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330:501 Systems Analysis

Problem Set 1 SOLUTIONS

S & B: Ch.[1], prob. 1:
We have $\rho(z,x) \leq \rho(x,y) + \rho(y,z)$. Let $x = y$ and we get $\rho(z,y) \leq \rho(y,y) + \rho(y,z)$. But $\rho(y,y) = 0$ so $\rho(z,y) \leq \rho(y,z)$. Now just swapping z and x in the original equation we get $\rho(x,z) \leq \rho(z,y) + \rho(y,x)$. Letting $x = y$ we get $\rho(y,z) \leq \rho(z,y)$. Since both are always true we must have $\rho(z,y) = \rho(y,z)$. **QED**.

S & B: Ch.[1], prob. 2:
We have $\rho(x,z) \leq \rho(x,y) + \rho(y,z)$. Let $z = x$ to get $\rho(x,x) \leq \rho(x,y) + \rho(y,x) = 2\rho(x,y)$ which implies $\rho(x,y) \geq 0$ since $\rho(x,x) \geq 0$. **QED**.

S & B: Ch.[1], prob. 4:
Easy to see in 2 dimensions with a piece of a larger radius disk within a smaller radius disk. Let $M = \bar{B}_2((0,0),0.5)$. Now let $\bar{B}_1((0,0.1),0.8) = \{(x,y)|rho((x,y),(0,0.1)) \leq 0.8)\}$. Notice the set $\bar{B}_1$ is completely contained in $\bar{B}_2$ because the only admissible points in M are contained in $\bar{B}_2$, but the radius of $\bar{B}_1$ is greater than that of $\bar{B}_2$. Another example is M as the non-negative reals with $\bar{B}_2(0.5,0.5)$ and $\bar{B}_1(0.1,0.8)$. Draw both and it will be obvious.

S & B: Ch.[1], prob. 6:
$\rho(A(x),A(y)) = |\ln x - \ln y| = |ln\frac{x}{y}|$. Might try to use MVT for any given x and y to get $\rho(A(x),A(y)) = |A'(\chi)||x-y|$ for some χ on $[x,y]$ However, since $x, y \in \Re$, A' is unbounded. $\rightarrow$ UNBOUNDED. Also could just look at part of $\ln x$ near $x = 0$ and notice that even if $x = y + \epsilon$, $|\ln\frac{x}{y}|$ could be HUGE.

S & B: Ch.[1], prob. 7:
Here's a simple example which does not require us to prove a particular space is complete. Use

$$\rho(x,y) = \begin{cases} 1 & x \neq y \\ 0 & x = y \end{cases}$$

and any old set (see example 1.4). We choose the set $[-1,-\epsilon] \cup [\epsilon, 1]$ with $\epsilon > 0$ and let $A(x) = -x$. $\rho(A(x),A(y)) = \rho(x,y)$ so it satisfies the condition, but you can see that $x_n = A(x_{n-1})$ bounces back and forth between $\pm x_0$. So no limit point.

S & B: Ch.[1], prob. 8:
Same as above only $A(x) = x$ with an infinity of limit points.

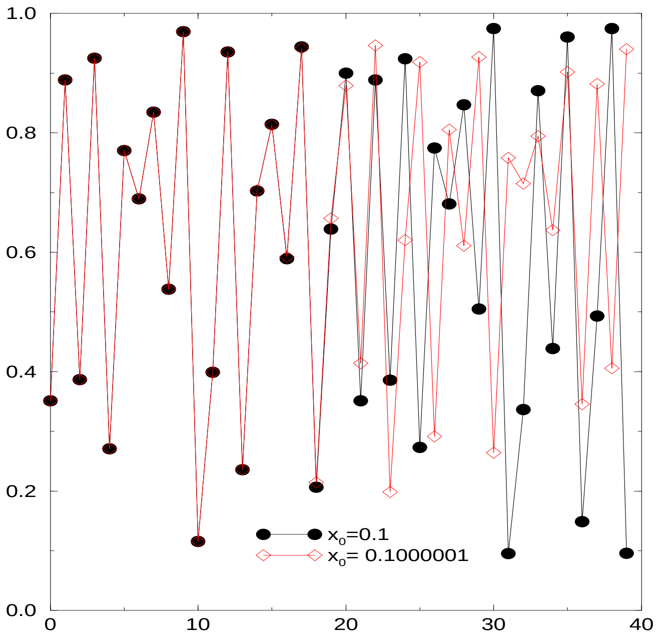

Figure 1: Plot of sequences for chaotic mapping.

Extra Problem:

- $R(A) = [0, b/4]$
- For $b < 1$ we have $|A(x) - A(y)| = b|x - y + y^2 - x^2| = b|x - y||1 - (x + y)|$. Since $b|1 - (x + y)| < 1$, we have $A()$ is a contraction mapping.
- NOPE! Just use $b \geq 1$ and proceed as above. However, just because a mapping is not a contraction, does NOT mean it doesn't have limit points. Try $b = 2$ which has limit point 0.5. (try it numerically!).
- Wierd, right? We start one one-millionth away and get very different sequences. The two sequences are plotted in the figure. This mapping is one of the simpler *chaotic mappings*. Chaos is generally annoying when you want to control a system completely since if you change the input even slightly (and precision is always finite because of noise) you can get wildly different results.