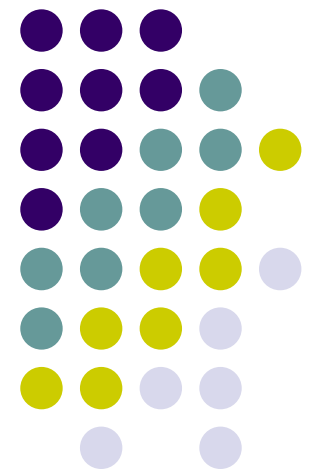


# Reducing The Computational Cost of Bayesian Indoor Positioning Systems

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WINLAB Research Review  
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# Motivation

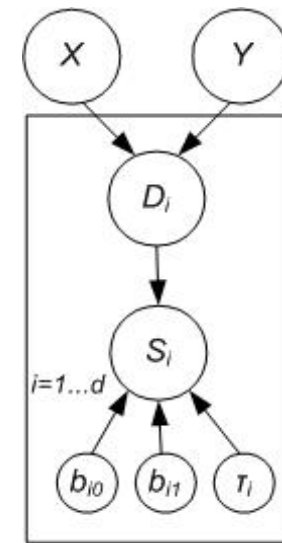
- Bayesian networks (BNs)
  - Have recently been used for location estimation in wireless networks
    - Networks M1, M2, M3, A1 [Madigan'05, Elnahrawy]
  - Can incorporate radio properties
    - Received signal strength (RSS)
    - Angle of Arrival of the signal (AoA)
  - Attractive approach
    - Similar performance with smaller training set when compared to other solutions
    - Training set: Radio properties measured at specific locations



# Motivation (cont'd)

- M1 network
  - $X, Y$ : latent variables (location)
  - $S_i$ : observable data (RSS)
  - $b_{0i}, b_{1i}, \tau_i$ : stochastic variables
  - Linear regression model for RSS

- $S_i = b_{0i} + b_{1i} \log(1 + D_i)$



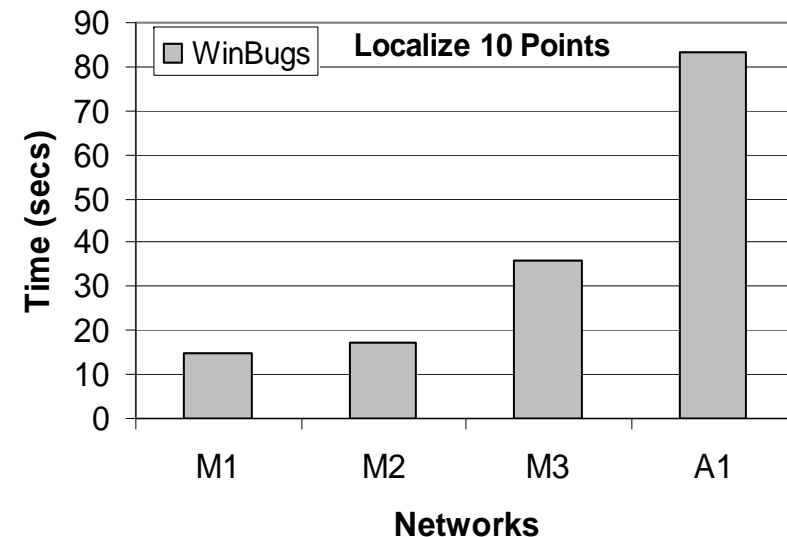
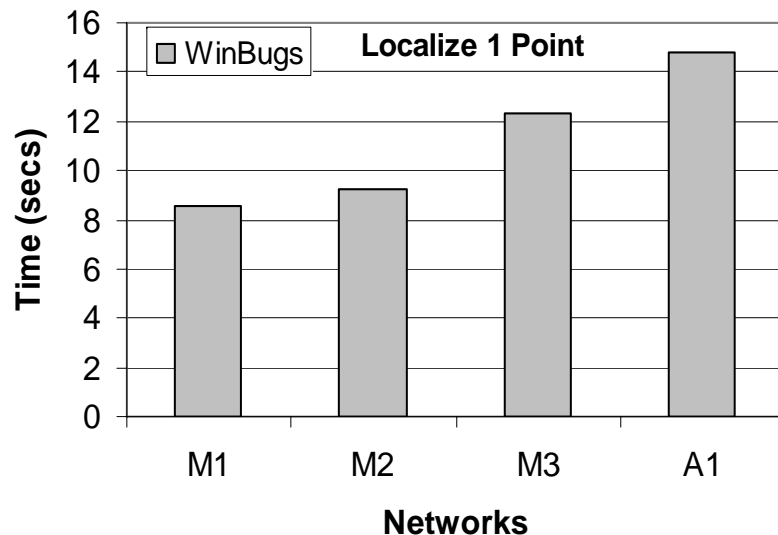
M1

- We want to compute the probability density function (PDF) of  $X, Y$  given the training set



# Motivation (cont'd)

- Computational cost of using BNs with statistical packages like WinBugs is large
- Platform: Pentium 4 PC, 2.80 GHz, 1 GB RAM, Windows XP
- Training set size: 253



- Can we be faster?



# Talk Outline

- Motivation
- Our Approach & Contributions
- Experimental Results
- Analytic Analysis
- Conclusions & Future Work



# Our Approach

- We have implemented our own solvers
- Our Bayesian networks have no closed-form solution
  - We use Markov Chain Monte Carlo (MCMC) simulation
  - MCMC explores the PDF of the variables in a BN using sampling



# MCMC methods

- Gibbs sampling

- It draws a new value for a variable  $v$  from its “full conditional”,  $f(v) \equiv p(v | V \setminus v)$ 
  - Conditional PDF given all the other quantities are fixed at their current value

$$p(v | V \setminus v) \propto p(v | pa(v)) \prod_{w \in child(v)} p(w | pa(w))$$

- E.g. Conjugate sampling, Slice sampling

- Metropolis algorithm

- Draws candidate values for  $v$  from a proposal distribution



# Contributions

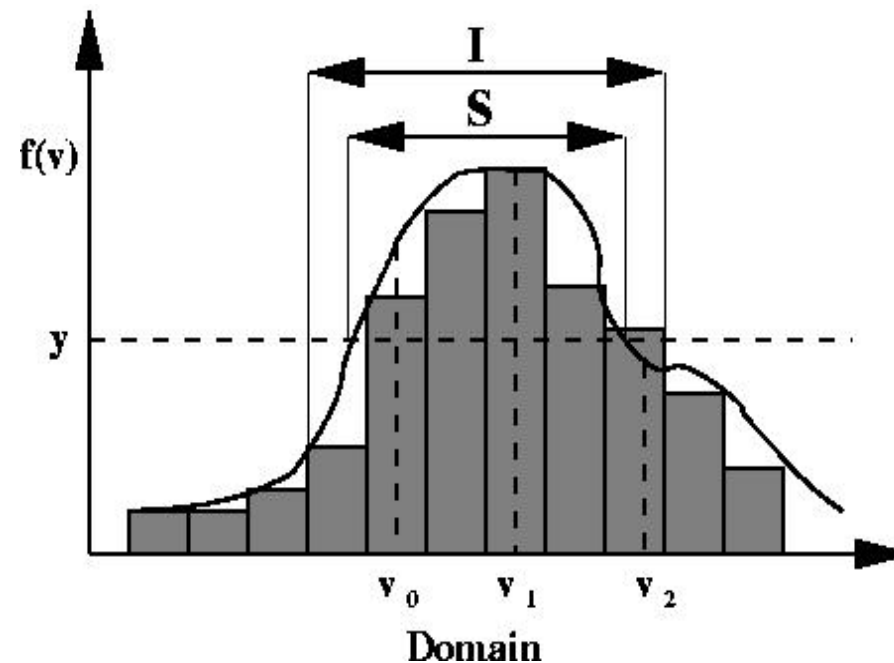
- Low latency in localization
  - 1 or 10 points  $\Rightarrow$  0.5 sec
  - 51 points with no location info in the training set  $\Rightarrow$  6 secs
  - Over 10 times faster than WinBugs
- Full conditionals are flat
  - Most efficient algorithm: Whole domain sampling
- Analytic model
  - How flat a distribution should be in order to use whole domain sampling





# Slice Sampling

- Goal: Compute the histogram of a variable  $v$  given we can compute only  $f(v)$
- Challenge: unknown shape of  $f$





# Slice Sampling (cont'd)

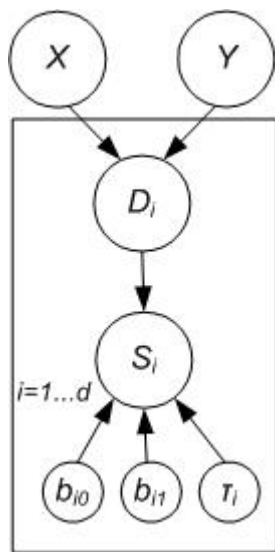
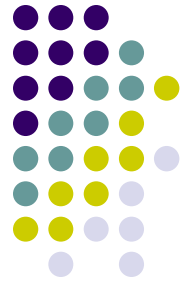
- Defines an interval  $S$  given the current value  $v_0$  of a variable  $v$ 
  - Hard to estimate the edges of  $S$ . It is approximated by  $I$
- Schemes to find  $I$ 
  - Whole domain  $\Rightarrow$  **Whole Domain Sampling**
  - Step out
  - Double out
- The new value of  $v$  is chosen from  $S \cap I$
- As  $I$  approximates  $S$  better, the fewer the rejections (i.e. values not in  $S \cap I$ )



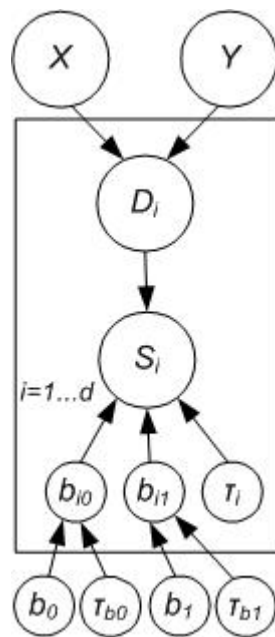
# Tradeoffs of Estimating I

- Whole Domain
  - Pros: Easy to compute  $I$
  - Cons: Potentially, a lot of rejections
- Step Out
  - Pros: A few rejections
  - Cons: Many evaluations of  $f(v)$  to compute  $I$
- Double Out: Intermediate solution

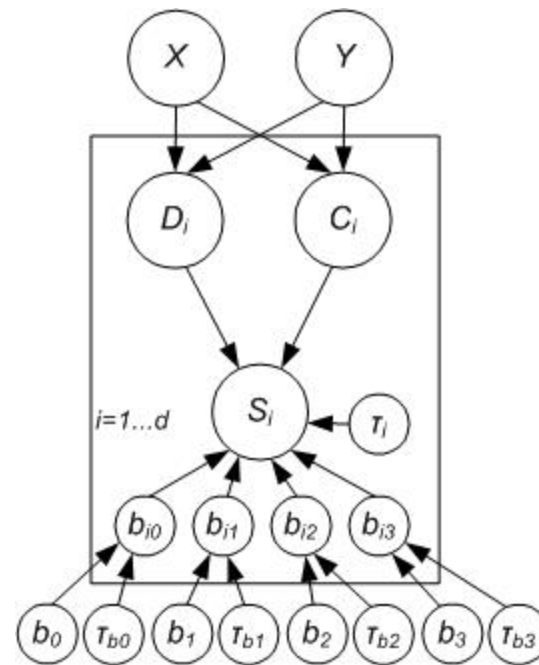
# Localization Models



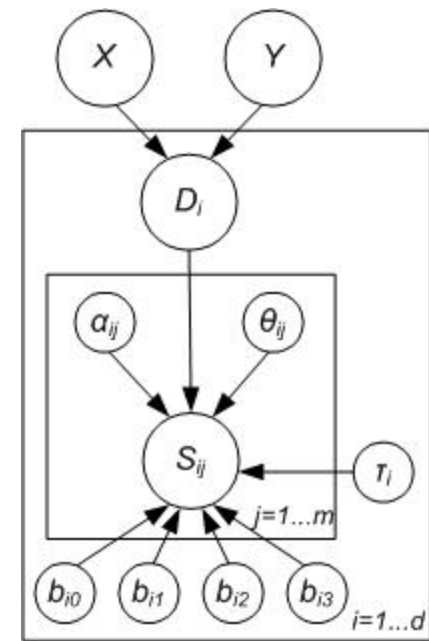
M1



M2



M3



A1



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# MCMC Algorithms

- Met algorithms
  - Metropolis for X, Y, angle
  - Conjugate sampling for the other variables
- Slice algorithms
  - Slice sampling for X, Y, angle
  - Conjugate sampling for the other variables

Algorithm	Description
met wd	Metropolis with proposal distribution uniform over the whole domain of X, Y (angle)
met sd = k (or sd=k, l)	Metropolis with proposal distribution Gaussian whose standard deviation = k ft (l rands for angle)
slice wd	Slice sampling over the whole domain X, Y (angle)
slice so = k (or so=k, l)	Slice sampling by stepping out with $w = k$ ft ( $w=l$ rands for angle)
slice do = k (or do=k, l)	Slice sampling by doubling out with $w = k$ ft ( $w = l$ rands for angle)
slice2d	2d slice sampling by updating (X, Y) together and using the whole domain of X, Y (1d for angle over whole domain)

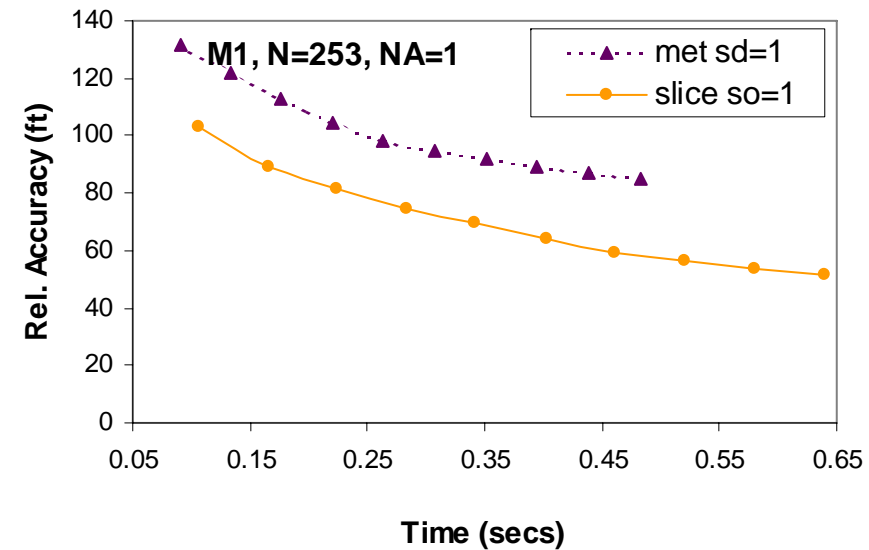
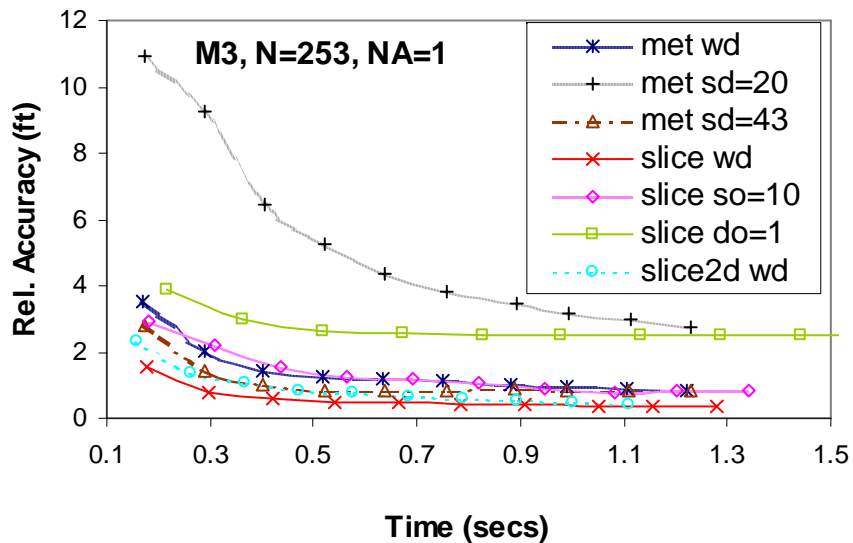
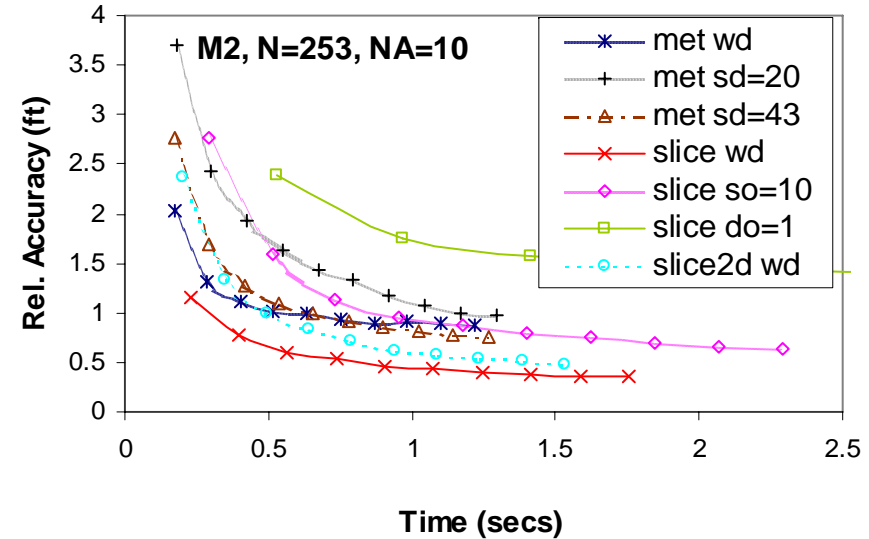
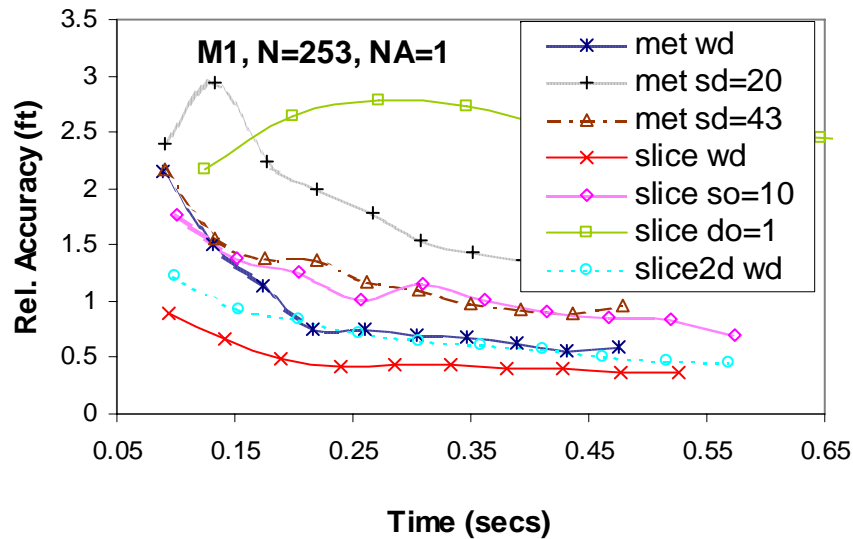


# Comparing Algorithms

- Metric: **Relative Reference**
  - Euclidean distance of the mean of our results to the ones from WinBugs
  - WinBugs results
    - 10000 iterations burnin
    - 100000 iterations additional
- Intuition: The results of our solver should converge to the statistics of a well-tested solver after a long run



# Comparing Algorithms (cont'd)

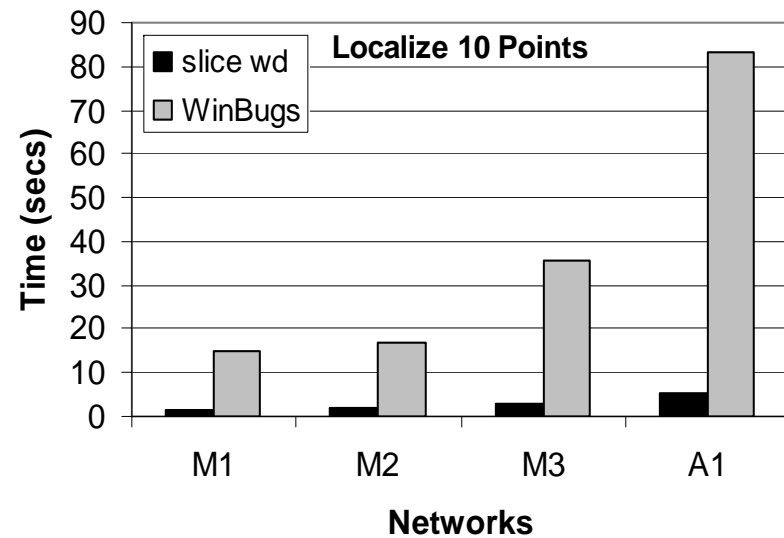
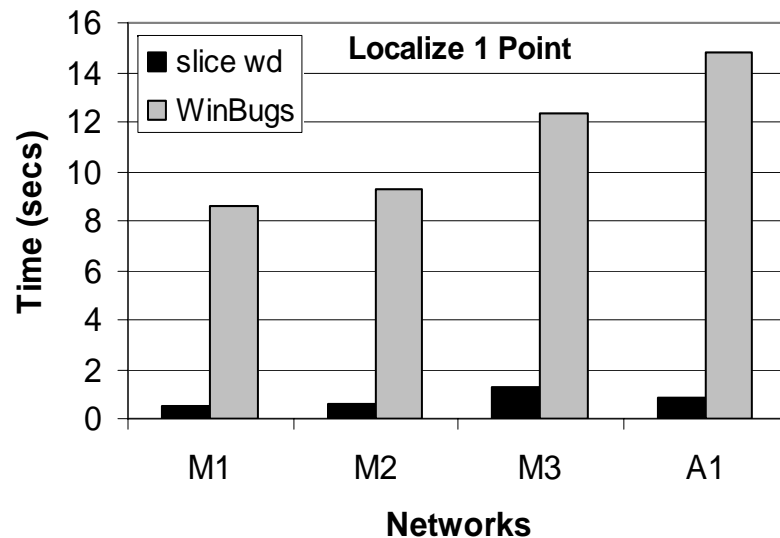






# Comparing Against WinBugs

- Whole Domain sampling is faster than WinBugs by a factor that ranges:
  - Localize 1 point: 9.8 (M1) – 17.9 (A1)
  - Localize 10 points: 9.1 (M1) – 16.1 (A1)





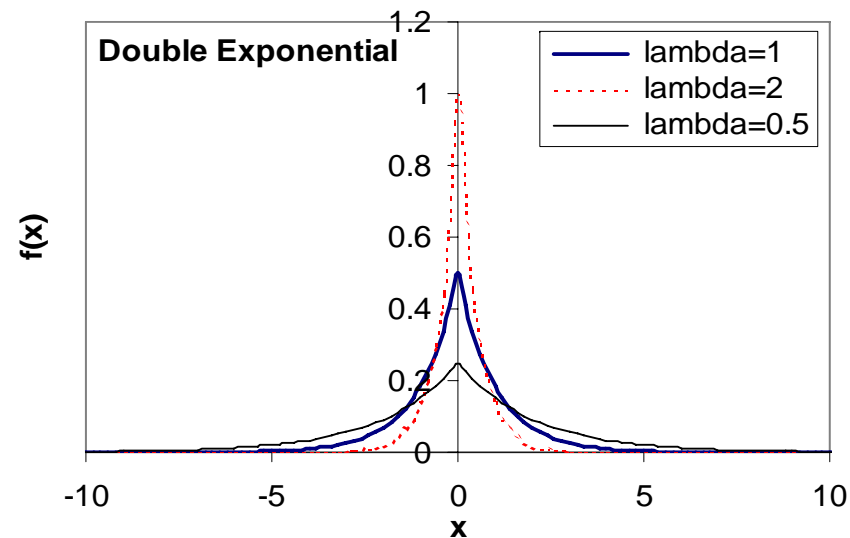
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# Analytic Analysis

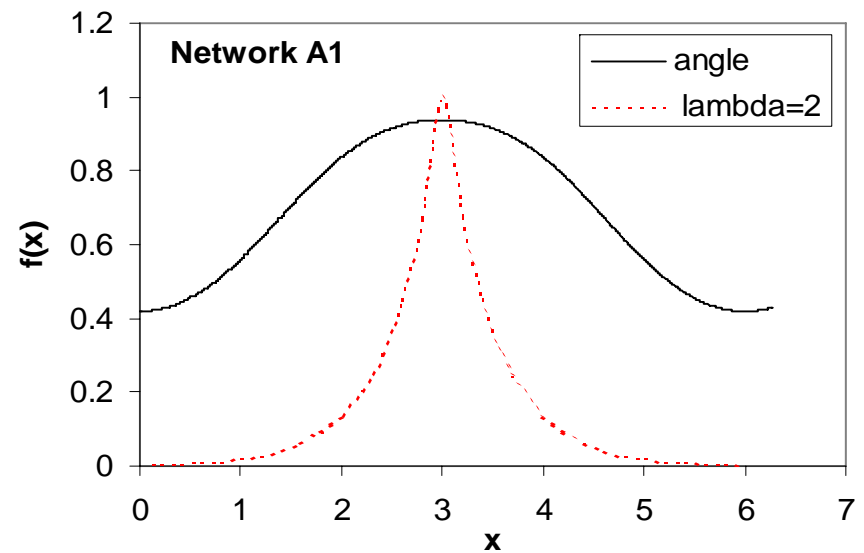
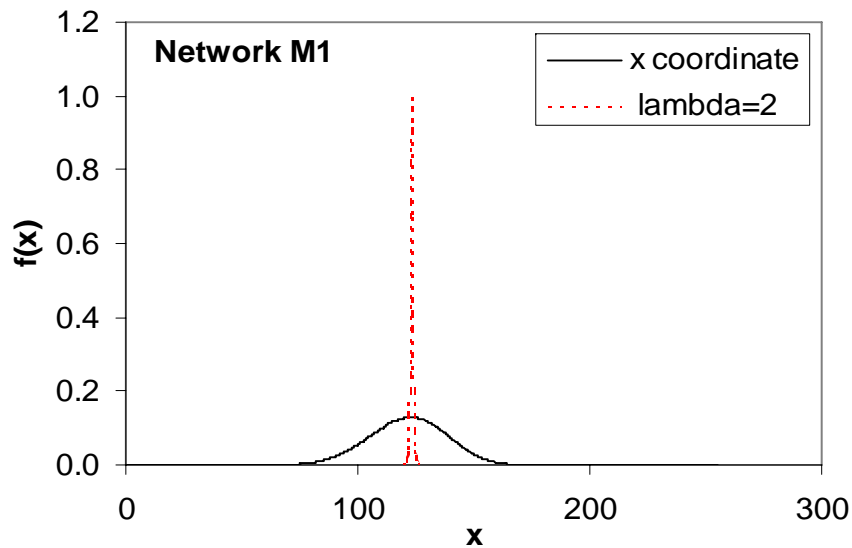
- When whole domain sampling is computationally more efficient than step out?
- We use the double exponential distribution
  - PDF:  $H(x; \lambda) = \frac{\lambda e^{-\lambda x}}{2}, \lambda > 0$
  - $\lambda$  determines how peaky the distribution is





# Comparison of flatness

- Analytic model: when  $\lambda < 2$  whole domain is faster
- **Full conditionals fall into this regime**





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# Conclusions & Future Work

- Conclusions
  - Low latency in localization
  - Whole domain sampling has the best performance
    - Relative accuracy vs. time
    - Requires no tuning
  - Full conditionals are flat
- Future Work
  - Parallel implementation of solvers



**Thank you!**