



# RUTGERS

School of Engineering  
Department of Electrical and Computer Engineering

**332:322**

**Principles of Communications Systems**  
Linear System Assessment

**Spring 2011**

*There are four questions. You have the class period to answer them and can use whatever resources you have available (except using a "life line" by calling someone on your cell phone or accessing the solutions on the web. Place all answers on the test sheet. This is an assesment exam and will NOT count toward your final grade. It's only purpose is for me to determine just how brilliant (or woefully incompetent) you are. :) Try to have fun and exercise what Wade (Prof. Trappe) taught you last term. Depending upon the outcome, Wade might make a special guest appearance for applause (or egg-throwing). :)*

1. The Fourier transform of  $x(t)$  is  $X(f)$ . Prove that the Fourier transform of  $x(t - t_0)$  is  $e^{-j2\pi ft_0} X(f)$ .

2. The energy in a signal  $x(t)$  is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Prove that if  $X(f)$  is the Fourier transform of  $x(t)$ , then we must also have

$$E = \int_{-\infty}^{\infty} |X(f)|^2 df$$

You may use the fact that

$$\int_{-\infty}^{\infty} e^{j2\pi vt} dt = \delta(v)$$

3. A signal  $x(t)$  of finite energy is applied to a square-law device whose output is defined by  $y(t) = x^2(t)$ . The spectrum,  $X(f)$ , of  $x(t)$  is limited to the frequency interval  $-W \leq f \leq W$ . Show that the spectrum,  $Y(f)$ , of  $y(t)$  is limited to  $-2W \leq f \leq 2W$ .

4. Show that the overall system function  $H(s)$  for the feedback system in FIGURE 1 is given by  $H(s) = \frac{F(s)}{1-F(s)G(s)}$ .

Figure 1: Feedback system for problem 4.

