Power Allocation and Signaling Strategies for Cooperative Relay Networks

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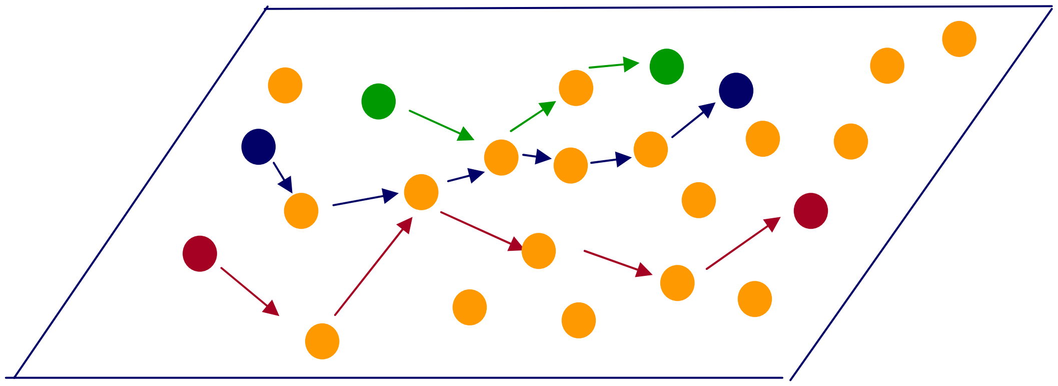
How To Cooperate?



- One way to communicate in a wireless network: multihop through logical point-to-point links
  - Other signals considered to be interference
- Multiuser information theory: we can do better
- What is the best we can do?
  - Capacity region characterizing all achievable rates is an open problem

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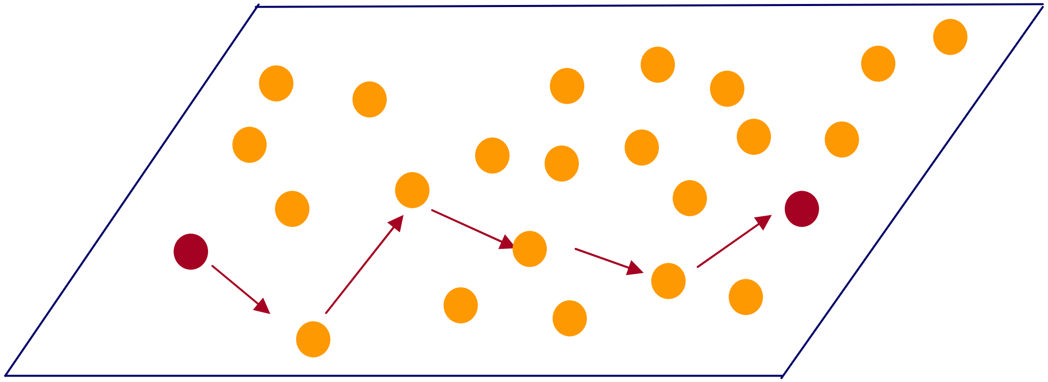
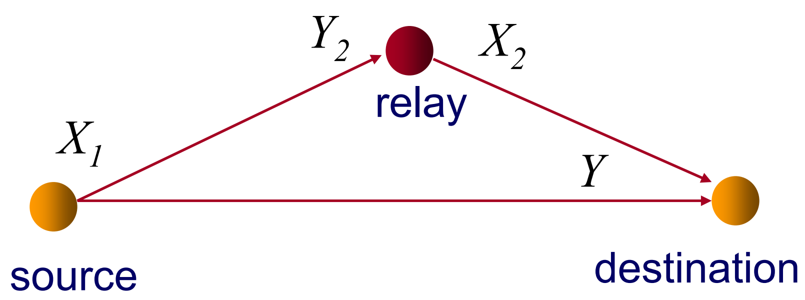
Multiple–Relay Channel



- Simpler scenario: Single source-destination
- Capacity is an open problem

Single-Relay Channel



- Causal Signaling:

  $x_{2,i} = f_i(y_{2,i-1}, \ldots, y_{2,1})$

- Introduced by van der Meulen in 1968
- Capacity is an open problem
- Fundamental ideas proposed by Cover and El Gamal in 1979
  - Exploited: Unreliably received signals, coherent combining

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Proposed Relay Strategies

- **Decode-and-forward (DF):** *[Cover & El Gamal], [Willems], [Xie & Kumar]*

  Relay decodes, re-encodes and transmits

  - Multiple-relay network: *multihopping [Gupta & Kumar],[Xie & Kumar], [Kramer, Gastpar & Gupta]*

- **Compress-and-forward (CF):** *[Cover & El Gamal]*

  Relay transmits a compressed version of its channel output

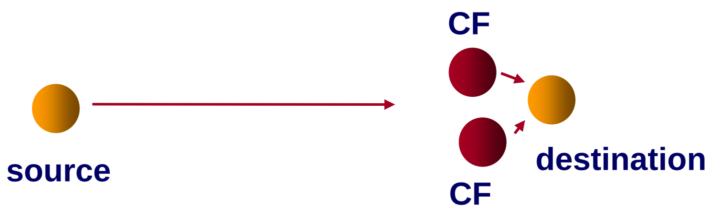
  - Wyner-Ziv coding used to exploit side information at the destination
  - Multiple-relay network: *[Gastpar, Kramer & Gupta], [Kramer et. al.]*

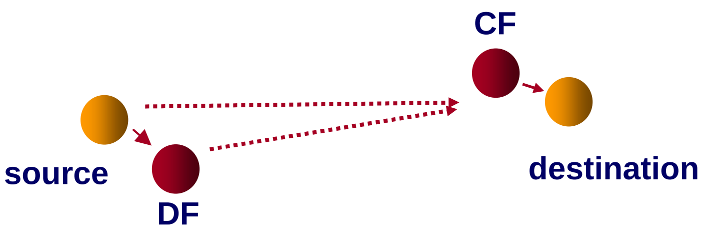
- **Amplify-and-forward (AF):**

  Relay transmits scaled version of its channel output

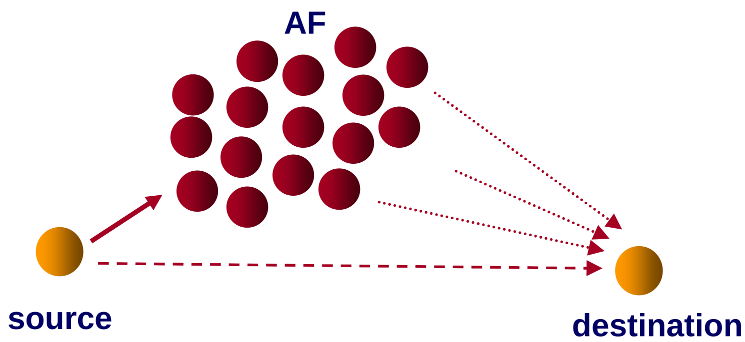
Capacity Results for the Multiple-Relay Channel

- Antenna-clustering capacity *[Gastpar, Kramer & Gupta]*





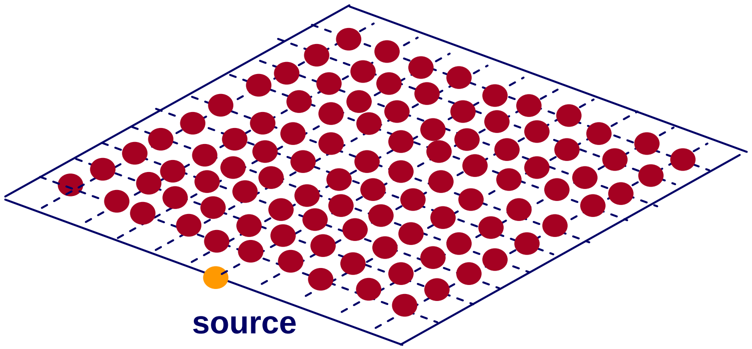
- Asymptotic capacity *[Gastpar & Vetterli]*



- Different:
  - Relay strategy
  - Relay positions

Motivation

- Sensor network:
  - Large number of nodes
  - Low-powered nodes
  - Very low data rates
  - Large bandwidth resources



How do we choose
- Relay strategy
- Set of relays
- Power allocation
- Bandwidth allocation

for

Multicast/Broadcast
- Multihop Transmission
- Decode-and-Forward
- Orthogonal Channels

Single Destination

Energy-Efficient Cooperative Broadcast

Broadcast at rate *R* to all network nodes such that an energy-efficiency metric is optimized

1. Minimize ***Total Transmit Power*** $\sum_i p_i$
2. Maximize ***Network Lifetime***

Duration of a data session until the first node battery is drained *[Chang & Tassiulas, Marks et al., Kang et al.]*

- Equivalent to: Minimize ***Maximum Transmit Power***

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Problem Solution

- In what order and with what powers nodes transmit?

- Determining optimum powers for a given order is easy
- Determining optimum order:
  - For minimum energy problem is NP-complete
  - For maximum lifetime simple!

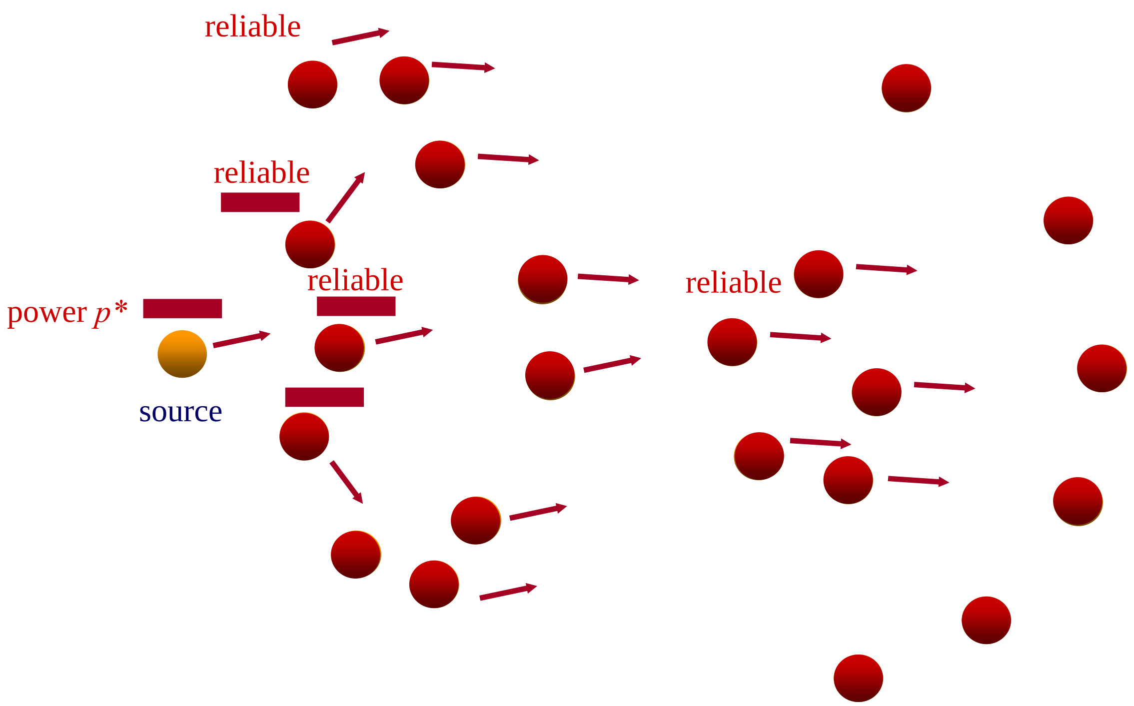
Minimum Total Power

$$\min \Sigma_i p_i$$
$$\mathbf{H}(\mathbf{X})\mathbf{p} \geq \mathbf{1} P_T$$
$$\mathbf{p} \geq \mathbf{0}$$

Maximum Lifetime

$$\min \max \; p_i$$
$$\mathbf{H}(\mathbf{X})\mathbf{p} \geq \mathbf{1} P_T$$
$$\mathbf{p} \geq \mathbf{0}$$

Optimal Solution for the Max Lifetime Problem



- MLAB algorithm finds the optimum *p**
- Node transmits with *p** ***as soon as*** it becomes reliable
- All nodes transmit with the same power *p**
- Nodes die at the same time

Outline

Sensor networks: Data rates are very low, bandwidth is free

How do we choose
- Set of relays
- Relay strategy
- Power allocation
- Bandwidth allocation

For

Multicast/Broadcast
- Multihop Transmission
- Decode-and-Forward
- Orthogonal Channels
- ASAP
- Exploit unreliable receptions

Single Destination

	AF	DF
cc	(shaded)	
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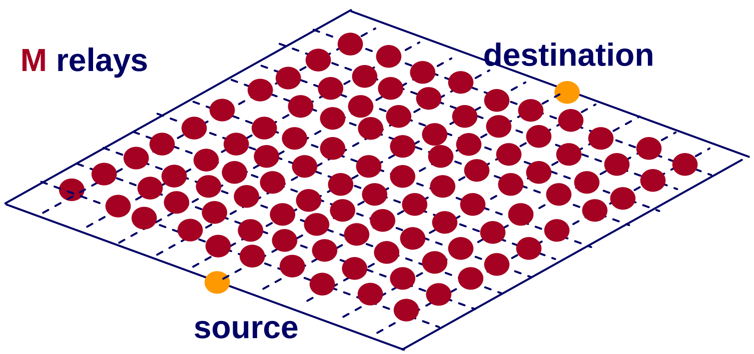
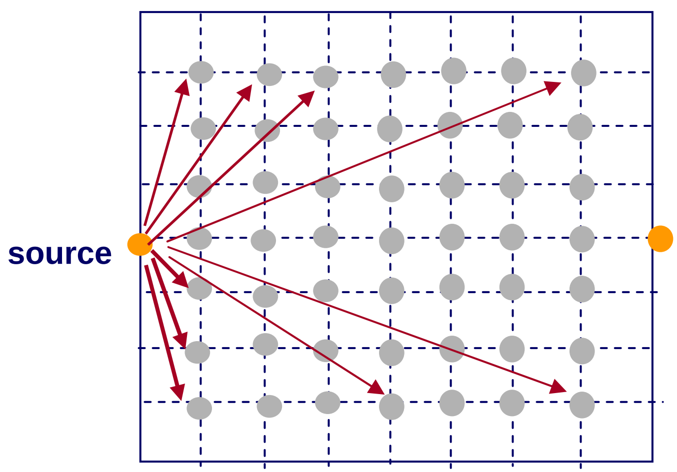
Amplify-and-Forward: Two-Hop Protocol

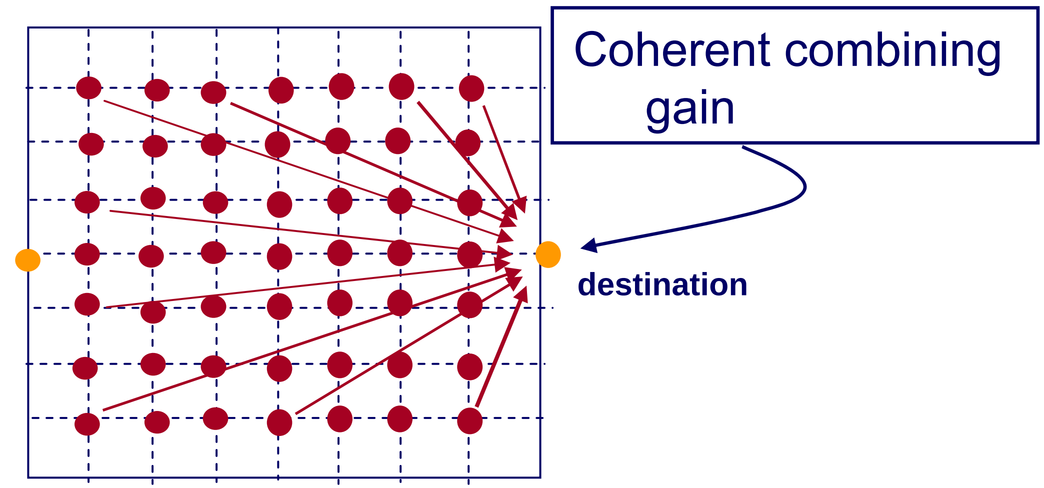


- Rate increases as $log(M)$ *[Gastpar & Vetterli]*
- Power efficiency increases as $\sqrt{M}$ *[Dana & Hassibi]*
  - Dense network
- Two-hop protocol

first hop: source transmits



second hop: relays transmit noisy copies of the source input



Two-Hop Model

	AF	DF
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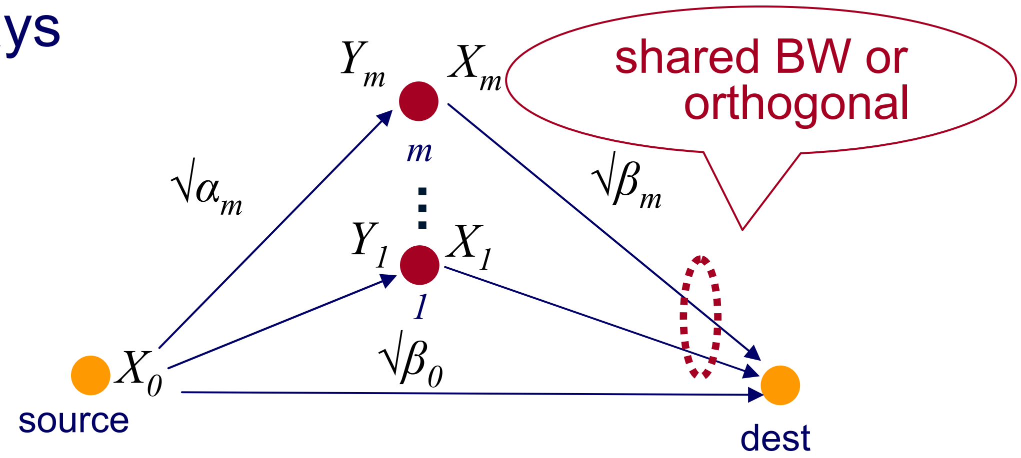
- No communication among relays



- Received at relay m:

$$Y_m[n] = \sqrt{\alpha_m} X_0[n] + Z_m[n]$$

- Amplify-and-forward at relay m:

$$X_m[n] = \sqrt{b_m} Y_m[n - N/2]$$

- Source power: $E[X_0^2] \leq P_0$
- Relay power: $E[X_m^2] \leq P_m$

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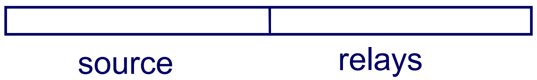
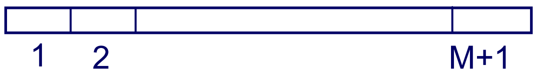
Amplify-and-Forward: Achievable Rate

- Shared bandwidth



- Orthogonal channels



- Rate per user dimension

$$R_{AF} = \frac{1}{2}\log\left(1 + P_0\left(\beta_0 + G(\mathbf{P})\right)\right)$$

AF gain

- AF gain

Shared bandwidth

$$G(\mathbf{P}) = \frac{\left(\sum_{m=1}^{M}\sqrt{\frac{\alpha_m \beta_m P_m}{\alpha_m P_0 + 1}}\right)^2}{\sum_{m=1}^{M}\frac{\beta_m P_m}{\alpha_m P_0 + 1} + 1}$$

Orthogonal channels

$$G(\mathbf{P}) = \sum_{m=1}^{M}\frac{\alpha_m \beta_m P_m}{\alpha_m P_0 + \beta_m P_m + 1}$$

	AF	DF
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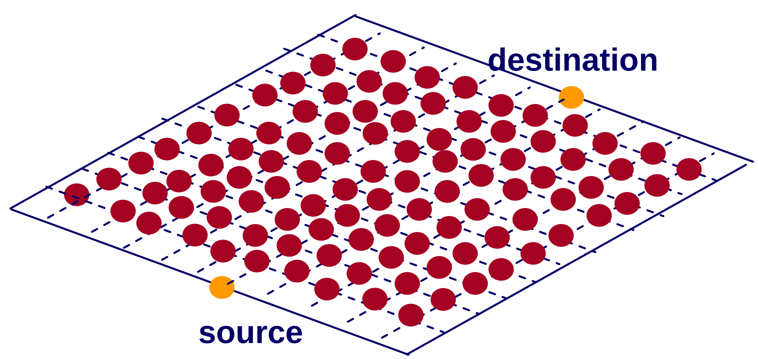
Questions

- Best choice of relays and the optimum relay power allocation?
- Optimum bandwidth?

- Are these the same for DF and AF?



	AF	DF
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Optimum Bandwidth?

- DF: wideband regime
  - Rate increases linearly with power

$$\lim_{W\to\infty} W \log_2\left(1+\frac{p}{WN_0}\right) = \frac{p}{N_0 \ln 2}$$

  - Energy per bit $E_b = \frac{p}{r}$ ⟶ $\left(\frac{E_b}{N_0}\right)_{\min}$ wideband regime

- AF: no benefit from relays in the wideband regime *[Schein]*

$$\lim_{W\to\infty} 2WR_{AF} = \frac{\beta_0 p_0}{N_0 \ln 2}$$

p_0 – total source power

→ Rate achieved by source transmission in the wideband regime

> • AF rate $2WR_{AF}$ maximized for a finite bandwidth $W=W^*$

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	AF	DF
cc		
⊥		

The AF Bandwidth/Power Relay Problem

- Maximize the achievable rate under the total power constraint

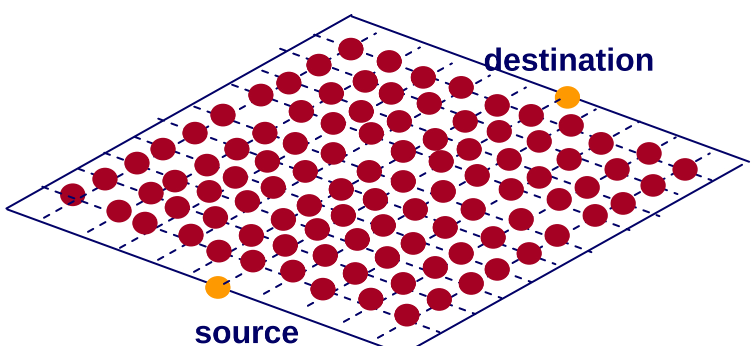
$$\max_{W,\mathbf{P}} 2WR_{AF}(\mathbf{P})$$

$$s.t. \quad 2W\sum_{m=0}^{M} P_m \leq p,$$

$$\mathbf{P} \geq 0,$$

$$W \geq 0$$

- Relays signal in:
  - Shared bandwidth
  - Orthogonal channels



	AF	DF
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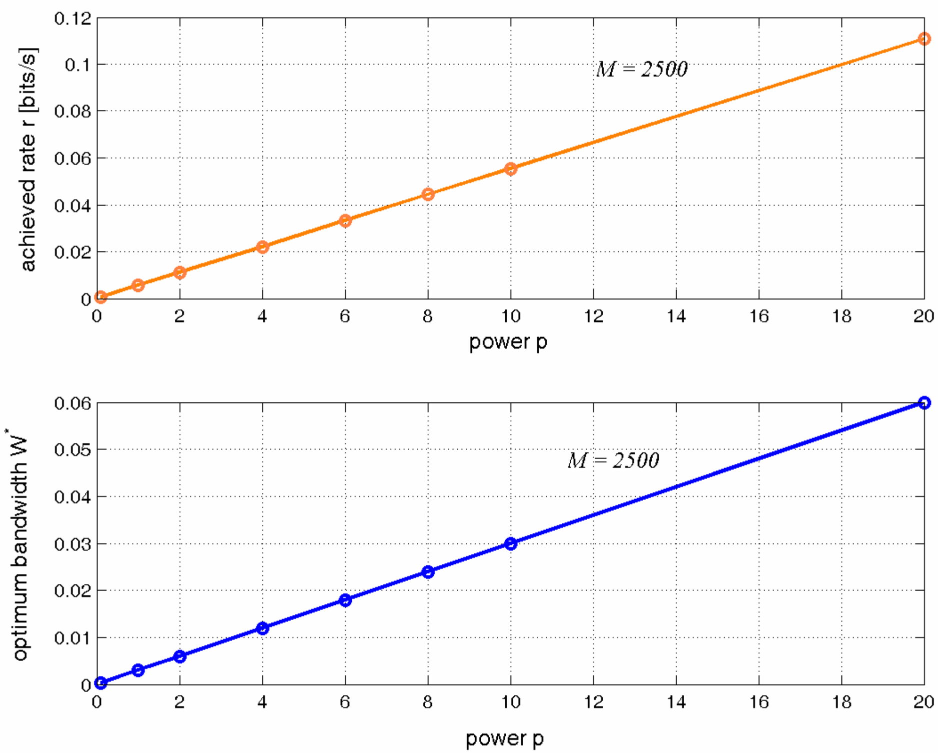
Solution to the AF Relay Problem

- From the K-T conditions:

$(P_0^*, P_1^*, \ldots, P_M^*)$ independent from p and W^*

The maximum rate r^*, power p and the optimum bandwidth W^* have a linear relationship



- Numerical results: Achieved rate and optimum bandwidth for different values of p

- What are the optimum relay powers $(P_1^*, P_2^*, \ldots, P_M^*)$?

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Relay Power Allocation: Shared Bandwidth

	AF	DF
cc	(shaded)	
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- Relay power allocation problem equivalent to maximizing the AF gain

$$\max_{\mathbf{P}} \; G(\mathbf{P})$$
$$\sum_{m=1}^{M} P_m \le P_R \,,$$
$$\mathbf{P} \ge 0$$

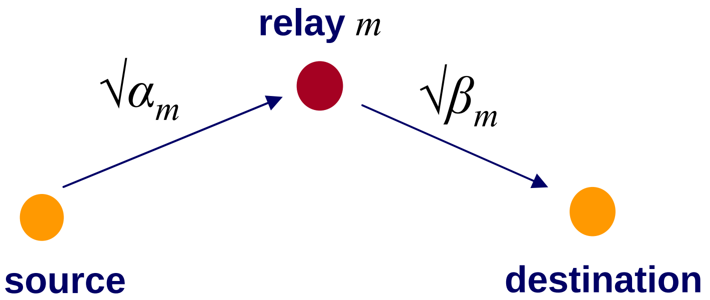
$$G(\mathbf{P}) = \frac{\left(\sum_{m=1}^{M} \sqrt{\frac{\alpha_m \beta_m P_m}{\alpha_m P_0 + 1}} \right)^2}{\sum_{m=1}^{M} \frac{\beta_m P_m}{\alpha_m P_0 + 1} + 1}$$

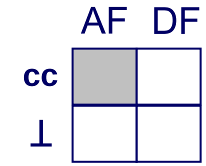
$$P_R = \frac{p}{2W} - P_0$$

- Optimum relay powers:

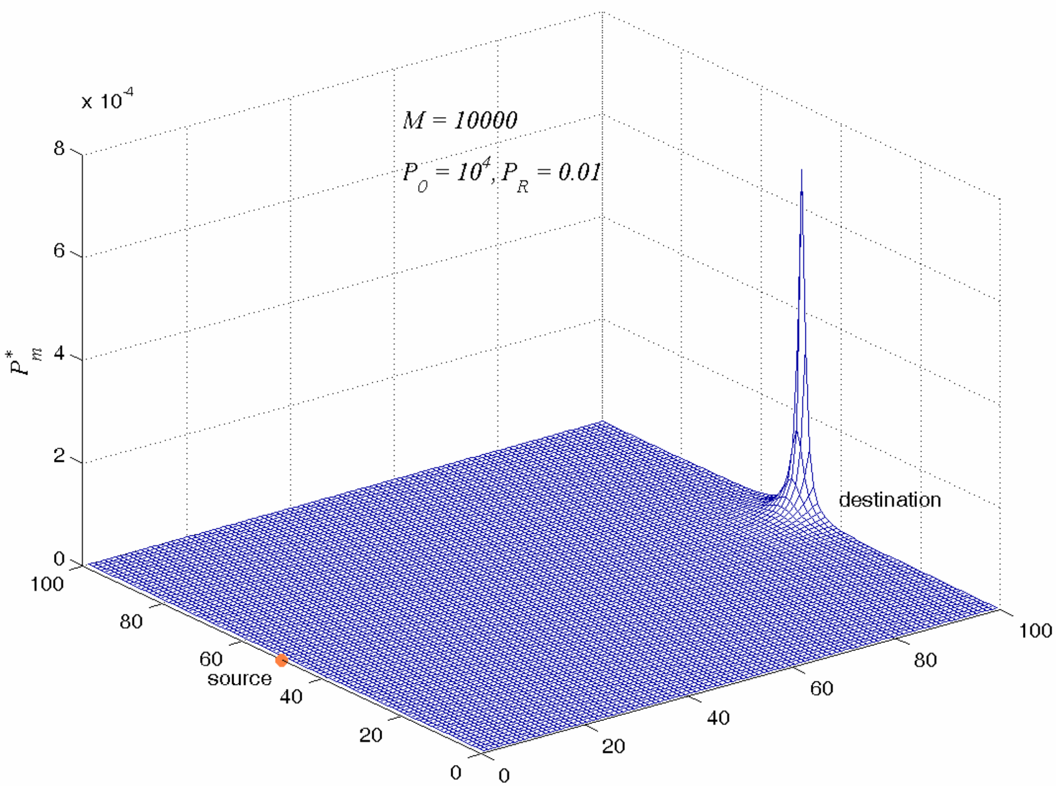
$$P_m^* = \frac{P_R \delta_m}{\sum_{k=1}^{M} \delta_k}$$

δ_m depends on α_m, β_m





Numerical Results: Relay Powers



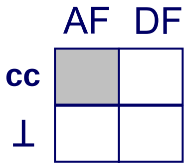
- **High SNR at the relays**
  Large source power, small relay power

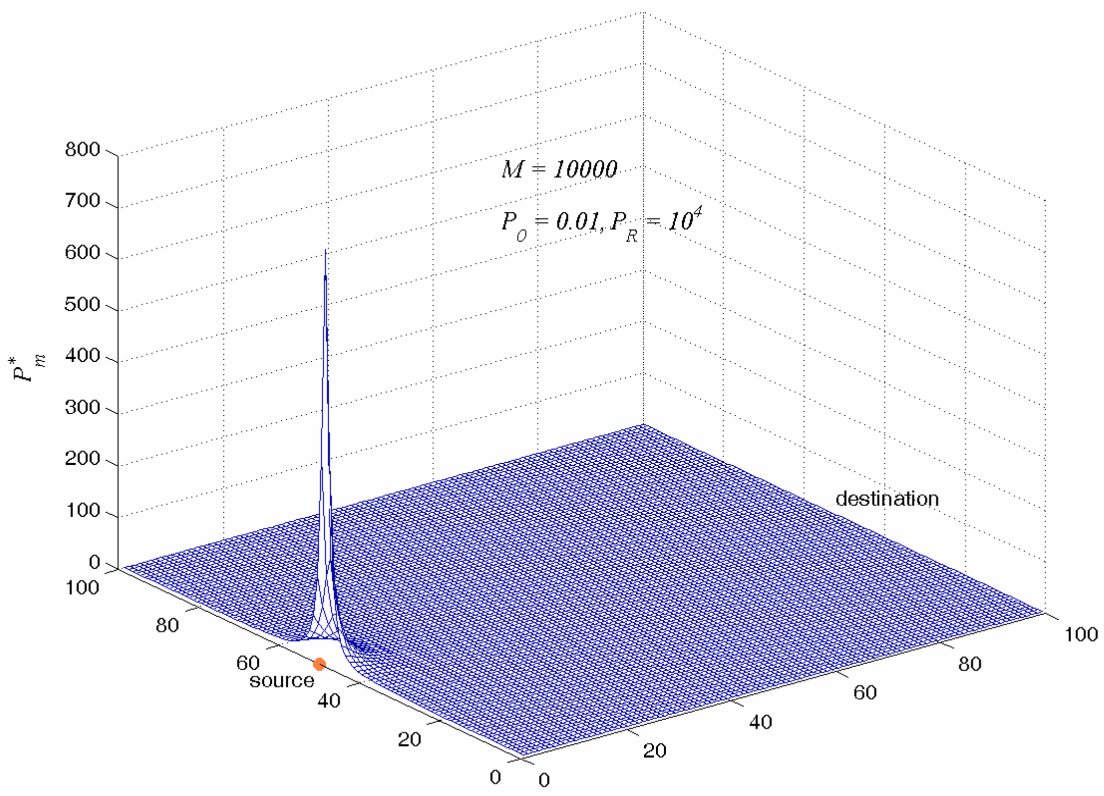
→ Improve MAC side: use relays with strong channel to the destination

→ Powers:

$$P_m^* = \frac{P_R \beta_m}{\sum_{k=1}^{M} \beta_k}$$

Numerical Results: Relay Powers

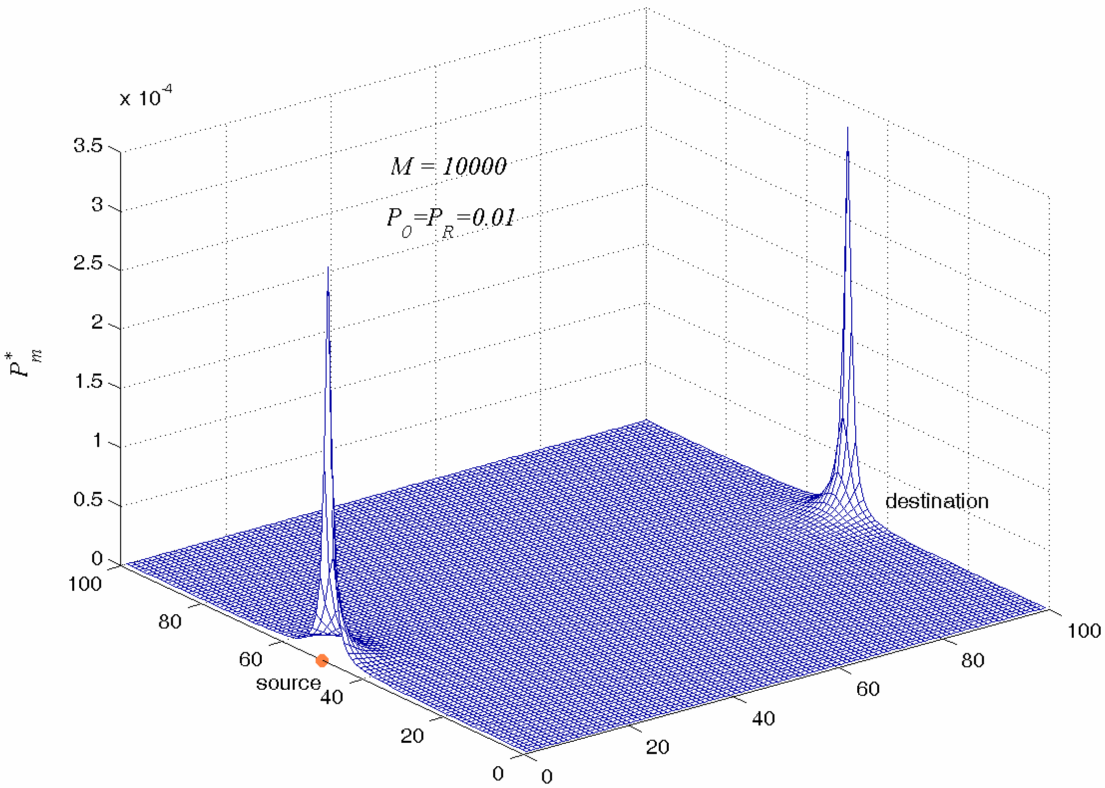




- **Low SNR at the relays**
  Small source power, large relay power

→ Improve BC side:
use relays with strong source-relay channel

Numerical Results: Relay Powers



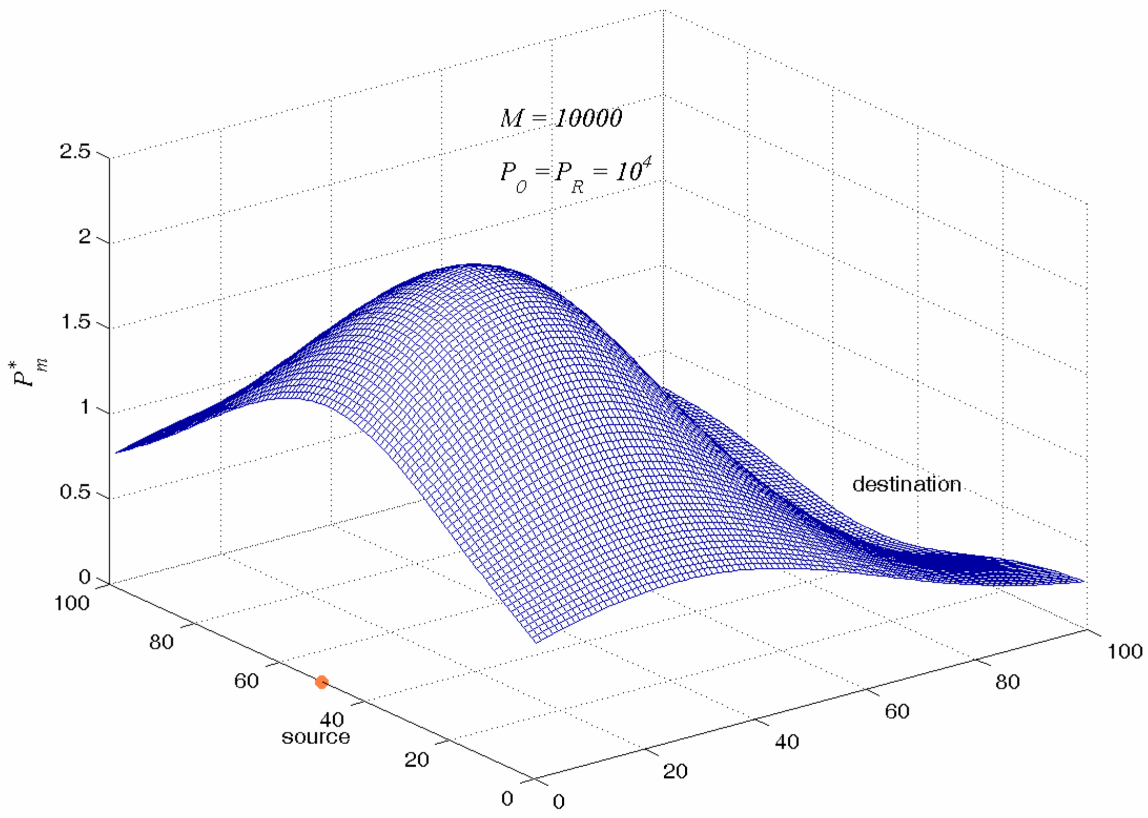
- Low SNR regime

→ Powers:

$$P_m^* = \frac{\alpha_m \beta_m P_R}{\sum_{k=1}^{M} \alpha_k \beta_k}$$

Numerical Results: Relay Powers

	AF	DF
cc		
⊥		



- High-SNR regime (small bandwidth)

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Numerical Results: Relay Powers

	AF	DF
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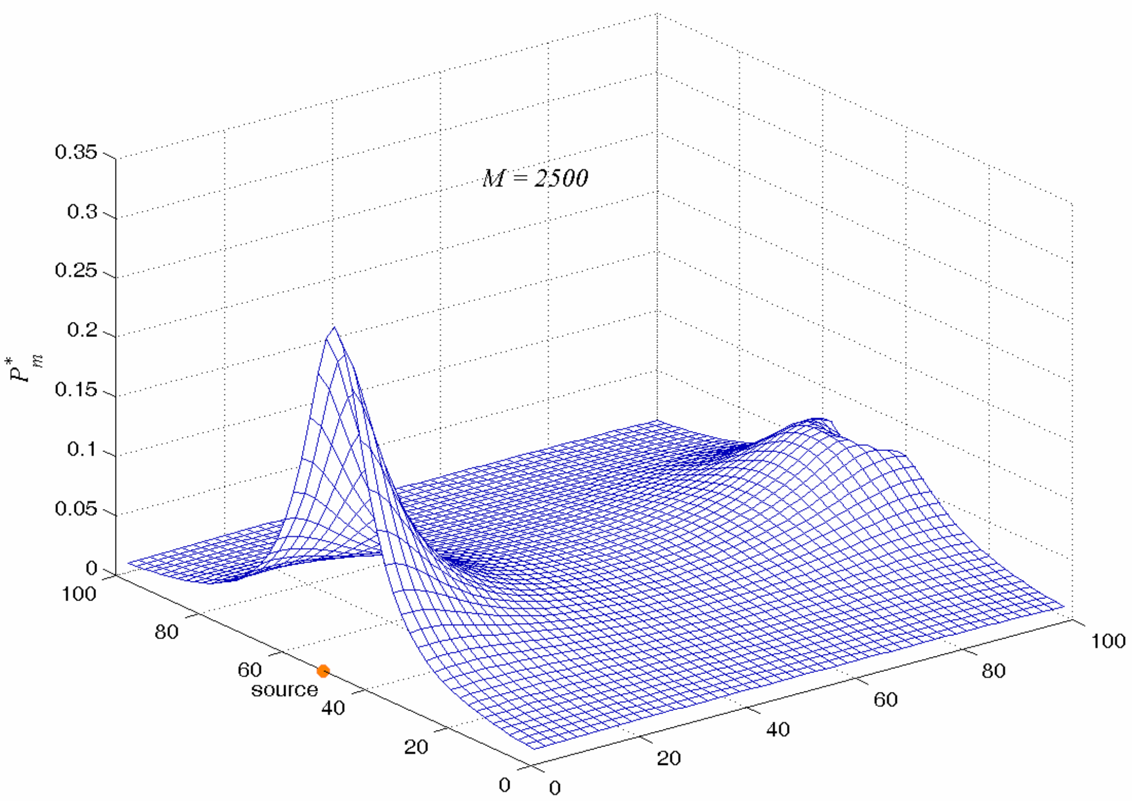
- Optimum relay powers for (P_0^*, W^*)



- Easy to find (W^*, P_0^*) numerically

$$P_m^* = \frac{P_R^* \delta_m}{\sum_{k=1}^{M} \delta_k}$$

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	AF	DF
cc		
⊥	(shaded)	

Orthogonal Channels: Relay Power Allocation

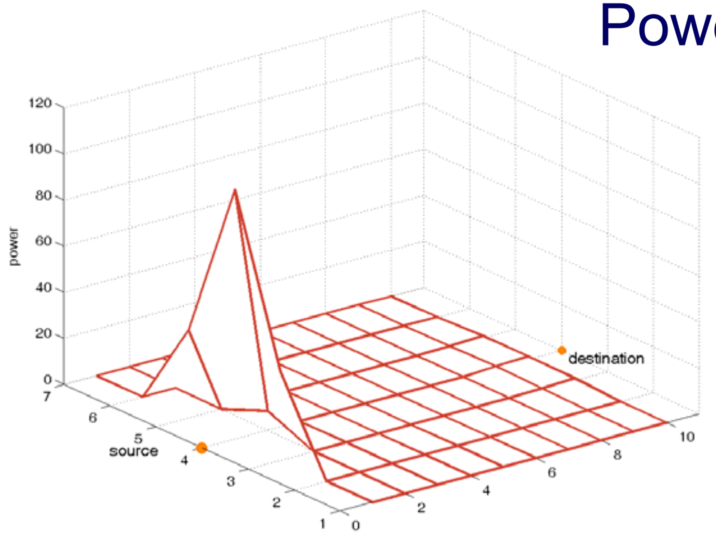
- Optimum relay powers:

$$P_k^* = \frac{\alpha_k}{\gamma_k}\left[\frac{1}{\sqrt{\eta}} - \frac{1}{\gamma_k}\right]^+ \qquad \gamma_k = \sqrt{\frac{\alpha_k \beta_k}{\alpha_k P_0 + 1}}$$

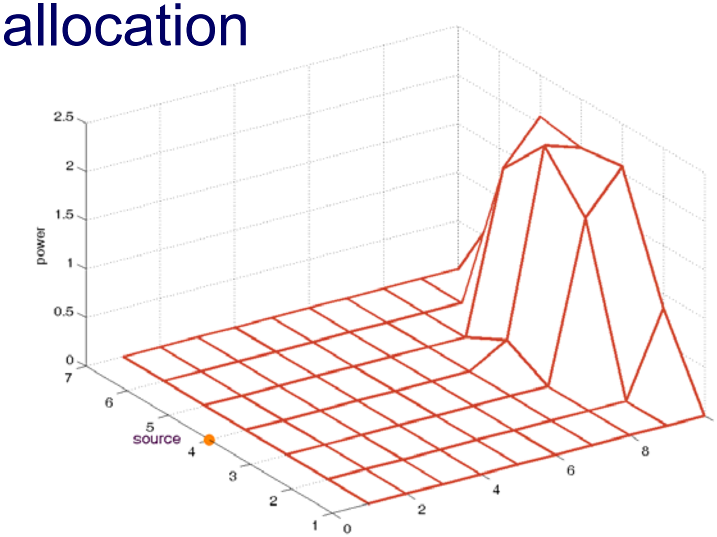
- Nodes with higher γ_k chosen to relay

- The optimum choice of relays strongly depends on P_0

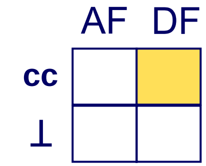
Power allocation



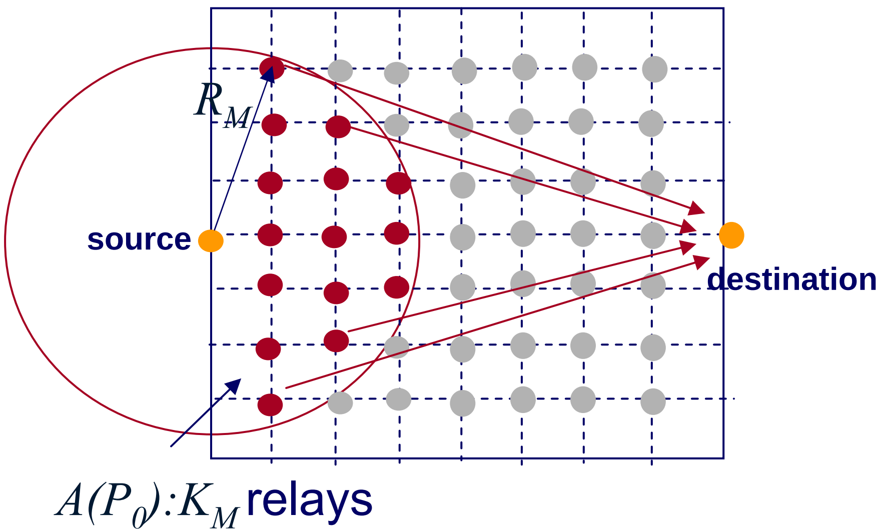
small P_0



large P_0



Decode-and-forward: Shared bandwidth



- Source transmits with power P_0
- Set of K_M relays coherently transmit
- Rate constrained by the worst source-relay link

$R_M = \max_{m \in A(P_0)} d_m$ - radius of reliable transmission

- Rate $R_{DF} = \min\{R_{BC}, R_{MAC}\}$

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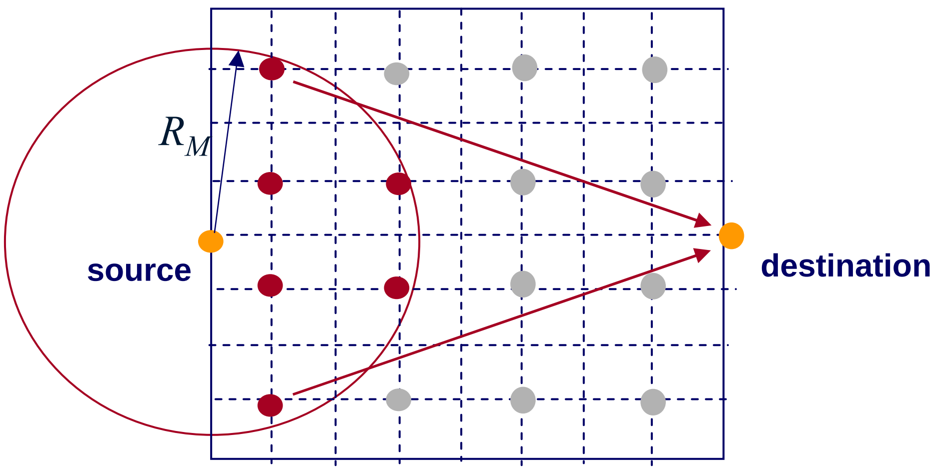
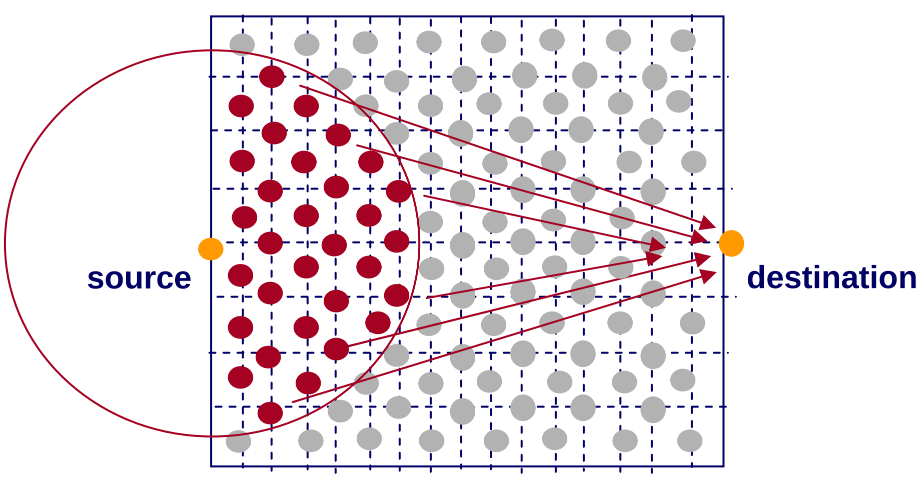
	AF	DF
cc		(highlighted)
⊥		

DF as the number of nodes increases





- As the node density increases:
  - R_{MAC} increases
  - To allow $R_{BC} = R_{MAC}$:
    - $\rightarrow$ Increase R_{BC}
    - $\rightarrow$ Decrease R_M
- Both rates increase
- At the source
  - "Beamforming cluster"
  - No "dead zone"

	AF	DF
cc		
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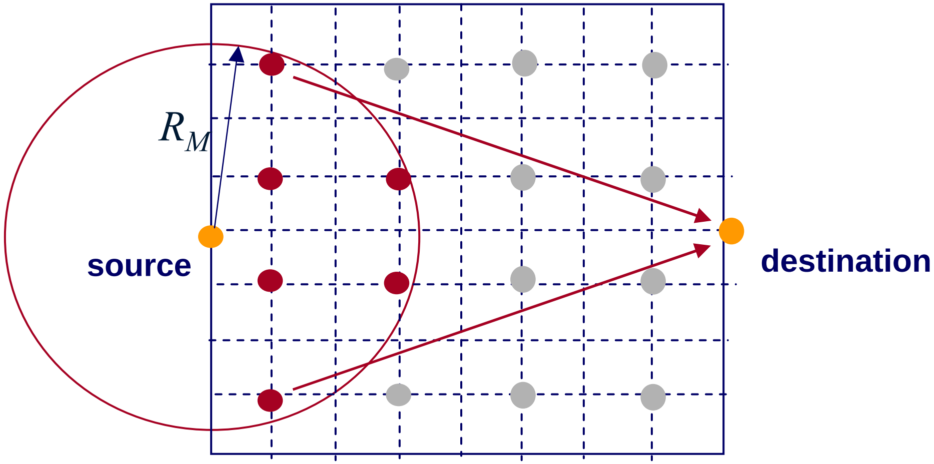
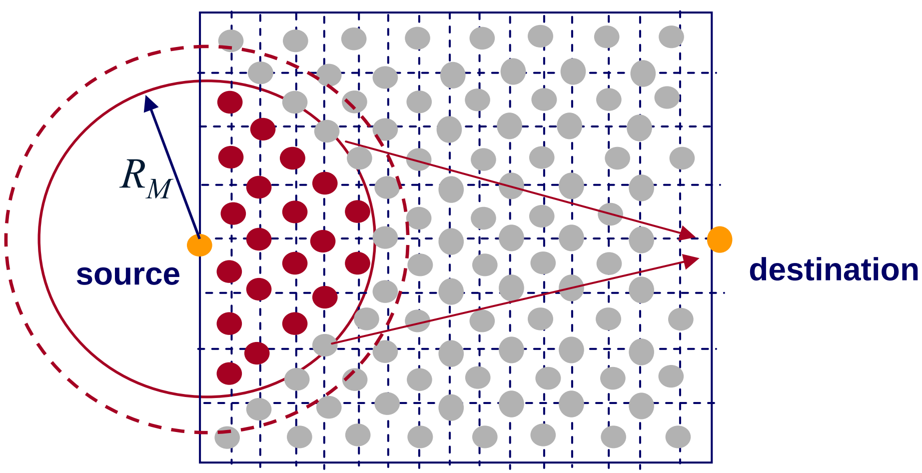
DF as the number of nodes increases





- As the node density increases:
  - R_{MAC} increases
  - To allow $R_{BC} = R^l_{MAC}$:
    - → Increase R_{BC}
    - → Decrease R_M
- Both rates increase
- At the source
  - “Beamforming cluster”
  - No “dead zone”

DF vs. AF comparison – dense network

	AF	DF
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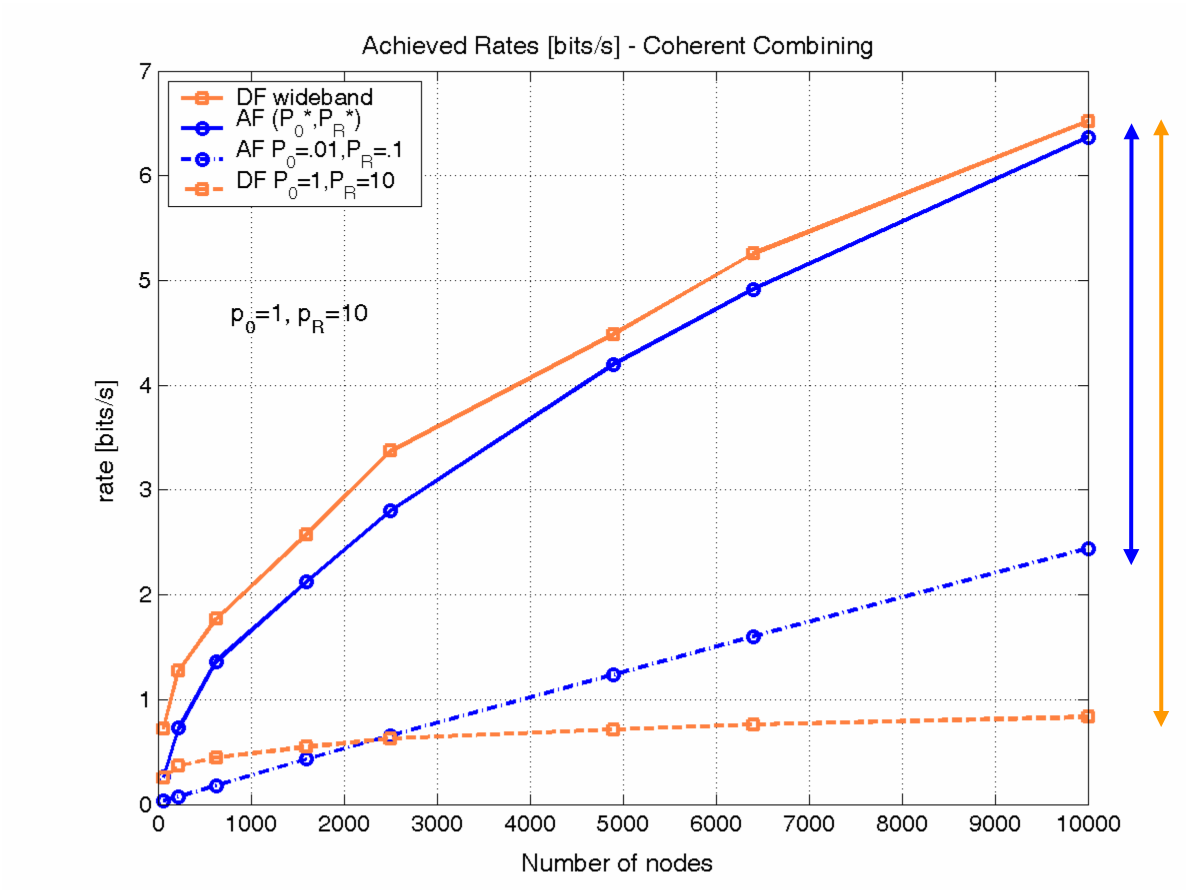

Bandwidth optimization

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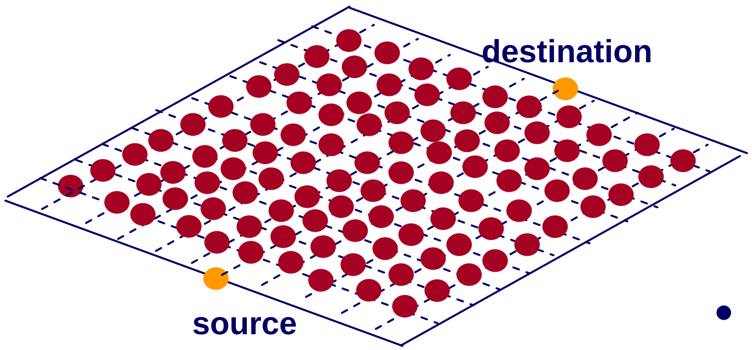
The DF strategy: Orthogonal channels



- Relay through only one node

$$k^* = \arg \min_{k \in U} \left[\frac{1}{\alpha_k} + \frac{1}{\beta_k} - \frac{\beta_0}{\alpha_k \beta_k} \right]$$

$$U = \{ i \mid \alpha_i > \beta_0 \, , \; \beta_i > \beta_0 \}$$

- Consequence of the two-hop limitation

- No gain from large number of nodes

	AF	DF
cc		
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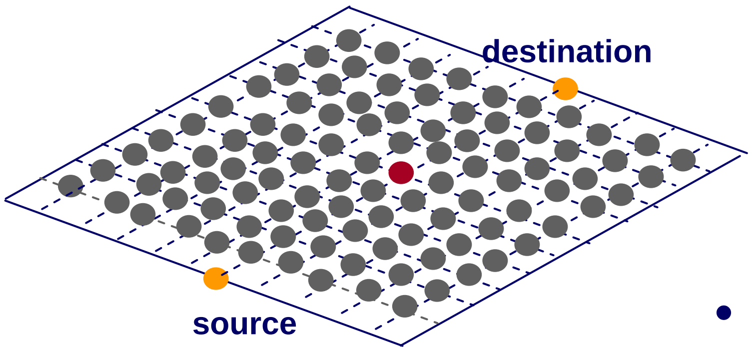
The DF strategy: Orthogonal channels



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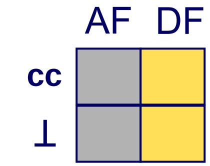
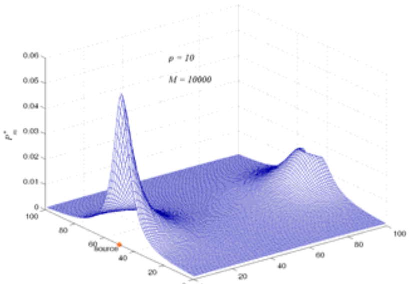
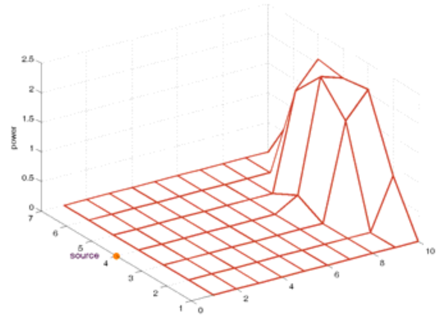
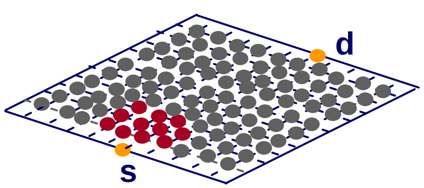


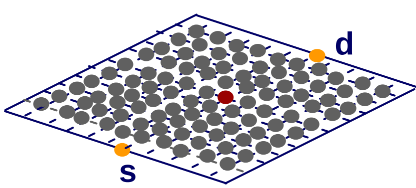
2-Hop Summary

- Characterized the optimum bandwidth for amplify-and-forward
  - Operate in the linear regime
- AF in a shared bandwidth outperforms orthogonal signaling
- Different relay strategies employ different set of relays









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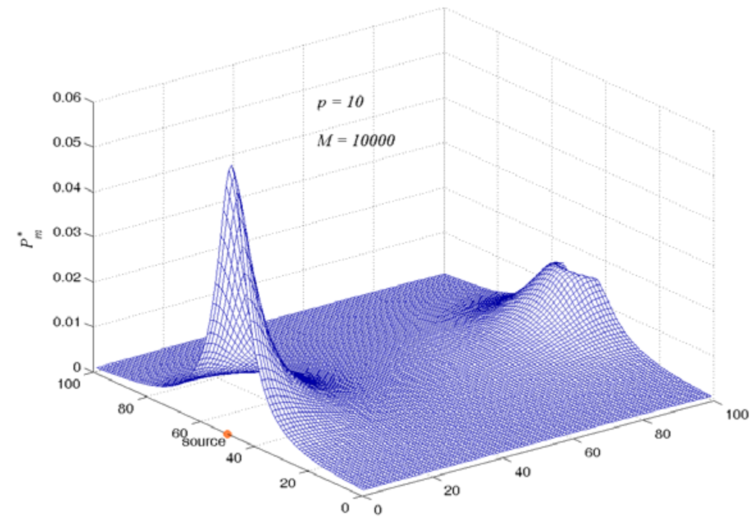
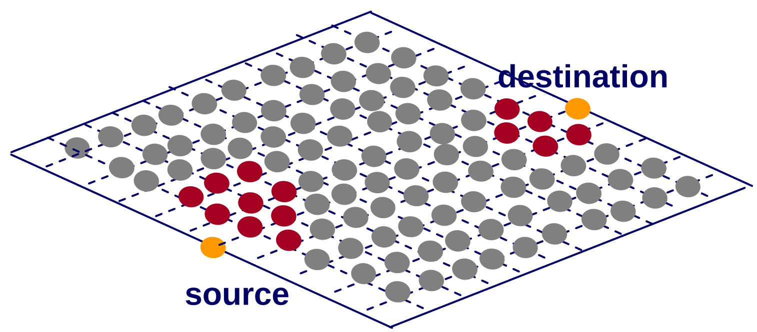
Discussion

- Transmitter and Receiver clusters
- 3-Hop System
  - MIMO system between transmit cluster and receiver cluster
- What are the gains?
- What are the relay strategies?

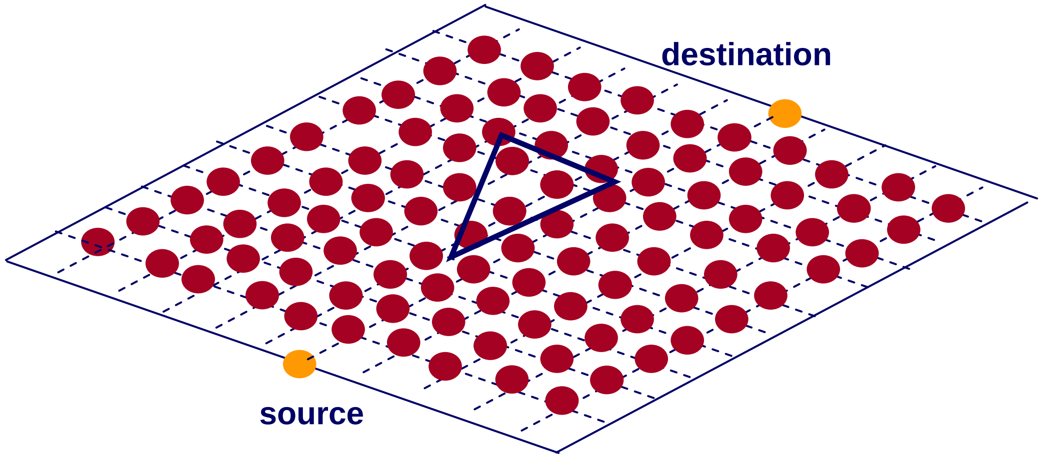




- Working on: capacity results for channel models that incorporate node cooperation
  - Analytical solutions

Discussion

- Consider networks with large number of nodes
- Consider their building blocks



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