(Error Control) Coding for Wireless Networks

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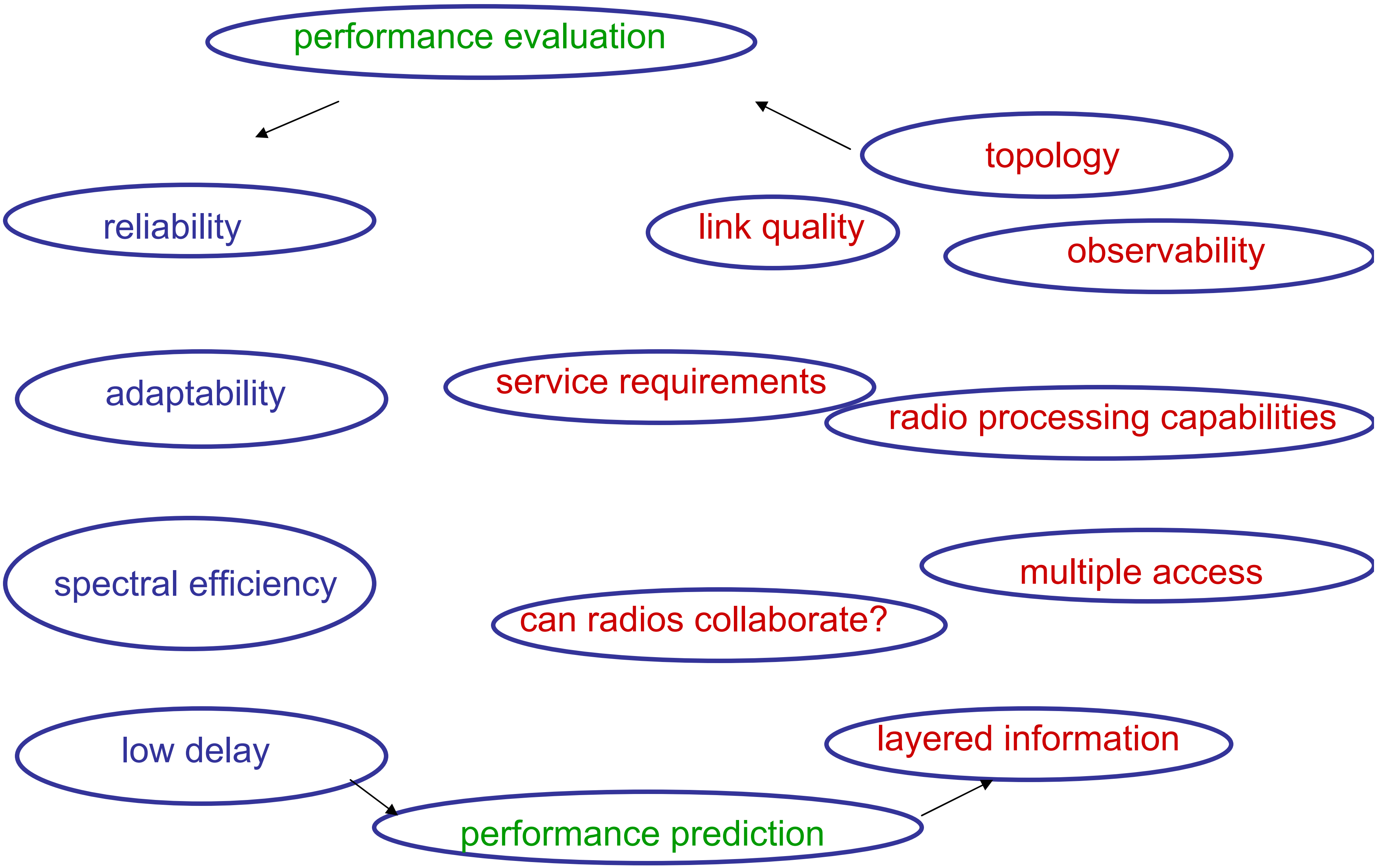
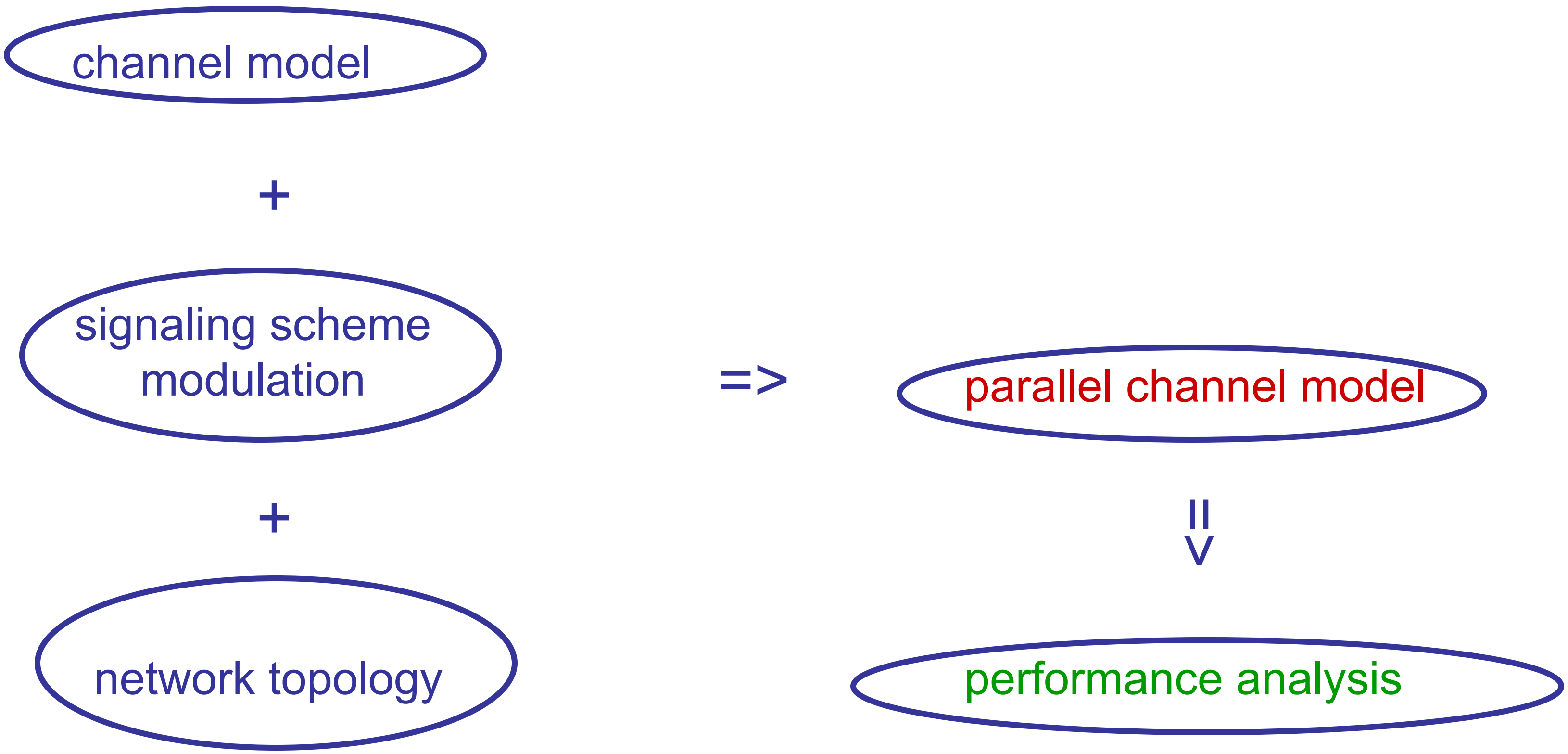
Wireless Networks: Performance Analysis



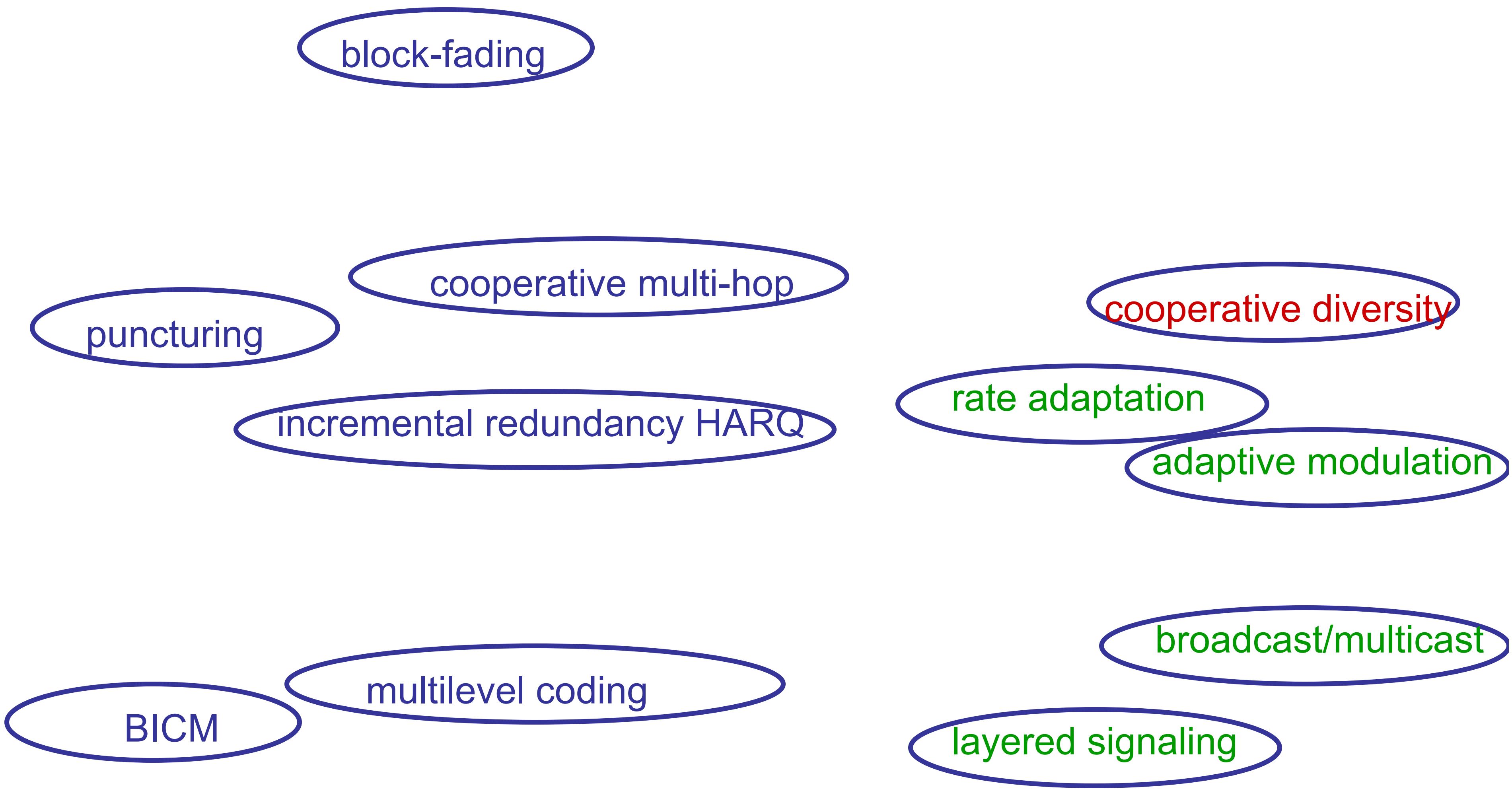
Parallel Channel Modeling



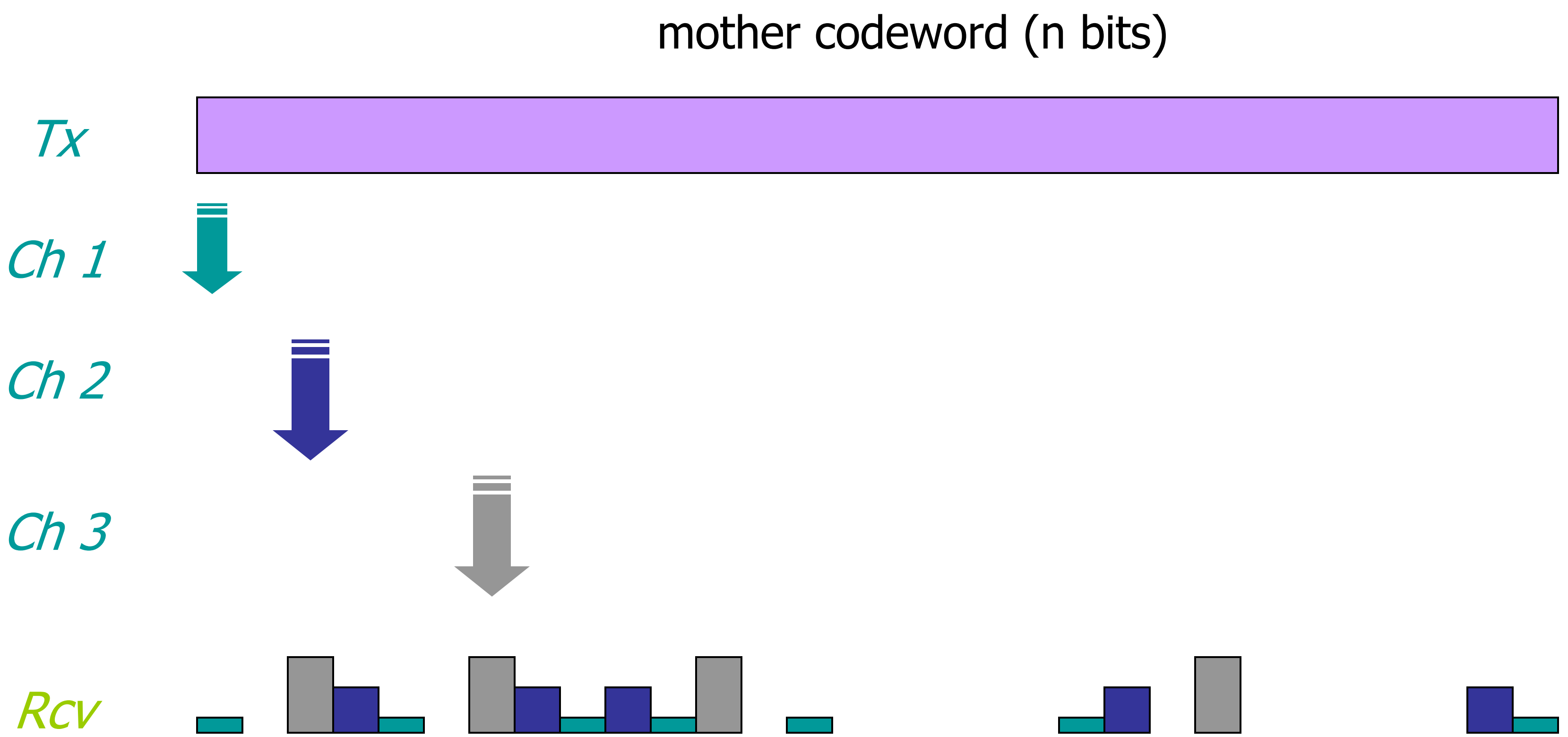
Parallel Channels: Models

- block-fading
- multi-path
- multi-carrier
- frequency hopping
- time hopping
- collision channel
- cooperative multi-hop
- puncturing
- incremental redundancy HARQ
- MIMO
- orthogonal signaling
- multilevel coding
- BICM
- unequal error protection
- frequency diversity
- spatial diversity
- time diversity
- multi-user diversity
- cooperative diversity
- rate adaptation
- adaptive modulation
- multiplexing
- broadcast/multicast
- layered signaling

Parallel Channels: Performance of Good Codes for



Codeword transmission over parallel channels



On the Performance of Good Codes over Parallel Channels

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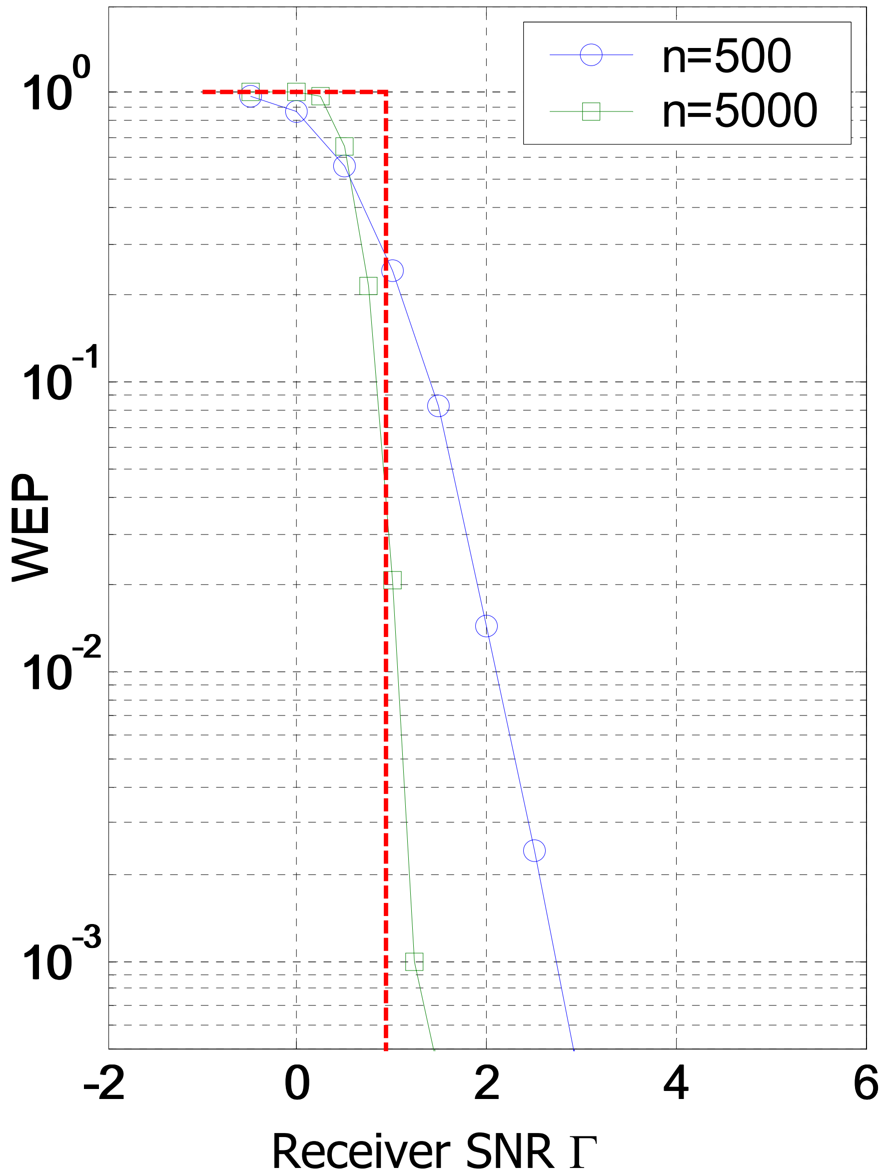
What is a "good" code?

- good codes: MacKay 99
  - a sequence of binary linear codes $X = \{X(n_i)\}_{i=1}^{\infty}$
  - achieve arbitrarily small word error probability (WEP) over a noisy channel at a nonzero threshold rate.
  - include turbo codes, LDPC codes, and RA codes

- capacity achieving codes
  - good codes
  - rate threshold is equal to the channel capacity

Threshold behavior of good codes



- an example of turbo codes
  - R=0.7
  - n=500, 5000
  - binary input AWGN channel

- code goodness implies there exists a threshold Γ_0

$$\lim_{n\to\infty} P_W^{X(n)}(\Gamma) = 0, \quad \Gamma > \Gamma_0$$

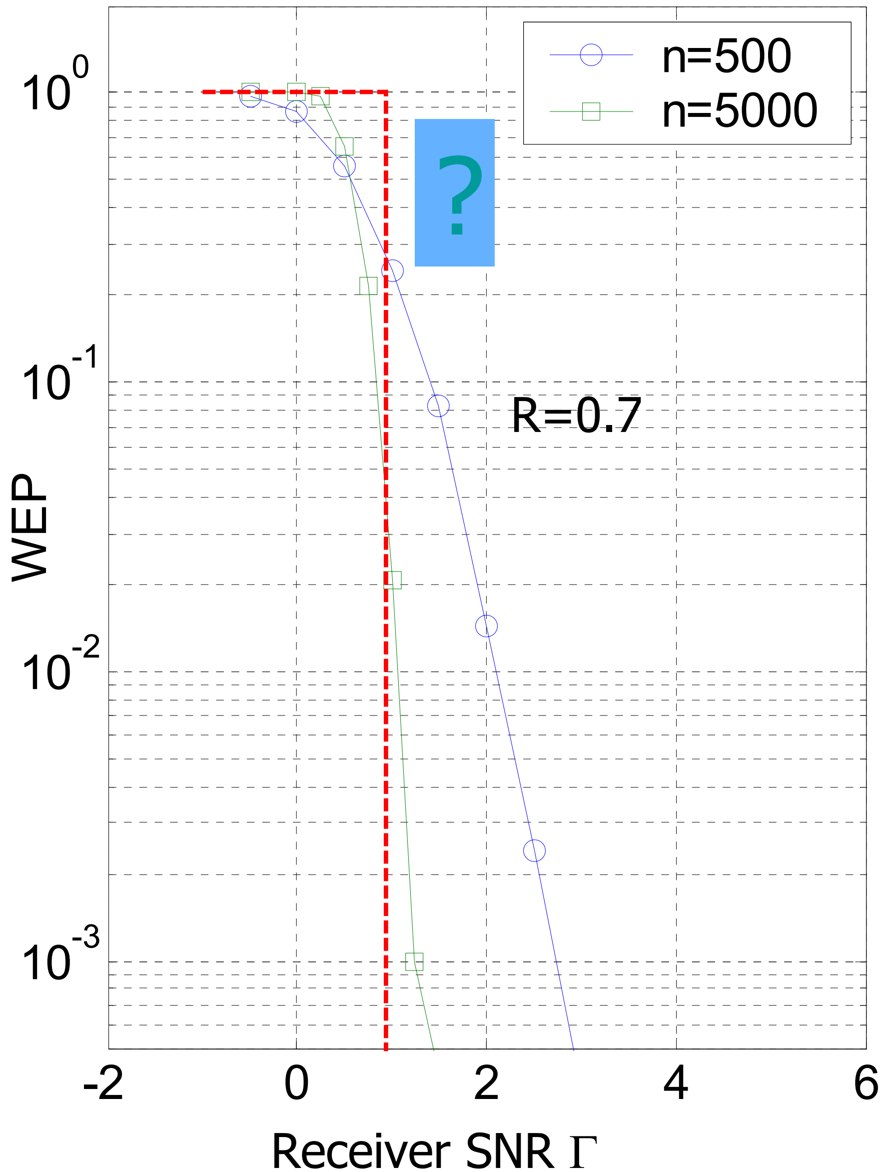
- Γ: received SNR.

Code Goodness (Liu etal. 2004)

codebook design requirement
(transmitting a codeword **x**)

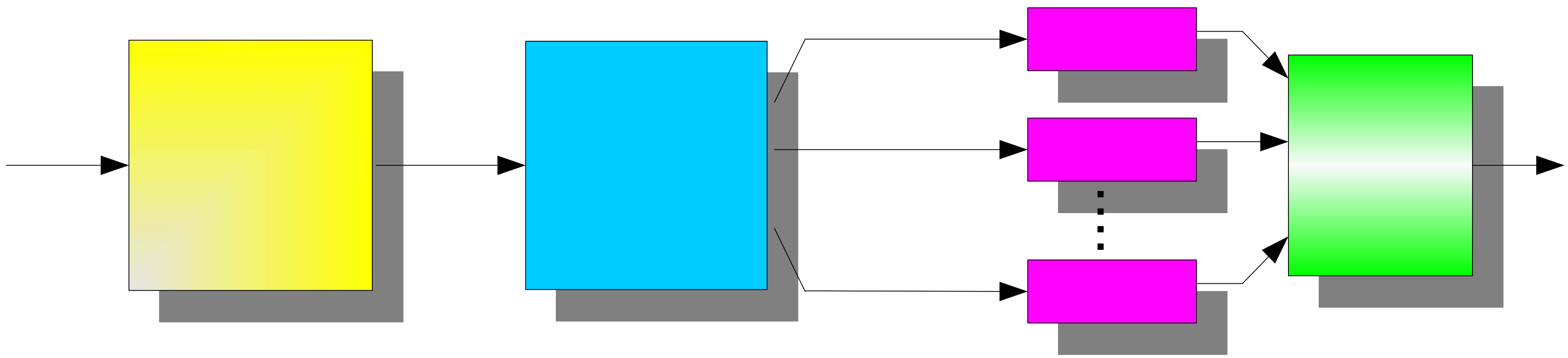
- distance from **x** to other codewords is large ⇨ $d_{\min}^{X(n)} \to \infty$
- the number of **x**'s neighbors is small ⇨ $\limsup_{n\to\infty} S^{X(n)}(f_n) < \infty$
  (low weight spectrum slope of a good code)

Threshold calculation



- single channel
  - Richardson and Urbanke iterative decoding
  - Jin, McEliece, et. al. typical pair decoding
  - Sason and Shamai maximum likelihood (ML) decoding
  - Ashikhmin, et. al. (Exit chart)
- parallel channel model?

Parallel channel model

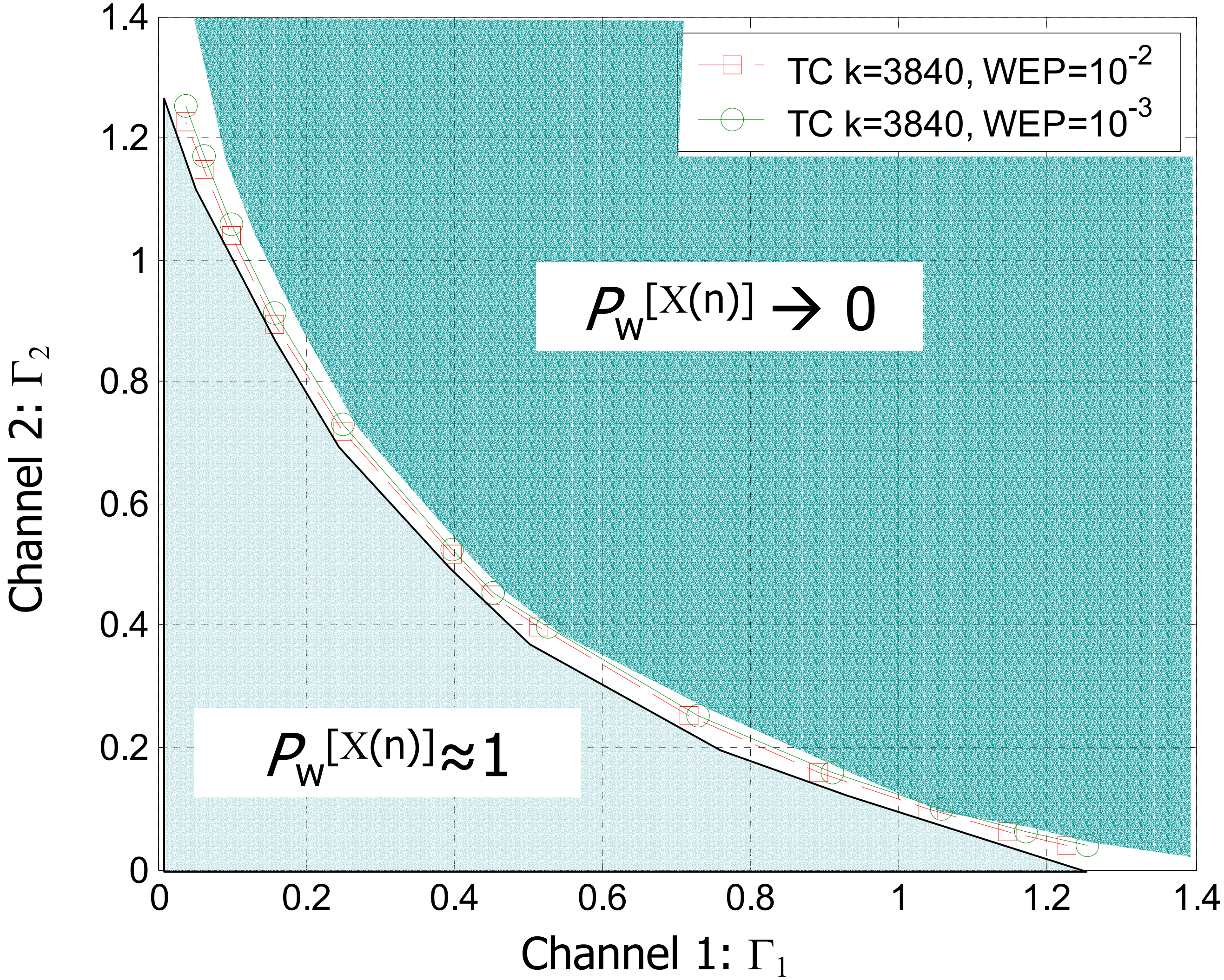


□ α_j : (asymptotic) assignment rate $\sum_{j=1}^{J} \alpha_j = 1$

- codeword is partitioned and transmitted over parallel channels
- under what channel conditions will the communication be reliable?

Reliable channel region

Turbo codes with R=1/3, two parallel AWGN channel, $\alpha_1=\alpha_2=1/2$



Union Bhattacharyya (UB) threshold

UB reliable channel region

effective Bhattacharyya distance

if

$$-\ln \bar{\gamma} > c_0^{[X]}$$

then average ML decoding WEP $\lim_{n\to\infty} P_W^{[X(n)]} = 0$

- $c_0^{[X]} = \max_{0\le\delta\le 1} \limsup_{n\to\infty} \frac{r^{[X(n)]}(\delta)}{\delta}$ UB threshold
- $r^{[X(n)]}(\delta) = \frac{\ln \overline{A}^{[X(n)]}(\lfloor \delta n \rfloor)}{n}$ normalized weight spectrum
- $\bar{\gamma} = \sum_j \alpha_j \gamma_j$ average Bhattacharyya noise parameter

Two code parameter description

if $-\ln \bar{\gamma} > c_P^{[\mathrm{X}]}$ and $\bar{I} > R + \zeta_P^{[\mathrm{X}]}$

then average ML decoding word error probability

$$\lim_{n \to \infty} P_W^{[\mathrm{X}(n)]} = 0.$$

$\bar{I} = \sum_{j=1}^{J} \alpha_j I_j$ average channel mutual infomation

Single parameter description: simple threshold (Liu etal. 2004)

$$-\ln \bar{\gamma} > c_{*}^{[\mathrm{X}]}$$

if

then the average ML decoding word error probability

$$\lim_{n\to\infty} \overline{P_{W}}^{[\mathrm{X}](n)} = 0$$

$$c_{*}^{[\mathrm{X}]} = \min_{P}\left\{ c_{P}^{[\mathrm{X}]} :\ 1 - \exp(-c_{P}^{[\mathrm{X}]}) \geq R + \zeta_{P}^{[\mathrm{X}]} \right\}$$

UB threshold vs simple threshold

- an example of turbo code

  (R=1/7, k=768, and J=3 RSC encoders)

  - UB threshold $c_0^{[X]} = 0.21 \Rightarrow -6.77dB$
  - simple threshold $c_*^{[X]} = 0.17 \Rightarrow -7.70dB$

Puncturing and Block Fading Channel

- punctured simple code threshold

$$c_*^{[C_P]}(\tau) = \ln \frac{\tau}{\exp(-c_*^{[C]}) + (1-\tau)}$$

  - $1-\tau$: punctured rate

- simple threshold bound on block fading channel coding

$$\overline{P}_W^{[\mathrm{X}(n)]}(\bar{\gamma}) = \mathbf{E}\left[P_W^{[\mathrm{X}(n)]}(\bar{\gamma})\right] \le P\left\{-\ln \bar{\gamma} \le c_*^{[\mathrm{X}]}\right\} + o(1)$$

  - decoding is done with full channel state information (CSI)

Adaptive Modulation for Variable-Rate Turbo-BICM

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This work has been supported in part by the NSF Grant SPN-0338805.

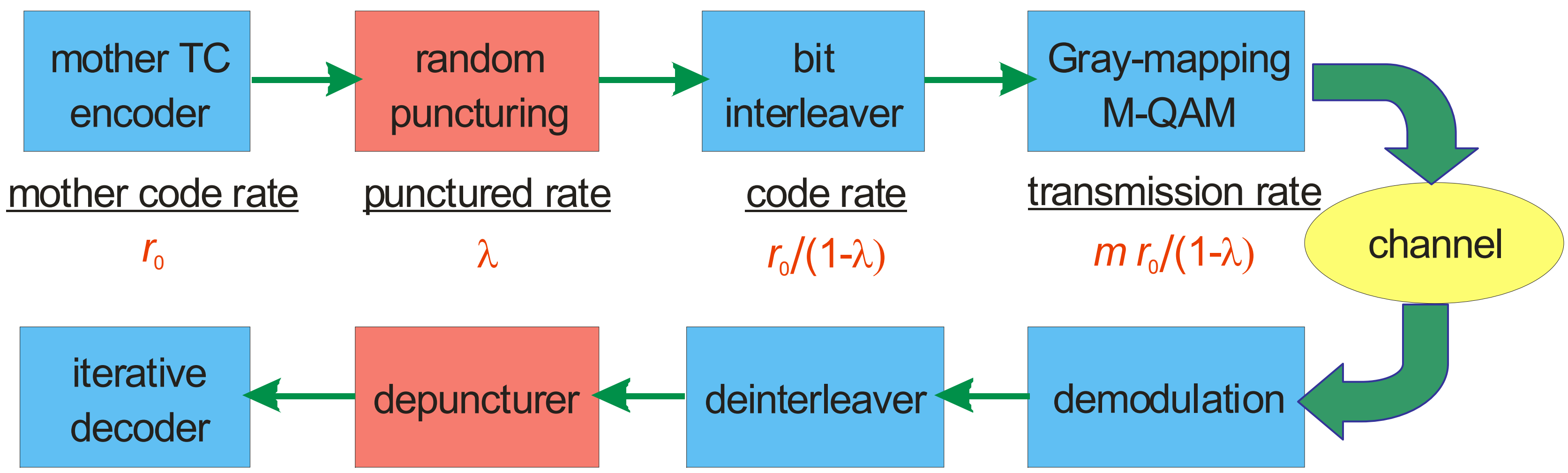
Motivation

- key requirements in 4G or B3G communications systems:
  - a wide range of data rates according to economic and service demands
  - QoS for packet oriented services

- high-speed wireless systems: bandwidth efficient turbo coding scheme
  - Turbo BICM -- *Goff 94'* [simplicity]
  - Parallel concatenated TCM -- *Benedetto 95'*
  - Turbo-TCM -- *Roberton, 98'*

- channel fluctuating in the wireless propagation environment
  - communication reliability and error prediction
  - lack of closed-form expressions for error probability of turbo coded modulation

System model



Rate threshold for VR-Turbo-BICM

Theorem:

For a VR-Turbo-BICM coding scheme using a mother code ensemble $[\mathcal{C}]$ of rate r_0 employed over an AWGN channel with a channel SNR ρ we define the rate threshold

$$r^{\circ}(\rho, m) = \begin{cases} 0 & \rho < \eta(m) \\ b(m) \cdot m\bar{I}(\rho, m) & \eta(m) \le \rho \le \varsigma(m) \\ r_m^{\infty}[1 - \bar{\gamma}(\rho, m)] & \rho > \varsigma(m) \end{cases}$$

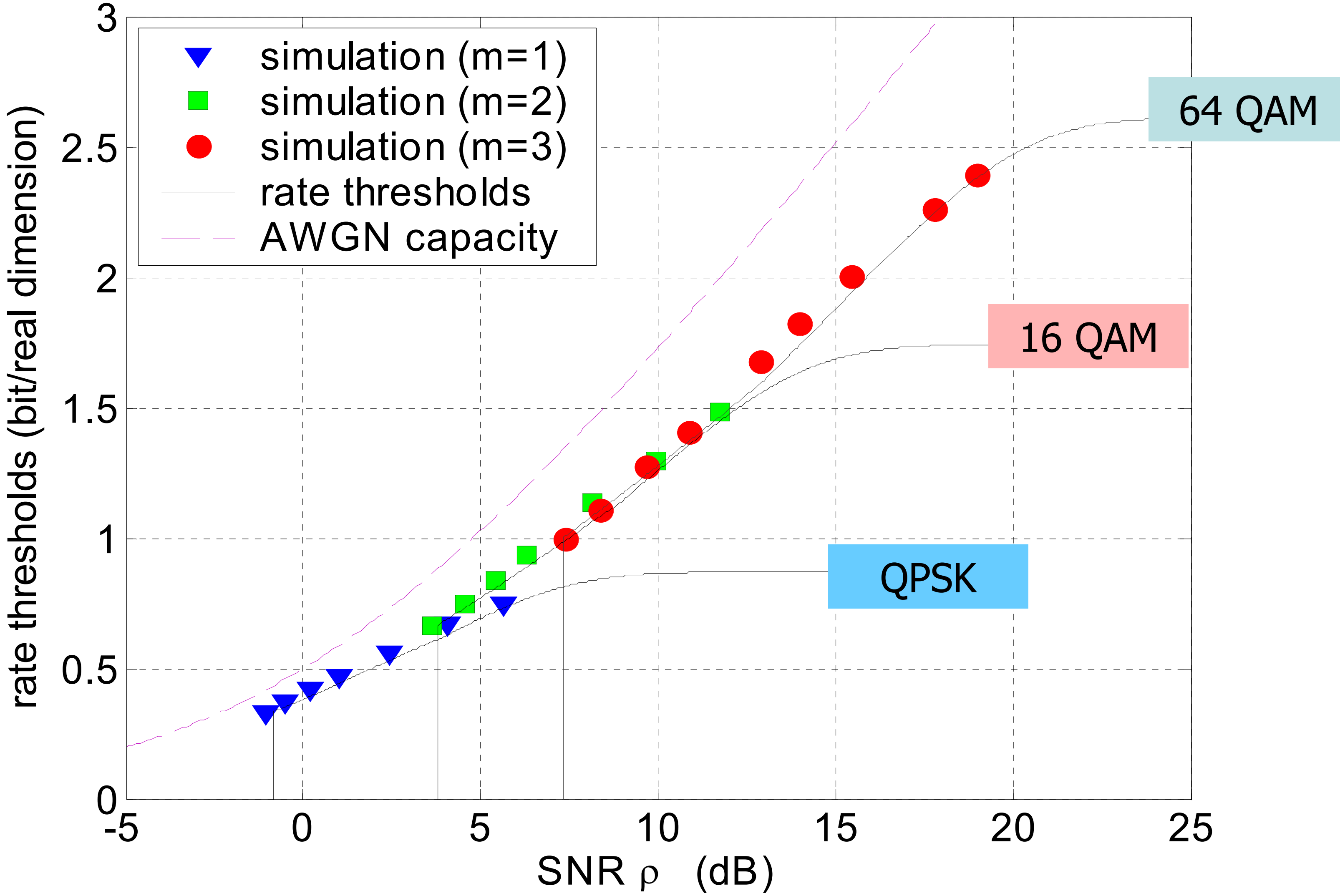
If

$$r \le r^{\circ}(\rho, m)$$

then the average ML decoding word error probability approaches zero.

Rate Threshold vs SNR

$r^{\circ}(\rho, m)$ vs. simulation results (FER=0.01)



Adaptive Turbo-BICM in slow fading

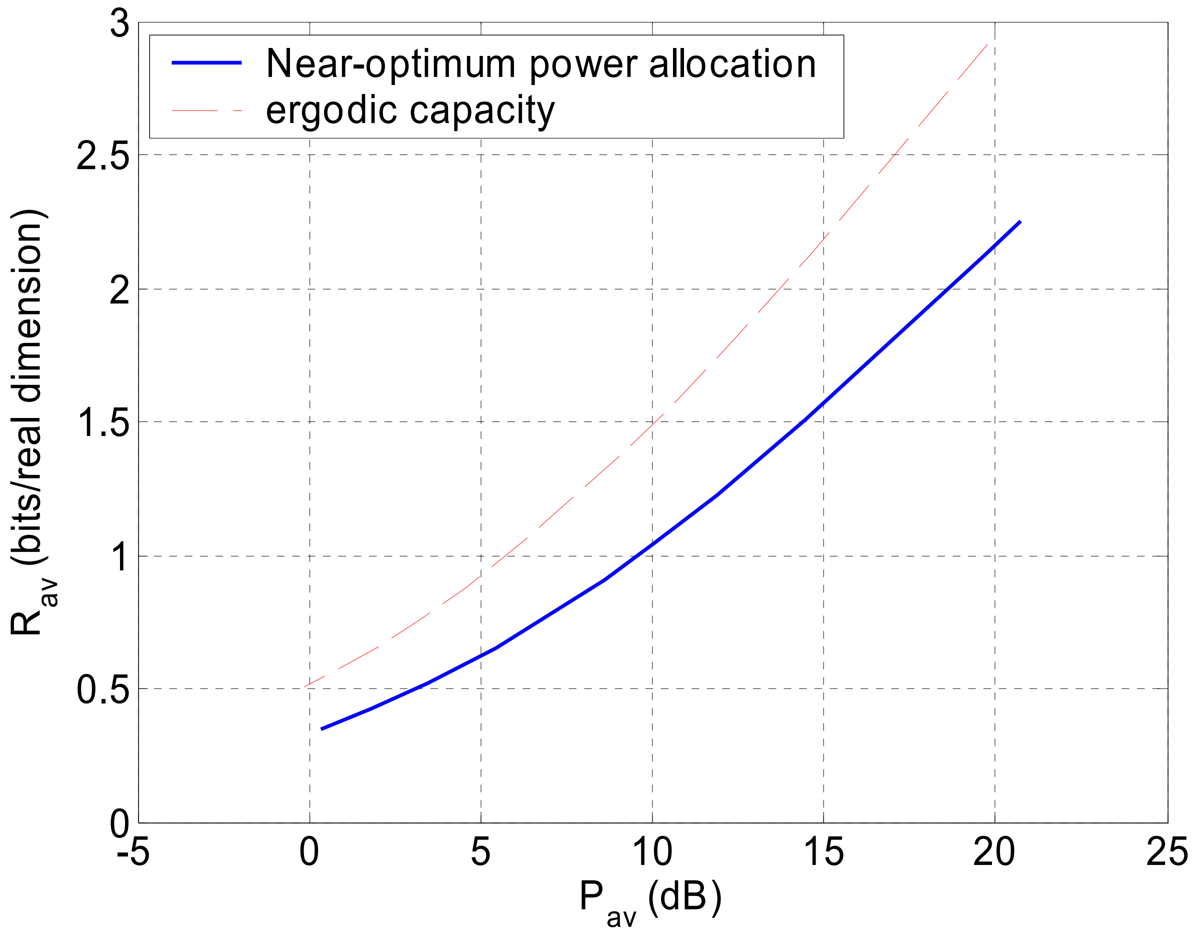
Allocation Problem:

given an average power constraint P_{av}, the optimum power and modulation index maximize the expected rate threshold

$$\max_{P(n),m(n)} E\left[r^{\circ}\left(P(g),m(g)\right)\right]$$

subject to $E\left[P(g)\right] \le P_{av}$

$$P(g) \ge 0,\ m(g) \in \{0,1,2,3,\ldots\}.$$



Cooperative Diversity with Incremental Redundancy (IR) Turbo Coding for Quasi Static Wireless Networks

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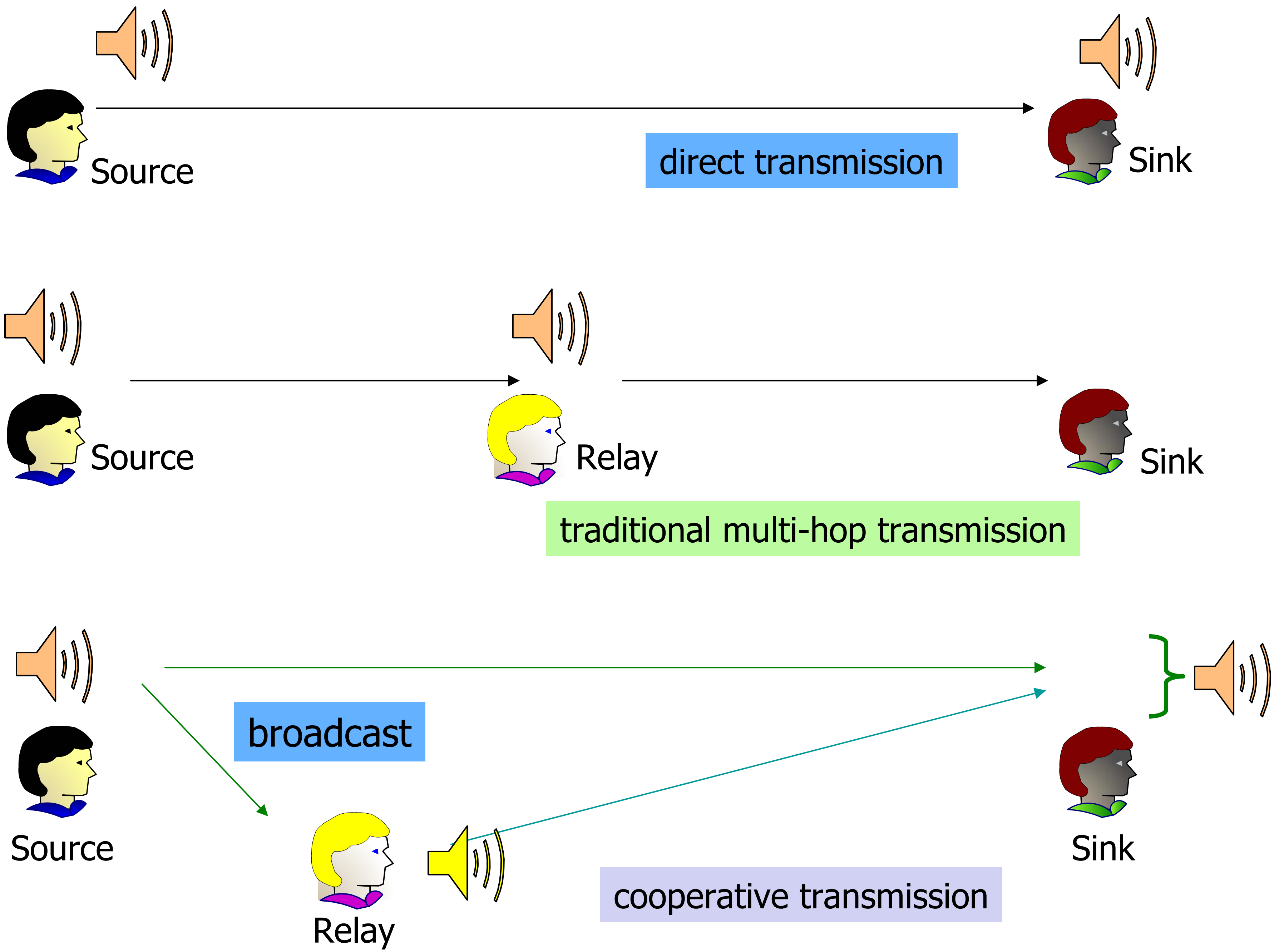
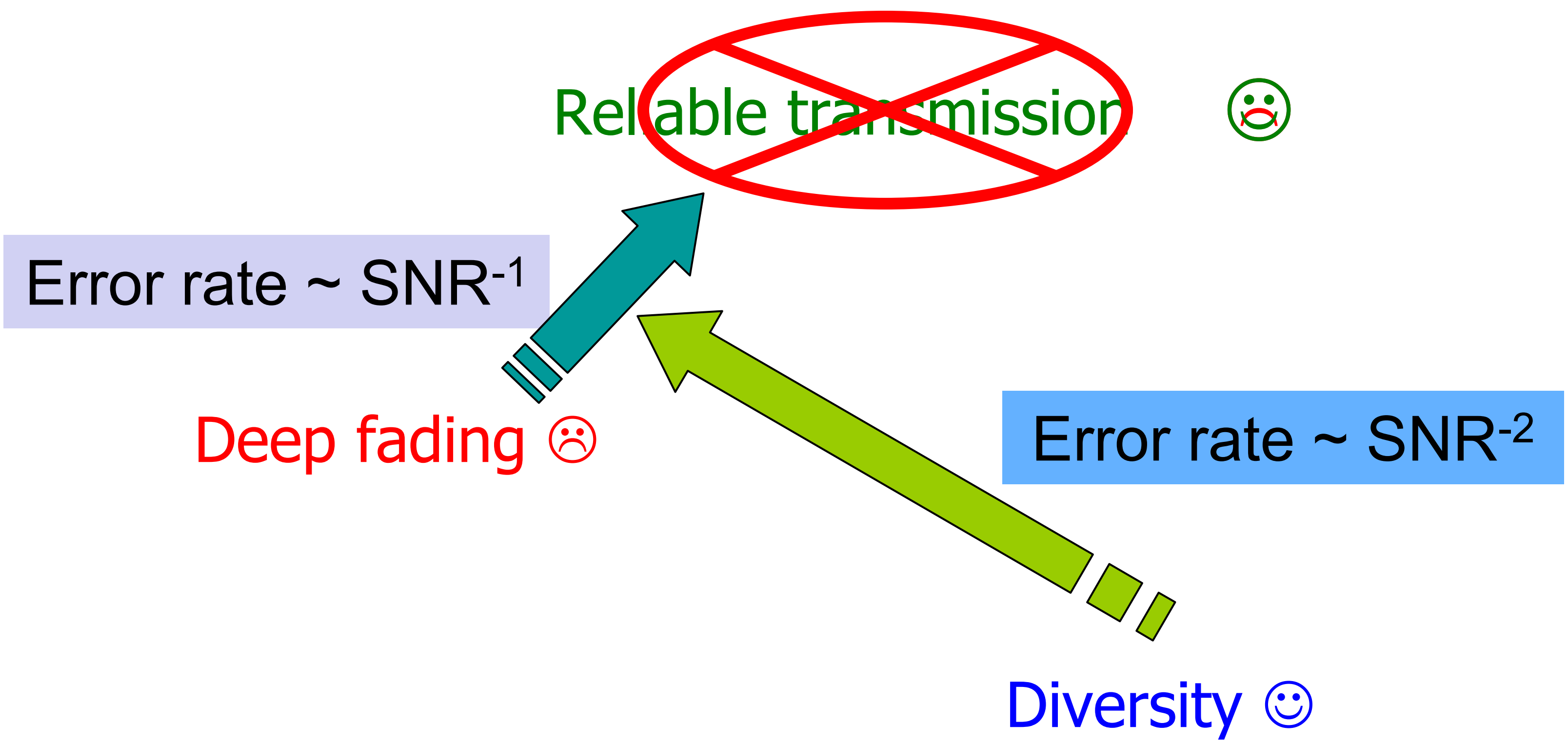

Source
direct transmission
Sink
Source
Relay
Sink
traditional multi-hop transmission
broadcast
Source
Relay
Sink
cooperative transmission

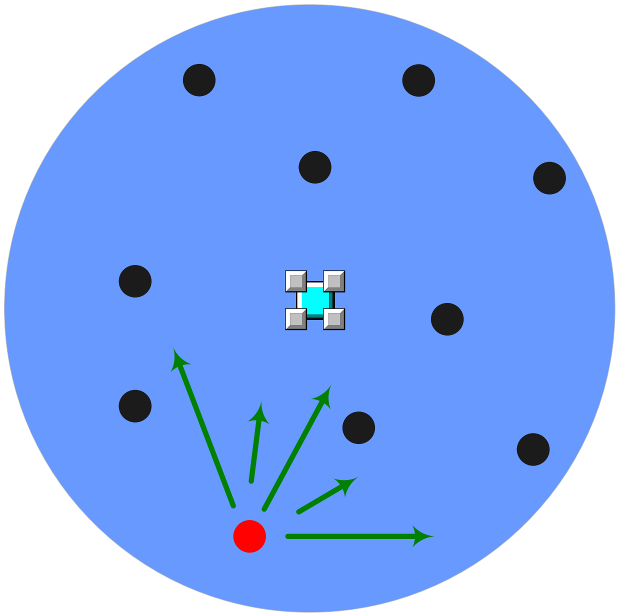
Cooperation benefit: reliability

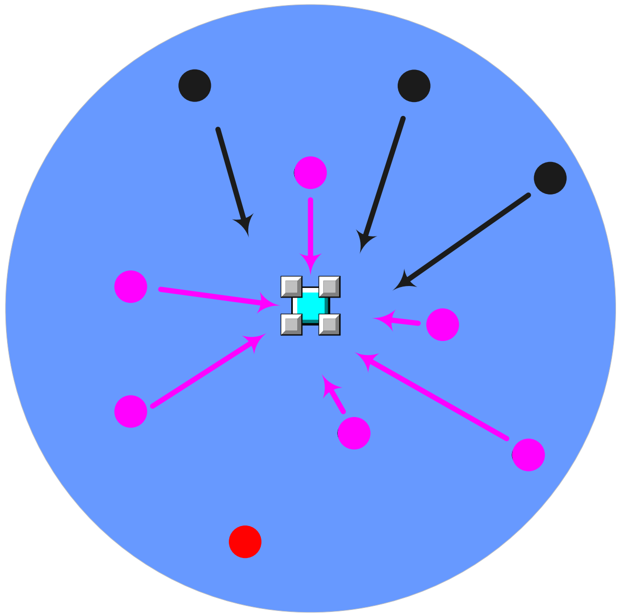
wireless communications



M-user Cooperation

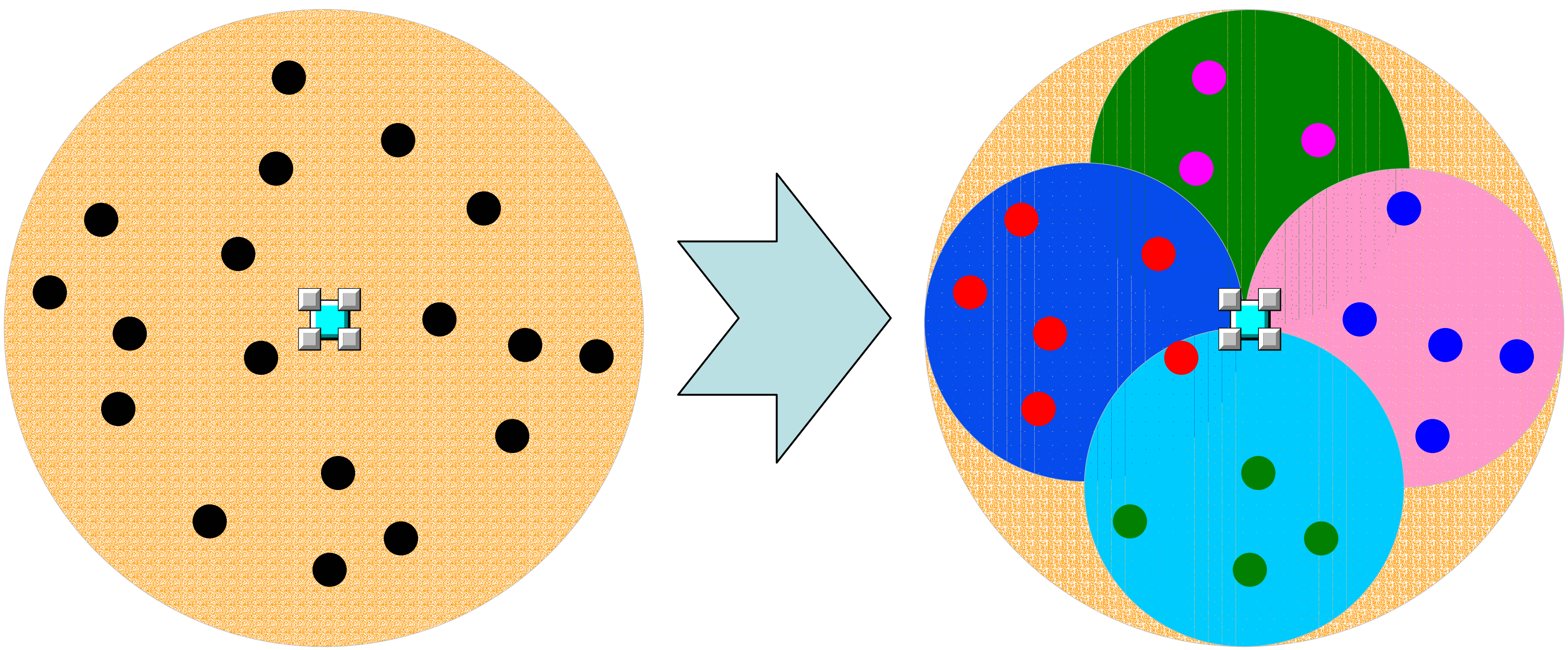
- M users share radio channel
- orthogonal frequency-division multiple access scheme
- 2 Hops scheme (for each user)



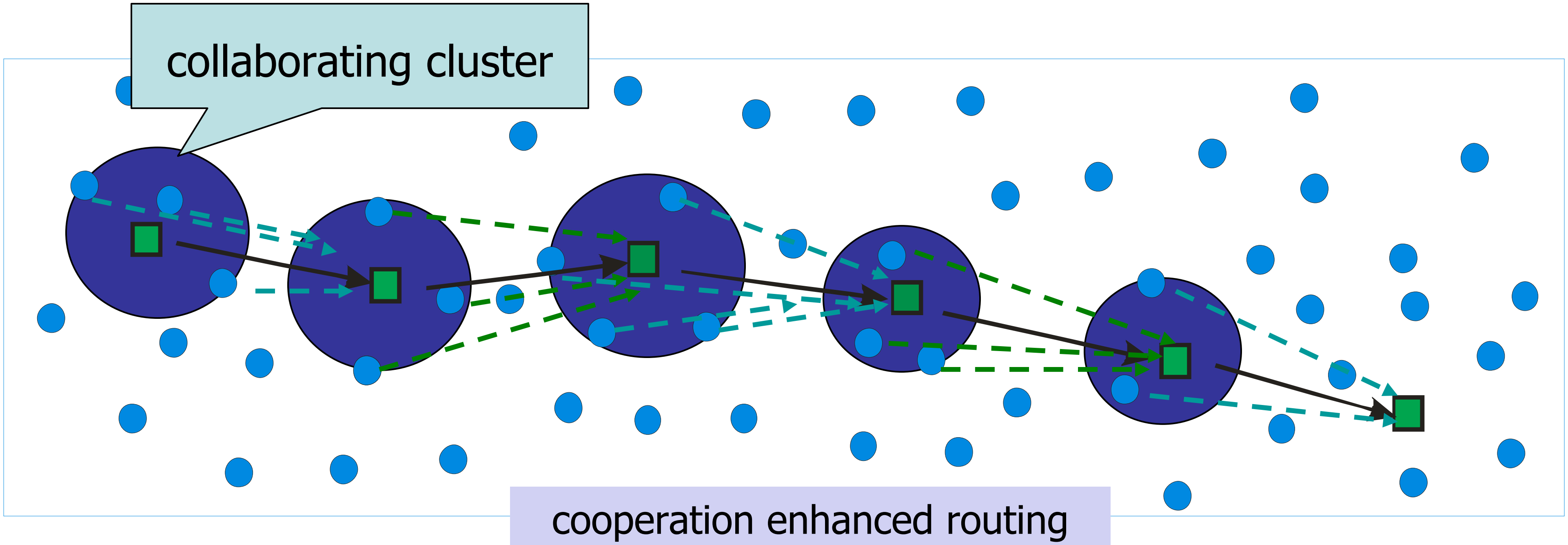


M-user Cooperation (cont.)

- Large SNR (asymptotic result)
  - Using cooperative coding each user can achieve **full** (M) diversity gain
- Medium and low SNR (how to get benefit ? cooperation criteria …)
  - the user-to-destination channel quality is good (same to two user case)
  - partners are sufficiently close (cluster behavior)



Wireless cooperative routing in networks with quasi-static fading



Frame error rate of the cooperative routing scheme

- simple threshold upper bound

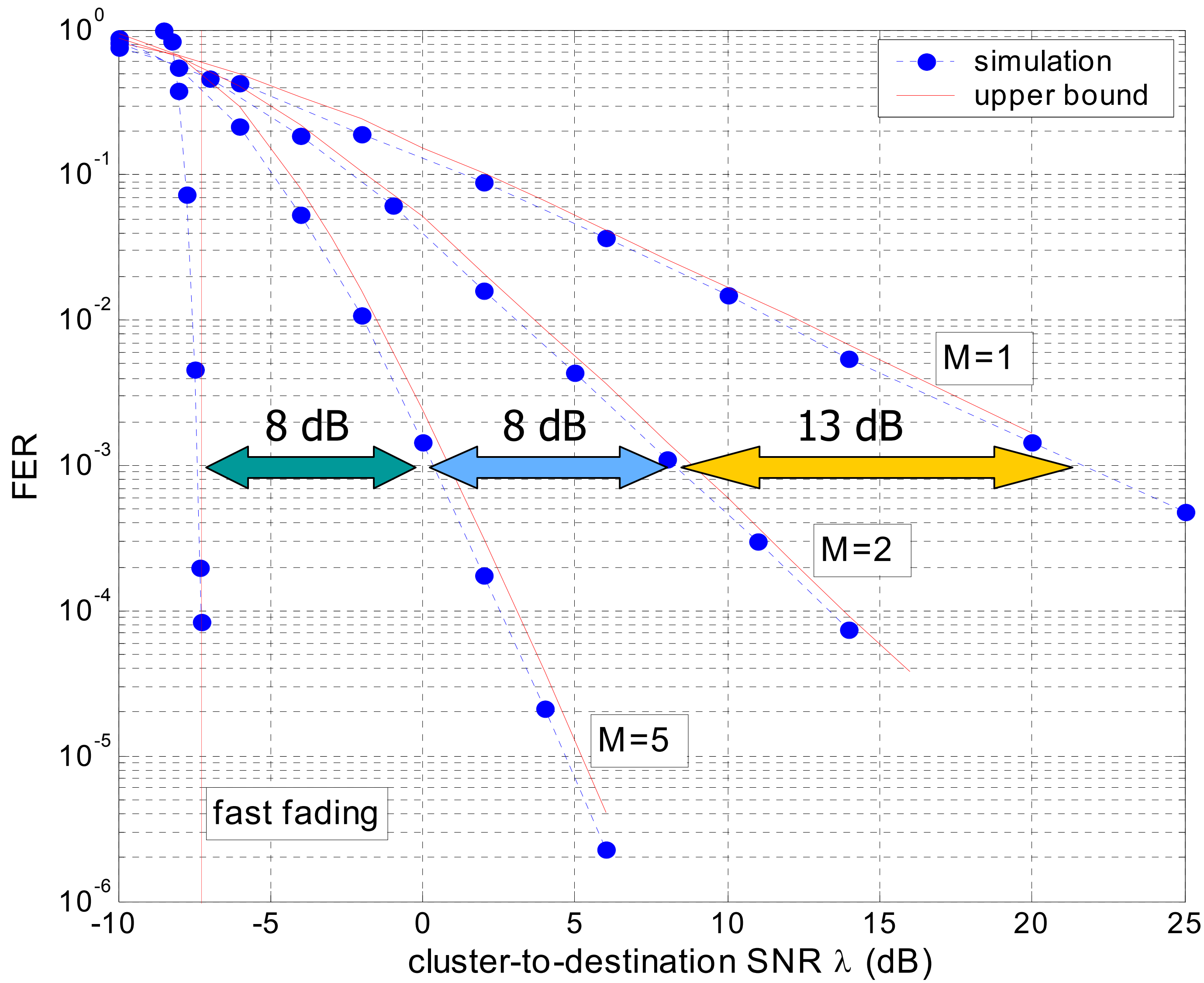
$$\mathsf{FER}^{(M)} \leq \sum_{k=0}^{M-1} \binom{M-1}{k} P(|\mathsf{F}| = k) \cdot P\left\{\theta(\mathsf{F}) \leq c_*^{[\mathsf{TC}]} \mid |\mathsf{F}| = k\right\}$$

  – θ(F): effective cluster-to-destination SNR

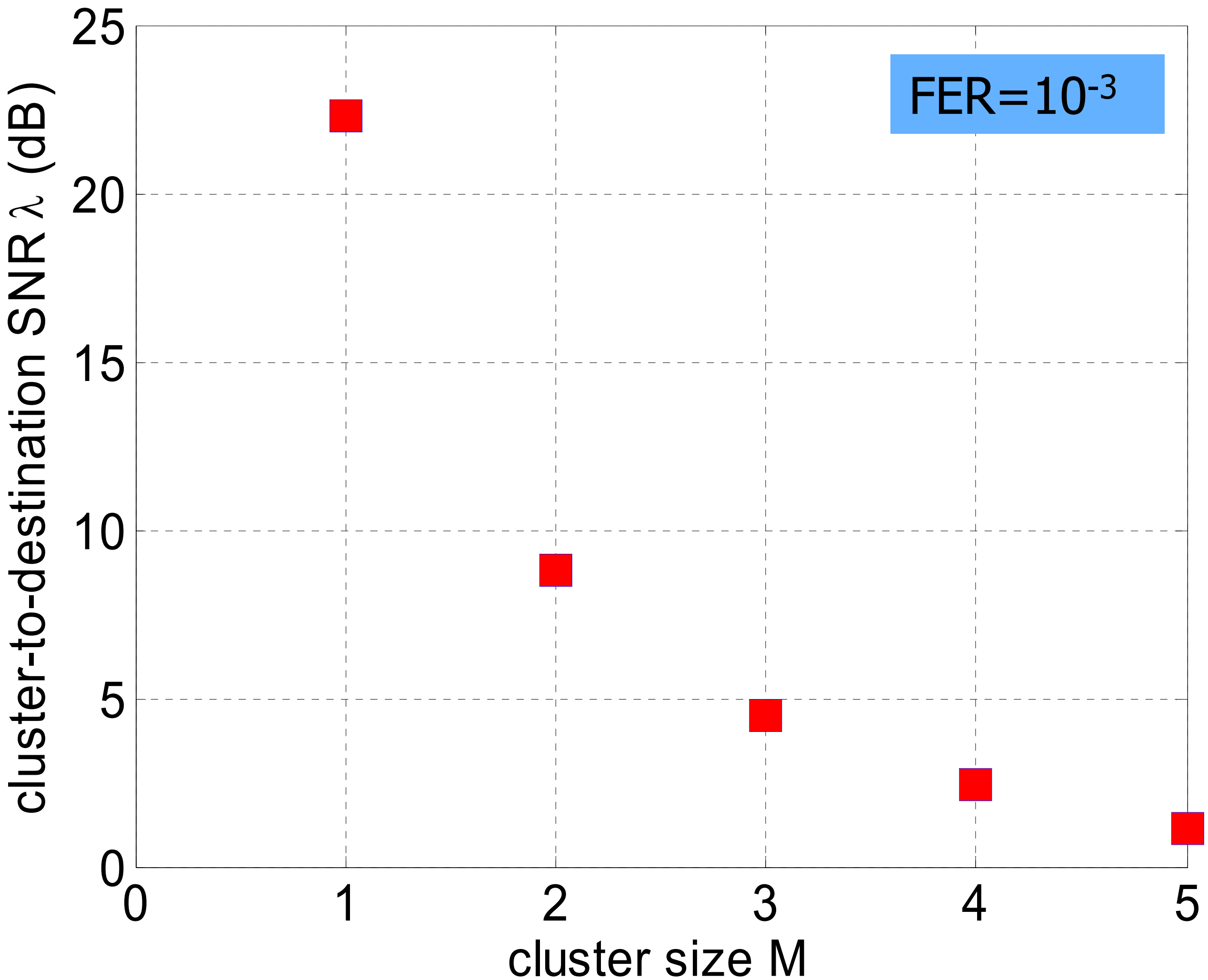
- asymptotic upper bound (small $c_*^{[TC]}$ and large ρ, λ)

$$\mathsf{FER}^{(M)} \leq_{\lambda,\rho,c_*^{[\mathsf{C}]}} \sum_{k=0}^{M-1} \binom{M-1}{k} \frac{(M \cdot c_*^{[\mathsf{TC}]})^M}{(M-k)(k+1)!} \rho^{-M+(k+1)} \lambda^{-(k+1)}$$

Diversity Gain vs M



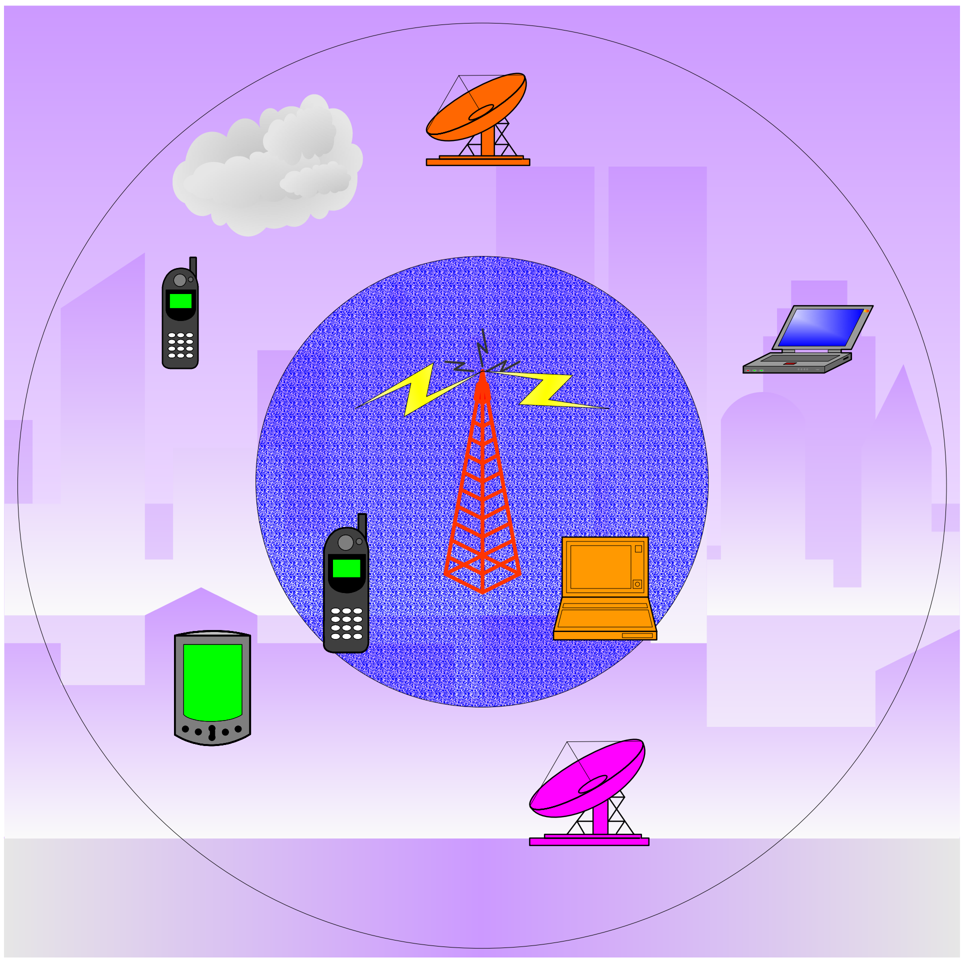
Cluster-to-destination SNR vs. M



Rate Design for Layered Broadcast using Punctured LDPC Codes and Multilevel Coding

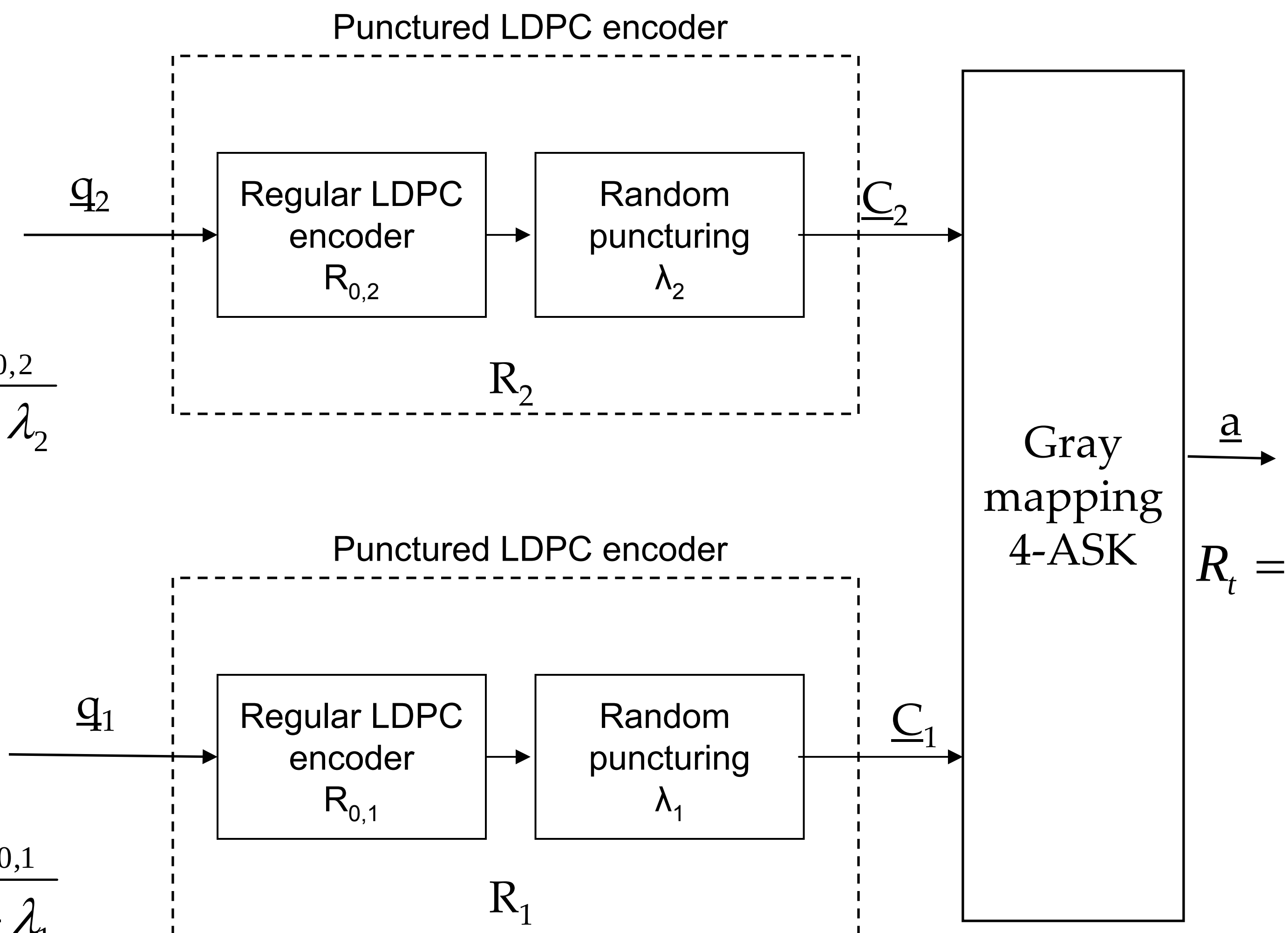
Ahmed Turk and Predrag Spasojević

Coding for Broadcast



- superposition structure
  - successive refineable sources
  - hierarchical channel coding
- code rate selection
  - different channel conditions
  - unequal error protection

Multilevel Broadcast System Encoder



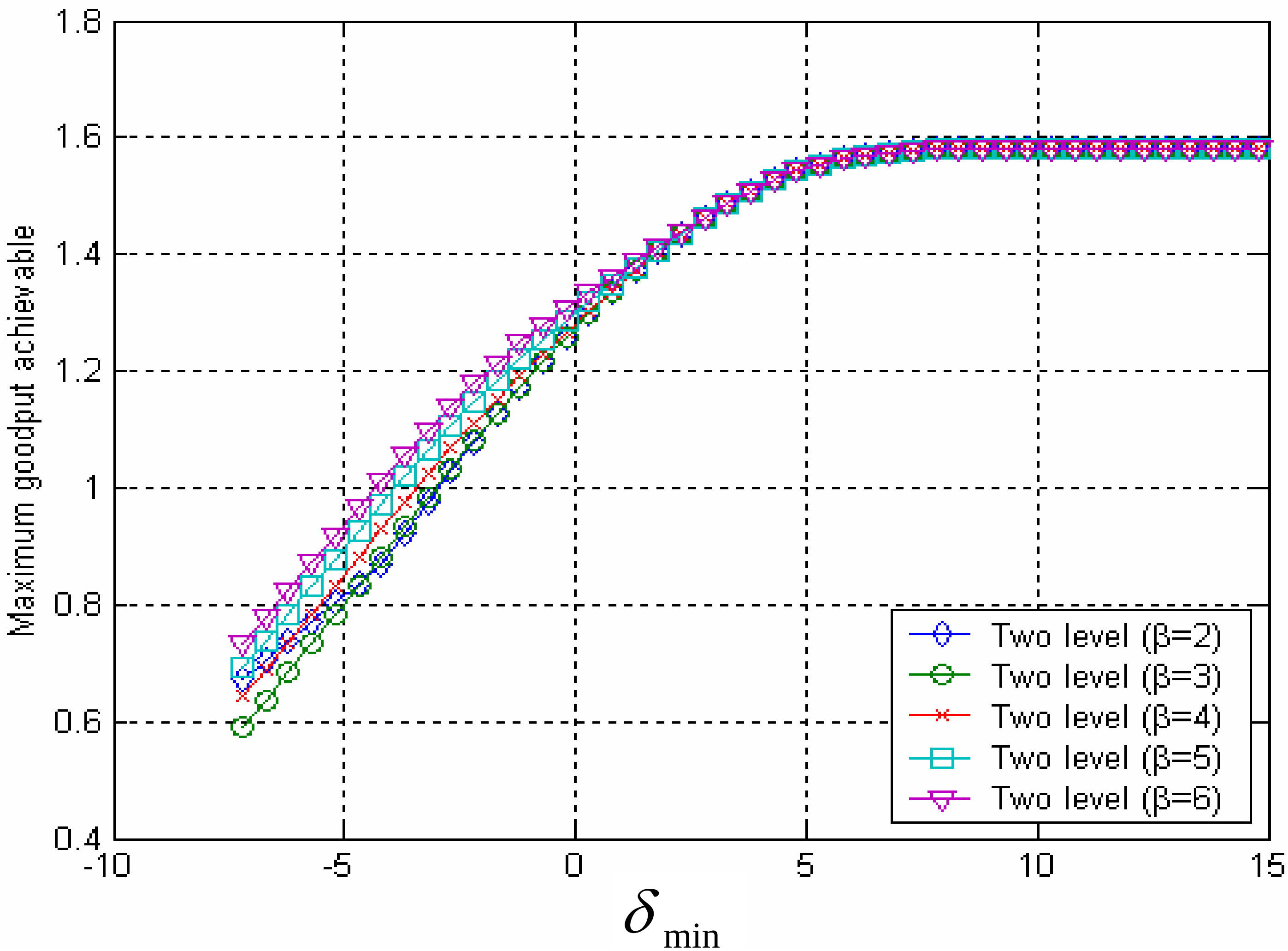
$$R_2 = \frac{R_{0,2}}{1-\lambda_2}$$

$$R_1 = \frac{R_{0,1}}{1-\lambda_1}$$

$$R_t = R_1 + R_2$$

Goodput vs worst user SNR

(n,3,4) mother code in each level



Incremental Multi-Hop based on Punctured Turbo Codes

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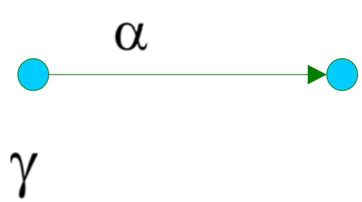
System Model

- one-dimensional multi-hop network with P nodes

- equal distance between the neighboring nodes

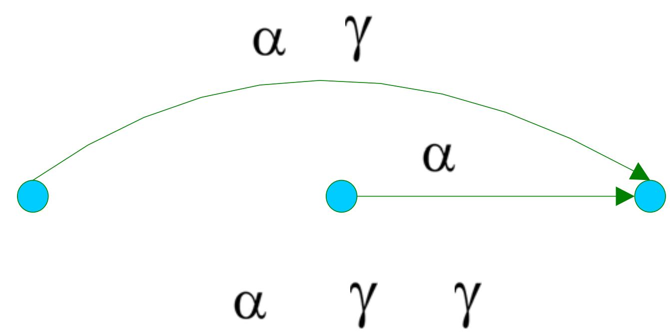
$$d_{1,2} = d_{2,3} = \cdots = d_{P-1,P} = d_0$$

Performance Analysis (1)



Step 1

$$\alpha_1 > \frac{1-\exp(-c_0^{[X]})}{1-\gamma(1)}$$

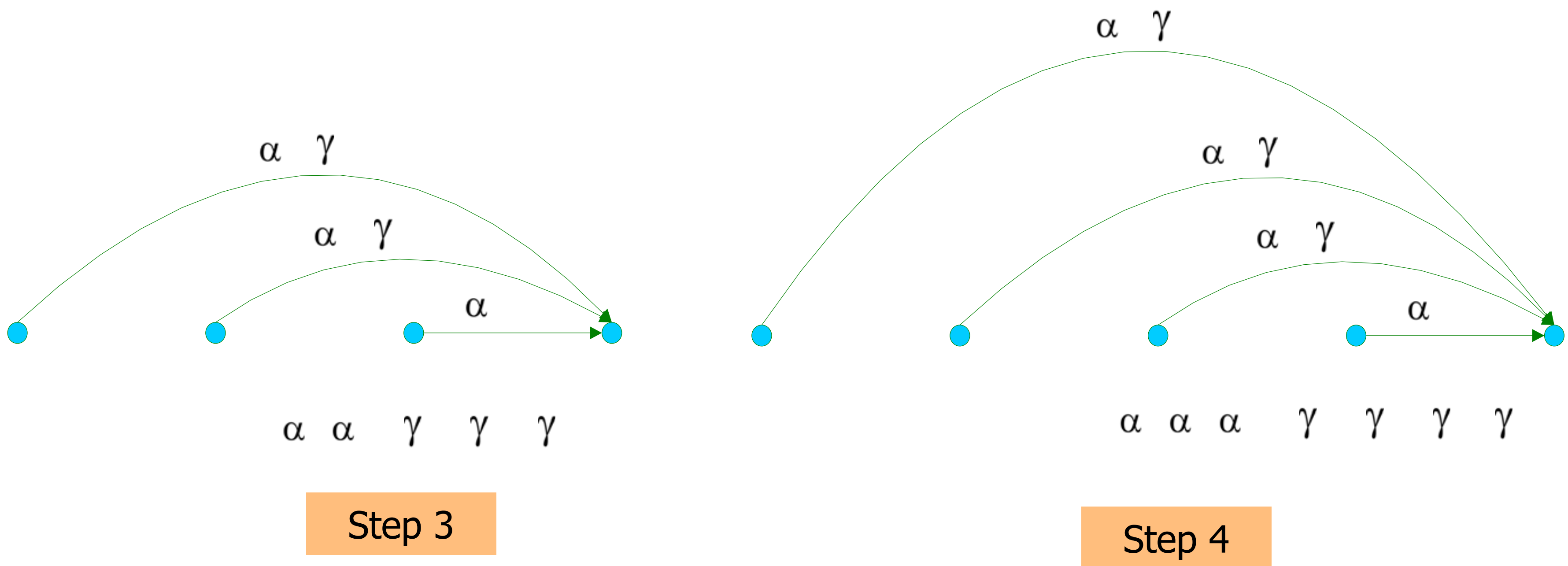


Step 2

Given: α_1

$$\alpha_2 > \frac{1-\exp(-c_0^{[X]})-\alpha_1\left[1-\gamma(2)\right]}{1-\gamma(1)}$$

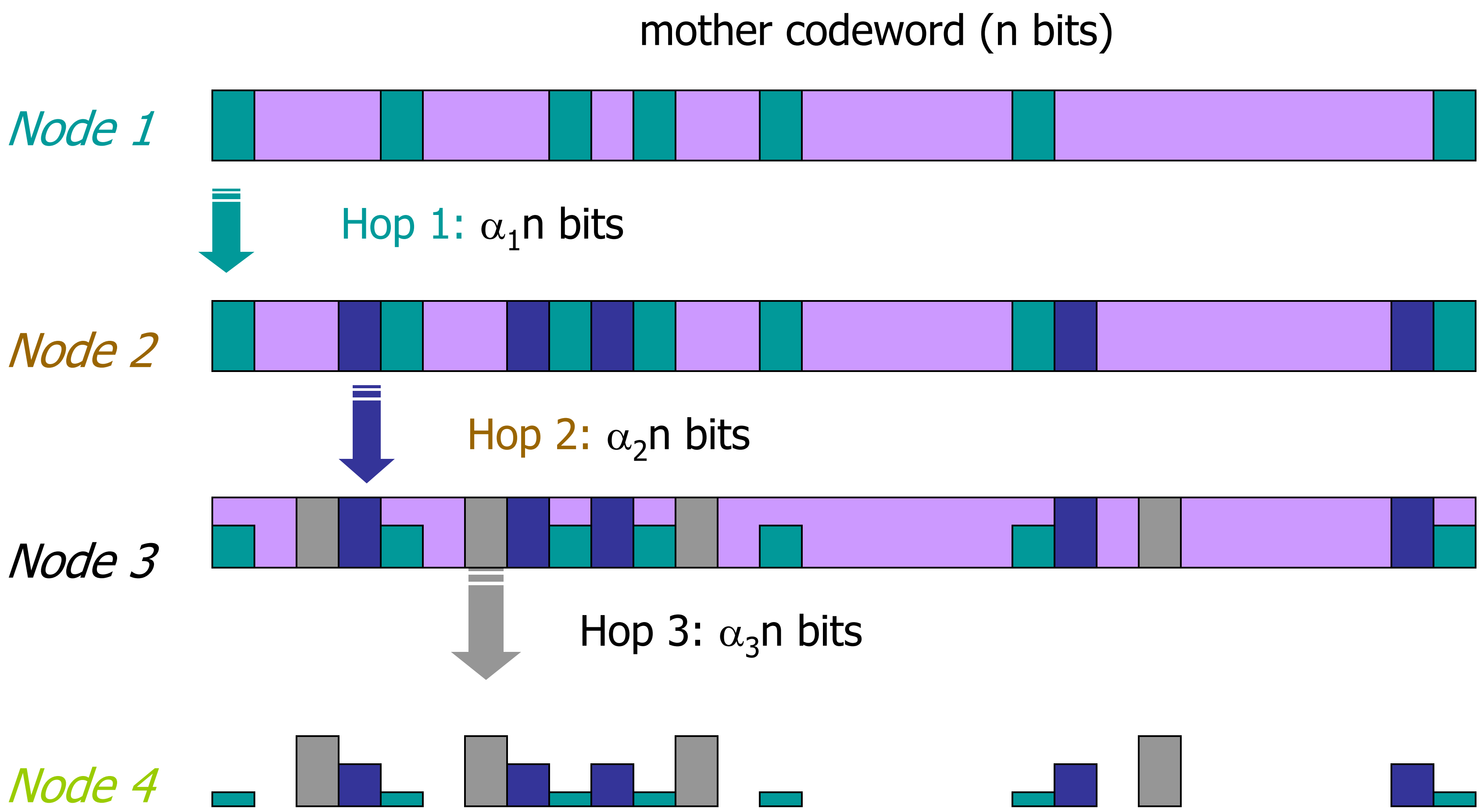
Performance Analysis (cont.)



Given: $\alpha_1, \cdots, \alpha_{j-1}$

$$\alpha_j > \frac{1 - \exp(-c_0^{[X]}) - \sum_{k=1}^{j-1} \alpha_k \left[1 - \gamma(j-k+1)\right]}{1 - \gamma(1)}$$

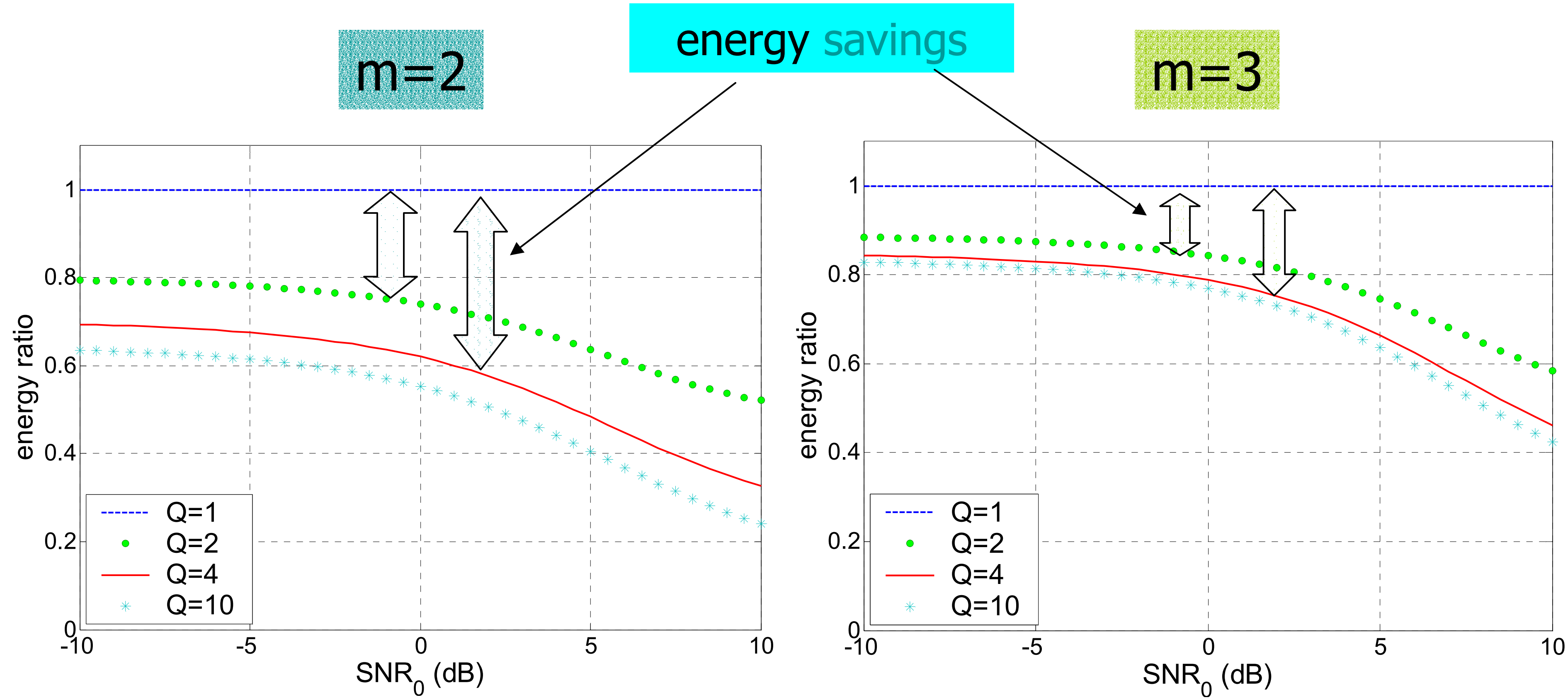
IR multi-hop transmission scheme (2)



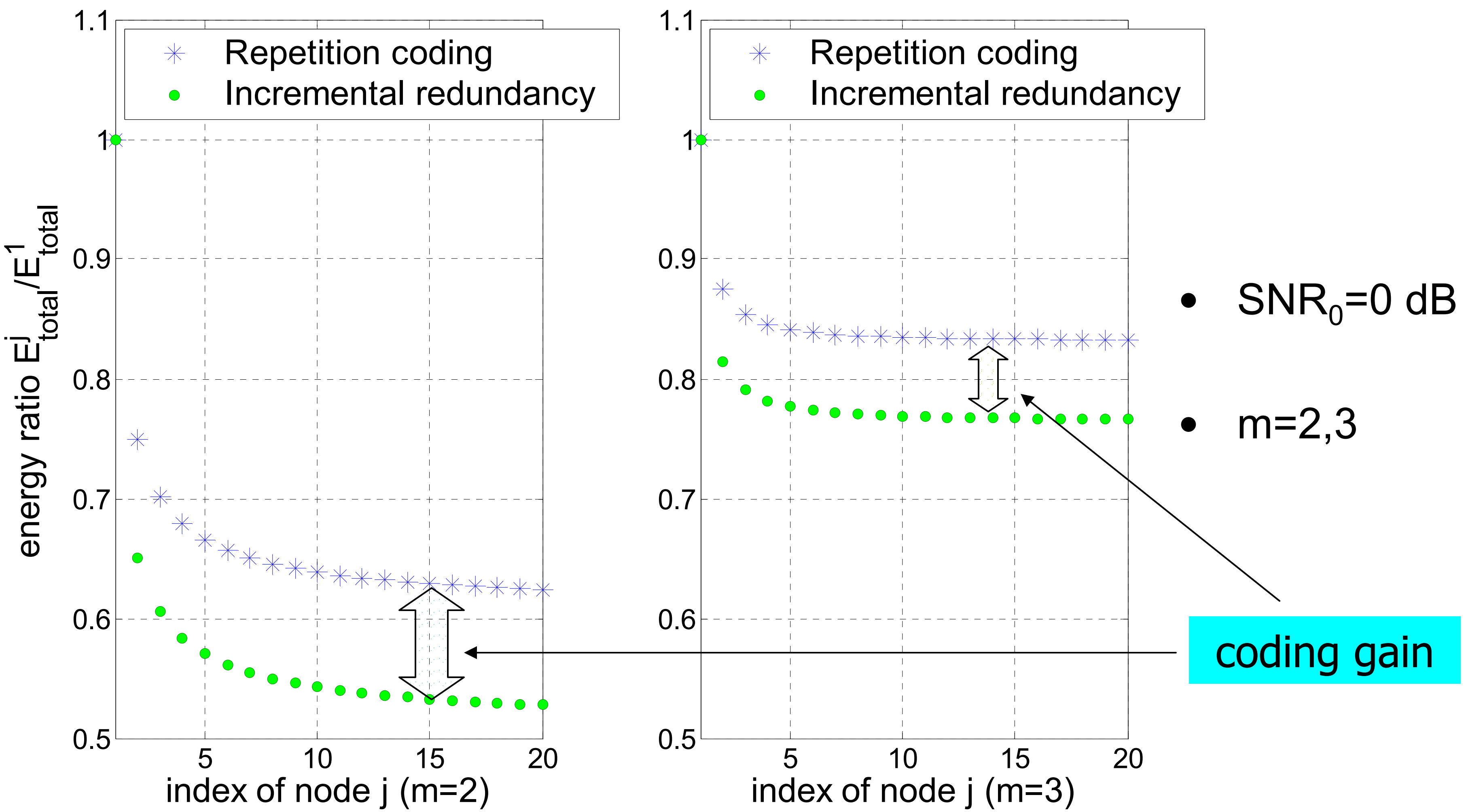
Energy Savings

- energy ratios: $$\eta = \frac{E_{total}^{(Q)}}{E_{total}^{(1)}} = \frac{\alpha^{(Q)}}{\alpha^{(1)}} = \frac{1-\gamma(1)}{Q-\sum_{j=1}^{Q}\gamma(j)}$$
- High SNR_0: $$\lim_{SNR_0 \to \infty} \eta = \frac{1}{Q}$$
- low SNR_0: $$\lim_{SNR_0 \to 0} \eta = \frac{1}{\sum_{j=1}^{Q} j^{-m}}$$

Traditional vs. IR Multi-hop Transmissions



Repetition Coding vs. IR Schemes



- SNR_0=0 dB
- m=2,3