Is User-Cooperation in Wireless Networks Always Beneficial?

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WIRELESS INFORMATION NETWORK LABORATORY

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Outline

Motivation

- Cooperation in wireless networks promises improved sharing of time and bandwidth resources
  - Advent of cognitive radio promises to make this a reality
- Does cooperation come with conflict?
  - Do all users always gain from cooperation?
  - Are shared resources utilized better when users cooperate?
- Can cooperative protocols induce the formation of disjoint groups of users that are "stable"?
  - Coalitional game theory can provide answers.

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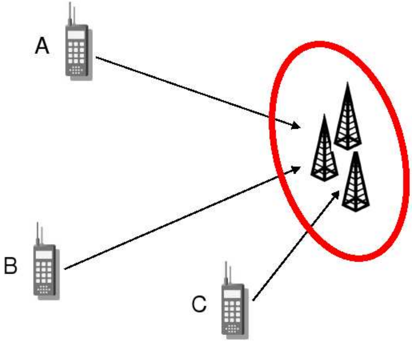
A simple example

- 3 users A, B and C communicating with their receivers (assume co-located)
- Receivers can cooperate by jointly decoding their received signals.
- Suppose sum-rate achieved by a coalition is apportioned equally.
  - What cooperative behavior emerges?
  - What coalitions are formed?



Equal Rate Splitting

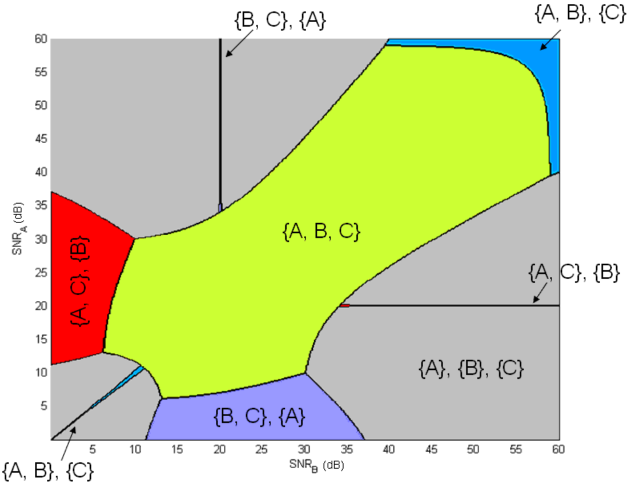


Figure: Stable coalition structures when recd. SNR of user 3 is fixed while those of A and B are varied.

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Coalitional Game Theory: Overview

A coalitional game with transferable utility $\langle \mathcal{S}, v \rangle$

- finite set of players $\mathcal{S}$
- value function $v : \mathcal{G} \rightarrow \mathbb{R} \quad \forall \mathcal{G} \subseteq \mathcal{S}$

Payoff: Share of the value $v(\mathcal{G})$ to each player.

Characteristic function form game: $v(\mathcal{G})$ is unaffected by the "strategy" of users not in $\mathcal{G}$.

When $v(\mathcal{G})$ can be flexibly apportioned between cooperating players, the game is said to have transferable utility (TU).

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Properties of Coalitions under TU

A coalitional game with TU is cohesive if
$\sum_{i=1}^{K} v(\mathcal{G}_k) \leq v(\mathcal{S})$ for every partition $\{\mathcal{G}_1, \ldots, \mathcal{G}_K\}$ of $\mathcal{S}$.

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A game with TU is superadditive if for any two disjoint coalitions $\mathcal{G}_1$ and $\mathcal{G}_2$ we have $v(\mathcal{G}_1 \cup \mathcal{G}_2) \geq v(\mathcal{G}_1) + v(\mathcal{G}_2)$.

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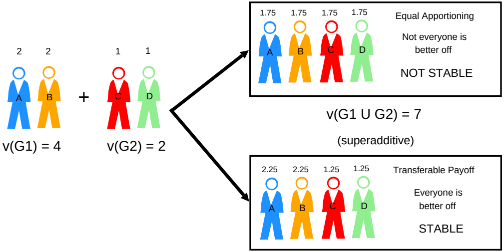

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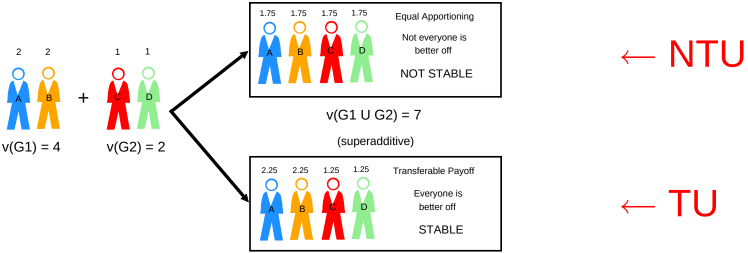

Core & Stability

The core $C(v)$ of a coalitional game is the set of feasible payoffs for which no coalition $\mathcal{G}$ has incentive to defect by achieving a greater payoff for all its members.

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The core can be empty!

Empty core $\Rightarrow$ No stable form of cooperation.

An example with an empty core

A simple 3-player game:

$$\begin{aligned} \mathcal{S} &= \{1,2,3\} \\ v(\{i\}) &= 0, i = 1,2,3. \\ v(\mathcal{G}) &= \alpha, \forall |\mathcal{G}| = 2 \\ 0 < & \alpha < 1 \\ v(\mathcal{S}) &= 1 \end{aligned}$$

Any feasible payoff profile in the core must satisfy:

$$\begin{aligned} x_1 &\geq v(\{1\}) = 0 \\ x_2 &\geq v(\{2\}) = 0 \\ x_3 &\geq v(\{3\}) = 0 \\ x_1 + x_2 &\geq v(\{1,2\}) = \alpha \\ x_2 + x_3 &\geq v(\{2,3\}) = \alpha \\ x_3 + x_1 &\geq v(\{1,3\}) = \alpha \\ x_1 + x_2 + x_3 &= v(\mathcal{S}) = 1 \end{aligned}$$

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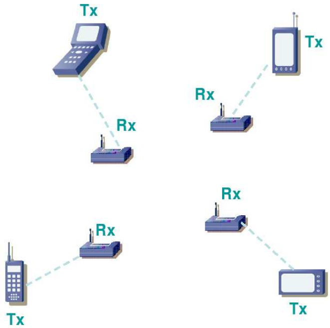
- Existence of a non-empty core $\iff$ feasibility of an LP.
- Core is non-empty only if $\alpha \leq \frac{2}{3}$.
  - Game is superadditive. Superadditivity does not guarantee non-empty core.

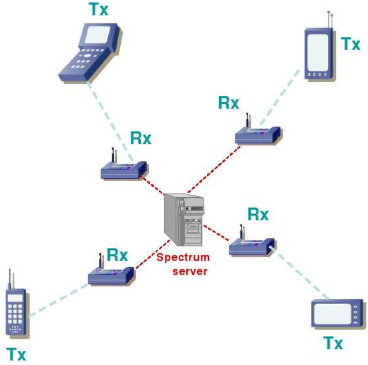
Outline

Links on shared spectrum: Interference channel



- When different types of devices/networks coexist, Tx cooperation may not always be possible.
  - Rx cooperation may be the only feasible way
  - Central entity required → Spectrum server [Ileri & Mandayam 2005], [Raman, Yates & Mandayam, 2005].

Receiver cooperation through joint decoding



- A spectrum server serves as a central entity that enables disparate devices to jointly decode their received signals.

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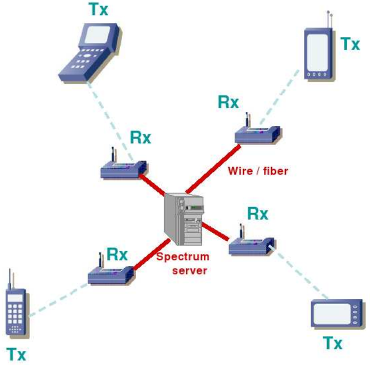

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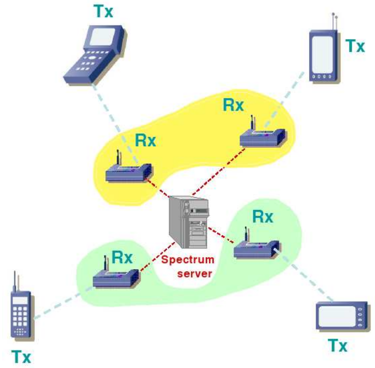

- Receiver cooperation turns IC into a Gaussian SIMO-MAC.
- Under TU, if value $v(\mathcal{G})$ = max information-theoretic sum-rate for users in $\mathcal{G}$
  - Are there stable coalitions?
  - Does the grand coalition form?

Setting up the receiver cooperation game

- Transmitters use Gaussian signalling.

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- $C_{\mathcal{G}}$ = Capacity region of the SIMO-MAC formed by a coalition of links $\mathcal{G}$ is given by $C_{\mathcal{G}} = \left\{ \underline{R}_{\mathcal{G}} : \sum_{k \in A} R_k \leq I(X_A ; Y_{\mathcal{G}} | X_{\mathcal{G}\ A} ; \forall A \subset \mathcal{G}) \right\}$

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- Value $v(\mathcal{G}) = \max_{\underline{R}_{\mathcal{G}}} \sum_{i \in \mathcal{G}} R_i = \max_{p_{X_{\mathcal{G}}}} I(X_{\mathcal{G}}; Y_{\mathcal{G}})$.
- We are interested in maximum sum-rate.
    - Let $D(C_{\mathcal{G}})$ denote the dominant face of the capacity region $C_{\mathcal{G}}$ where $\sum_{i \in \mathcal{S}} R_i = v(\mathcal{S})$

Receiver cooperation - Results

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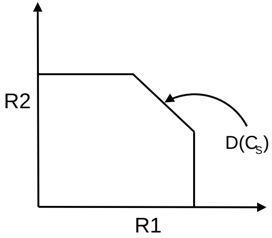
The core of the receiver cooperation IC coalitional game is non-empty. In fact, every point on the dominant face $D(C_{\mathcal{S}})$ of the capacity region $C_{\mathcal{S}}$ of the grand coalition belongs to the core.

[Mathur, Sankaranarayanan & Mandayam, ISIT 2006]

Fair Allocations - Bargaining for Rates



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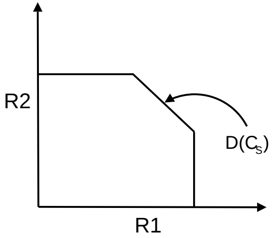

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- Nash Bargaining Solution over IC performance

$$\underline{R}_{\mathcal{S}}^{NBS} = \arg \max_{\{\underline{R}_{\mathcal{S}} : R_m > R_m^{IC}\}} \prod_{m=1}^{M} \left(R_m - R_m^{IC} \right)$$

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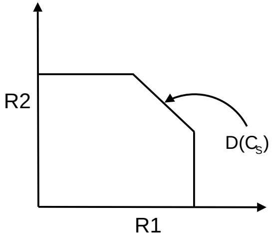

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- Proportional fairness

$$\underline{R}_\mathcal{S}^{PF} = \arg \left\{ \max_{\{\underline{R}_\mathcal{S} \in \mathcal{C}_\mathcal{S}\}} \prod_{m=1}^{M} R_m \right\}$$

An example topology

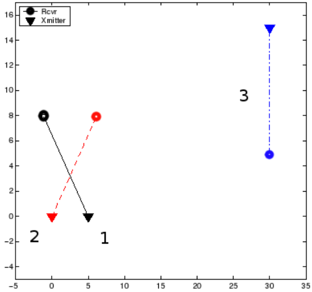

Figure: A skewed topology.

Coalition Structure	R_1	R_2	R_3	Sum-rate
Transferable utility Allocation Strategies (NBS and PF)				
$\{1,2,3\}_{NBS}$	1.4391	1.4346	1.0671	3.9408
$\{1,2,3\}_{PF}$	1.4372	1.4365	1.0671	3.9408
Non-Transferable Payoff Strategy (ER)				
$\{1,2,3\}$	1.3136	1.3136	1.3136	3.9408
$\{1,2\},\{3\}$	1.4174	1.4174	0.9355	3.7703
Stable ER Coalition: $\{1,2\},\{3\}$				

Table: Rate allocations NBS, PF and ER (NTU)

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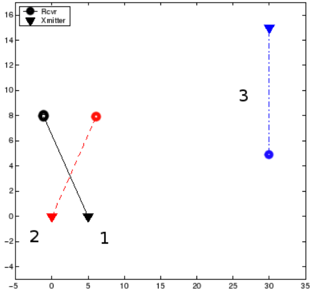

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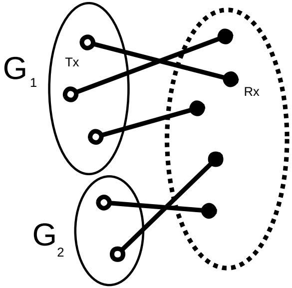
Transmiter cooperation



- Transmitters are allowed to cooperate:
  - Through ideal noise-free inter-user links.
  - Cooperating transmitters do joint encoding: by appropriately structuring their transmit covariance matrices.
- All receivers always jointly decode their recd. signals.
- Transmitters can form coalitions.

Coalitions of transmitters - The value of a coalition

- Such cooperation turns the channel into a MIMO-MAC.
- Virtual MIMO with individual power constraints.
- Value of a coalition $\mathcal{G}$ of transmitters can be defined again as maximum sum-rate achievable by $\mathcal{G}$.

$$v(\mathcal{G}) = \max_{\mathbf{X}_{\mathcal{G}}} I(\mathbf{X}_{\mathcal{G}}; \mathbf{Y}_{\mathcal{S}})$$

- Interference seen by $\mathcal{G}$ depends on whether signals from users outside $\mathcal{G}$ coherently combine at Receivers.
- $v(\mathcal{G})$ depends on the actions of players outside $\mathcal{G}$!
- Tx cooperation game not of characteristic function form!
- Difficult to analyze the game in the present form.

Transmitter Jamming Game

- Users in $\mathcal{G}^c$ jam the coalition $\mathcal{G}$ by jointly transmitting the worst case interference signal $\mathbf{X}_{\mathcal{G}^c}$ [La & Anantharam, 2002]

$$v(\mathcal{G}) = \min_{X_{\mathcal{G}^c}} \max_{X_{\mathcal{G}}} \left\{ \log \frac{|\mathbf{I} + \mathbf{H}_{\mathcal{G}}\mathbf{Q}_{\mathcal{G}}\mathbf{H}_{\mathcal{G}}^{\dagger} + \mathbf{H}_{\mathcal{G}^c}\mathbf{Q}_{\mathcal{G}^c}\mathbf{H}_{\mathcal{G}^c}^{\dagger}|}{|\mathbf{I} + \mathbf{H}_{\mathcal{G}^c}\mathbf{Q}_{\mathcal{G}^c}\mathbf{H}_{\mathcal{G}^c}^{\dagger}|} \right\}$$

$$\mathbf{Q}_{ii} \leq P_i \quad i = 1, \ldots, N$$

$$\mathbf{H}_{\mathcal{G}} = \text{Channel from users in } \mathcal{G} \text{ to receivers.}$$
$$\mathbf{H}_{\mathcal{G}^c} = \text{Channel from users in } \mathcal{G}^c \text{ to receivers.}$$
$$\mathbf{Q}_{\mathcal{G}} = \text{Transmit covariance matrix of coalition } \mathcal{G}.$$
$$\mathbf{Q}_{\mathcal{G}^c} = \text{Transmit covariance matrix of users in } \mathcal{G}^c.$$

- The log function above is strictly concave in $\mathbf{Q}_{\mathcal{G}}$, strictly convex in $\mathbf{Q}_{\mathcal{G}^c}$.
  - $v(\mathcal{G})$ has a saddle point [Diggavi & Cover, 2001].

Transmiter cooperation - Results

Theorem

The coalitional game with TX cooperation is cohesive.

Proof: Follows from saddle point property.

Corollary

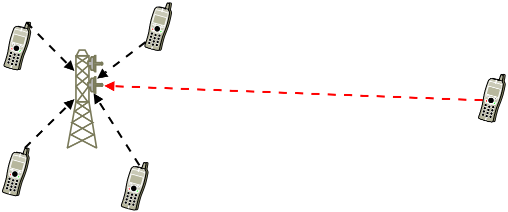
Since the game has TU, the grand coalition is the only candidate coalition structure for the core.

- The core cannot be guaranteed to be non-empty
  - We have counter-examples to show that the grand coalition cannot always be guaranteed to be stable.

Example with an empty core

Consider a cooperative MAC (1 receiver) with 5 users with channel gains given by

$$\mathbf{H} = \begin{pmatrix} 4.6 \times 10^{-2} \\ 5.4 \times 10^{-3} \\ \color{red}{2.3 \times 10^{-6}} \\ 1.2 \times 10^{-3} \\ 1.5 \times 10^{-2} \end{pmatrix}$$



- Recall: Existence of a non-empty core $\equiv$ feasibility of an LP
- Check feasibility of the LP for this example
- Infeasible LP $\Rightarrow$ Core is empty $\Rightarrow$ grand coalition not stable
- Further, cohesiveness $\Rightarrow$ no stable coalition exists.

We find when there are users with widely disparate link gains, cooperation need not be stable.

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  - The grand coalition is always stable.
  - Grand coalition maximizes sum-rate.
- Transmitter cooperation as a jamming game:
  - Grand coalition is the only candidate for the core
  - Not always guaranteed to have stable forms of cooperation.
- When there are costs to cooperation, cohesiveness cannot be guaranteed and disjoint stable coalitions may result.
  - Coalitional games in linear multiuser detectors. [Mathur, Sankaranarayanan & Mandayam, CISS 2006]