Power Control in a Multicell CDMA Data System Using Pricing

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WIRELESS INFORMATION NETWORK LABORATORY

Outline

- Introduction
  - Motivation
  - Microeconomics: utility, game, price
- Utility
- Multicell power control game
- Efficiency through pricing
- Numerical Results

Introduction

- Cellular voice telephony
  - Successful evolution of technology and business of systems
  - Effective Radio Resource Management for system quality and efficiency
- Wireless multimedia communications
  - Voice and data have different quality of service (QoS) objectives
  - RRM techniques for voice not necessarily efficient for data
- Concepts and mathematics of microeconomics/game theory for RRM in wireless networks
  - Utility, Non-cooperative games, Nash equilibria, Pareto efficiency, Pricing
  - Focus on power control

Microeconomics Concepts

- Utility
  - The level of satisfaction received from consumption of resources
- Non-cooperative game
  - Player chooses strategy/action to maximize own utility in a distributed fashion
- Nash equilibrium
  - Fixed point reached as a result of non-cooperative game
  - Does not necessarily exist
- Pareto efficiency
  - Describes socially desirable solution
  - Nash equilibrium not necessarily Pareto efficient
- Pricing
  - Promotes choice of more Pareto efficient strategies/actions

Utility: Voice versus Data

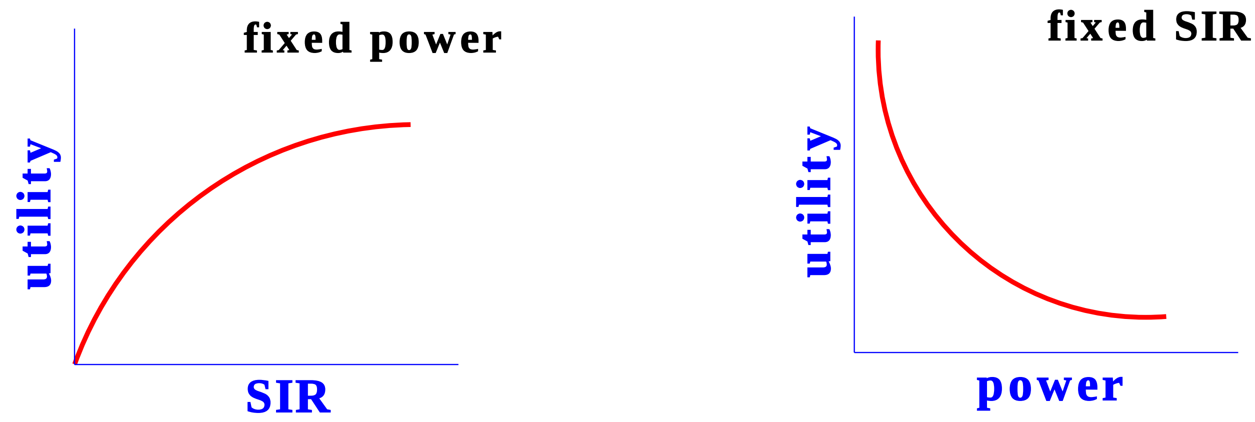
- Utility function measures quality of service
- Power control for voice
  - Studied extensively (e.g. [Grandhi][Zander][Yates])
  - Objectives
    - Provide each signal with adequate QoS
    - Avoid unnecessary interference
    - Minimize battery drain
  - Service considered unacceptable below SIR target, however no extra benefit above target
    - Implicit assumption of a step utility function as a function of SIR

Utility: Voice versus Data

- Several specific definitions possible in wireless networks
- Different from voice due to different QoS objectives for data traffic
- QoS measure for wireless data in (Goodman,Mandayam, IEEE Pers. Comm., April 00)
- Factors affecting utility of wireless data systems?
  - Signal-to-interference ratio (SIR)
    - Frame Success Rate: data intolerant to errors
    - Throughput: rate of reception of correct data
  - Power consumption
    - Battery life: Inversely proportional to power drain
  - SIR and power strongly interdependent:

Properties of Data Utility Function

- Low SIR ⇒ High error rate ⇒ Low utility
- High SIR ⇒ High throughput ⇒ High utility
- Very high SIR ⇒ Utility approaches constant asymptotically
- High transmit power ⇒ Fast battery drain ⇒ Low utility



System Model

- K base stations serving N terminals where each terminal j
  - transmits L information bits in M bit frames
  - transmits at R bits/second over W Hz with AWGN (σ^2)
  - is located d_{ij} meters from base station i with path gain h_{ij}
- SIR for terminal j at base station i:

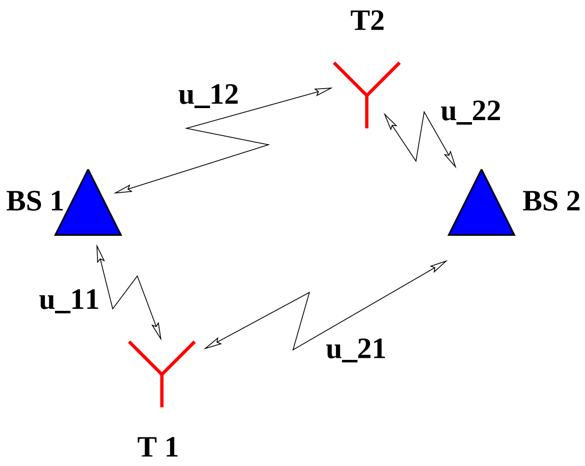
$$\gamma_{ij} = \frac{W}{R} \frac{h_{ij} p_j}{\sum_{k=1, k \neq j}^{N} h_{ik} p_k + \sigma^2} \quad . \qquad (1)$$

The Utility Function

- Utility is the number of bits transmitted successfully per unit energy
  - Utility of terminal j obtained at base station i is

$$u_{ij}(p_j, \mathbf{p}_{-j}) = \frac{RLf(\gamma_{ij})}{Mp_j} \quad \text{[bits/Joule]} \qquad (2)$$

    * $\mathbf{p}_{-j}$ vector of transmit powers except user j
    * $f : R_+ \longrightarrow [0,1]$ is the Efficiency Function
      · Approximates the frame success rate (FSR)
      · Depends on modulation format, channel coding



Efficiency Function

- Efficiency function: Approximation of Frame Success Rate
- Assuming independent bit errors

$$\mathrm{FSR} = (1 - \mathrm{BER})^M \tag{3}$$

- FSR > 0 if $p = 0$. With FSR, $\lim_{p\to 0} u = \lim_{p\to 0} \frac{RLf(\gamma)}{Mp} = \infty$
- Efficiency function approximated as

$$f(\cdot) = (1 - 2\mathrm{BER})^M \tag{4}$$

which has the property $\lim_{p\to 0} u = \lim_{p\to 0} \frac{RLf(\gamma)}{Mp} = 0$

Multicell Power Control Game

- In MCPG based on Maximum Received Signal Strength (MRSS),

$$\max_{p_j \in P_j} \; u_j(p_j, \mathbf{p}_{-j}) = u_{a_j j}(\mathbf{p}) \quad \text{for all } j = 1, \cdots, N \tag{5}$$

  - assigned base station $a_j = \arg\max_i h_{ij} \equiv \arg\min_i d_{ij}$

- First order necessary optimality condition,

$$f(\gamma_{a_j j}) - \gamma_{a_j j} f'(\gamma_{a_j j}) = 0 \tag{6}$$

  - $\gamma_{a_j j} = \tilde{\gamma}$ for all j and $\tilde{\gamma}$ is unique.

- Similar to target SIR based power control for speech communications, but
  - Value of $\tilde{\gamma}$ dictated by system (modulation, frame length)
  - Target SIR determined by considerations of subjective speech quality
  - Also, Nash of MCPG-MRSS inefficient

Nash Equilibrium in MCPG-MRSS

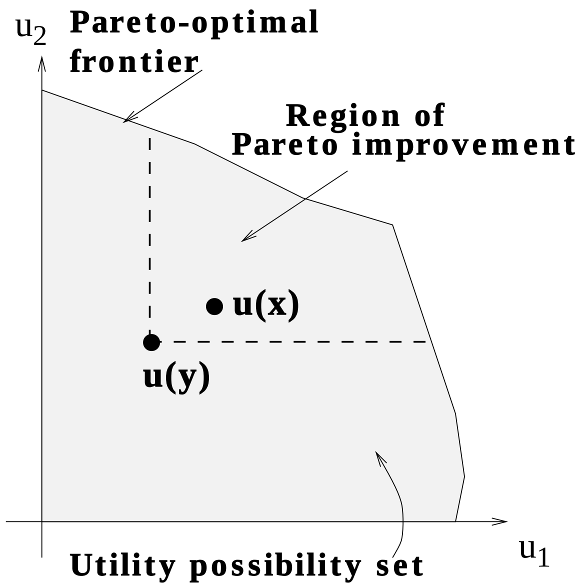
- What happens as a result of distributed self-optimizing behavior?
  - Nash equilibrium: no single user can improve its utility by unilateral change in its power
- Nash equilibrium exists in MCPG-MRSS due to quasiconcave utility functions and compact strategy spaces
- At the Nash equilibrium for MCPG-MRSS,

$$p_j = \min\left(\overline{p}, \frac{\tilde{\gamma}(\sum_{k \neq j} h_{a_j k} p_k + \sigma^2)}{G h_{a_j j}}\right) \quad (7)$$

  - Terminals with better channel (high $h_{a_j j}$) achieve $\tilde{\gamma}$
  - Terminals with poor channel quality (low $h_{a_j j}$) do best with $\overline{p}$

Efficiency of the MCPG Equilibrium

- Power vector **x** is more Pareto efficient than **y**
  - if $u_j(\mathbf{x}) \geq u_j(\mathbf{y})$ for all j and
  - if $u_j(\mathbf{x}) > u_j(\mathbf{y})$ for some j



- Force all terminals to reduce powers at equilibrium of MCPG-MRSS

  $\Rightarrow$ All terminals receive increased utility

  $\Rightarrow$ MCPG-MRSS equilibrium is Pareto inefficient

  - Fixed target type power control not efficient for data communications
  - Introduce pricing to encourage lower power

MCPG with Pricing under MRSS Assignment

- In MCPGP-MRSS, terminal j optimizes net utility:

$$\max_{p_j \in P_j} \quad u_j^c(\mathbf{p}) = u_{a_j j}(\mathbf{p}) - c_{a_j} p_j \tag{8}$$

  – c_{a_j} is the pricing factor imposed by base station a_j

- Nash equilibrium exists in MCPGP-MRSS

  – Due to supermodularity of the utility functions.

- Iterative power updates result in Nash equilibrium

  – Equilibrium utilities with pricing, higher than MCPG-MRSS

  – Equilibrium transmit powers with pricing, lower than MCPG-MRSS

Joint Power Control and Base Station Assignment

- Each terminal j solves

$$\max_{p_j, a_j} \quad u_{a_j j}(p_j, \mathbf{p}_{-j}). \tag{9}$$

- We find that joint problem equivalent to

$$\max_{p_j} \quad u_j(p_j, \mathbf{p}_{-j}) = u_{a_j j}(\mathbf{p}) \tag{10}$$

where $a_j = \arg\max_i \, u_{ij}(\mathbf{p}) \equiv \arg\max_i \, \gamma_{ij}$

- Resulting assignment referred to as Maximum SIR (MSIR)
- MCPG-MSIR has Nash equilibrium ($u_j(\mathbf{p})$ in (10) is quasiconcave)
  - Equilibrium is inefficient

MCPG with Pricing under MSIR Assignment

- In MCPGP-MSIR, terminal j optimizes net utility:

$$\max_{p_j} \quad u_j^c(\mathbf{p}) = u_j(\mathbf{p}) - c_{a_j} p_j \tag{11}$$

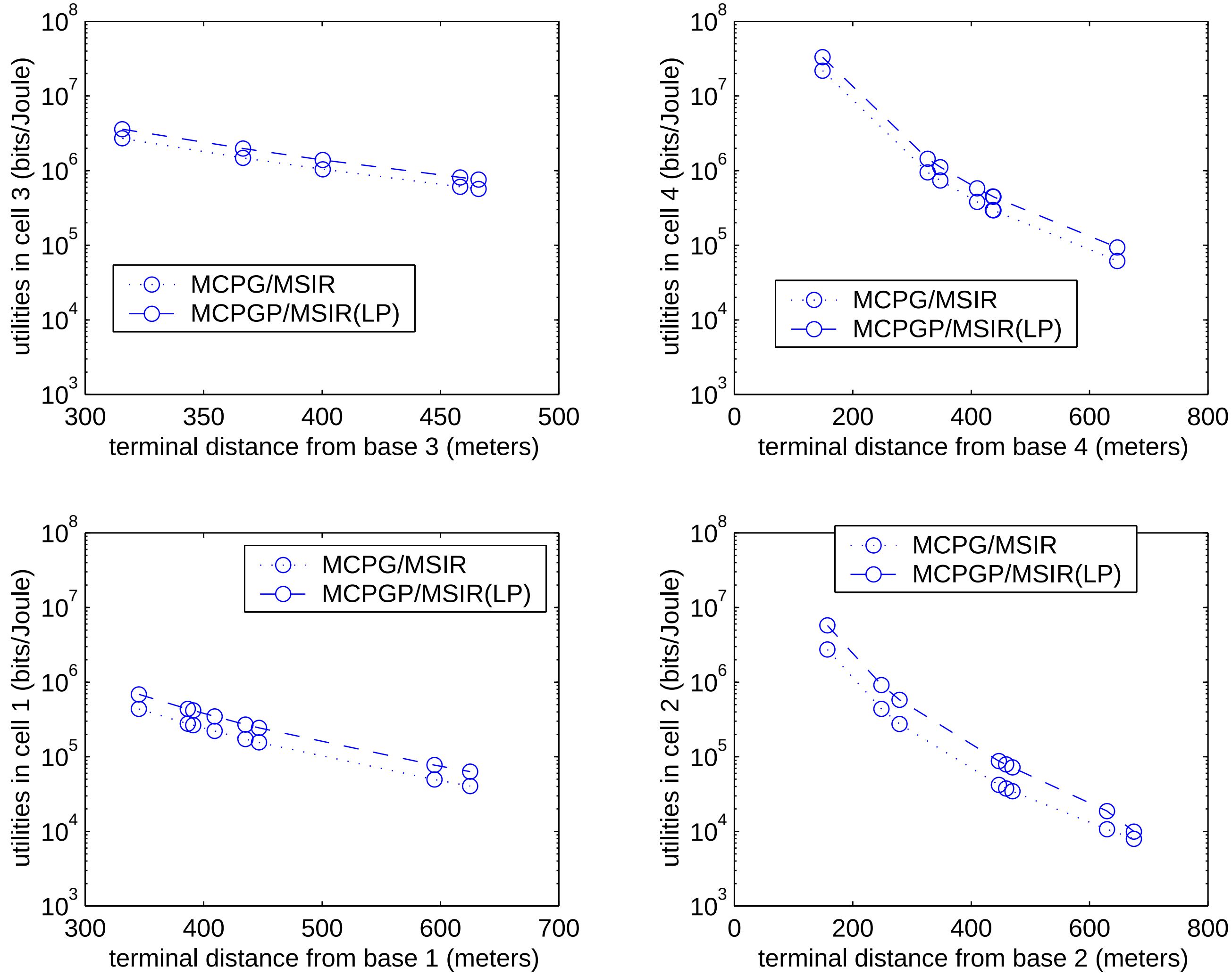
where $a_j = \max_i \gamma_{ij}$

- Experiments suggest existence of equilibrium
- Heuristic **local pricing (LP)** scheme:
  - $c_i = \alpha N_i$ where N_i is the number of terminals in cell i
  - In experiments $\alpha = R$

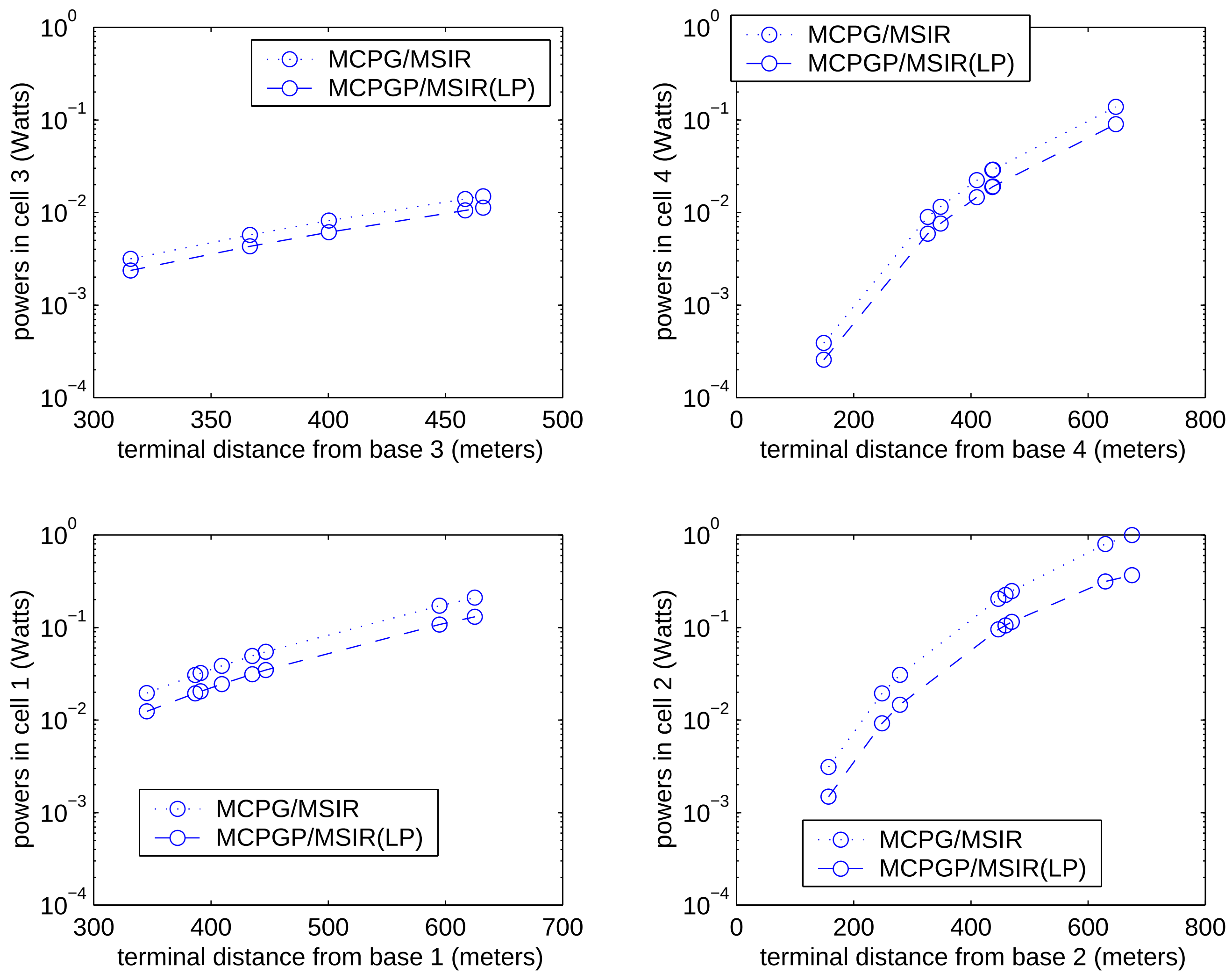
Numerical Results

N, number of terminals	28
K, number of base stations	4
M, total number of bits per frame	80
L, number of information bits per frame	64
W, spread spectrum bandwidth	10^6 Hz
R, bit rate	10^4 bits/second
σ^2, AWGN power at the receiver	5×10^{-15} Watts
modulation technique	non-coherent FSK
$\overline{p}$, maximum power constraint	1 Watt

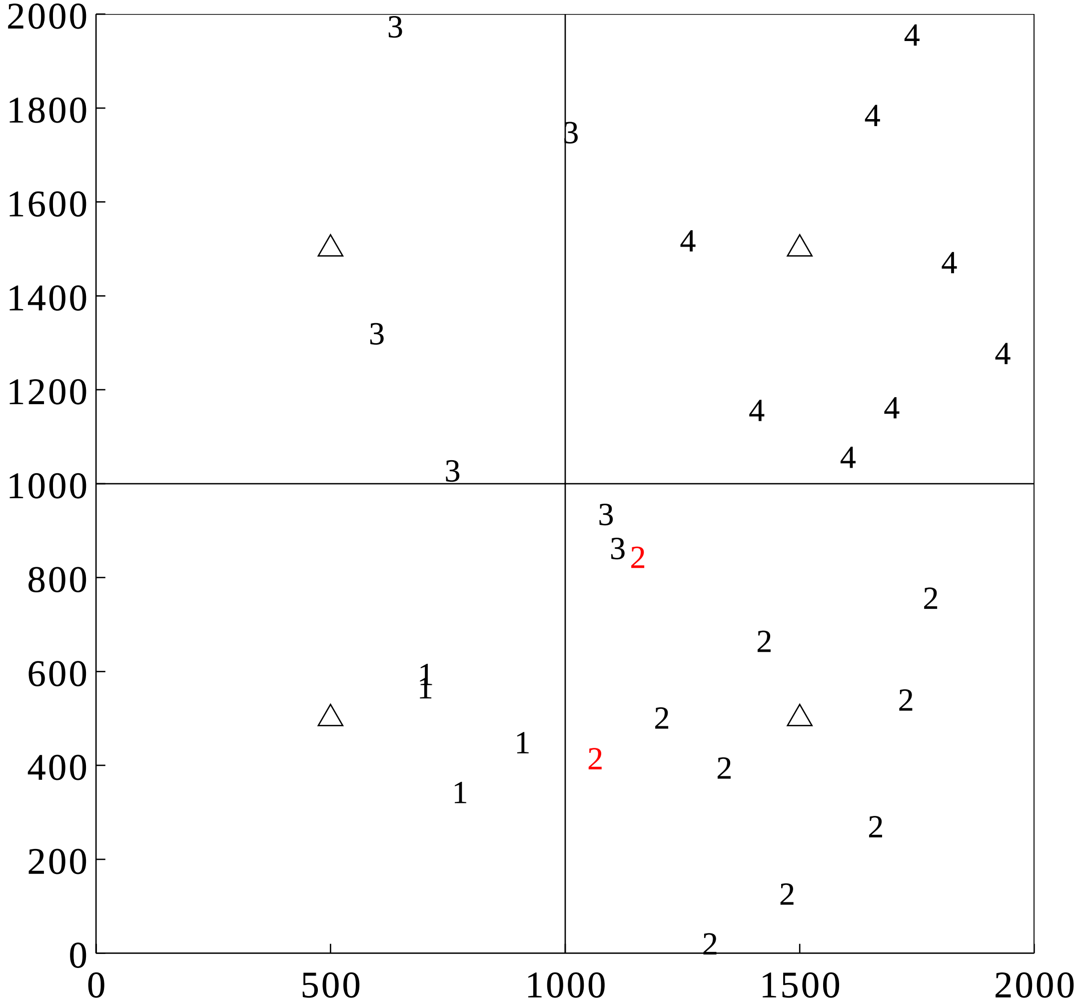
Equilibrium Utilities with MSIR



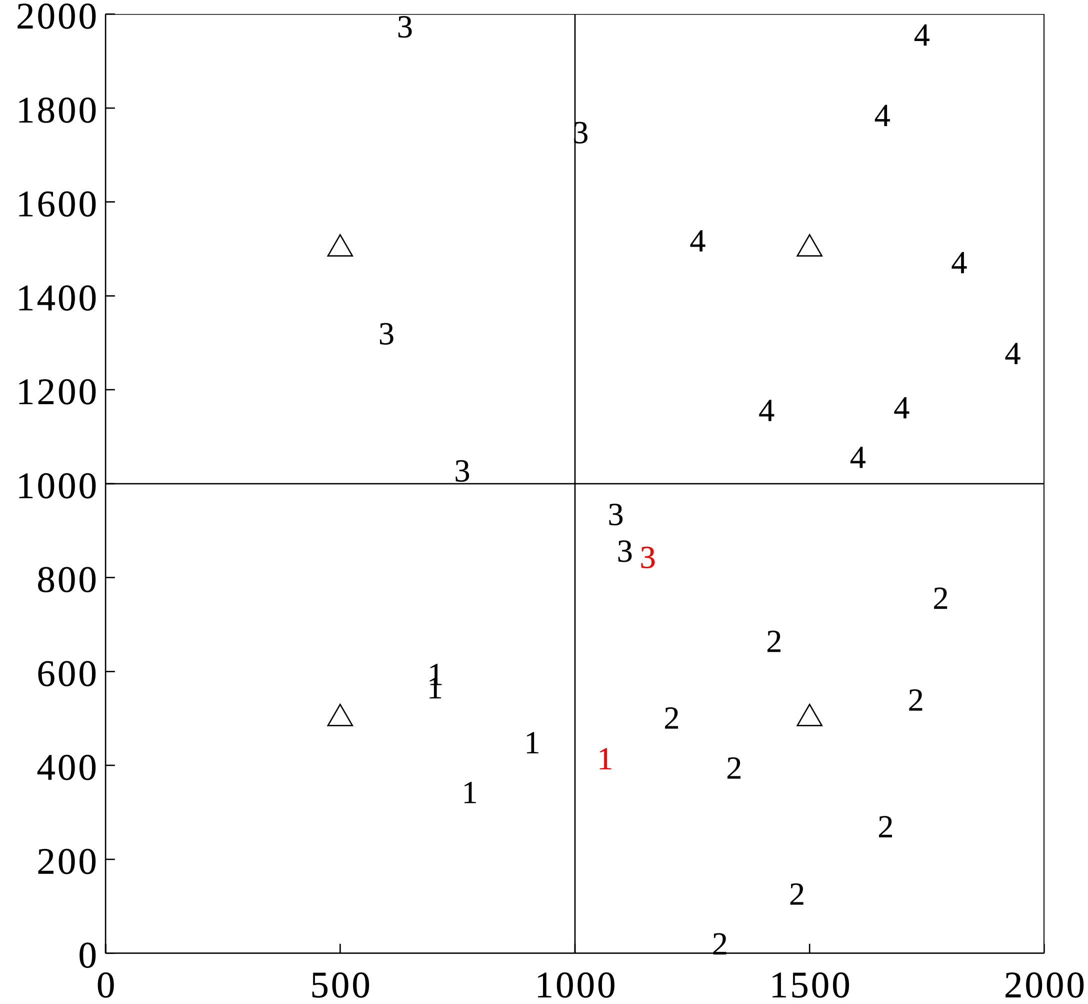
Equilibrium Transmit Powers with MSIR



Equilibrium Base Assignment for MCPG-MSIR



Equilibrium Base Assignment for MCPGP-MSIR



Summary

- Studied power control for utility maximization in wireless multicell data networks
  - Leads to voice type power control
  - Inefficient for data
- Pricing improves efficiency
  - Benefit due to decreased power
- In addition to resulting in increased utilities and decreased transmit powers, pricing may also help relieve loaded cells