Code Optimization in CDMA Systems

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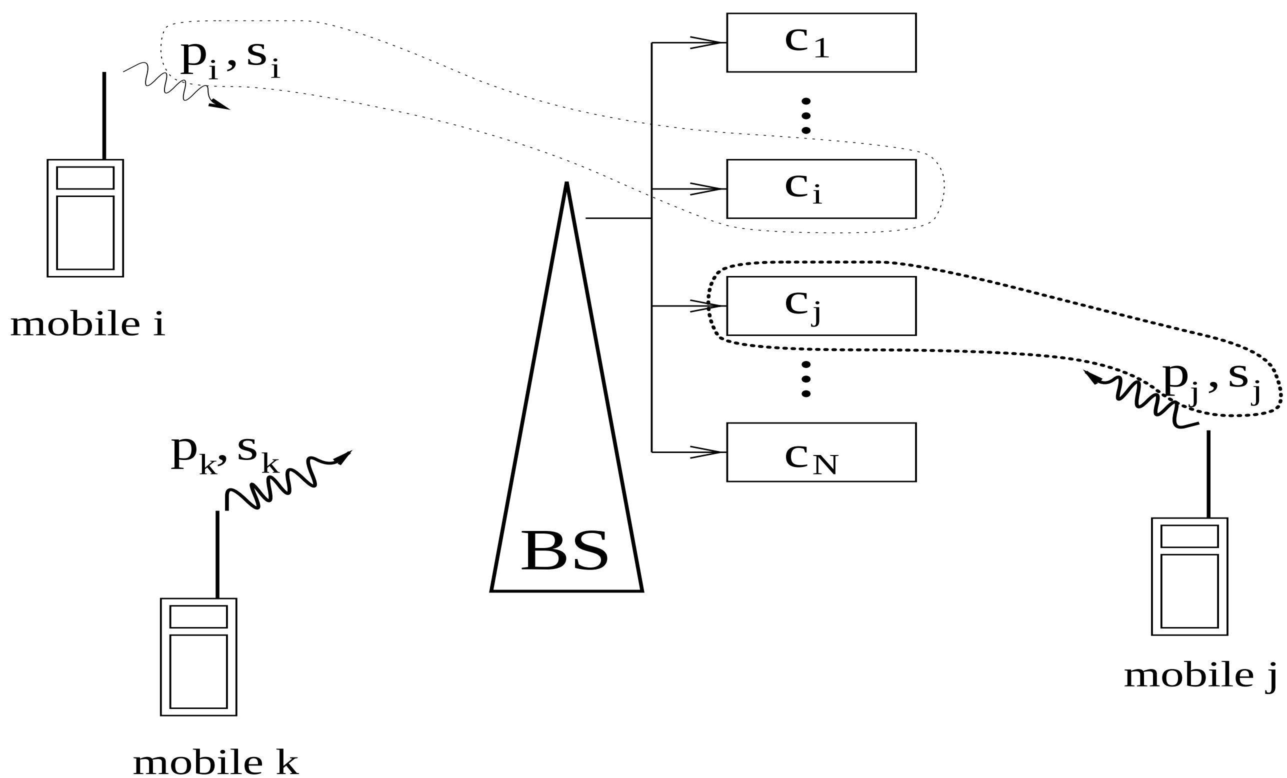
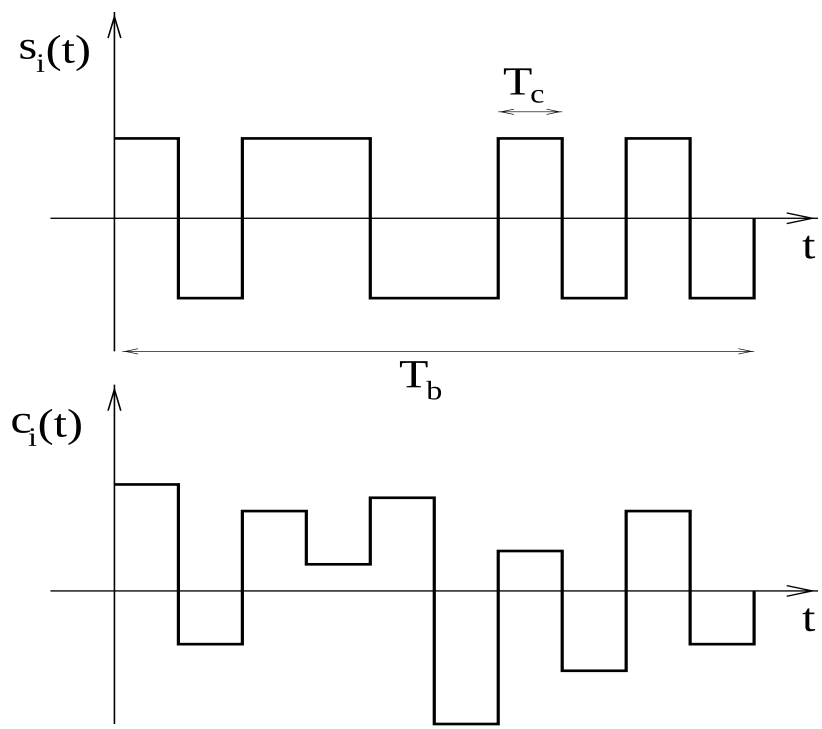
Introduction

- Each user has a power (p_i), a signature sequence ($\mathbf{s}_i$) and a receiver filter ($\mathbf{c}_i$)



- What are the jointly optimum p_i, $\mathbf{s}_i$, $\mathbf{c}_i$ for all i?

From Continuous Signals to Vectors



Processing gain = L

$$L = \frac{T_b}{T_c}$$

$$s_i = \begin{bmatrix} +1 \\ -1 \\ +1 \\ +1 \\ -1 \\ -1 \\ +1 \\ -1 \\ +1 \\ -1 \end{bmatrix}$$

- Chip sampled received signal: $\mathbf{r} = \sum_{i=1}^{N} \sqrt{p_i} a_i \mathbf{s}_i + \mathbf{n}$
- Receiver filtering: $y_i = \mathbf{r}^\top \mathbf{c}_i$
- This work is generalizable to any communication system where transmit waveforms are representable in a finite dimensional vector space.

Conventional CDMA System

- Generate $\mathbf{s}_i$ randomly (no signature optimization)
- Use matched filters $\mathbf{c}_i = \mathbf{s}_i$ (no filter optimization)
- Control transmitter power (SIR-based power control)

$$p_i(n+1) = \frac{\gamma_i^*}{\gamma_i(n)} p_i(n)$$

- Power control converges to componentwise smallest power vector where all users satisfy their SIR requirements
- Further improvement can be achieved if the receiver filters are designed to <u>suppress interference</u>

Joint Power and Receiver Filter Optimization

- S. Ulukus and R. D. Yates, "Adaptive Power Control and MMSE Interference Suppression," ACM J. on Wireless Networks.
- Fixed $\mathbf{s}_i$ (no signature optimization)
- Choose powers (p_i) and linear receiver filters ($\mathbf{c}_i$) jointly optimally
- Iterative algorithm
  - For fixed powers, update filters to minimize the MSE (equivalently maximize the SIR) – *MMSE multiuser detectors*
  - For fixed receiver filters update powers in the usual way

$$p_i(n+1) = \frac{\gamma_i^*}{\gamma_i(n)} p_i(n)$$

- Results depend only on the signature sequences; further improvement can be achieved if the signatures are designed to <u>avoid interference</u>

Review: Information Theoretic CDMA Capacity

- User i has signature sequence $\mathbf{s}_i$; let $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_N]$.
- Sum capacity of a general multiaccess channel

$$C_{\text{sum}} = \max_{(R_1,\cdots,R_N)\in\mathcal{C}} \sum_{i=1}^{N} R_i$$

- C_{sum}: maximum total number of bits N users can transmit on the uplink.
- For a single cell synchronous CDMA system, with $p_i = p$ for all i, (Verdú)

$$C_{\text{sum}} = \frac{1}{2}\log\left[\det\left(\mathbf{I}_L + \frac{p}{\sigma^2}\mathbf{S}\mathbf{S}^\top\right)\right]$$

Maximization of the Sum Capacity

- For matrices $\mathbf{A}_{K\times M}$ and $\mathbf{B}_{M\times K}$

$$\det(\mathbf{I}_K + \mathbf{AB}) = \det(\mathbf{I}_M + \mathbf{BA})$$

- CDMA sum capacity becomes

$$C_{\text{sum}} = \frac{1}{2}\log\left[\det\left(\mathbf{I}_L + \frac{p}{\sigma^2}\mathbf{S}\mathbf{S}^\top\right)\right] = \frac{1}{2}\log\left[\det\left(\mathbf{I}_N + \frac{p}{\sigma^2}\mathbf{S}^\top\mathbf{S}\right)\right]$$

- To maximize the sum capacity (Rupf, Massey)
  - If $N \leq L$, $\mathbf{S}^\top\mathbf{S} = \mathbf{I}_N$ (N orthonormal sequences)
  - If $N > L$, $\mathbf{S}\mathbf{S}^\top = \frac{N}{L}\mathbf{I}_L$ (N Welch Bound Equality sequences)

Review: Network Capacity

- **Network capacity:** Maximum number of admissible users given processing gain L and SIR target β

- N users are admissible if there are positive powers p_i and signature sequences $\mathbf{s}_i$ such that $\mathrm{SIR}_i \geq \beta$

- Network capacity with MMSE receivers (Viswanath, Anantharam, Tse)

$$N < L\left(1 + \frac{1}{\beta}\right)$$

- The maximum is achieved with

Equal received powers:	$p_i = p$ for all i
WBE signature sequences:	$\mathbf{S}\mathbf{S}^\top = (N/L)\mathbf{I}_L$

Network Capacity II

- MMSE receiver for the *i*th user is

$$\mathbf{c}_i = \sqrt{p_i}\mathbf{B}^{-1}\mathbf{s}_i$$

- When $p_i = p$ for all i, $\quad \mathbf{B} = p\mathbf{S}\mathbf{S}^\top + \sigma^2\mathbf{I}_L$
- With WBE sequences $\mathbf{S}\mathbf{S}^\top = \frac{N}{L}\mathbf{I}_L$

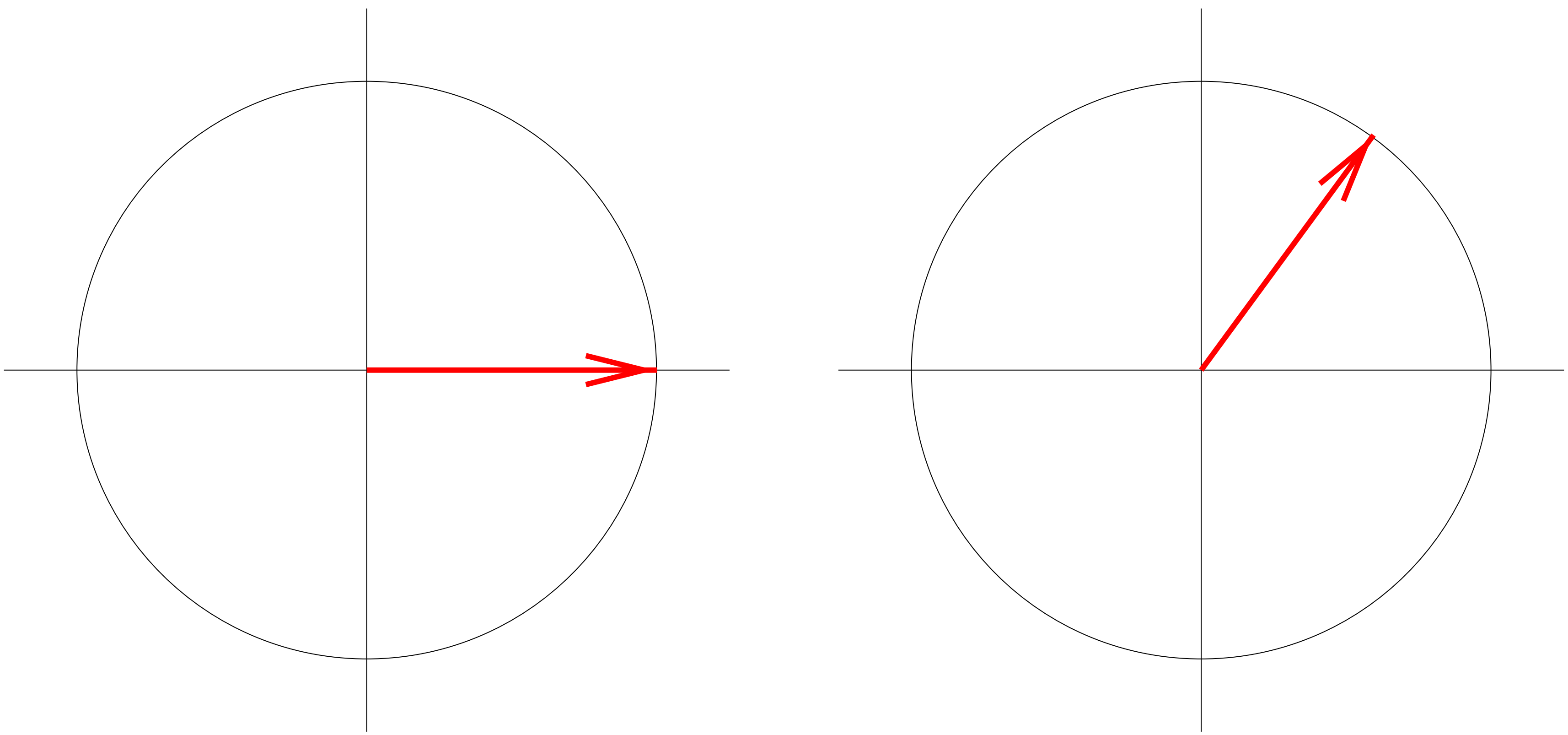
$$\mathbf{c}_i = \alpha_i \mathbf{s}_i$$ ***scaled matched filters!***

- Network capacity <u>with matched filters</u> (Viswanath, Anantharam, Tse)

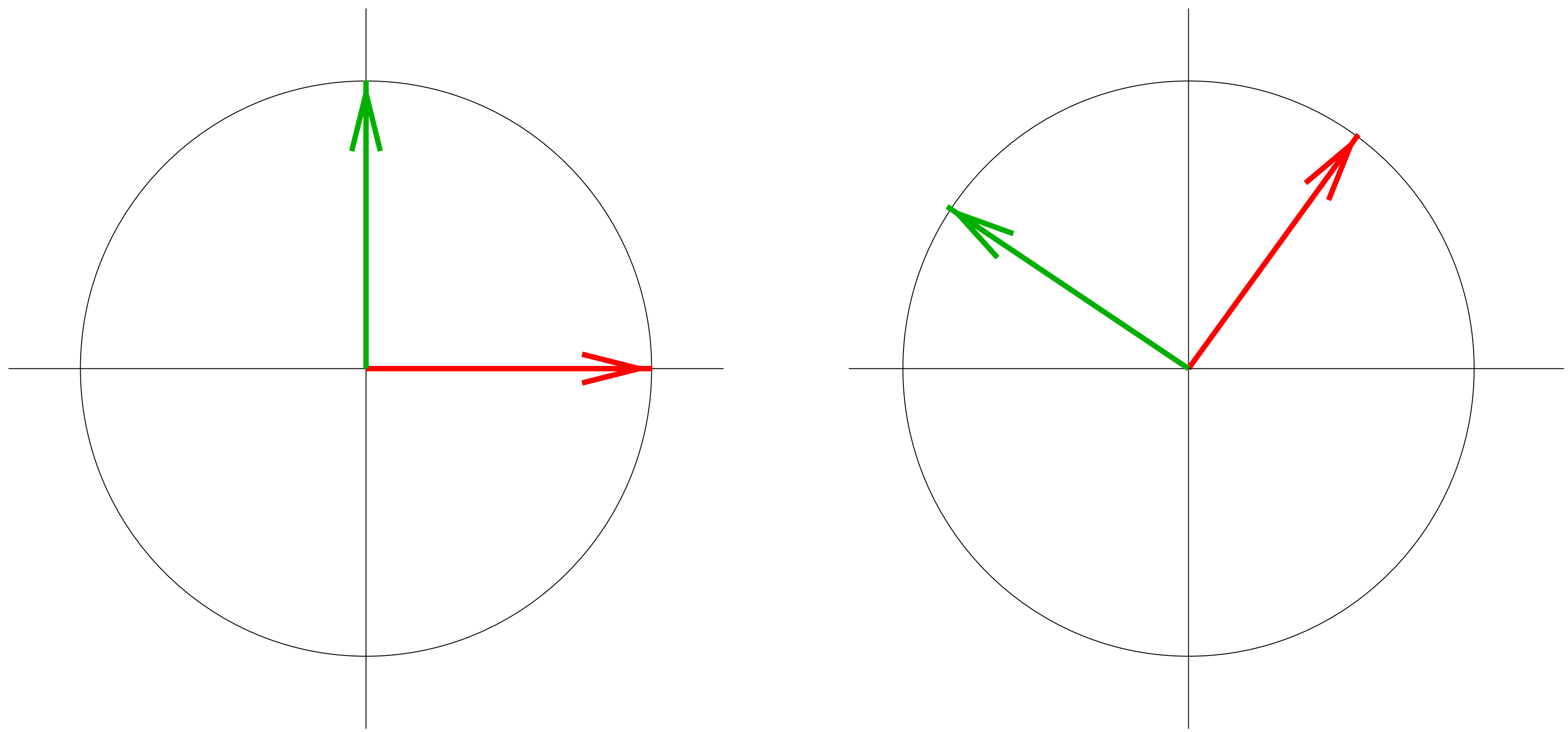
$$N < L\left(1 + \frac{1}{\beta}\right)$$

- Max is achieved with equal rec'd powers and WBE sequences
- "Power / signature sequence / receiver filter optimization" problem actually has two *degrees of freedom*: powers and signatures.

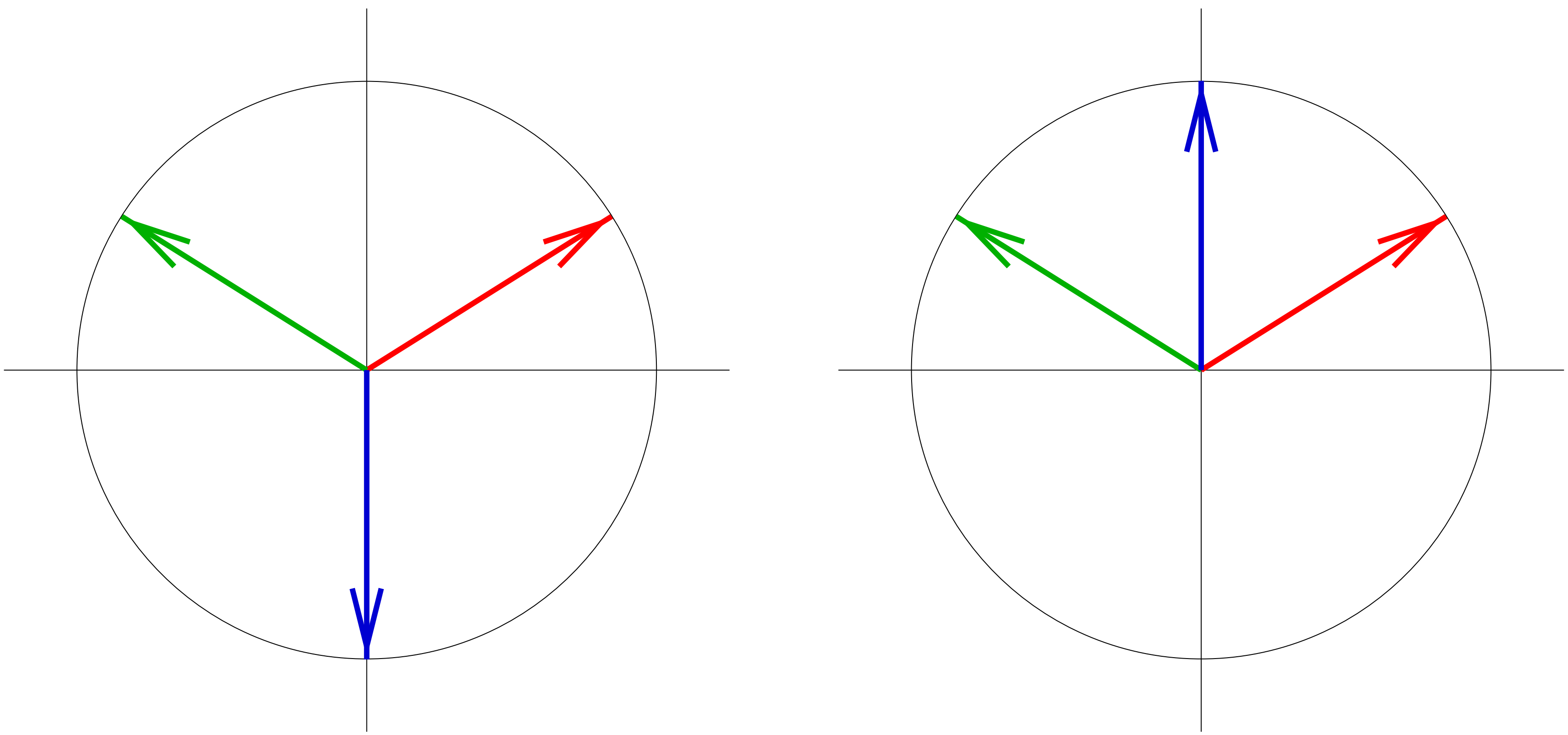
Simple example, $L = 2$, $N = 1$



Simple example, $L = 2$, $N = 2$



Simple example, $L = 2$, $N = 3$



Welch's Bound

- Originally a lower bound for $\max_{i,j}(\mathbf{s}_i^\top \mathbf{s}_j)^{2k}$ in a set of unit energy vectors
- The derivation uses the lower bound:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}(\mathbf{s}_i^\top \mathbf{s}_j)^{2k} \geq \frac{N^2}{\binom{L+k-1}{k}}$$

- For $k = 1$, a lower bound for Total Squared Correlation:

$$\mathrm{TSC} = \sum_{i=1}^{N}\sum_{j=1}^{N}(\mathbf{s}_i^\top \mathbf{s}_j)^{2} \geq \frac{N^2}{L}$$

- If $N < L$, the bound is loose. For N orthonormal vectors, $\mathrm{TSC} = N$
- If $N > L$, the bound is achieved iff $\mathbf{S}\mathbf{S}^\top = \frac{N}{L}\mathbf{I}_L$ (Massey, Mittelholzer)

WBE Sequences, Minimum TSC, and Optimality

- Minimum TSC sequences:
  - Orthonormal sequences for $N \leq L$
  - WBE sequences for $N > L$
- For a single cell CDMA system, minimum TSC sequences maximize
  - Information theoretic sum capacity
  - Network capacity
- Goal: A simple algorithm which converges to a set of minimum TSC sequences.

WBE Sequences and Minimum TSC

- Optimal signatures are minimum TSC signatures
- Starting point for TSC reduction:

$$\text{TSC} = \underbrace{(\mathbf{s}_k^\top \mathbf{s}_k)^2}_{1} + 2\mathbf{s}_k^\top \left(\sum_{j \neq k} \mathbf{s}_j \mathbf{s}_j^\top \right) \mathbf{s}_k + \sum_{i \neq k} \sum_{j \neq k} (\mathbf{s}_i^\top \mathbf{s}_j)^2$$

- Many ways to reduce TSC:
  - e.g. choose s_k to be eigenvector of $\sum_{j \neq k} \mathbf{s}_j \mathbf{s}_j^\top$ with min eigenvalue

An Iterative TSC Reduction Algorithm

- Method: Replace $\mathbf{s}_k$ with

$$\mathbf{c}_k = \frac{\mathbf{A}_k^{-1}\mathbf{s}_k}{(\mathbf{s}_k^\top \mathbf{A}_k^{-2}\mathbf{s}_k)^{1/2}}$$

where $\mathbf{A}_k = \sum_{j \neq k} \mathbf{s}_j \mathbf{s}_j^\top + a^2 \mathbf{I}_L$.

- $\mathbf{c}_k$ is a generalized normalized MMSE filter for user k.
- Practical Implementation:
  - Use a blind adaptive MMSE detector for each user.
  - When receiver filter for user k converges, transmit filter coefficients back to the transmitter

Algorithm Properties

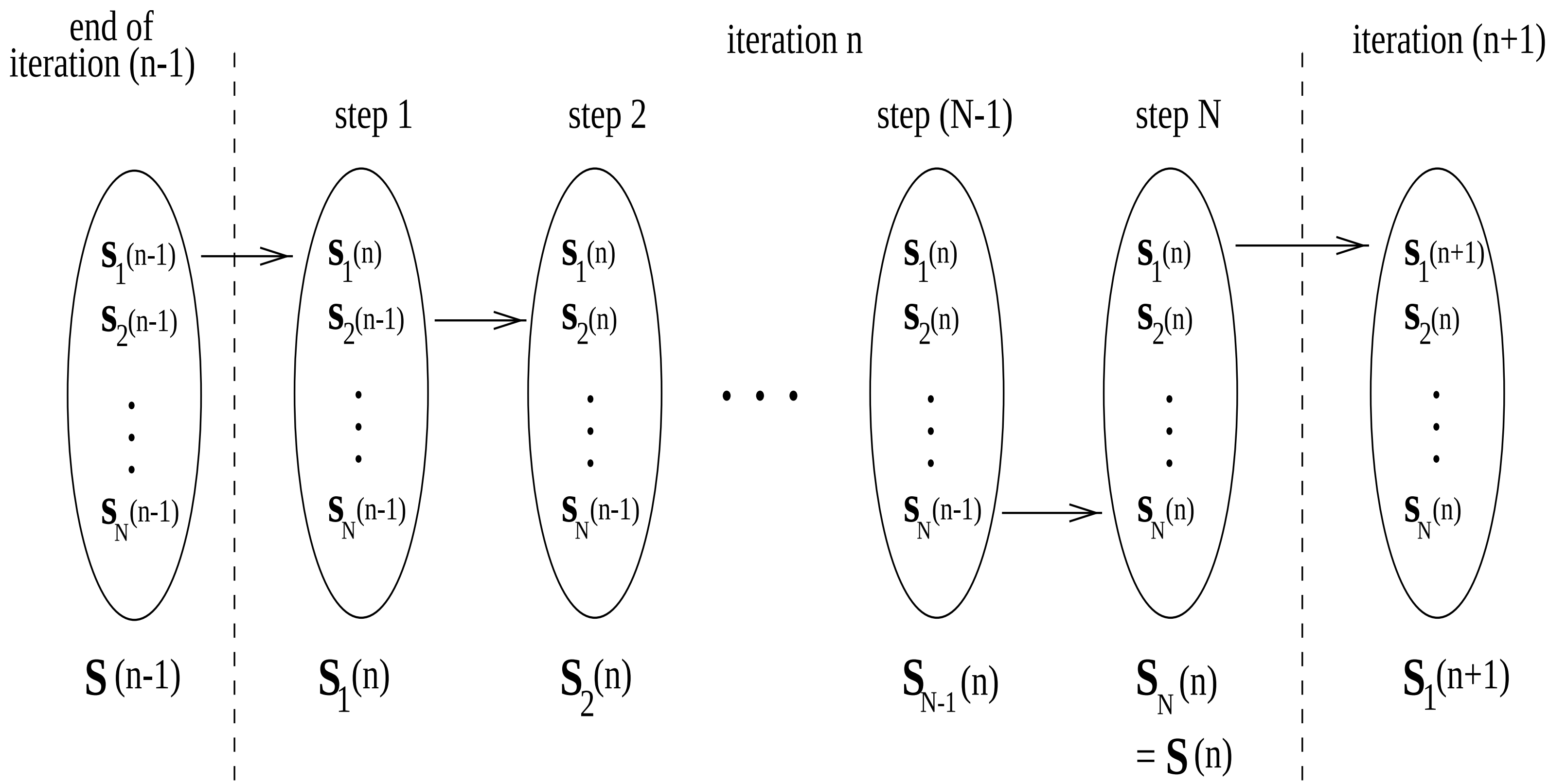
- Algorithm: Replace $\mathbf{s}_k$ with MMSE filter $\mathbf{c}_k$

Old Signatures: $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_{k-1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \cdots, \mathbf{s}_N]$

New Signatures: $\mathbf{S}' = [\mathbf{s}_1, \cdots, \mathbf{s}_{k-1}, \mathbf{c}_k, \mathbf{s}_{k+1}, \cdots, \mathbf{s}_N]$

- Theorem: $\text{TSC}(\mathbf{S}') \leq \text{TSC}(\mathbf{S})$ and $\text{TSC}(\mathbf{S}') = \text{TSC}(\mathbf{S})$ iff $\mathbf{c}_k = \mathbf{s}_k$.

The Iterative Algorithm



- $\mathrm{TSC}(n-1) \geq \mathrm{TSC}_1(n) \geq \cdots \geq \mathrm{TSC}_{N-1}(n) \geq \mathrm{TSC}_N(n) = \mathrm{TSC}(n)$

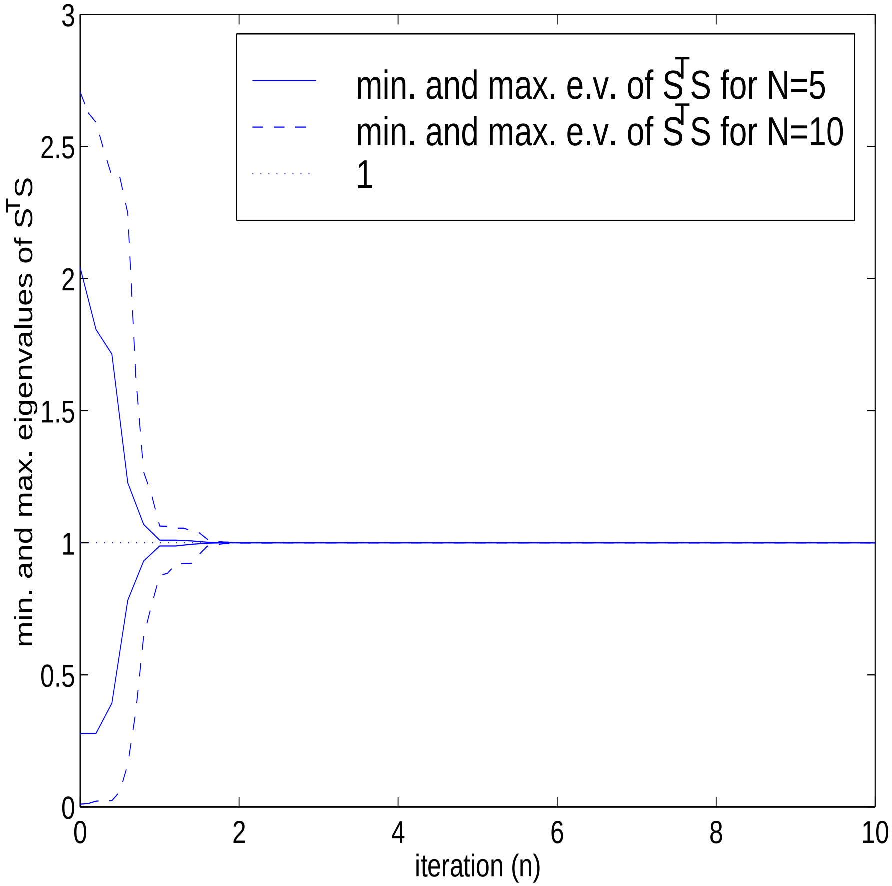
Algorithm Convergence

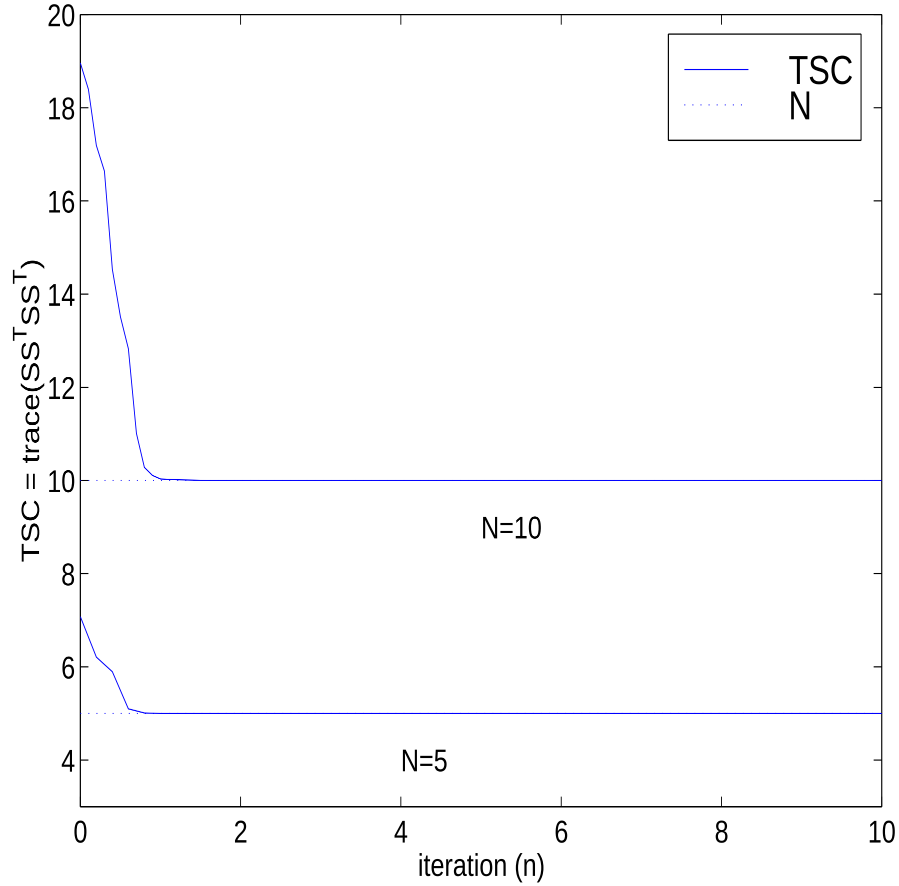
- $\mathrm{TSC}(n)$ is decreasing and lower bounded $\Longrightarrow$ $\mathrm{TSC}(n)$ converges
- $\mathrm{TSC}(n)$ converges $\Longrightarrow \mathbf{S}(n) \to \mathbf{S}$
- Does TSC reach global minimum?

Properties of the Fixed Point

- Let $\mathbf{S} = \lim_{n\to\infty} \mathbf{S}(n)$
- $N \leq L$: $\mathbf{S}$ converges to $\mathbf{S}^\top \mathbf{S} = \mathbf{I}_N$
  - <u>Sufficient condition:</u> initial signature sequences $\mathbf{S}(0)$ are linearly independent
- $N > L$: $\mathbf{S}$ converges to $\mathbf{S}\mathbf{S}^\top = \frac{N}{L}\mathbf{I}_L$
  - <u>Sufficient condition:</u> L of N initial signature sequences $\mathbf{S}(0)$ are linearly independent
  - <u>Necessary condition:</u> $\mathbf{S}(0)$ does not have orthogonal subsets
- For both cases sufficient conditions can be relaxed

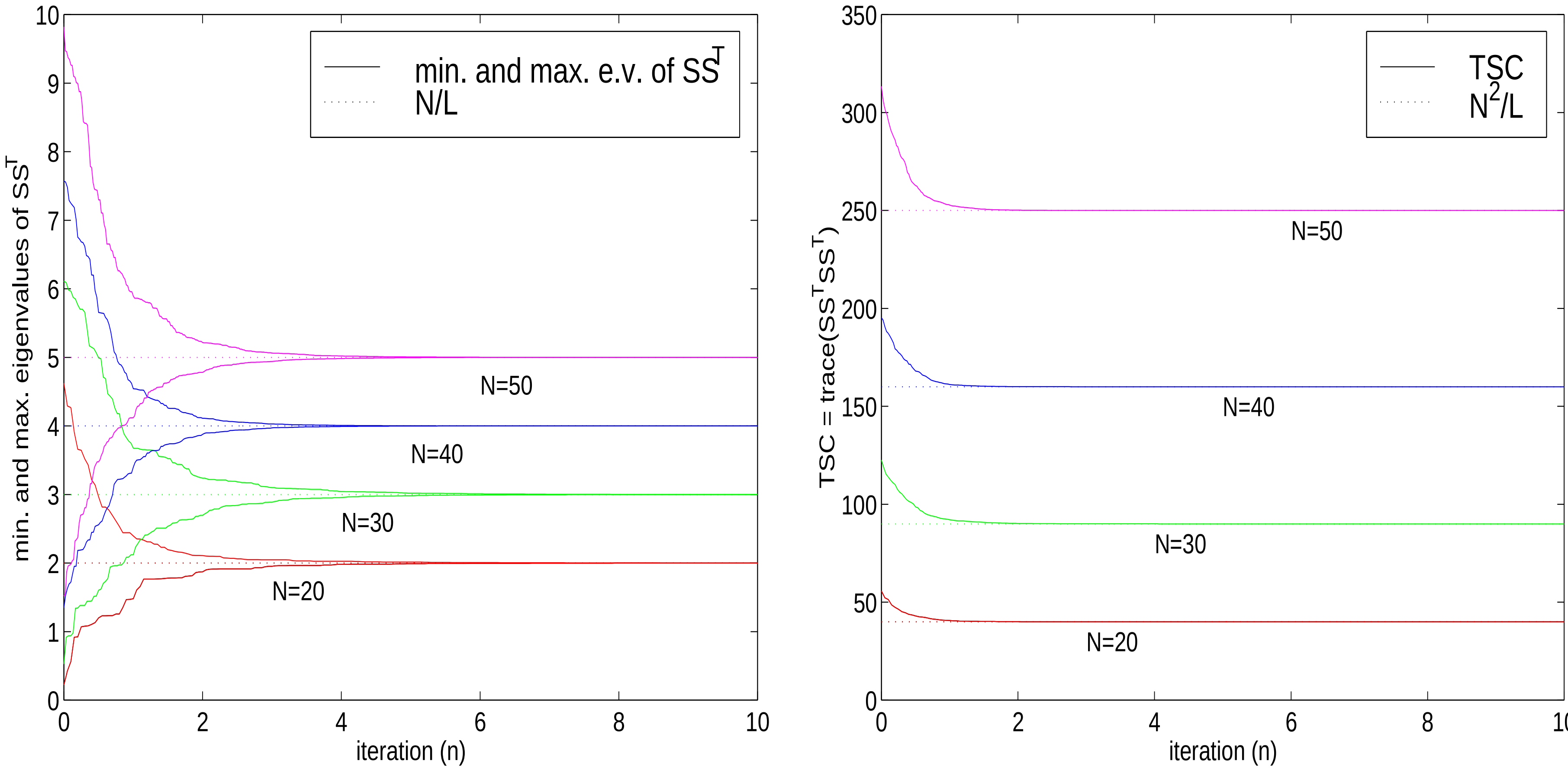
Min/Max Eigenvalues of $\mathbf{S}^\top(n)\mathbf{S}(n)$ and TSC(n)





$\underline{N \leq L \text{ case:}}$ $N = 5, 10$ and $L = 10$

Min/Max Eigenvalues of $\mathbf{S}(n)\mathbf{S}^\top(n)$ and $\mathbf{TSC}(n)$



$\underline{N > L \text{ case:}}$ $N = 20, 30, 40, 50$ and $L = 10$

Conclusions

- This work developed a distributed algorithm for adapting users' signatures that can be used for ***interference avoidance***.
- The algorithm is shown to converge to a set of users' signatures that are optimal both in terms of information theoretic capacity and network capacity.
- The algorithm can be implemented using feedback to the transmitter from an adaptive MMSE receiver.

Work in Progress

- Extensions to asynchronous systems.
- Analysis of multipath channels
- Multiple receivers (multicell systems)
- Implementation based on blind adaptive detectors.
- Effectiveness in unlicensed environments.