THE STATE UNIVERSITY OF NEW JERSEY

RUTGERS

College of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2003**

Problem Set 1

1. Derive the convolution intergal from first principles (as outlined in class) given by $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$, where $h(t)$ is the impulse response of a LTI system, $x(t)$ the input and $y(t)$ the output.

2. For each of the systems described by the input output relationships below, determine which of the following properties apply to the system: Memoryless(M), causal(C), linear(L), time-invariant(TI), stable(S). Justify your answers.

   (a) $y(t) = \sin(t+1)x(t)$

   (b) $y[n] = x[2-n] + 1$

3. Consider a continuous time system with the following input $x(t)$ and impulse response $h(t)$,

$$x(t) = \begin{cases} 1 & 0 < t < 2 \\ -1 & 2 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

   and $h(t) = \exp(-2t)u(t)$

   (a) Compute the output of the system $y(t) = x(t) * h(t)$

   (b) Is the system stable? Is the system causal?

4. A signal $x(t)$ is periodic with period $T = 10^{-3}$. The Fourier series coeffecients for x(t) are given by

$$a_k = \begin{cases} (\frac{1}{2j})^k & k > 0 \\ 0 & k = 0 \\ (\frac{1}{-2j})^{-k} & \text{otherwise} \end{cases}$$

   .
   Find the average power in the signal x(t), $\frac{1}{T}\int_0^T |x(t)|^2$. Hint:Use Parseval's relationship!

5. Let $x(t) = \exp(-at)u(t)$,where u(t) is the unit step function. Find the Fourier transform of the following signals.

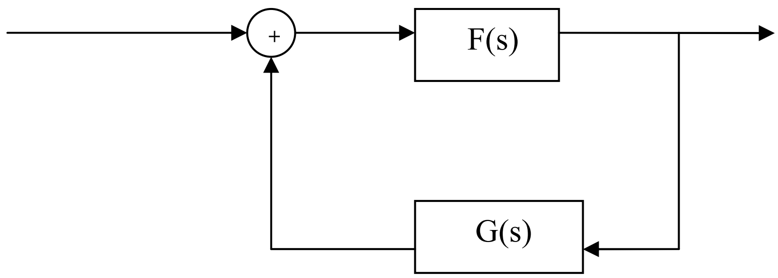
   (a) $x(t)$

   (b) $y(t) = x(t+5)$

   (c) $z(t) = x(t)\sin(2\pi 40t)$

6. Evaluate the Fourier transform of the damped sinusoidal wave $g(t) = \exp(-t)\sin(2\pi f_c t)u(t)$, where $u(t)$ is the unit step function.

7. Show that the overall system function $H(s)$ for the feedback system in FIGURE 7 is given by $H(s) = \frac{F(s)}{1-F(s)G(s)}$.



8. A signal $x(t)$ of finite energy is applied to a sqaure-law device whose output is defined by $y(t) = x^2(t)$. The spectrum of $x(t)$ is limited to the frequency interval $-W \leq f \leq W$. Hence show that the spectrum of $y(t)$ is limited to $-2W \leq f \leq 2W$.