Unquantized and Uncoded Channel State Information Feedback on Wireless Channels

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Abstract

In this paper we consider a system where a mobile terminal obtains the downlink channel state information (CSI) and feeds it back to the base station using an uplink feedback channel. If the downlink channel is an independent Rayleigh fading channel, then the CSI may be viewed as an output of a complex independent identically distributed Gaussian source. Further, if the uplink feedback channel is AWGN, it can be shown that that unquantized and uncoded (UQ-UC) CSI transmission (that incurs zero delay) is optimal in that it achieves the same minimum mean squared error (MMSE) distortion as a scheme that optimally (in the Shannon sense) quantizes and encodes the CSI while incurring infinite delay. Since the UQ-UC transmission is suboptimal on correlated wireless channels, we propose a simple linear CSI feedback receiver that can be used in conjunction with the UQ-UC transmission while still retaining the attractive zero-delay feature. We provide bounds on the performance of the UQ-UC scheme and also explore the performance in multiple antenna multiuser wireless systems.

***Keywords*: Channel state information, distortion, correlated wireless channels, ARMA process, MMSE receiver.**

I. Introduction

The tremendous capacity gains due to transmitter optimization in multiple antenna multiuser wireless systems [1]–[6] rely heavily on the availability of the channel state information (CSI) at the transmitter. This requires reliable feedback of CSI that will also have to be fast and frequent owing to ever increasing demands on mobility.

In this paper we consider a system where a mobile terminal obtains the downlink CSI and feeds it back to the base station using an uplink feedback channel. If the downlink channel is an independent Rayleigh fading channel, then the CSI may be viewed as an output of a complex independent identically distributed (*iid*) Gaussian source. Further if the uplink feedback channel is AWGN, it can be shown that unquantized and uncoded (UQ-UC) CSI transmission (that incurs zero delay) is optimal in that it achieves the same minimum mean squared error (MMSE) distortion as a scheme that optimally (in the Shannon sense) quantizes and encodes the CSI while incurring infinite delay. Results on the optimality of unquantized and uncoded transmission have also been discussed in other contexts in [7]–[9]. Since the UQ-UC transmission is suboptimal on correlated wireless channels, we propose a simple linear CSI feedback receiver that can be used in conjunction with the UQ-UC transmission while still retaining the attractive zero-delay feature. Furthermore, we describe an auto regressive-moving average (ARMA) correlated channel model and present the corresponding performance bounds for the UQ-UC CSI feedback scheme. We explore the performance limits of such schemes in the context of multiple antenna multiuser wireless systems.

II. Background

Consider the communication system in Figure 1. The system is used for transmission of unquantized and uncoded outputs (i.e., symbols) of the source. The source is complex, continuous in amplitude and discrete in time (with the symbol period T_{sym}). We assume that the symbols x are zero-mean with the unit variance. The average transmit power is P, while the channel introduces the additive zero-mean noise n with the variance N_0.

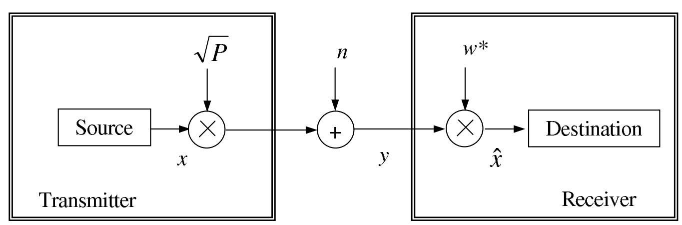

Fig. 1. Unquantized and uncoded transmission that achieves the MMSE distortion of the transmitted signal.

At the receiver, the received signal y is multiplied by the conjugate of w. Consequently, the signal $\hat{x}$ at the destination is

$$\hat{x} = w^* y = w^* \left(\sqrt{P} x + n\right) \tag{1}$$

and $\hat{x}$ is an estimate of the transmitted symbol x. We select the coefficient w to minimize the mean squared error (MSE) between $\hat{x}$ and x. Thus,

$$w = \arg\min \; \mathrm{E}|\hat{x} - x|^2 = \arg_v \min \; \mathrm{E}|v^* \left(\sqrt{P} x + n\right) - x|^2. \tag{2}$$

Consequently,

$$w = \frac{\sqrt{P}}{P + N_0} \quad (3)$$

and the corresponding mean squared error is

$$\min \mathrm{E}|\hat{x} - x|^2 = \frac{1}{1 + \frac{P}{N_0}}. \quad (4)$$

The MSE corresponds to a measure of distortion between the source symbols and estimates at the destination.

Let us now relate the above results to the transmission scheme that applies optimal quantization and channel coding. Based on the Shannon rate distortion theory [10], for the given distortion D^*, average number of bits per symbol at the output of the optimal quantizer is

$$R = \log_2\left(1 + \frac{1 - D^*}{D^*}\right). \quad (5)$$

Note that the optimal quantizer that achieves the above rate incurs infinite quantization delay. For the AWGN channel, the maximum transmission rate is

$$C = \log_2\left(1 + \frac{P}{N_0}\right). \quad (6)$$

As in the case of the optimal quantizer, the optimal channel coding would incur infinite coding delay. Furthermore, optimal matching (in the Shannon sense) of the quantizer and the channel requires that

$$R = C \Rightarrow D^* = 2^{-C} = \frac{1}{1 + \frac{P}{N_0}}. \quad (7)$$

The above distortion is equal to the MSE for the UQ-UC transmission scheme given in (4) (see also [8]). The above result points to the optimality of the UQ-UC scheme (while it incurs zero delay) when the source is *iid* Gaussian and the channel is AWGN.

III. UQ-UC CSI FEEDBACK

In the following we consider a communication system presented in Figure 2. It consists of a base station transmitting data over a downlink channel. A mobile terminal receives data, and transmit the CSI of the downlink channel state h_{dl} over an uplink channel. If the uplink channel is AWGN and the downlink channel is *iid* Rayleigh, then the above UQ-UC scheme is optimal for transmission of the downlink CSI over the uplink channel (with zero delay). This follows directly from the discussions in Section II.

We now illustrate an example of how this UQ-UC CSI feedback scheme could be applied in the context of a CDMA communication system. The functional blocks of the mobile terminal are depicted in Figure 3. Based on the pilot assisted estimation, the mobile terminal obtains an estimate of the downlink channel h_{dl} (see [11]). The downlink channel h_{dl} is the CSI to be transmitted on the uplink. The estimate $\bar{h}_{dl}$ modulates (i.e., multiplies) a Walsh code that is specifically allocated as a CSI feedback carrier. The second Walsh code is allocated to the conventional uplink data transmission. The

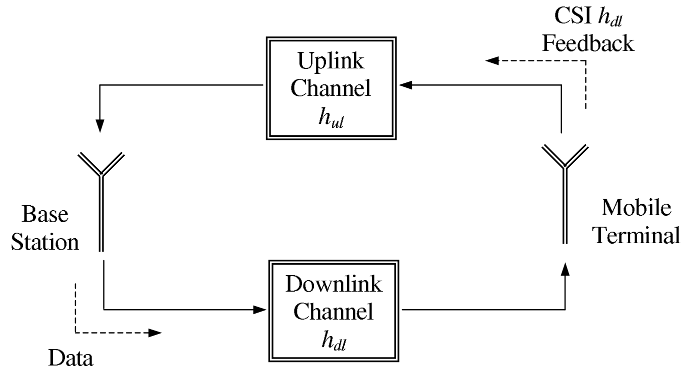

Fig. 2. Communication system with CSI feedback.

uplink pilot is also transmitted allowing the base station to obtain an estimate $\bar{h}_{ul}$ of the uplink channel h_{ul}.

At the time instant i, the uplink received signal corresponding to the CSI feedback is

$$y(i) = h_{ul}(i)\bar{h}_{dl}(i) + n(i) \quad (8)$$

where $n(i)$ is the AWGN on the uplink, with the distribution $\mathcal{N}_{\mathcal{C}}(0, N_0/P_{ul}^{csi})$ (all for the time instant i). P_{ul}^{csi} is the average power of the uplink CSI feedback. For simplicity, in the following we will assume that the estimate $\bar{h}_{dl}(i)$ is perfect, i.e., $\bar{h}_{dl}(i) = h_{dl}(i)$. Using the received signal in (8) and the estimate $\bar{h}_{ul}(i)$, the CSI feedback receiver at the base station will estimate the transmitted CSI $h_{dl}(i)$. One possible solution of the CSI feedback receiver is proposed in the following section.

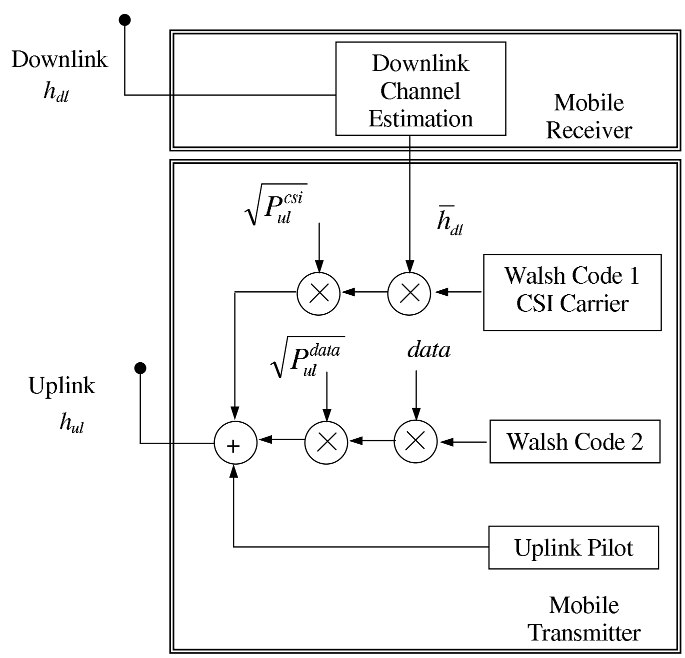

Fig. 3. CDMA mobile terminal that applies the UQ-UC CSI feedback.

IV. UQ-UC CSI Feedback on Correlated Channels

Before proposing the CSI feedback receiver we analyze how typical characteristics of the downlink and uplink channel bound the performance of the above CSI feedback scheme.

A. Performance Bounds

We first assume the uplink and downlink channel states are independent (which is typical in FDD wireless systems). Both the uplink and downlink channels are varying in time and are assumed to be ergodic. If the scheme shown in Figure 1 is now applied on the CSI feedback channel (explicitly accounting for the uplink channel state h_{ul}), then it follows that the MSE is

$$MSE_{uq-uc}^{ub} = \mathrm{E}_{h_{ul}}\left[\frac{1}{1+\frac{|h_{ul}|^2 P_{ul}^{csi}}{N_0}}\right]. \quad (9)$$

Clearly this serves as an upper bound on the MSE achieved by any additional processing that accounts for both the downlink and the uplink CSI feedback channel being correlated channels. Furthermore, the ergodic capacity of the uplink channel is

$$\bar{C}_{ul} = \mathrm{E}_{h_{ul}}\left[\log_2\left(1+\frac{|h_{ul}|^2 P_{ul}^{csi}}{N_0}\right)\right]. \quad (10)$$

We will model the downlink channel as an Lth order auto regressive-moving average (ARMA) process as

$$h_{dl}(i) = \sum_{j=1}^{L} c_j h_{dl}(i-j) + c_0\, n_{dl}(i) \quad (11)$$

where $n_{dl}(i)$ is a complex random variable with distribution $\mathcal{N}_{\mathcal{C}}(0,1)$. The coefficients c_j ($j = 0, \cdots, L$) determine the correlation properties of the channel. $n_{dl}(i)$ is the innovation sequence. For completeness, in the appendix we describe how to obtain the coefficients c_j ($j = 0, \cdots, L$) for the given correlation between the downlink channel states. Furthermore, in the appendix we describe an approximation of Jake's model using the above ARMA model.

Let us assume that the above model and previous channel states $h_{dl}(i-j)$ ($j = 1, \cdots, L$) are known at the CSI feedback transmitter and receiver. In this idealized case, having the innovation $n_{dl}(i)$ transmitted over the uplink CSI feedback channel, the receiver can estimate the channel state $h_{dl}(i)$. Similar to the derivations in (5) - (7), the optimal quantization and channel coding of the innovation $n_{dl}(i)$ results in its MSE

$$\mathrm{E}|\hat{n}_{dl}(i) - n_{dl}(i)|^2 = 2^{-\bar{C}_{ul}} \quad (12)$$

where $\hat{n}_{dl}(i)$ is an estimate of $n_{dl}(i)$ at the receiver and $\bar{C}_{ul}$ is given in (10). Based on (11) and (12), it can be shown the MSE of $h_{dl}(i)$ is lower bounded as

$$\mathrm{E}|\hat{h}_{dl}(i) - h_{dl}(i)|^2 \geq c_0^2 2^{-\bar{C}_{ul}}. \quad (13)$$

However, we expect this bound to be loose since it is obtained using idealized knowledge of the previous channel states and also a channel coding scheme that achieves the ergodic capacity of the uplink channel. Note that the bounds in (9) and (13) correspond to ergodic and mutually independent uplink and downlink channels where the downlink obeys the model in (11).

B. Linear CSI Feedback Receiver Design

In order to exploit the correlations while applying the UQ-UC CSI feedback scheme, we propose the following linear receiver.

Based on the uplink received signal in (8) we form a temporal K-dimensional received vector as

$$\mathbf{y}(i) = [y(i)\, y(i-1) \cdots y(i-K+1)]^{\mathrm{T}}. \quad (14)$$

The uplink receiver then estimates the downlink CSI $h_{dl}(i)$ as

$$\hat{h}_{dl}(i) = \mathbf{w}^{\mathrm{H}}\mathbf{y}(i) \quad (15)$$

where $\mathbf{w}$ is a linear filter that is derived from the following MMSE optimization

$$\mathbf{w} = \arg_{\mathbf{v}} \min\ \mathrm{E}|\mathbf{v}^{\mathrm{H}}\mathbf{y}(i) - h_{dl}(i)|^2. \quad (16)$$

For the given estimates of the uplink channel $\bar{\mathbf{h}}_{ul}(i) = [\bar{h}_{ul}(i)\, \bar{h}_{ul}(i-1) \cdots \bar{h}_{ul}(i-K+1)]^{\mathrm{T}}$ we define the following matrix

$$\mathbf{U} = \mathrm{E}_{\mathbf{y}|\bar{\mathbf{h}}_{ul}}\left[\mathbf{y}\,\mathbf{y}^{\mathrm{H}}\right] \quad (17)$$

and the vector

$$\mathbf{s} = \mathrm{E}_{h_{dl},\,\mathbf{y}|\bar{\mathbf{h}}_{ul}}\left[h_{dl}^{*}\,\mathbf{y}\right]. \quad (18)$$

Note that we have omitted the temporal index i since we assume a stationary system (i.e., the uplink channel, downlink channel and the AWGN are assumed to be stationary random processes). It can be shown that the linear MMSE receiver $\mathbf{w}$ is

$$\mathbf{w} = \mathbf{U}^{-1}\mathbf{s}. \quad (19)$$

As is evident from the equations (17) to (19), the linear transformation $\mathbf{w}$ takes into account the following correlations: (1) temporal correlations in the downlink channel, (2) temporal correlations in the uplink channel and (3) the correlations between the uplink and the downlink (as is in TDD systems).

Note that for $K = 1$ and the uplink and the downlink being mutually independent, the above receiver will achieve the upper bound in (9).

V. Numerical Results

We consider the case when the uplink and the downlink channels are mutually independent. Further, the downlink channel is modeled as an ARMA process whose coefficients are chosen to correspond to Jake's model for a carrier frequency of 2GHz and a mobile terminal velocity of 10kmph. For the uplink, we assume that the channel is Rayleigh with an average $SNR_{ul}^{csi} = 10\log(P_{ul}^{csi}/N_0) = 10$dB. In Figure 4 we show the MSE of the UQ-UC scheme with the linear CSI feedback receiver as a function of the CSI update period τ. τ is the absolute time difference between successive channel states $h_{dl}(i)$ and $h_{dl}(i-1)$. The corresponding lower and upper bounds are also shown. Figure 5 shows the MSE of the same scheme as a function of the CSI update period τ for different mobile terminal velocities. These results show that the linear receiver (for $K = L+1$) in combination with the UQ-UC transmission is able to exploit the temporal correlations in the channel and improve the performance relative to the lower bound. Note that when either the mobile terminal velocities are low or the CSI update period is small, the improvement is greater.

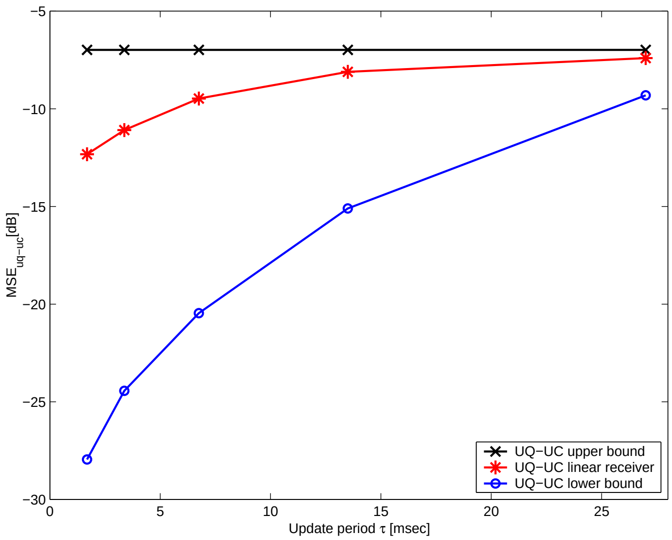

Fig. 4. MSE vs. CSI update period, f_c = 2GHz, v = 10kmph, average SNR_{ul}^{csi} = 10dB.

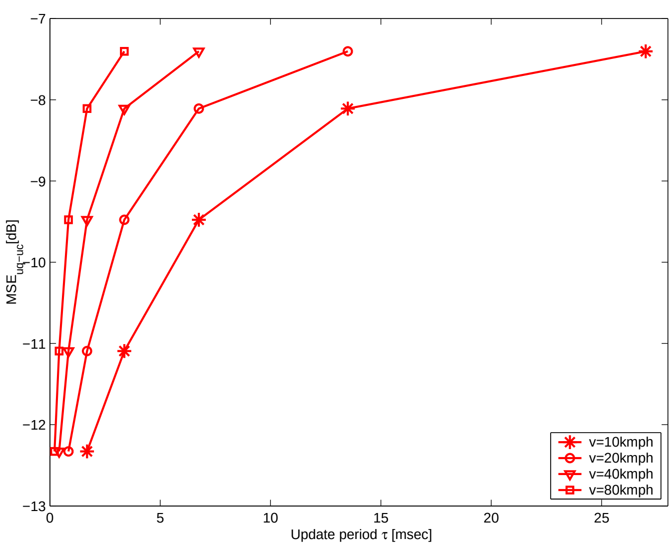

Fig. 5. MSE vs. CSI update period, f_c = 2GHz, different mobile terminal velocities 10, 20, 40, 80kmph, average SNR_{ul}^{csi} = 10dB.

UQ-UC CSI Feedback for Transmitter Optimization in Multiple Antenna Multiuser Systems

To illustrate the effects of the UQ-UC CSI feedback on system capacity we consider a communication system that consists of M transmit antennas and N single-antenna mobile terminals (see Figure 6). x_n is the information bearing signal intended for mobile terminal n and y_n is the received signal at the corresponding terminal (for $n = 1, \cdots, N$). The received vector $\mathbf{y} = [y_1, \cdots, y_N]^{\mathrm{T}}$ is

$$\begin{aligned} &\mathbf{y} = \mathbf{H}\mathbf{S}\mathbf{x} + \mathbf{n}, \\ &\mathbf{y} \in \mathcal{C}^N, \mathbf{x} \in \mathcal{C}^N, \mathbf{n} \in \mathcal{C}^N, \mathbf{S} \in \mathcal{C}^{M\times N}, \mathbf{H} \in \mathcal{C}^{N\times M} \end{aligned} \tag{20}$$

where $\mathbf{x} = [x_1, \cdots, x_N]^{\mathrm{T}}$ is the transmitted vector ($\mathrm{E}[\mathbf{x}\mathbf{x}^{\mathrm{H}}] = P_{dl}\, \mathbf{I}_{N\times N}$), $\mathbf{n}$ is AWGN ($\mathrm{E}[\mathbf{n}\mathbf{n}^{\mathrm{H}}] = N_0\, \mathbf{I}_{N\times N}$), $\mathbf{H}$ is the MIMO channel response matrix, and $\mathbf{S}$ is a transformation (spatial pre-filtering) performed at the transmitter. Note that the vectors $\mathbf{x}$ and $\mathbf{y}$ have the same dimensionality. Further, h_{nm} is the nth row and mth column element of the matrix $\mathbf{H}$ corresponding to a channel between mobile terminal n and transmit antenna m.

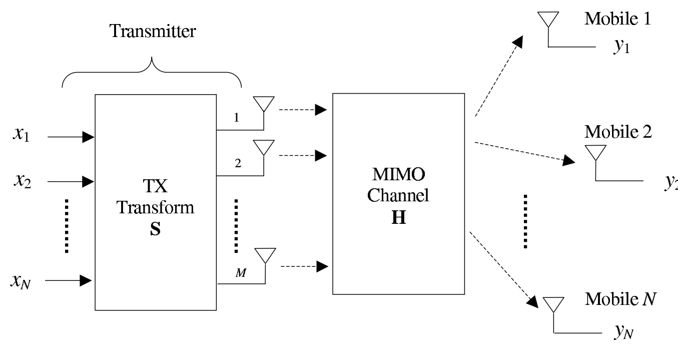

Fig. 6. System model consisting of M transmit antennas and N mobile terminals.

Application of the spatial pre-filtering results in the composite MIMO channel $\mathbf{G}$ given as

$$\mathbf{G} = \mathbf{H}\mathbf{S}, \qquad \mathbf{G} \in \mathcal{C}^{N\times N} \tag{21}$$

where g_{nm} is the nth row and mth column element of the composite MIMO channel response matrix $\mathbf{G}$. The signal received at the nth mobile terminal is

$$y_n = \underbrace{g_{nn}x_n}_{\text{Desired signal for user } n} + \underbrace{\sum_{i=1, i\neq n}^{N} g_{ni}x_i}_{\text{Interference}} + n_n. \tag{22}$$

In the above representation, the interference is the signal that is intended for other mobile terminals than terminal n. As said earlier, the matrix $\mathbf{S}$ is a spatial pre-filter at the transmitter. It is determined based on optimization criteria that we address later in the text and has to satisfy the following constraint

$$\mathrm{trace}\left(\mathbf{S}\mathbf{S}^{\mathrm{H}}\right) \leq N \tag{23}$$

which keeps the average transmit power conserved. We represent the matrix $\mathbf{S}$ as

$$\mathbf{S} = \mathbf{A}\mathbf{P}, \qquad \mathbf{A} \in \mathcal{C}^{M\times N}, \mathbf{P} \in \mathcal{C}^{N\times N} \tag{24}$$

where $\mathbf{A}$ is a linear transformation and $\mathbf{P}$ is a diagonal matrix. $\mathbf{P}$ is determined such that the transmit power remains conserved. We study the following zero-forcing (ZF) spatial pre-filtering scheme where $\mathbf{A}$ is represented by

$$\mathbf{A} = \mathbf{H}^{\mathrm{H}}(\mathbf{H}\mathbf{H}^{\mathrm{H}})^{-1}. \tag{25}$$

As can be seen, the above linear transformation is zeroing the interference between the signals dedicated to different mobile terminals, i.e., $\mathbf{HA} = \mathbf{I}_{N\times N}$. x_n are assumed to be circularly symmetric complex random variables having Gaussian distribution $\mathcal{N}_{\mathcal{C}}(0, P_{dl})$. Consequently, the maximum achievable data rate (capacity) for mobile terminal n is

$$R_n^{\mathrm{ZF}} = \log_2\left(1 + \frac{P_{dl}|p_{nn}|^2}{N_0}\right) \tag{26}$$

where p_{nn} is the nth diagonal element of the matrix $\mathbf{P}$ defined in (24). The elements of the matrix $\mathbf{P}$ are selected such that

$$\mathrm{diag}(\mathbf{P}) = [p_{11}, \cdots, p_{NN}]^{\mathrm{T}} = \arg \max_{\mathrm{trace}(\mathbf{APP}^{\mathrm{H}}\mathbf{A}^{\mathrm{H}})\leq N} \sum_{i=1}^{N} R_n. \tag{27}$$

For more details on the above spatial pre-filtering see [6] and [12].

To perform the above spatial pre-filtering, the base station obtains CSI corresponding to each downlink channel state h_{nm}. The CSI is obtained from each mobile terminal using the UQ-UC CSI feedback. In other words, at time instant i, terminal n $(n = 1, \cdots, N)$ is transmitting the corresponding CSI $h_{nm}(i)$ $(m = 1, \cdots, M)$ via the uplink CSI feedback channel. Relating to the analysis in the previous sections, each $h_{nm}(i)$ corresponds to different $h_{dl}(i)$. Instead of the ideal channel state $h_{nm}(i)$, the spatial pre-filter applies the estimate $\hat{h}_{nm}(i)$ obtained from the uplink CSI feedback receiver.

In Figure 7 we present downlink sum data rates for the downlink data $SNR_{dl} = 10\log(P_{dl}/No) = 10dB$, and $M = 3$ and $N = 3$. The rates are presented as a function of the mobile terminal velocity using the approximate Jake's model for carrier frequency 2GHz and the CSI update period $\tau =$ 2msec and spatially uncorrelated channels. Furthermore, the uplink CSI feedback channel is assumed to be *iid* Rayleigh with the average $SNR_{ul}^{csi} =$ 10dB. In addition, we present the rates for instantaneous ideal channel knowledge and a delayed ideal channel knowledge (2msec delay) which may correspond to a practical feedback scheme that quantizes and encodes the CSI. We note that under the UQ-UC CSI feedback with the linear receiver, the performance is better for channels with higher correlations (i.e., lower mobile terminal velocities). For the given update period $\tau =$ 2msec and moderate and higher velocities, the UQ-UC CSI feedback scheme is outperforming the case of the delayed ideal channel knowledge.

VI. Conclusion

In this paper we have considered a system where a mobile terminal obtains the downlink CSI and feeds it back to the base station using an uplink feedback channel. If the downlink channel is an independent Rayleigh fading channel, then the CSI may be viewed as an output of a complex independent identically distributed Gaussian source. Further, if the uplink feedback channel is AWGN, we have shown that unquantized and uncoded CSI transmission (that incurs zero delay) is optimal in that it achieves the same minimum mean squared error distortion as a scheme that optimally quantizes and encodes the CSI while incurring infinite delay. Furthermore, we presented the zero-delay UQ-UC CSI feedback scheme on correlated wireless channels. Since the UQ-UC transmission is suboptimal in this case, we have proposed a simple linear CSI feedback receiver that exploits the correlations while still retaining the attractive zero-delay feature. Furthermore, we described the ARMA correlated channel model and presented the corresponding performance bounds for the UQ-UC CSI feedback scheme. We have shown that the linear receiver exploits the temporal correlations in the channel; resulting in lower MSE values when either the mobile terminal velocities are low or the CSI update period is small. We explored the performance limits of the scheme in the context of downlink multiple antenna multiuser transmitter optimization.

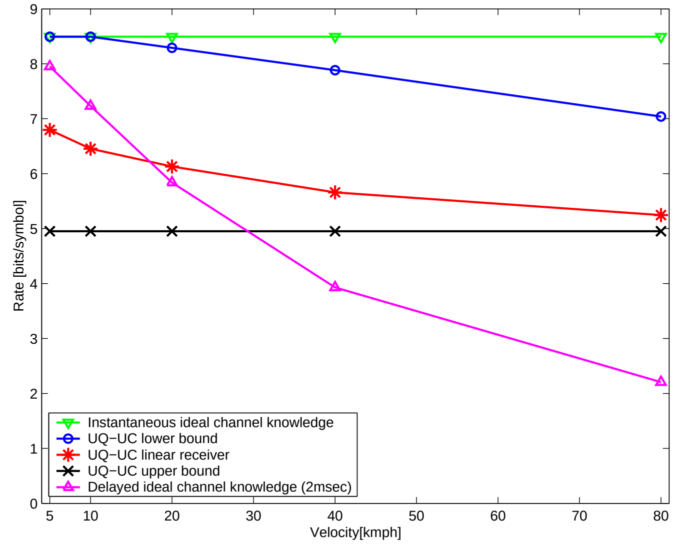

Fig. 7. Average downlink sum data rate vs. the mobile terminal velocity, for CSI update period $\tau =$ 2msec, $f_c =$ 2GHz, $M = 3$, $N = 3$, spatially uncorrelated, downlink data $SNR_{dl} =$ 10dB and uplink CSI feedback $SNR_{ul}^{csi} =$ 10dB.

Appendix

In this appendix we show how for the given correlation between the downlink channel states, the coefficients c_0 to c_L of the ARMA model in (11) are determined. The correlation between the downlink channel states is given as

$$\phi(k) = \mathrm{E}[h_{dl}(i)h_{dl}(i-k)^*] \text{ for } |k| \leq L \tag{28}$$

where $\phi(-k) = \phi(k)^*$, and for $|k| > L$, $\phi(k) = 0$. Further, based on the ARMA model in (11) we form a set of $2L$ linear equations

$$\phi(0) = \sum_{j=1}^{L} c_j\phi(-j) + c_0^2 \tag{29}$$

and

$$\phi(k) = \sum_{j=1}^{L} c_j \phi(k-j) \quad k = 1, \cdots 2L-1. \tag{30}$$

Let us define the following matrix

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \phi(1)^* & \phi(2)^* & \cdots & \phi(L)^* \\ 0 & \phi(0) & \phi(1)^* & \cdots & \phi(L-1)^* \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & \phi(L-1) & \phi(L-2) & \cdots & \phi(0) \\ 0 & 0 & \phi(L-1) & \cdots & \phi(1) \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \phi(L-1) \end{bmatrix} \tag{31}$$

and vectors

$$\mathbf{c} = [c_0^2 \; c_1 \; \cdots \; c_L]^{\mathrm{T}} \tag{32}$$

and

$$\mathbf{f} = [\phi(0) \; \phi(1) \; \cdots \; \phi(L) \; \underbrace{0 \; \cdots \; 0}_{L-1}]^{\mathrm{T}}. \tag{33}$$

The above system of linear equations can be rewritten as

$$\mathbf{f} = \mathbf{\Phi} \mathbf{c}. \tag{34}$$

By construction, the columns of the matrix Φ are linearly independent. Thus, the least squares solution of the above linear equation is

$$\tilde{\mathbf{c}} = (\mathbf{\Phi}^{\mathrm{H}} \, \mathbf{\Phi})^{-1} \, \mathbf{\Phi}^{\mathrm{H}} \, \mathbf{f}. \tag{35}$$

With $c_0 \geq 0$, the above solution determines the coefficients c_0 to c_L of the ARMA model in (11).

To approximate Jake's model using the finite length ARMA model in (11) we select elements of the vector $\mathbf{f}$ as

$$\phi(k) = J_0(2\pi f_d k \tau), \;\; k = 0, \cdots, L \tag{36}$$

where f_d is the maximum Doppler frequency and τ is the time difference between successive channel states $h_{dl}(i)$ and $h_{dl}(i-1)$. If the update of the CSI is performed at discrete time moments the update period τ should be such that

$$\tau < \frac{1}{2 f_d}. \tag{37}$$

Furthermore, the length L is selected as

$$L \geq \frac{4}{\tau f_d}. \tag{38}$$

The above assumptions provide a good approximation of Jake's model using the finite length ARMA model.

References

[1] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Transactions on Information Theory*, vol. 49, pp. 1691–1706, July 2003.

[2] D. Tse and P. Viswanath, "On the capacity region of the vector Gaussian broadcast channel," *IEEE International Symposium on Information Theory*, pp. 342–342, July 2003.

[3] S. Vishwanath, G. Kramer, S. Shamai, S. Jafar, and A. Goldsmith, "Capacity bounds for Gaussian vector broadcast channel," *The DIMACS Workshop on Signal Processing for Wireless Transmission*, October 2002. Rutgers University.

[4] E. Rashid-Farrohi, L. Tassiulas, and K. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Transactions on Communications*, vol. 46, pp. 1313–1323, October 1998.

[5] E. Visotsky and U. Madhow, "Optimum beamforming using transmit antenna arrays," *The IEEE Vehicular Technology Conference (VTC)*, vol. 1, pp. 851–856, May 1999.

[6] D. Samardzija and N. Mandayam, "Multiple antenna transmitter optimization schemes for multiuser systems," *The IEEE Vehicular Technology Conference (VTC)*, October 2003. Orlando.

[7] T. J. Goblick, "Theoretical limitations on the transmission of data from analog sources," *IEEE Transactions on Information Theory*, vol. 11, pp. 558–567, October 1965.

[8] T. Berger, "Shannon lecture: living information theory," *Presented at IEEE ISIT*, July 2002.

[9] M. Gastpar, B. Rimoldi, and M. Vetterli, "To code, or not to code: lossy source channel communication revisited," *IEEE Transactions on Information Theory*, vol. 49, pp. 1147–1158, May 2003.

[10] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley Publications, first ed., 1991.

[11] D. Samardzija and N. Mandayam, "Pilot assisted estimation of MIMO fading channel response and achievable data rates," *IEEE Transactions on Signal Processing, Special Issue on MIMO*, vol. 51, pp. 2882–2890, November 2003.

[12] D. Samardzija and N. Mandayam, "Downlink multiple antenna transmitter optimization on spatially and temporally correlated channels with delayed channel state information," *Conference on Information Sciences and Systems*, March 2004. Princeton University.