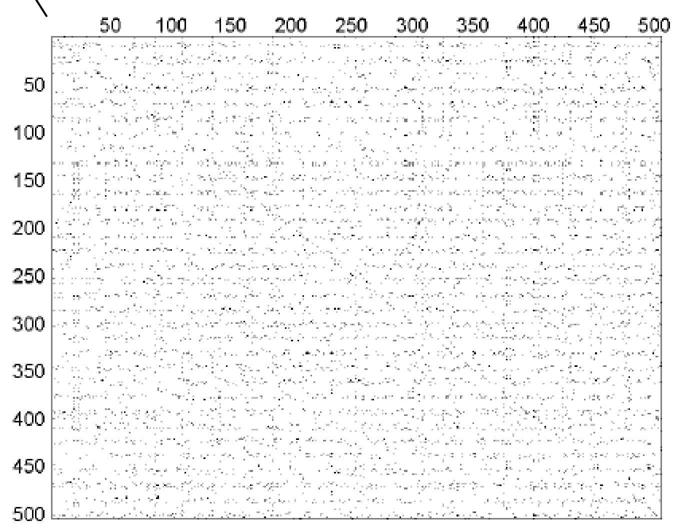


$\phi_0(r) = (1-r)_+^2$	C^0
$\phi_1(r) = (1-r)_+^4(4r+1)$	C^2
$\phi_2(r) = (1-r)_+^6(35r^2+18r+3)$	C^4
$\phi_3(r) = (1-r)_+^8(32r^3+25r^2+8r+1)$	C^6

$$s_k(\mathbf{x}) = \sum_{j=1}^{N} c_j \phi\left(\|\mathbf{x}-\mathbf{x}_j\|/\alpha_k\right)$$

Distance Matrix by Compactly Support Functions

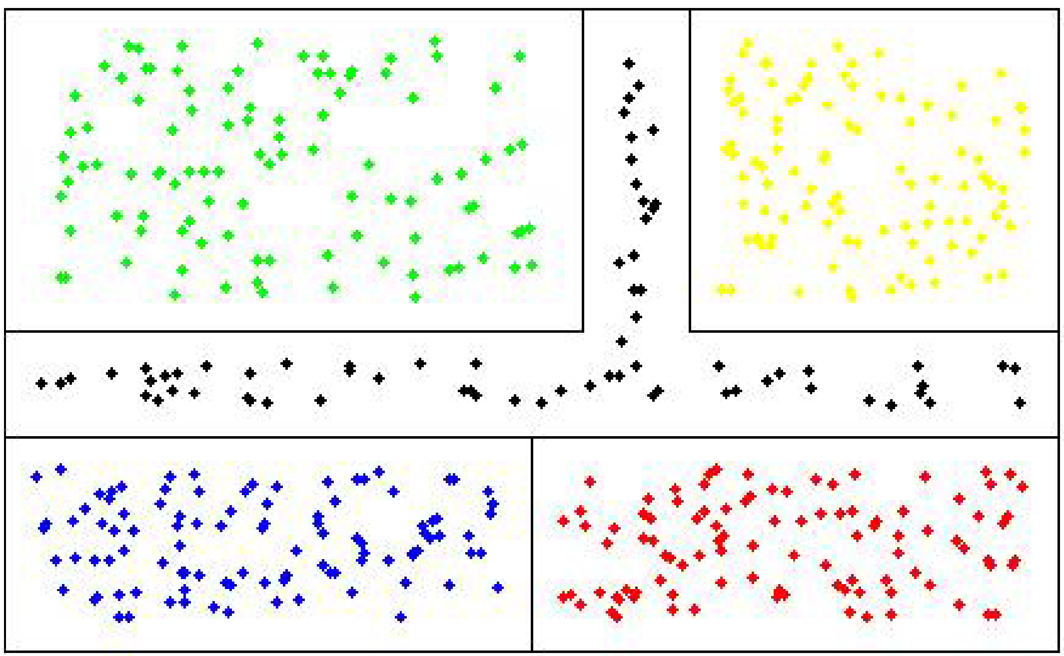
$$\begin{bmatrix} \phi(0) & \phi(||\mathbf{x}_1-\mathbf{x}_2||) & \cdots & \phi(||\mathbf{x}_1-\mathbf{x}_{N-1}||) & \phi(||\mathbf{x}_1-\mathbf{x}_N||) \\ \phi(||\mathbf{x}_2-\mathbf{x}_1||) & \phi(0) & \cdots & \phi(||\mathbf{x}_2-\mathbf{x}_{N-1}||) & \phi(||\mathbf{x}_2-\mathbf{x}_N||) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \phi(||\mathbf{x}_{N-1}-\mathbf{x}_1||) & \cdots & \cdots & \phi(0) & \phi(||\mathbf{x}_{N-1}-\mathbf{x}_N||) \\ \phi(||\mathbf{x}_N-\mathbf{x}_1||) & \cdots & \cdots & \phi(||\mathbf{x}_N-\mathbf{x}_{N-1}||) & \phi(0) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{N-1} \\ \alpha_N \end{bmatrix} = \begin{bmatrix} f(\mathbf{x}_1) \\ f(\mathbf{x}_1) \\ \vdots \\ f(\mathbf{x}_{N-1}) \\ f(\mathbf{x}_N) \end{bmatrix}$$

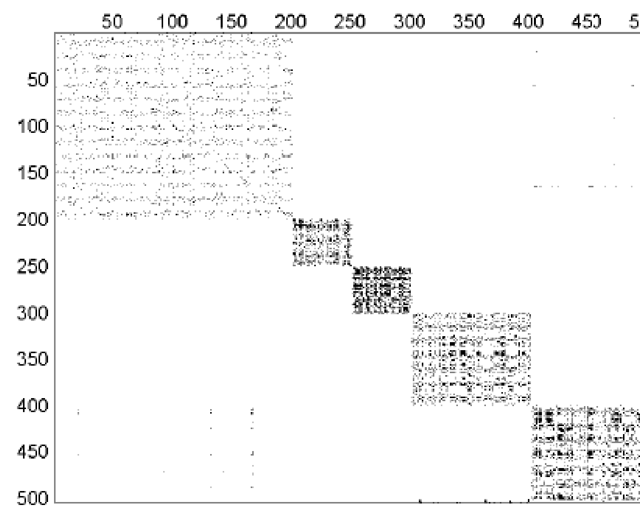


- Sparse Matrix
  - Unstructured: unpredictable complexity depends on the support radius
  - Worst case: non-sparse

Reduce Computational Complexity by Segmentation

- Data are usually spatially clustered in reality
  - Segment the area of interest based on the knowledge of scene and/or machine learning techniques





$$O(n^3) \rightarrow O(k(n/k)^3)$$

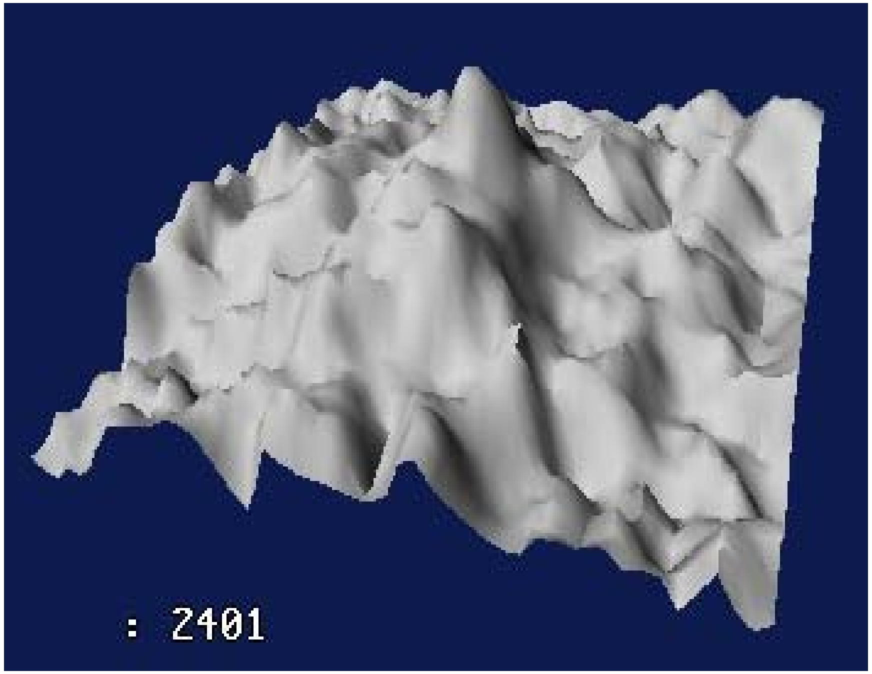
Summary

- Building a radio map in a random field is a joint optimization of sensor placement and reconstruction accuracy
- By balancing the uncertainty and estimation error, the reconstruction accuracy in regions close to radio sources can be significantly improved without impairing the overall performance
- The spectrum over the area of interest is approximated by interpolation. If sensors are locally clustered, the complexity of interpolation methods can be greatly reduced by segmentation

Thank You!

Questions?

$$\gamma = 2,\ \sigma_{\mathrm{dB}} = 4\,\mathrm{db},\ X_C = 10\,\mathrm{m}$$

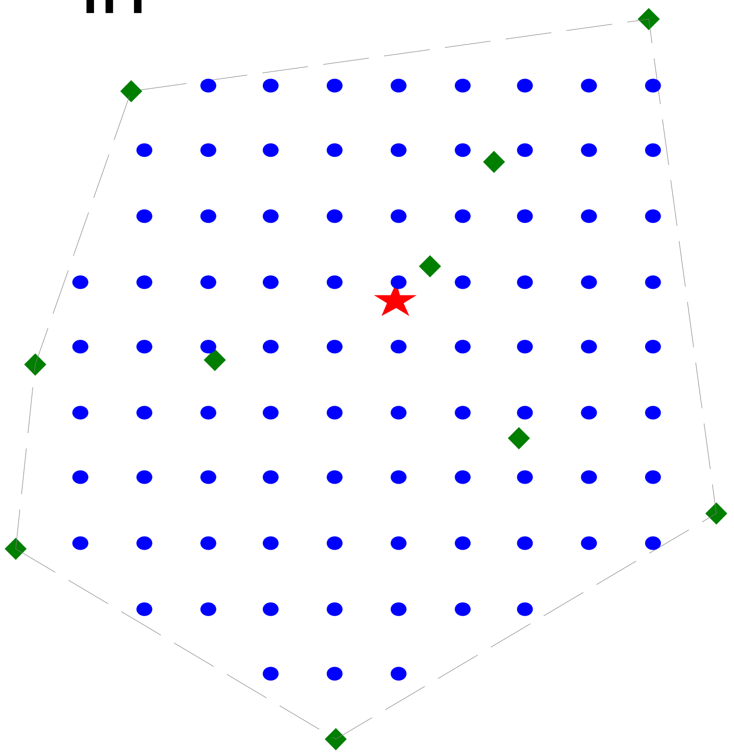


Application of Spectrum Map in Localization

- Weighted Centroid Localization

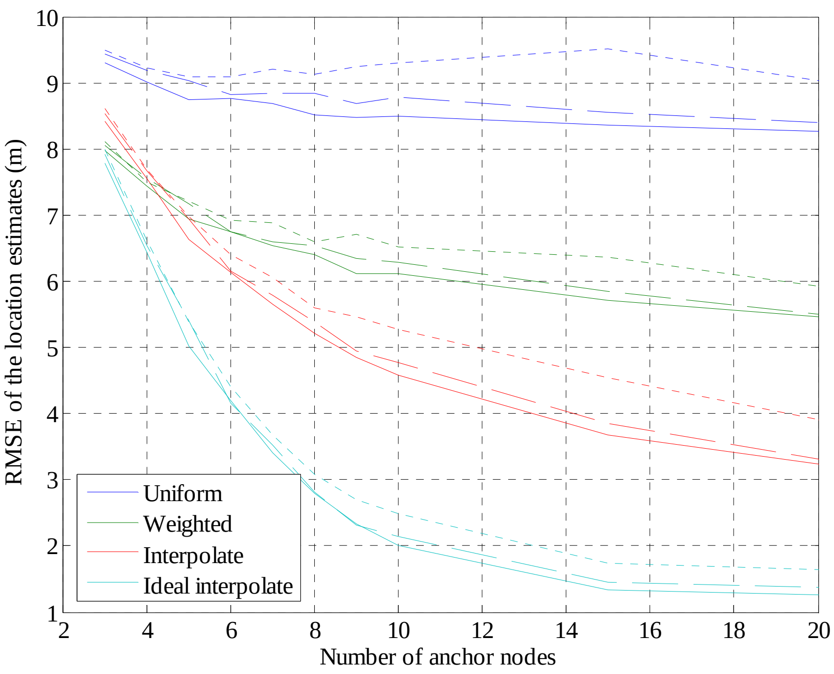
$$[x, y] = \sum_{i=1}^{N} \frac{P_i}{\sum_{j=1}^{N} P_j} [x_i, y_i]$$

- Radio Map "Plug-in"

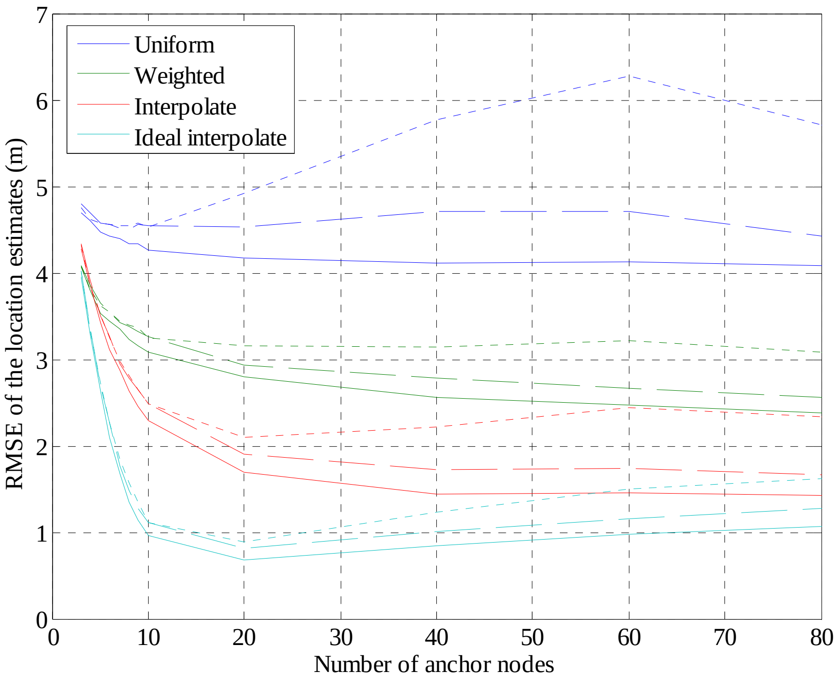


Enhanced Centroid Localization

- Random placement 100m x 100m, $\gamma = 2, \sigma_{dB} = 1\,dB$



N = 25



N = 100