School of Engineering
Department of Electrical and Computer Engineering

332:322 **Principles of Communications Systems** **Spring 2008**

Final Examination

There are 4 questions. You have three hours to answer them. Show all work. Answers given without work will receive no credit. GOOD LUCK!

1. *(50 points)* **Signal Space 101:** You are given two signals defined on $(-\frac{1}{2},\frac{1}{2})$: $\phi_1(t) = 1$ and $\phi_2(t) = \alpha t$, where α is a constant.

   (a) *(10 points)* Show that $\phi_1(t)$ and $\phi_2(t)$ are orthogonal.

   (b) *(10 points)* Does $\phi_1(t)$ have unit energy on $(-\frac{1}{2},\frac{1}{2})$? For what value of α does $\phi_2(t)$ have unit energy on $(-\frac{1}{2},\frac{1}{2})$?

   (c) *(10 points)* Let $f(t) = t^2 - 1$. Assume $\alpha = 2\sqrt{3}$ and let $g(t) = a\phi_1(t) + b\phi_2(t)$. Please find the a and b which minimize
   $$\int_{-\frac{1}{2}}^{\frac{1}{2}} (f(t) - g(t))^2 dt$$

   (d) *(10 points)* Plot $g(t)$ as a point in the signal space represented by $\phi_1(t)$ and $\phi_2(t)$.

   (e) *(10 points)* Let $h(t) = f(t) - g(t)$. Is $h(t)$ orthogonal to $\phi_1(t)$? How about to $\phi_2(t)$?

2. *(50 points)* **Cora Sees the Light:** Late at night, Cora lies in wait for her arch nemisis Marty the Squirrel. She's set up a tube through which Marty must run to forage. At the end of the tube is a remote-controlled mine and Cora holds the transmitter. When an animal runs through the tube, it glows. If it is Marty running through the tube, the glow duration is a random variable G with exponential distribution $f_{G|M}(g|M) = e^{-g}u(g)$. However Cora's cat, Cassandra, is also about at night. Cassandra is rather fat so her glow duration is distributed as $f_{G|C}(g|C) = \frac{1}{2}e^{-g/2}u(g)$. Cora measures the glow duration and has to make a decision about tripping the mine. Marty or Cassandra traversing the tube are assumed to be equally likely events.

   (a) *(20 points)* Derive a decision rule on g that minimizes the probability Cora blows up Cassandra instead of Marty, or misses Marty when he is there.

   (b) *(20 points)* Unbeknownst to Cora, Marty places catnip laced with caffeine at the tube entrance. Cassandra will eat the catnip and her glow time distribution will become $f_{G|C}(g|C) = 2e^{-2g}u(g)$. If Cora uses the decision rule of the previous part, what is the probability she blows up Cassandra?

   (c) *(10 points)* Cora decides Cassandra-be-damned and only seeks to maximize the probability she blows up Marty. What's her optimum decision rule? What's the probability Marty's blown up?

3. *(50 points)* **Signal Space 102:** Signals $s_1(t)$ and $s_2(t)$ are sent equiprobably over a channel which corrupts them with additive zero mean white Gaussian noise $w(t)$ of spectral height 1. That is, the received signal $r(t) = s_k(t) + w(t)$ where k is either 1 or 2. You are told $s_1(t) = u(t)$ and $s_2(t) = (u(t) - 2u(t-0.5))$ and that both signals exist only on $(0,1)$.

(a) *(10 points)* Sketch the signals $s_1(t)$ and $s_2(t)$ and derive an appropriate signal space for them. What is the dimension of your signal space? (EASY)

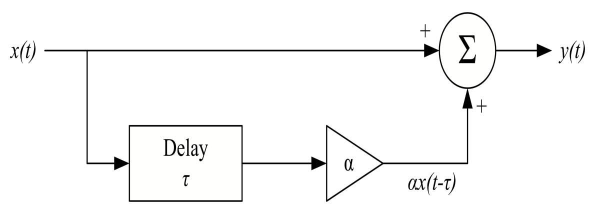
(b) *(10 points)* Plot the signal points corresponding to $s_1(t)$ and $s_2(t)$ in this signal space. (EASY)

(c) *(10 points)* Assuming a correlator receiver based on your signal space basis functions, derive a minimum probability of error decision rule and sketch the decision region in your signal space. (EASY)

(d) *(10 points)* Derive explicit expressions for $f_{R_1R_2|s_2}(r_1,r_2|s_1)$ and $f_{R_1R_2|s_1}(r_1,r_2|s_2)$. (NOT HARD)

(e) *(10 points)* If we set $s_2(t) = 0$ how does the decision rule change? Discuss quantitatively (or qualitatively for half credit) whether the probability of error increases or decreases compared to $s_2(t) = (u(t) - 2u(t-0.5))$.

4. *(50 points)* **Multipath Channel Equalization:** In wireless channels, radio (or sound) waves bounce all over the place and recombine at the receiver causing echoes which can degrade the signal. Your job is to derive an equalizer which takes the incoming signals and removes the echoes as well as possible. So, consider the channel and equalization filter shown in FIGURE 1



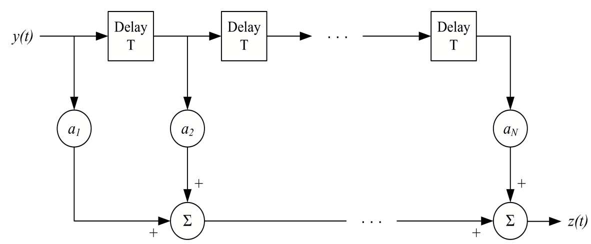


Figure 1: LEFT: Channel model. RIGHT: Equalization filter

$\tau \in \Re^+$ and $\alpha \in \Re$.

(a) *(10 points)* $x(t)$ is the channel input. What is the channel output $y(t)$ in terms of $x(t)$?

(b) *(10 points)* What is the impulse response of the channel, $h(t)$? What is the frequency response of the channel, $H(f)$?

(c) *(10 points)* Show that

$$z(t) = \sum_{k=1}^{N} a_k y(t-(k-1)T)$$

What is the impulse response, $g(t)$ of this transversal filter? **HINT:** Don't overthink this.

(d) *(10 points)* What is $Z(f)$ the Fourier transform of $z(t)$? What is $G(f)$ the Fourier transform of $g(t)$?

(e) *(10 points)* Assume $\tau = T$. What is the overall impulse response $q(t)$ of the channel filter followed by the transversal filter? Assume $|\alpha| < 1$ what values should be chosen for the a_k, $k = 1,2,\cdots N$ so that $q(t)$ is as close to a perfect channel as possible; i.e., $q(t) \approx \delta(t)$. Put more formally, we seek

$$\{a_k^*\} \arg\min_{\{a_k\}} \int_0^{kT} |q(t) - \delta(t)| dt$$

HINT: Try a hueristic approach before going formal. If you go formal, remember that $d|x|/dx = sgn(x)$ except at $x = 0$ where it's defined as 0.